Continuum Mechanics and Transport Phenomena
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Lecture – 107
Differential Total Energy Balance
Part 1

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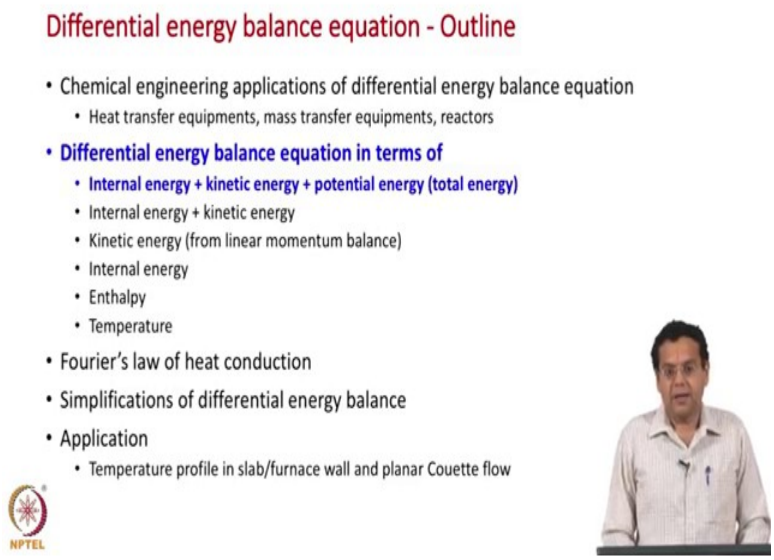

Having looked at the different chemical engineering applications of differential energy balance, now let us starts deriving the differential energy balance.

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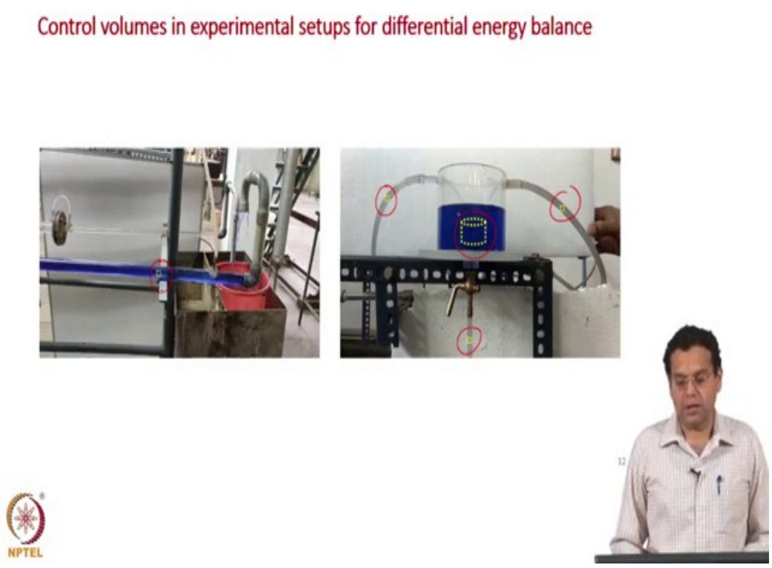

So, as usual we will start with the control volume in our familiar experimental setups. The left hand side shows the flow through the pipe and the right hand side set up shows flow through the tank. And, in the left hand side we have one inlet one outlet and in the case of the tank we have two inlets and then one outlet.

Now, what is a control volume which you should imagine over which you are deriving differential energy balance? In the case of the pipe and in the case of the tank we take a small control volume inside that domain and that is what is shown by the yellow color region in both the experimental setups. And, so these are our control volumes, they are inside the domain it could be in the pipe, it could be in the tank.

And for this control volume, we are going to take into account the rate of change of energy, energy flowing in flowing out and then heat added to the control volume, and then work done on the control volume. So, in sense we are going to apply the integral energy balance for this small control volume.

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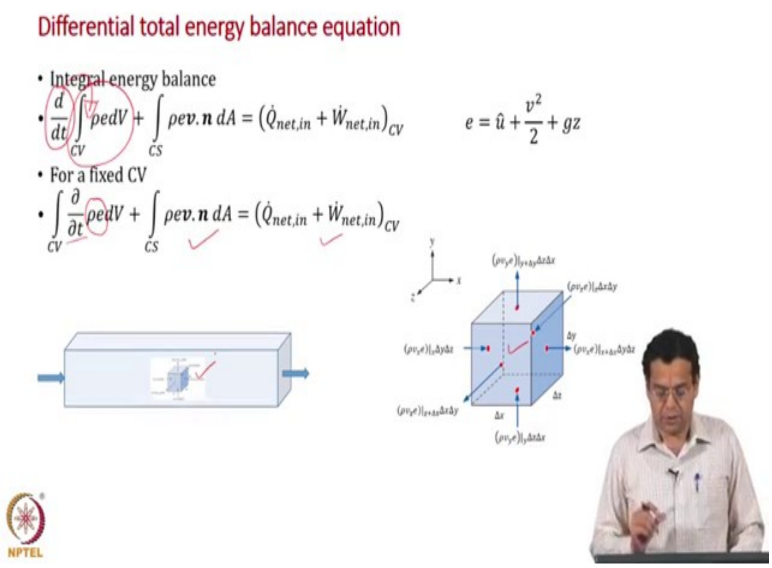

So, let us start with the integral form of energy balance equation, this is what we have been doing while deriving all the differential balance equations, we will start with the integral form of the corresponding balance equation. So, in this case we are starting with the integral energy balance equation, so let us write down the integral energy balance equation.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e v.n\, dA = ¿¿$$

$$e = \hat{u} + \frac{v^2}{2} + gz$$

The first term tells about the time rate of change of total energy, next term tells about the net rate at which energy leaves through the control surface. And the terms of the right hand side tell about net rate of heat addition, net rate of work done. Now, for a fixed control volume as we are done earlier we will bring the time derivative inside the integral sign.

$$\int_{CV} \frac{\partial}{\partial t}\rho e dV + \int_{CS} \rho e v.n\, dA = ¿¿$$

And now, it should be written as a partial derivative; the reason is that when that derivative was outside we integrated over the control volume. So, all the spatial variations have been taken into account and then only time remained as the only independent variable.

So, we wrote it as, $\frac{d}{dt}$, but now it is inside the integral sign ρ and e can vary with space, so now both spatial variation and time variation are there. And, so we represent as partial derivative and the remaining terms are same. Now, we are going to apply this integral energy balance equation for the control volume shown in the above slide image and how should we imagine, as we are done earlier and as seen in the experimental setups.

We should imagine this control volume inside a pipe and inside a domain, and because of our restriction to Cartesian coordinates we should imagine a pipe of rectangle cross section. Also like to mention that this small control volume could be in a solid as well we are talking about energy balance, so it could be let us say a conduction in a solid. So, this small control volume could be inside a pipe where that is a flow taking place or it could be a solid.

So, we are going to apply this differential energy balance for two examples, one for a solid and one in a fluid domain. So, our objective in the next few slides is to apply this integral energy balance equation for this small control volume.

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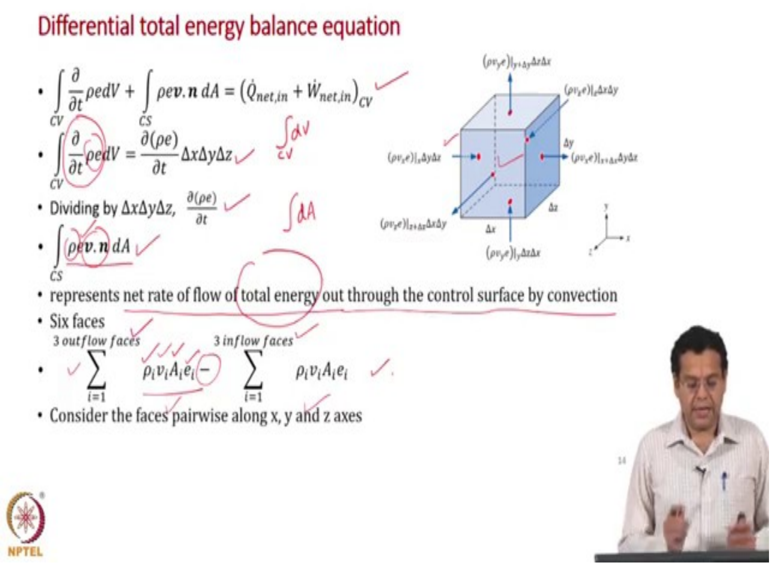

So, let us proceed that is the integral energy balance equation which we have seen in the previous slide written for fixed control volume,

$$\int_{CV} \frac{\partial}{\partial t} \rho e dV + \int_{CS} \rho e v . n \, dA = ¿¿$$

Now, let us take term by term and apply for this small control volume. Let us take the transient term

$$\int_{CV} \frac{\partial}{\partial t} \rho e dV = \frac{\partial(\rho e)}{\partial t} \Delta x \Delta y \Delta z$$

As we have done earlier we will take an average value of ρ and e within the small control volume. The control volume has dimensions of $\Delta x \Delta y \Delta z$ and, because we are taking on average value it is a constant inside the control volume. So, $\frac{\partial}{\partial t} \rho e$ can be taken outside the integral sign and then we are left with integral dV over the control volume and the volume of this control volume is, $\Delta x \Delta y \Delta z$.

And like to mention and specify that e here represents the total energy per unit mass which includes the internal energy per unit mass, the kinetic energy per unit mass, and the potential energy per unit mass. And then like all the other differential balance equations we will finally, divide by $\Delta x \Delta y \Delta z$ and shrink to a point. So, what we will do is divide now itself and keep the terms ready, so that finally, we can substitute in the integral energy balance equation and then take the limit finally.

$$\frac{\partial(\rho e)}{\partial t}$$

Now, let us proceed with the convection terms,

$$\int_{CS} \rho e v . n dA$$

Now, let us look at the significance once again. It represents net rate of flow of total energy out through the control surface by convection. So, it tells about the rate of flow of energy and it represents a net rate of flow of total energy out through the control surface by convection. Now, many terms are very familiar to us, we have looked at this several times in while deriving other conservation equations. Only difference is that now we are applying for total energy, what this tells us about the flow of energy.

And, now let us look at the control volume, the arrows represent the flow of energy entering and leaving to the respective faces. So, we have a six faces energy enters through the left face, the bottom face, and the rear face and it leaves through the right face, the top face, and the front face. And, now what we should do is express this integral expression for the six faces let us do that.

$$\int_{CS} \rho e v.n\, dA = \sum_{i=1}^{3\,inflow\ faces} \rho_i v_i A_i e_i - \sum_{i=1}^{3\,outflow\ faces} \rho_i v_i A_i e_i$$

This has to be written for six faces, 3 outlet face, and 3 inlet face and for the outflow face $v.n$ is positive and that is way it is positive here. And, for the inflow faces $v.n$ is negative and that has been taken care by us explicitly. So, $\int_{CS} \rho e v.n\, dA$ has been expressed for the six faces in this expression 3 outlet faces, 3 inlet faces.

Now, what we will do is consider the faces pair wise and apply this expression here we are considered the three outflow face and three inflow faces. So, we will take one outflow face, one inflow face let us say the x direction and then see how to express that.

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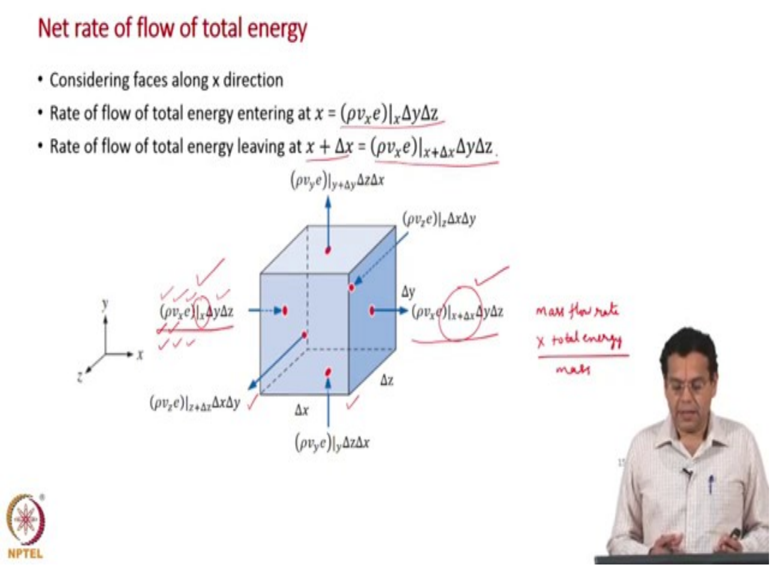

Now, let us look at the two faces along the x direction that inflow face and the out flow face that is the left face and the right face. Now,

The rate of flow of total energy entering at x = $\left(\rho v_x e\right) \vee ¿_x \Delta y \Delta z ¿$

Now, how do we interpret this, as we have been doing earlier you can interpret in two ways, first the easier one v_x is the velocity, we multiply by area, we get the volumetric flow rate. Multiplied by density we get the mass flow rate and then we multiplied by the total energy per unit mass then we get the rate of flow of total energy. So, velocity then multiply by area gives volumetric flow rate, multiply by density gives mass flow rate, then if you multiply with the total energy per unit mass, we get rate flow of total energy.

What is the other way of interpreting v_x is the volumetric flux, and then if you multiplied by ρ you get you get the mass flux, multiplied by area we once again get mass flow rate, multiplied by the total energy per unit mass we get the rate of flow of total energy the ways of interpreting. Now, ρ, v_x, e all can vary specially, and the left face is that x the right face is at $x+\Delta x$.

So, you have indicated that the whole term is evaluated at x, the whole terms is evaluated $x+\Delta x$, so these terms put together represent rate of flow of total energy entering at x and that is what is written here.

The rate of flow of total energy entering at $x+\Delta x = (\rho v_x e) \vee ¿_{x+\Delta x} \Delta y \Delta z ¿$

So, what we are discuses now is, how to express the rate of flow of total energy entering and then leaving through the faces of the x direction.

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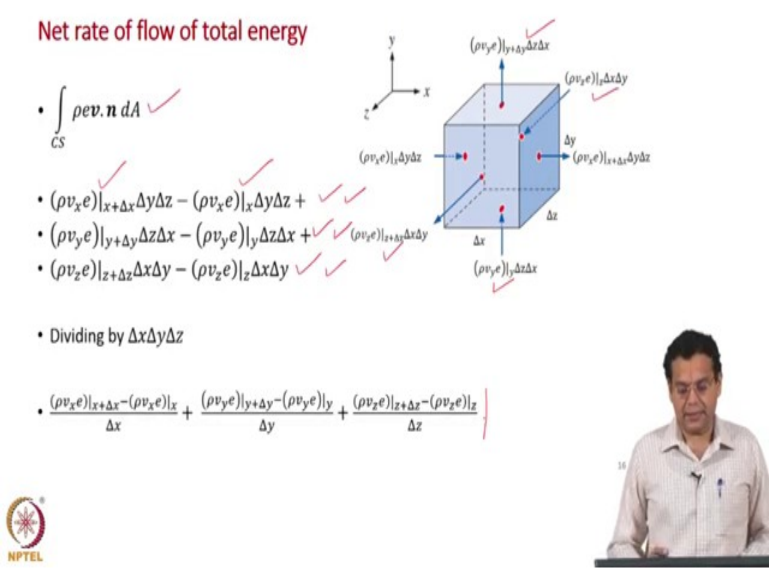

$$\int_{CS} \rho e v . n \, dA$$

So, now let us extend this for all the other three directions, what we have seen in the previous slide are these terms.

$$(\rho v_x e) \vee \text{¿}_{x+\Delta x} \Delta y \Delta z - (\rho v_x e) \vee \text{¿}_x \Delta y \Delta z \text{¿¿}$$

Now, what does this represent this represents rate of flow of energy leaving, and this represents rate of flow of energy entering when I say energy at I mean total energy.

So, these two terms put together represents net rate of flow of total energy leaving the control volume through the control surfaces in the x direction. So, let us write similarly for other directions there is energy entering at the bottom face and then energy leaving at the top face and those two terms are shown here.

$$(\rho v_y e) \vee \text{¿}_{y+\Delta y} \Delta x \Delta z - (\rho v_y e) \vee \text{¿}_y \Delta x \Delta z \text{¿¿}$$

Once again we should take as rate of flow of energy leaving minus rate of flow of energy entering, because this integral terms tells about net rate at which energy leaves the control surface. Similarly, in the z direction

$$(\rho v_z e) \vee \text{¿}_{z+\Delta z} \Delta x \Delta y - (\rho v_z e) \vee \text{¿}_z \Delta x \Delta y \text{¿¿}$$

So, as we are discussed earlier let us divide by $\Delta x \Delta y \Delta z$ and keep it ready, so that later on we can take limit $\Delta x \Delta y \Delta z \rightarrow 0$, so that becomes a point.

$$(\rho v_x e) \vee \text{¿}_{x+\Delta x} - \frac{(\rho v_x e) \vee \text{¿}_x}{\Delta x} + (\rho v_y e) \vee \text{¿}_{y+\Delta y} - \frac{(\rho v_y e) \vee \text{¿}_y}{\Delta y} + (\rho v_z e) \vee \text{¿}_{z+\Delta z} - \frac{(\rho v_z e) \vee \text{¿}_z}{\Delta z} \text{¿¿¿¿¿¿}$$

So, now we should note that they once again represent rate of flow of total energy, the net rate of flow of total energy leaving. But, now because we are divided by $\Delta x \Delta y \Delta z$, it is per unit volume and now it accounts for all the three directions.

So, here it is in terms of only rate of flow of energy and then net rate of flow of energy leaving that is for x direction and then for y direction, then for z direction. When we divide we get here as per unit volume basis, also accounting for all the three directions.

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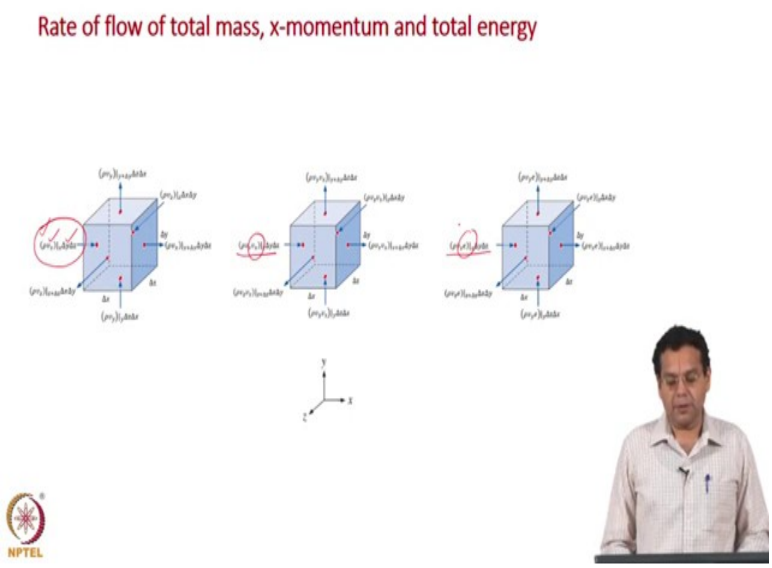

Now, like to compare this control volume with the control volume which we have discussed earlier while driving the differential mass balance and differential momentum balance and those control volumes are shown in the above slide image. So, this slide shows the three control volumes; the first one, the left side one which we use while deriving the differential mass balance. The second one, the middle one shows the control volume which are used while deriving the momentum balance, the third one, the right side one which we have seen just now. So, let us understand them, let us take one term, $\left(\rho v_x\right) \vee$ ¿$_{x+\Delta x} \Delta y \Delta z$ ¿ it represents the rate of flow of mass entering in the x direction.

And we interpreted that as the velocity into area the volumetric flow rate multiply by mass gives a mass flow rate, then when we discuss the momentum balance what did we do we took the same term, but now included v_x, $\left(\rho v_x v_x\right) \vee$ ¿$_{x+\Delta x} \Delta y \Delta z$ ¿. So, that now it represents the rate of flow of x momentum entering the x direction. Why is it? Without v_x it represented the rate flow of mass we multiplied by v_x which is the momentum per unit mass and then we got rate of flow of momentum entering in the x direction.

Now what is that we have done once again taking the same term and then included a total energy per unit mass, $\left(\rho v_x e\right) \vee$ ¿$_{x+\Delta x} \Delta y \Delta z$ ¿ and now it represents rate of flow of total energy entering in the x direction. So, it should be easy if you compare all the control volumes all the terms of similar physical significance they represent the particular property. They represent the corresponding property brought in and brought out in all the three directions because of convection.

So, if you understand that then from one control volume we should be able to draw the other two control volumes. And in fact, that is how these figures have been prepared the first figure was prepared then copied pasted added the, v_x, then replace the v_x with e and got the third figure.

So, that also it may look like a copying and pasting and replacing, but that has a physical significance behind it meaning that. All the three control volumes represent with these terms represent the flow of that particular property entering and leaving and all these terms represent the convection term.

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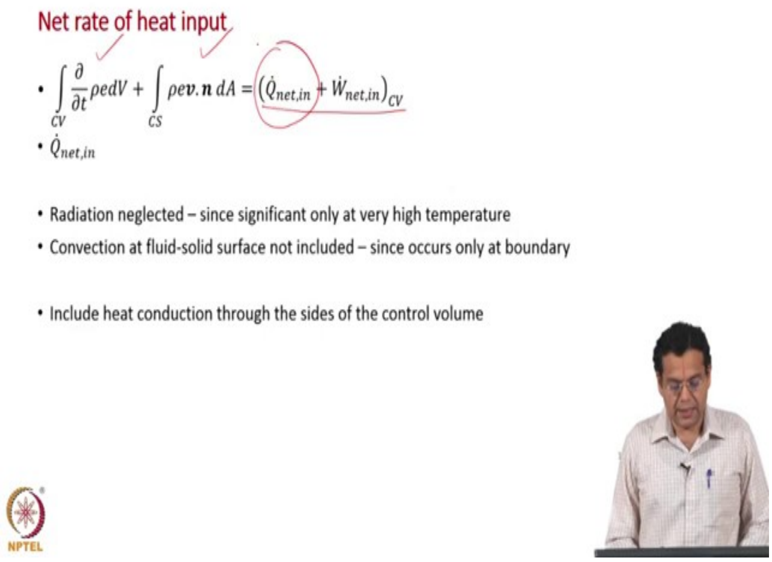

Now, let us go back to your integral form of energy balance equation let us see what is it we have done and where should we proceed further.

$$\int_{CV} \frac{\partial}{\partial t}\rho e dV + \int_{CS} \rho e v.n\, dA = ¿¿$$

Now, we have applied this integral energy balance equation for the small control volume, we have expressed the left hand side for the small control volume, both the transient term and the convection term.

Now, we will focus our attention on the right hand side and in the right hand side we will focus our term on the net rate of heat input or heat addition. So, what is that we are going to discuss, now how do we express that $\dot{Q}_{net,\in ¿¿}$ which represents net rate of heat put for the small control volume. Now, we know that there are three modes of heat transfer; conduction, convection, and radiation. Let us discuss what do we include here and what do we exclude here; in fact, first we discuss the exclusion, so that we can include whatever is required.

- First radiation: radiation is significant only at high temperatures, so we neglect in fact, not even high temperature it is at very very high temperature ok. Let us say 1000 Kelvin, 1000 degrees centigrade, order of such a high temperature radiation is important, we will restrict to not so high temperature, so let us neglect radiation.
- Now, convection which represents heat transfer occurring at a fluid solid surface is also not included.

The reason is that let us imagine this our fluid domain and our control volume is inside. So, when you are considering a control volume inside that is not at the boundary between a let us say a solid and a fluid, our region is inside. So, because we are considering a control volume inside the fluid domain we are not including the heat transfer by convection which happens at fluid solid surface. It does not mean you are ignoring that; remember we had a differential equations we discussed about boundary conditions.

So, when we solve the energy balance equation, these convection terms appear as boundary conditions. We may not discuss, we may not come across an example in this course such a boundary condition. But, in your heat transfer course you will come across a case where the differential energy balance equation is solved using the convective heat transfer as a boundary condition.

So, radiation is not included, because we are not considering very high temperature. Convection which represents heat transfer at fluid solid surface is not included in the differential energy balance equation, in the conservation equation it is not included. Because, that occurs only at solid fluid interface our control volumes somewhere inside. So, what is left out is heat transfer by conduction and let us include the heat transfer conduction through all the six faces of the control volume.

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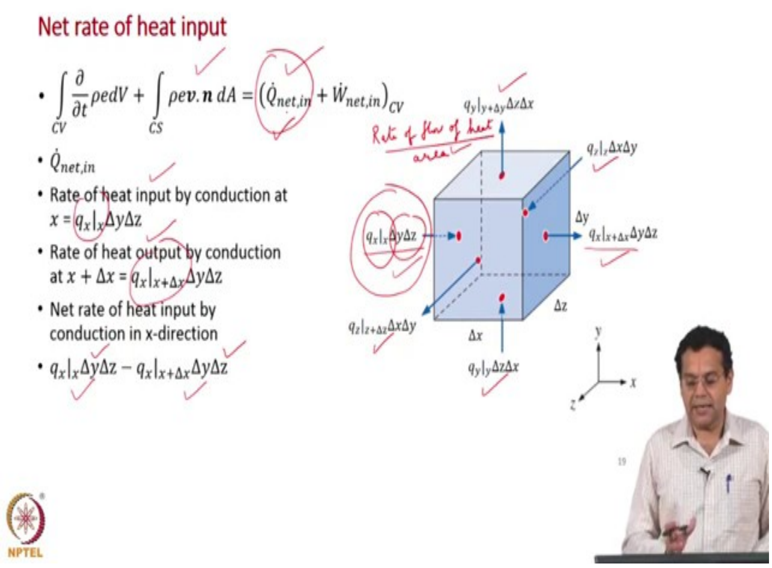

$$\int_{CV} \frac{\partial}{\partial t}\rho e dV + \int_{CS} \rho e v.n\, dA = ¿¿$$

So, now our objective is to represent this rate of heat input through the control surfaces by conduction only. And that is what is shown in this control volume through these different terms here. Here q_x represents the heat flux in the x direction, q_x represents the heat flux in the x direction due to conduction. And how do you represent heat flux we represent that as rate of flow of heat per unit area. And when you multiply by the area we get the rate of flow of heat, and this heat flux can vary with spatial location.

So we express that as $q_x \vee ¿_x ¿$ and similarly, $q_x \vee ¿_{x+\Delta x} ¿$. And, this represents rate of heat input by conduction and rate of heat output by conduction respectively.

So, our focus is on the net rate of heat input or heat addition,

The rate of heat input by conduction at the x = $q_x \vee ¿_x \Delta y \Delta z ¿$

Once again one specify that q_x represents heat flux in the x direction due to conduction and flux is always any quantity per time per area.

So, we have rate of flow of heat which has per time unit per unit area multiply by the area which is $\Delta y \Delta z$ we get the rate of flow of heat and because it is inflow we are writing it as rate of heat input by conduction. And, at the right face we have

The rate of heat output by conduction at the $x+\Delta x$ = $q_x \vee ¿_{x+\Delta x} \Delta y \Delta z ¿$

So, now

$$q_x \vee ¿_x \Delta y \Delta z - q_x \vee ¿_{x+\Delta x} \Delta y \Delta z ¿ ¿$$

The first term represents heat input and second term represents heat output what we want is net rate of heat input that is why we subtract output from input. So, we have input minus output representing net input, this is in contrast to the convection term which was outflow minus inflow.

So, this has to be kept in mind when we did the convection term it was net outflow. So, we always throughout all the conservation equations we have been considering it as outflow minus inflows, so that it represents net outflow. But, now on the right hand side we have net rate of heat input, so we should take the term as input minus output or heat input minus heat output. So, that does that term represents net rate of heat input ok, this convention has to be kept in mind, so that we take care of the sign correctly.

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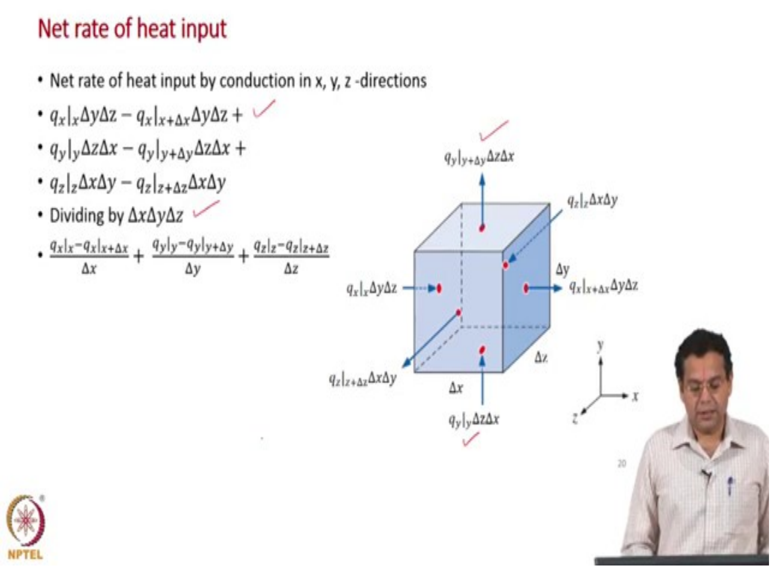

So, let us do it for the other directions, the net rate of heat input by conduction in the x, y, z directions you have to just repeat it for other directions with the corresponding area. So,

$$q_x \vee ¿_x \Delta y \Delta z - q_x \vee ¿_{x+\Delta x} \Delta y \Delta z ¿ ¿$$

$$q_y \vee ¿_y \Delta x \Delta z - q_y \vee ¿_{y+\Delta y} \Delta x \Delta z ¿¿$$

$$q_z \vee ¿_z \Delta y \Delta x - q_z \vee ¿_{z+\Delta z} \Delta y \Delta x ¿¿$$

So, as for the other terms on the left hand side here also we will divide by $\Delta x \Delta y \Delta z$ and keep it ready with us.

$$q_x \vee ¿_x - \frac{q_x \vee ¿_{x+\Delta x}}{\Delta x} + q_y \vee ¿_y - \frac{q_y \vee ¿_{y+\Delta y}}{\Delta y} + q_z \vee ¿_z - \frac{q_z \vee ¿_{z+\Delta z}}{\Delta z} ¿¿¿¿¿¿$$

So, here the significance is net rate of heat input by conduction and includes all the three directions. When you divide by, $\Delta x \Delta y \Delta z$; now these terms represent net rate of heat input per unit volume taking into account x, direction y, direction and x direction. So, here the significance is net rate of heat put per unit volume.