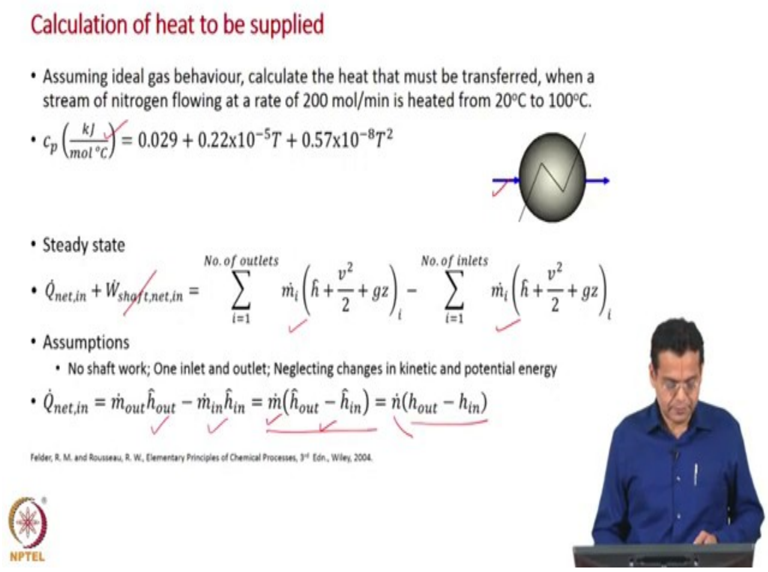
Continuum Mechanics and Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 105
Integral Energy Balance : Examples

Example: (Refer Slide Time: 00:14)



Now, what we will do is discuss a few applications of the Integral Energy Balance equation what we have derived. First example is finding out a heating load of a heater, second is the calculation of power required for compression. These two examples are exactly same as you would have done in a process calculation course, idea is to show that what I discussed is more general form compared to what I have used or in fact, what you are solved as a integral energy balance equation just to make that connection so, that you know that conceptually they are same. Third example of course, a transient energy balance filling of a tank will be considered. So, first two examples are exactly what you do in a process calculation course.

Let us read the example, assuming ideal gas behavior, calculate the heat that must be transferred, when a stream of a nitrogen a flowing at a rate of 200 mol per minute is heated from 20 degree centigrade to 100 degree centigrade. This example is from the book Elementary Principles of Chemical Processes by Felder and Rousseau an excellent book for process calculation. We are given the specific heat capacity

$$c_p\left(\frac{kJ}{mol^0 C}\right)=0.029+0.22\,x\,10^{-5}T+0.57\,x\,10^{-8}T^2$$

Solution:

So, of course, it is steady state. So, let us write the steady state integral energy balance equation

$$\dot{Q}_{net,\in ¿}+\dot{W}_{shaft,net,\in ¿}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i ¿$$

Assumptions are, there is no shaft work it is just getting heated, there is only one inlet, one outlet and as we have discussed the temperature changes about 80 degree centigrade and we can easily neglect changes in kinetic and potential energy. So, the integral balance gets simplified to

$$\dot{Q}_{net,\in ¿}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\hat{h}_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\hat{h}_i ¿$$

We have only one outlet and only one inlet. So,

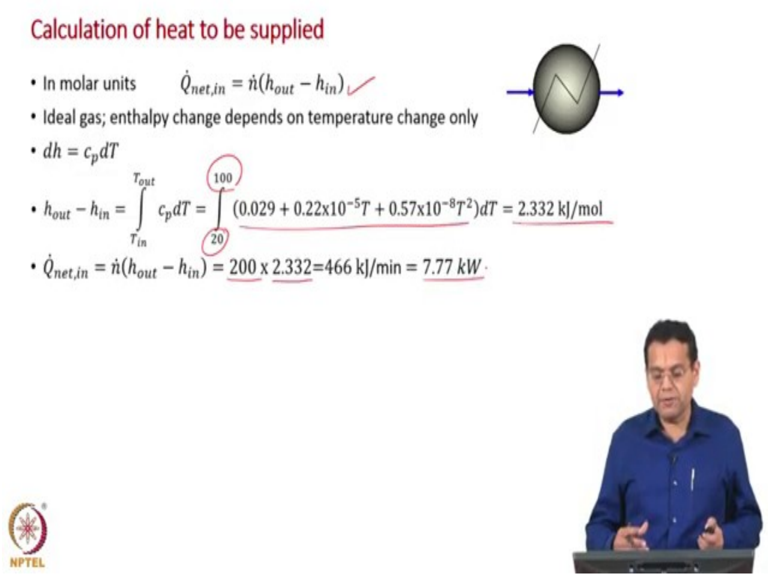
$$\dot{Q}_{net,\in ¿}=\dot{m}_{out}\hat{h}_{out}-\dot{m}_{¿}\hat{h}_{¿} ¿$$

Now, in terms of the question the flow rate is given in terms of mole per minute and of course, c_p is also in terms of kilo joule per mole degree centigrade. So, let us rewrite this equation in terms of molar units.

$$\dot{Q}_{net,\in ¿}=\dot{m}_{out}\hat{h}_{out}-\dot{m}_{¿}\hat{h}_{¿}=\dot{m}\left(\hat{h}_{out}-\hat{h}_{¿}\right)=\dot{n}\left(h_{out}-h_{¿}\right) ¿$$

So, $\dot{n}$ now represents the molar flow rate and $h_{out}-h_{¿}$ now represents change in enthalpy in terms of let us say Joule per mole.

(Refer Slide Time: 04:03)



So, in molar units

$$\dot{Q}_{net,in} = \dot{n}(h_{out} - h_{in})$$

Now we will have to evaluate the change in enthalpy, we will assume as a question says ideal gas behavior in which case the enthalpy change depends on temperature change only. So,

$$dh = c_p dT$$

Now, let us integrate

$$h_{out} - h_{in} = \int_{T_{in}}^{T_{out}} c_p \, dT = \int_{20}^{100} (0.029 + 0.22\,x\,10^{-5} T + 0.57\,x\,10^{-8} T^2)\, dT$$

Now simple integration will give us

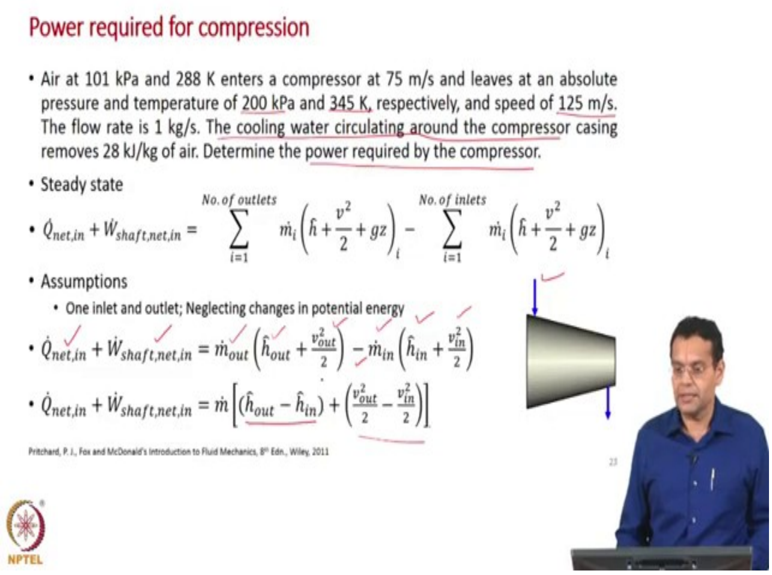
$$h_{out} - h_{in} = 2.332 \frac{kJ}{mol}$$

So, let us substitute in this equation

$$\dot{Q}_{net,in} = \dot{n}(h_{out} - h_{in}) = 200\,x\,2.332 = 466 \frac{kJ}{min} = 7.77\,kW$$

That is the heat to be supplied ok, at typical process calculation example to emphasize that that equation is nothing, but a integral energy balance equation. We have we had derived a very very general form, but when we simplify we get the equation which I used.

Example: (Refer Slide Time: 05:39)



Let us look at another example this example is from Fox and McDonald and let us read the example, air at 101 kilo Pascal and 288 Kelvin enters a compressor at 75 meters per second and leaves at an absolute pressure and temperature of 200 kilo Pascal and 345 Kelvin respectively and speed of 125 meters per second. Look at the velocities now, as I told you for compressors and the velocities are much higher. So, inlet velocity is 75, exit is 125 and of course, there is a pressure change and temperature change as well.

The flow rate is 1 kg per second and because of compression there is lot of heat release will have to remove that. So, the cooling waters circulating around the compressor casing removes 28 kilo Joule per kg of air. Determine the power required by the compressor.

Solution:

So, this also a typical example which comes in a process calculation course once again of course, steady state operation. Let us write down the steady state integral energy balance equation

$$\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i$$

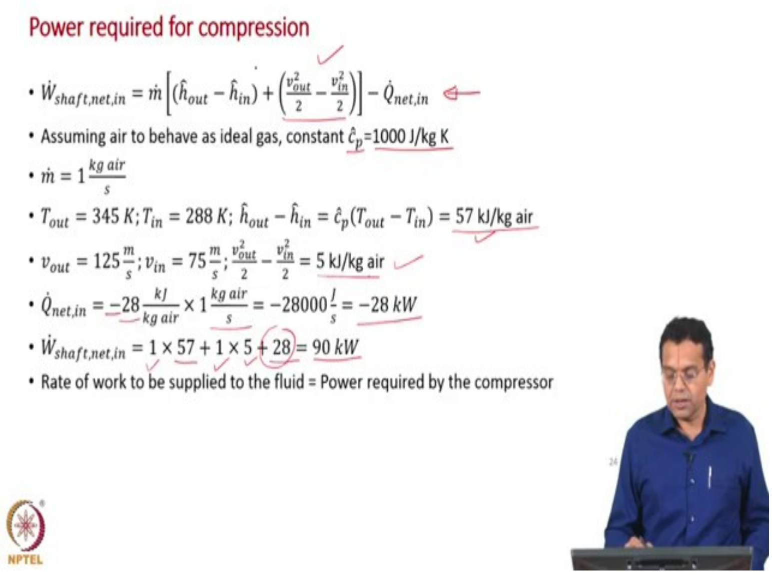
Now, let us simplify, the assumptions are, one inlet and one outlet like in the last case and we neglect changes in potential energy not in kinetic energy, we cannot in neglect, the velocities are high, differences also high, we have seen that for 100 meters per second order of inlet velocity even at 10 meters per second change in velocity can cause significant change in kinetic energy if not very very high, it can cause.

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \dot{m}_{out}\left(\hat{h}_{out} + \frac{v_{out}^2}{2}\right) - \dot{m}_{in}\left(\hat{h}_{in} + \frac{v_{in}^2}{2}\right)$$

Because it is only one inlet one outlet $\dot{m}$ is same. So, let us take out that and write as

$$\dot{Q}_{net,in} + \dot{W}_{shaft,net,in} = \dot{m}\left[\left(\hat{h}_{out} - \hat{h}_{in}\right) + \left(\frac{v_{out}^2}{2} - \frac{v_{in}^2}{2}\right)\right]$$

(Refer Slide Time: 09:10)



Now, we will have to evaluate the different terms and we will write that equation for the rate of shaft work that is what is to be found out.

$$\dot{W}_{shaft,net,in} = \dot{m}\left[\left(\hat{h}_{out} - \hat{h}_{in}\right) + \left(\frac{v_{out}^2}{2} - \frac{v_{in}^2}{2}\right)\right] - \dot{Q}_{net,in}$$

Now, we will assume air to behave as ideal gas and in this case we are let us assume the specific heat capacity is a constant value and we have used

$$\hat{c}_p = 1000\frac{J}{kg.K}$$

$$\dot{m}=1\frac{kg\,air}{s}$$

$$T_{out}=345\,K\,;T_{¿}=288\,K$$

So, the difference in enthalpy is

$$\hat{h}_{out}-\hat{h}_{¿}=\hat{c}_{p}\left(T_{out}-T_{¿}\right)=57\frac{kJ}{kg}$$

So, it is specific enthalpy change. We have given the outlet velocity as 125 meters per second inlet velocity as 75 meters per second. So, we can also find out what is the kinetic energy change per unit mass,

$$v_{out}=125\frac{m}{s}\,;v_{¿}=75\frac{m}{s}$$

$$\frac{v_{out}^{2}}{2}-\frac{v_{¿}^{2}}{2}=5\frac{kJ}{kg\,air}$$

As we have discussed it is not negligible it is not very significant also for this conditions it is roughly about one tenth of the enthalpy change.

The $\dot{Q}_{net,\in¿¿}$ is the rate at which heat is added, remember it is heat removal here. So,

$$\dot{Q}_{net,\in¿=-28\frac{kJ}{kg\,air}x1\frac{kg\,air}{s}=-28000\frac{J}{s}=-28\,kW¿}$$

We had to pay attention to the minus sign the way in which we had defined is $\dot{Q}_{net,\in¿¿}$ is net rate at which energy is added by heat transfer into the control volume. So, in this case because heat is being removed it is minus 28 kilo Watts.

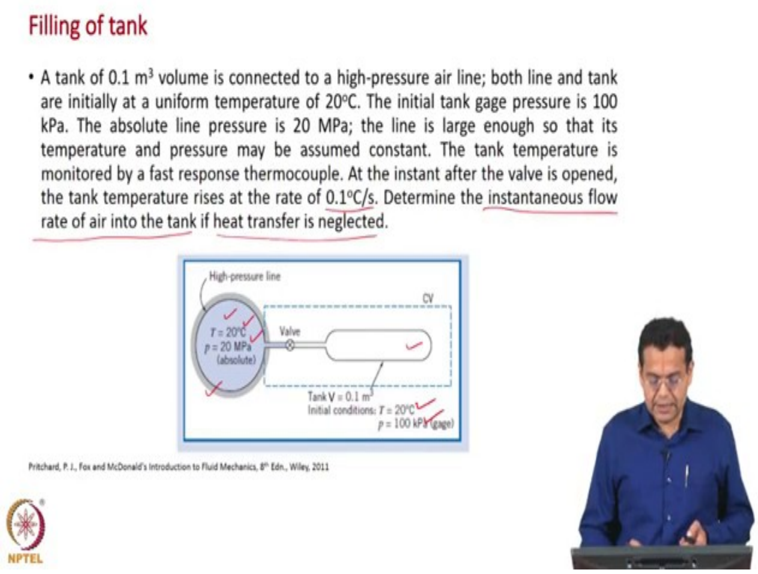
So, let us substitute all of them in the equation. So,

$$\dot{W}_{shaft,net,\in¿=\dot{m}\left[\left(\hat{h}_{out}-\hat{h}_{¿}\right)+\left(\frac{v_{out}^{2}}{2}-\frac{v_{¿}^{2}}{2}\right)\right]-\dot{Q}_{net,\in¿¿}¿}$$

$$\dot{W}_{shaft,net,\in¿=1x57+1x5+28=90\,kW¿}$$

So, the rate of shaft work is 90 kilo Watts and of course, that is the rate of work to be supplied to the fluid and that is the power required by the compressor. So, once again this example the kinetic energy changes or not very significant they are not very high, but they are also not negligible.

Example: (Refer Slide Time: 12:41)



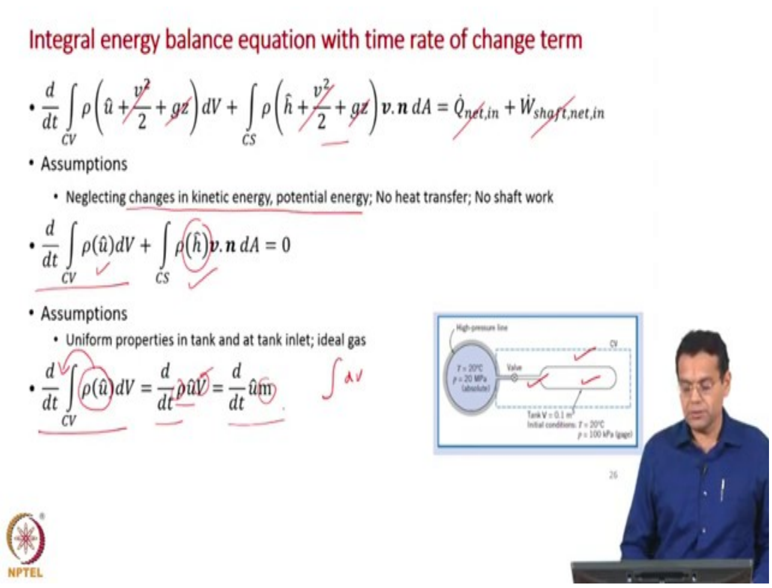
The last example based on the transient energy balance, let us read the example, a tank of 0.1 meter cubed volume is connected to a high pressure airline; both line and tank are initially at a uniform temperature of 20 degree centigrade. The initial tank gauge pressure is 100 kilo Pascal. The absolute line pressure is 20 mega Pascal, the line is large enough so that it is temperature and pressure may be assumed constant. This line is very large and we are connecting a say a small tank and so, we neglect the changes in the temperature and pressure in the line.

The tank temperature is monitored by a fast response thermocouple, thermocouple is used for measuring temperature and should respond fast and the instant after the valve is open. So, you want to fill this tank you are opening the valve and so the temperature starts to rise inside the tank and that temperature rise is given as 0.1 degree centigrade per second. So, at the instant after the valve is opened the tank temperature rises at the rate of 0.1 degree centigrade per second that is why we need a fast response thermocouple. Determine the instantaneous flow rate of air into the tank is heat transfer is neglected we neglect heat transfer.

So, nice question it is difficult to measure the mass flow rate, but easy to measure the temperature rise just put a thermocouple inside and then you can note down the rate of change of temperature, from that using a conservation equation namely integral energy balance equation we are able to find out what is the rate at which mass enters the control volume namely the tank here. It says instantaneous and it says initial etcetera because the value keeps changing the temperature keeps changing pressure keeps changing in the tank.

Of course, the for the high pressure line we have assumed to be constant because very large, but these values keep changing the temperature, pressure, the mass flow rate keeps changing the tank as it is getting filled up. That is why at that moment the valve is open let us say at sometime T equal to 0 we can calculate what is the instantaneous mass flow rate and these values corresponds to that initial condition and this example a very nice example from Fox and McDonald.

Solution: (Refer Slide Time: 15:53)



So let us start with the integral energy balance equation, specifically this time with the time rate of change term that is main attention or main focus of this example.

$$\frac{d}{dt}\int_{CV}\rho\left(\hat{u}+\frac{v^2}{2}+gz\right)dV+\int_{CS}\rho\left(\hat{h}+\frac{v^2}{2}+gz\right)v.n\,dA=\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}$$

So, now, all the terms are there in the integral energy balance equation. Now, let us write down the assumptions we will neglect changes in kinetic energy, potential energy, there is no heat transfer as per the question there is no shaft also.

So, in the control volume, there is no shaft work, there is no heat transfer as per the question and with neglect a changes in kinetic and potential energy. So, let us see how the equation gets simplified,

$$\frac{d}{dt}\int_{CV}\rho(\hat{u})dV+\int_{CS}\rho(\hat{h})v.n\,dA=0$$

When we say neglecting changes in kinetic energy and potential energy, two implications are there one is change with respect to time. So, that is why we are neglecting in the transient term also and then changes between inflow and outflow of course, here there is no outflow, but changes between inflow and out flow there again the kinetic energy changes, potential energy changes are negligible.

So, both in the transient term and the convection term we do not consider the kinetic energy and potential energy terms of course, right hand side there is no heat transfer term, no shaft work etcetera. So, as I told you the idea of this question is to mainly focus on the transient term. So, left hand side we have the transient term the internal energy is what plays a role, remember in the convection term as we have discussed and emphasized also what plays a role is the enthalpy and the convection term has got simplified.

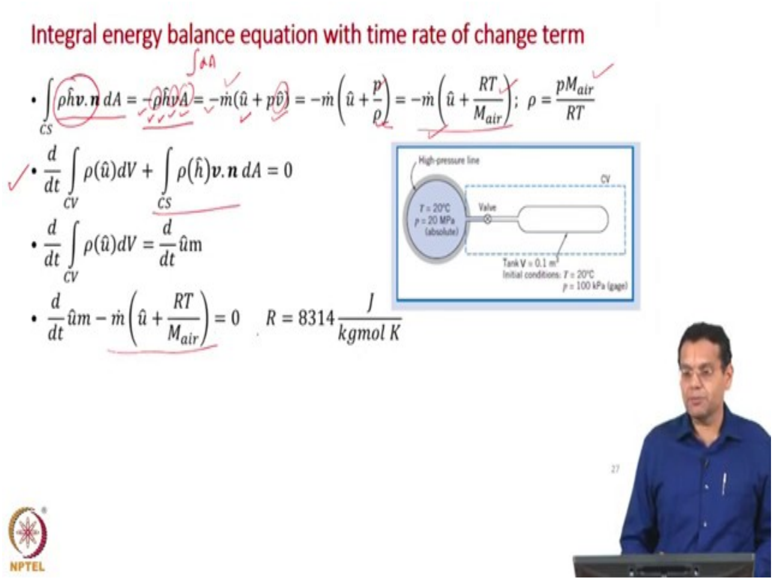
So in this simplified form you can say that the first term tells about rate of change of internal energy in the control volume and second term tells you net rate at which enthalpy leaves the control volume through the control surface in this simplified form and want to emphasize once again internal energy in transient term enthalpy in the convection term. Let us make some assumptions to simplify further we will assume uniform properties in the tank when I say in the tank inside the tank and also at the inlet you will also assume ideal gas behavior.

Now, if you take the first term the transient term

$$\frac{d}{dt}\int_{CV}\rho(\hat{u})dV=\frac{d}{dt}\rho\hat{u}V=\frac{d}{dt}\hat{u}m$$

Because the properties are uniform you can take $\rho\hat{u}$ outside the integral sign So, the first term in left hand side has got simplified to the above expression.

(Refer Slide Time: 19:53)



Now, let us take the convection term let us let us see how do we simplify that, the convection term we have

$$\int_{CS} \rho\left(\hat{h}\right)v.n\,dA = -\rho\hat{h}vA = -\dot{m}\left(\hat{u} + \frac{p}{\rho}\right) = -\dot{m}\left(\hat{u} + \frac{RT}{M_{air}}\right); \rho = \frac{pM_{air}}{RT}$$

Once again as we have assumed we will assume the properties to be uniform across the area. So, I can take out all the terms outside the integral sign and because it is inflow we know that $v.n = -v$ where v is the magnitude.

Now, ρvA is the mass flow rate so, which is denoted as $\dot{m}$ we have a negative sign and enthalpy because we have internal energy on the left hand side. We will express enthalpy in terms of internal energy as I told you is an example where it discussing about the medium is gas. So, we will have to distinguish between enthalpy internal energy changes.

So, let us substitute the two simplified terms in the integral energy balance equation before there is also simplified.

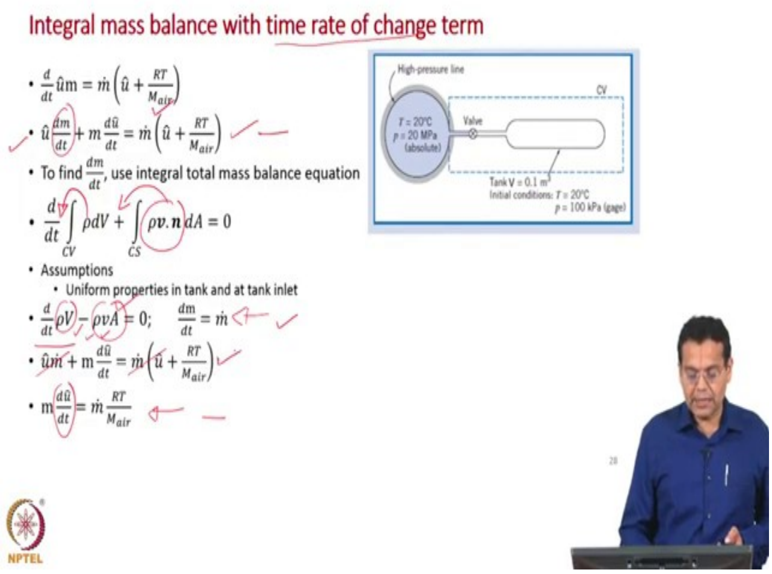
$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u}\right)dV + \int_{CS} \rho\left(\hat{h}\right)v.n\,dA = 0$$

So, let us substitute in this equation the transient term is

$$\frac{d}{dt}\hat{u}m-\dot{m}\left(\hat{u}+\frac{RT}{M_{air}}\right)=0$$

Remember R is 8314 $\frac{J}{kg\,mol\,K}$, because density is in $\frac{kg}{m^3}$.

(Refer Slide Time: 22:55)



That is the equation you have seen in the last slide

$$\frac{d}{dt}\hat{u}m=\dot{m}\left(\hat{u}+\frac{RT}{M_{air}}\right)$$

Let us apply product rule to the left hand side

$$\hat{u}\frac{d}{dt}m+m\frac{d}{dt}\hat{u}=\dot{m}\left(\hat{u}+\frac{RT}{M_{air}}\right)$$

Now, we will have to find an expression for $\frac{d}{dt}m$. So, what we will do is, to use the integral total mass balance equation with it time rate of change term that is why the title of the slide says integral mass balance with time rate of change term.

Let us write the integral total mass balance equation.

$$\frac{d}{dt}\int_{CV}\rho dV+\int_{CS}\rho v.n\,dA=0$$

So, that is way it is a good example where we use both the integral mass balance equation and the energy balance equation, that is the integral mass balance equation. We will assume uniform properties in the tank and at the tank inlet, which means that

$$\frac{d}{dt}\rho V-\rho vA=0$$

$$\frac{d}{dt}m=\dot{m}$$

So, how do we interpret, very simple rate of change of mass in the tank is equal to rate at which mass enters the tank very simple interpretation of course, very well known to us, but you have done more formally here. So, let us substitute this relationship in the equation. So,

$$\hat{u}\dot{m}+m\frac{d}{dt}\hat{u}=\dot{m}\left(\hat{u}+\frac{RT}{M_{air}}\right)$$

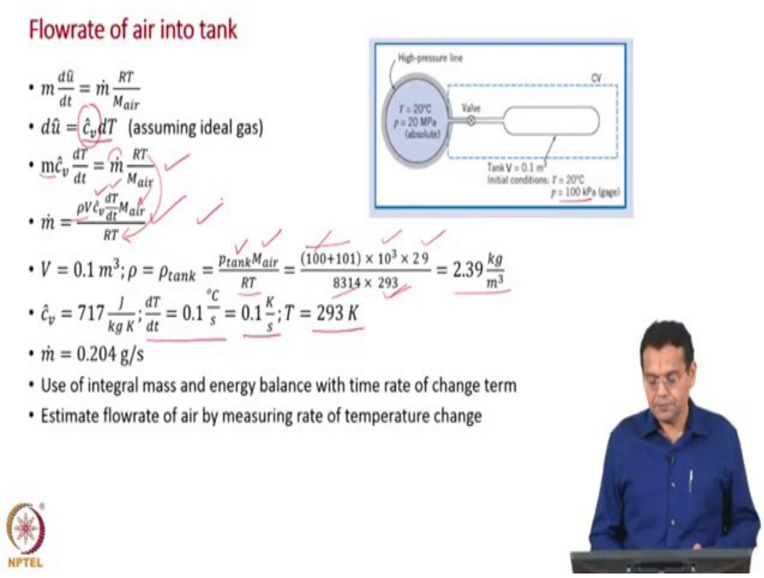
So, now left hand side we have $\hat{u}\dot{m}$, right hand side we have once again $\hat{u}\dot{m}$ they cancel each other. So, above equation gets simplified to

$$m\frac{d\hat{u}}{dt}=\dot{m}\frac{RT}{M_{air}}$$

Remember even intuitively the final equation cannot have an internal energy term standing alone or enthalpy terms standing alone why is that they are all absolute values of internal energy enthalpy do not have a meaning, always only change of internal energy as a meaning change of enthalpy has a meaning.

So, look at this equation we had $\hat{u}$ certainly they cannot they cannot appear in the final equation, they will appear only in terms of rate of change of internal energy or some change of enthalpy etcetera. Suppose, if your equation final equation somewhere or somewhere enthalpy has such if at all should be relative to some reference. Also once again like to mention is rate of change of internal energy not enthalpy because it is in the accumulation term.

(Refer Slide Time: 26:52)



So, now we are almost few more steps to find out the flow rate of air into the tank that is equation which we have written in the last slide.

$$m\frac{d\hat{u}}{dt}=\dot{m}\frac{RT}{M_{air}}$$

Now how do we express $d\hat{u}$, we assume air to behave as an ideal gas

$$d\hat{u}=\hat{c}_v dT$$

This is what I have being trying to emphasize the change in internal energy is related to change in temperature through specific heat capacity at constant volume. If we for example, wrongly assume that in the transient term also you have enthalpy then we would have wrongly used $\hat{c}_p$ here, suppose if it were liquid then does not matter because $\hat{c}_p$ and $\hat{c}_v$ are same for liquids or almost same for liquids. So, let us substitute. So, left hand side becomes

$$m\hat{c}_v\frac{dT}{dt}=\dot{m}\frac{RT}{M_{air}}$$

So, $\dot{m}$ is the unknown. So, let us keep that on the left hand side and bring all other variables to the right hand side,

$$\dot{m} = \frac{m\,\hat{c}_v \frac{dT}{dt} M_{air}}{RT} = \frac{\rho V\,\hat{c}_v \frac{dT}{dt} M_{air}}{RT}$$

Now, let us list down all the data given in the problem,

$$V = 0.1\,m^3 ; \rho_{tank} = \frac{p_{tank} M_{air}}{RT} = \frac{(100+101)\,x\,10^3\,x\,29}{8314\,x\,293} = 2.39\frac{kg}{m^3}$$

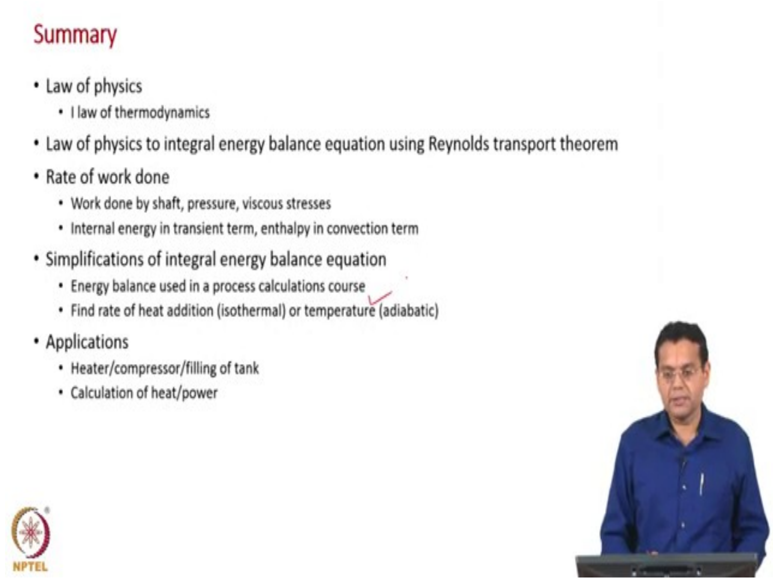
$$\hat{c}_v = 717\frac{J}{kg\,K} ; \frac{dT}{dt} = 0.1\frac{{}^0C}{s} = 0.1\frac{K}{s} ; T = 293\,K ;$$

So, if we substitute all these values we will get

$$\dot{m} = 0.204\frac{g}{s}$$

So, very good example we have used both the integral mass balance and the energy balance including the transient term. Practically how do you measure the flow rate entering the tank very difficult, now indirectly we are measuring the flow rate I would indirectly measuring the flow rate. I would say you are measuring rate of change of temperature that is easy to measure using a conservation equation assumptions of course, new are I would say estimating the rate at which estimating the rate of mass inflow into the tank and that is, this value is at that instant as time progresses that keeps changing.

(Refer Slide Time: 31:25)



So let us summarize the this part of the lecture on integral energy balance equation started the law of physics first law of thermodynamics. Then from the law of physics we went to the integral energy balance equation using a Reynolds transport theorem. We discussed in detail about the rate of work done which could be by a shaft, by pressure, viscous stresses and work done by pressure got added to the internal energy in the convection term. So, we emphasize that the transient term as internal energy the convection term has enthalpy.

We looked at several levels of simplifications of the internal energy balance equation and we arrived the equation which is usually used in a process calculation. Of course how do you use that equation, mostly if you are finding heat to be supplied, also in a process calculation course you would have done some adiabatic calculations where Q is 0 and you would have used a same integral energy balance equation to find out the temperature of course, we are not done any such example you would have done certainly several examples.

So, either the heat addition is a unknown or the temperature is unknown, but same integral energy balance equation. We looked at few applications heater, compressor, filling of a tank, mainly our objective was to find out at least under steady state conditions to find out the heat and power requirements.