Continuum Mechanics and Transport Phenomena
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Lecture – 116
Non Isothermal Planar Couette Flow

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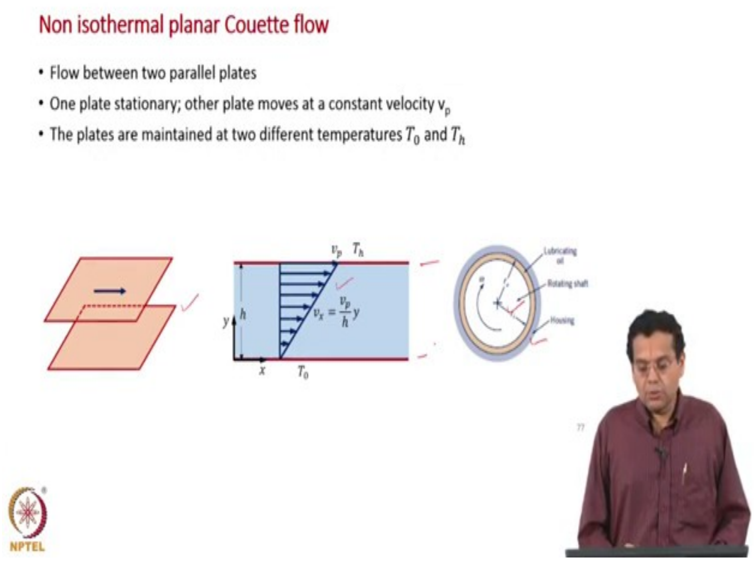

Now, let us proceed to the next example, very familiar example. The flow between two parallel plates, the top plate set in motion which is a planar Couette flow. We have solved the velocity profile earlier, but now we are going to take the two plates to be at two different temperatures. Earlier we had the two plates the top plate was set in motion, but we never even talked about the temperature. Because, we are focusing on the Navier stokes equation our attention was on the velocity profile.

But now, we are considering a case where the top plate is moving bottom plate is stationary, but now they are at two different temperatures; bottom plate is a T_0, top plate is at T_h, the top is a higher temperature than bottom plate. Now, what is an application? We said the flow between the two parallel plates approximates flow between two coaxial cylinders and that happens in a viscometer or in a shaft barring. And by considering the two plates to be at two different temperatures we are allowing for the rotating shaft and the housing to be at two different temperatures, which is more realistic than assuming them to be at the same temperature. What is the objective, to find out the temperature profile in this region.

So, let us proceed with that flow between two parallel plates; one plate stationary, other plate moves at a constant velocity v_p till that it is same as the earlier description. Now, this condition is different the plates are maintained at two different temperatures T_0 and T_h.

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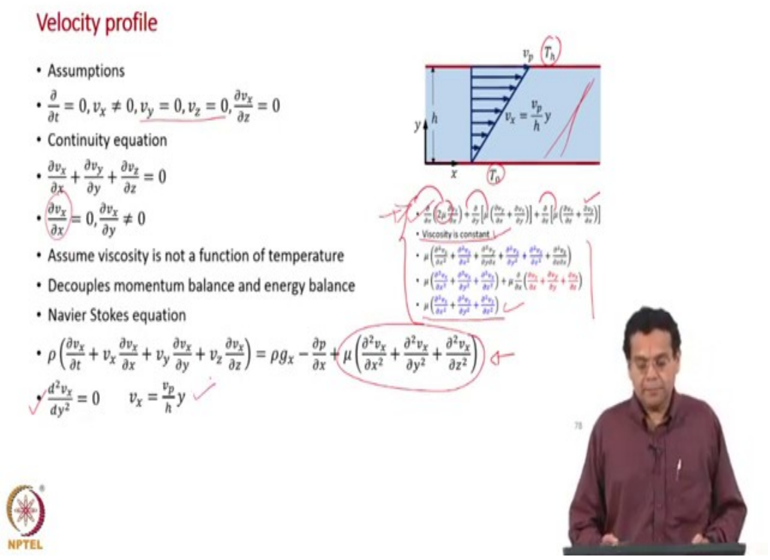

So, now this is almost a recall slide, but with some more discussion. The first step is to find out the velocity profile, we already found out the velocity profile for this planar Couette flow. So, in that sense it is recall, but some more discussion is required. So, let us proceed with that, what are the assumptions we made? It is steady state and then we have only x component of velocity, there is no y component of velocity, there is no z component of velocity. And then we also said the x component of velocity does not vary in the z direction.

$$\frac{\partial}{\partial t}=0, v_x \neq 0; v_y=0; v_z=0; \frac{\partial v_x}{\partial z}=0$$

And we use the continuity equation

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

And when we used these conditions $v_y=0; v_z=0$, we concluded that based on the continuity equation

$$\frac{\partial v_x}{\partial x}=0; \frac{\partial v_x}{\partial y} \neq 0$$

This means that the x velocity does not vary along the x direction. Based on this, we arrived that the x velocity varies only along the y direction.

Now, till this the discussion is same, but to proceed further which mean that to solve for the velocity profile, we earlier used the Navier-Stokes equation. Now to proceed further, we need to make an assumption, what is that? We are going to assume the viscosity is not a function of temperature. Now question arises why this assumption is required? Earlier the two plates were at the same temperature or we did not even mention that.

Now, they are at two different temperatures; so, there is going to be a temperature profile here. Now if viscosity varies, if viscosity is a function of temperature, then viscosity will also vary between the two plates, which means that first, we will have to solve the energy balance get the temperature profile and then solve the linear momentum balance equation including the variation of viscosity with space. So, by assuming viscosity as not function of temperature we have decouple the momentum balance in energy balance equation, what does it mean? We can solve momentum balance separately energy balance separately.

Just to quickly recall, assumption of viscosity not to depend on temperature is required reason is temperature varies between the two plates. So, viscosity will also vary between the two plates, if you assume it to depend on temperature, and which means that first we will have to solve the energy balance then come to the momentum balance to solve for velocity profile. But, we are avoiding all the difficulties to make the problem simple by assuming viscosity is not a function of temperature. So, that we can solve the momentum balance energy balance separately that is the first point.

Now, what is the second point let us write down Navier-Stokes equation. And then we will see what is the other implication of assuming viscosity not to be a function of temperature,

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$$

That is the Navier-Stokes equation. What is shown here at the last steps in the derivation of the Navier-Stokes equation. The right hand side alone is shown in the right hand side only the viscous terms alone are shown or the momentum flux terms alone shown.

Now, this was the step we had and then we assumed viscosity as a constant, then we did simplification and arrived at this expression which was in terms of the Laplacian of the velocity. But now, if viscosity is not a constant then this expression is not valid, we will have to use this terms as such why is that? Because, we assumed viscosity as a constant what does it means? Viscosity not varying with spatial location we could take out the viscosity outside of this spatial derivative.

But now if we assume viscosity to depend on temperature, viscosity will also vary spatially. So, I cannot take it out and get this expression, I will have to solve the linear momentum balance equation with those terms as such. So, to simplify that as well, where we assumed the viscosity to be constant; so, two implications; one decouples the momentum balance and energy balance equation we can solve them separately. Second to enable us to use the Navier-Stokes equation itself, otherwise we will have to take the form of linear momentum balance equation which includes the special variation of viscosity.

So, once we have justified this or made assumptions such that the Navier-Stokes equation can be used, then if you apply the usual simplifications we arrived at this differential equation for the velocity profile. And if you solve we get the linear velocity profile, but now we should know the assumptions behind this equation.

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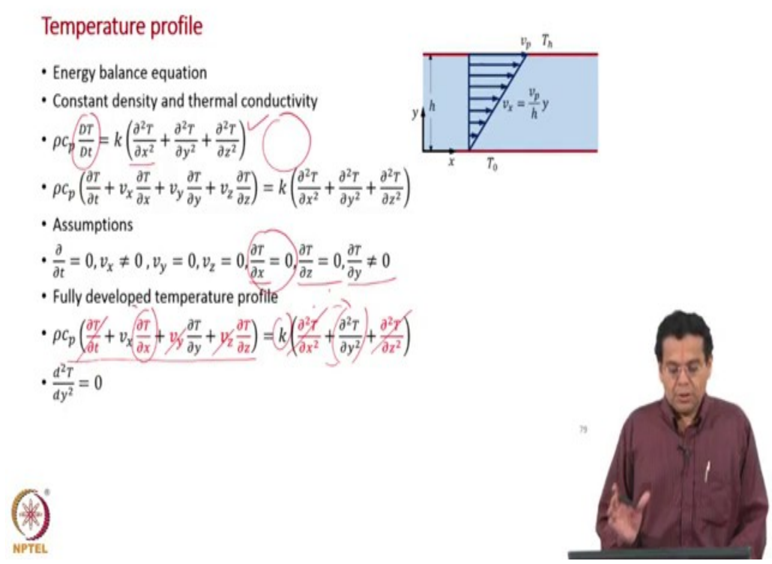

Now, let us proceed towards solving the temperature profile. Let us write down the energy balance equation. Now, we will assume the density is constant and the thermal conductivity is also a constant, and so, this is the relevant energy balance equation.

$$\rho c_p \frac{DT}{Dt} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

Once again we looked at the several levels of simplification; if you refer the particular discussion we will find out that this is the equation applicable for the case of constant density and constant thermal conductivity.

To quickly recall, thermal conductivity is constant so, we could take out and we could get a second order derivative for temperature. We get another term here which was, effect of temperature on density because we have assumed density to be constant that term does not appear. And left hand side we have capital $\frac{DT}{Dt}$, remember because we are applying for fluids in motion, when you applied for solid it was $\frac{\partial T}{\partial t}$.

Now, let us expand the substantial derivative, so that we can see how to simplify that

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

Now assumptions, steady state so, $\frac{\partial T}{\partial t} = 0$ there is only x component of velocity which I have already discussed. There is no component of velocity along y direction, z direction already discussed.

$$\frac{\partial}{\partial t} = 0;\ v_x \neq 0;\ v_y = v_z = 0;\ \frac{\partial T}{\partial x} = 0;\ \frac{\partial T}{\partial z} = 0;\ \frac{\partial T}{\partial y} \neq 0$$

But now to proceed further, we need to make more assumptions what is that? Temperature does not vary along the flow direction and then temperature does not vary along z direction. So, temperature will vary only along y direction. The whole idea is to make sure that there is only one spatial dependency, the temperature or velocity, we make it to vary along one direction. So, that we result in a ordinary differential equation. For the present case, this assumption is called as fully developed temperature profile for the present case.

So, now, let us look at the left hand side, because it is steady state

$$\rho c_p \left(0 + v_x 0 + 0 \frac{\partial T}{\partial y} + 0 x 0\right) = k\left(0 + \frac{\partial^2 T}{\partial y^2} + 0\right)$$

So, the entire left hand side vanishes or putting it the other way, we have made assumption such that the substantial derivative vanishes, what is the reason? As I told you we want to retain only one spatial dependence and make the problem simpler. So, it becomes

$$\frac{d^2 T}{d y^2} = 0$$

So, energy balance once again becomes very simple, but we should be aware of the assumptions which you have made to arrive at this equation.

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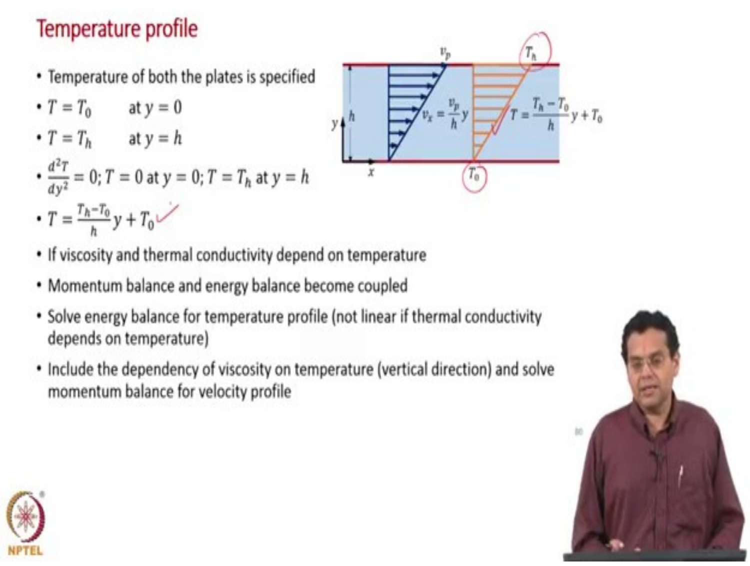

So, now we will have to solve the second ordinary differential equation. The temperature of both the plates are specified; so

$$T = T_0 \; at \; y = 0$$

$$T = T_h \; at \; y = h$$

So, we have

$$\frac{d^2T}{dy^2}=0$$

This subject to these two boundary conditions, which have at this stage it is similar to the equation which you have seen for the case of slab. So, we get the same linear temperature profile

$$T=\frac{T_h-T_0}{h}y+T_0$$

So, that is the temperature profile shown here varying linearly from T_0 to T_h. So, same temperature profile only the directions is different, there we considered a slab like this here we considered two plate like this. But, this temperature profile as we have seen as several assumptions behind it.

Now, just to discuss we have already discussed about this, just to I would say summarize; suppose, if you have taken viscosity and thermal conductivity to depend on temperature what will happen, momentum balance energy balance become coupled. First you have to solve the energy balance equation for the temperature profile. Now because thermal conductivity depends on temperature we will not get a linear temperature profile. Then once you know the temperature variation, you know the viscosity variation also along the y direction.

So, include the dependency of viscosity and temperature; viscosity directly depends on temperature because temperature depends on y viscosity indirectly depends on y and then solve the momentum balance equation for the velocity profile not the Navier-Stokes equation. So, that way it becomes very involved, that is why to suit the level of this course. We have made several of the assumptions and the profile I would said is not very interesting just linear. And looks as if we have the same profile even when we have flow and when you have a solid, but we should keep in mind the assumptions here.

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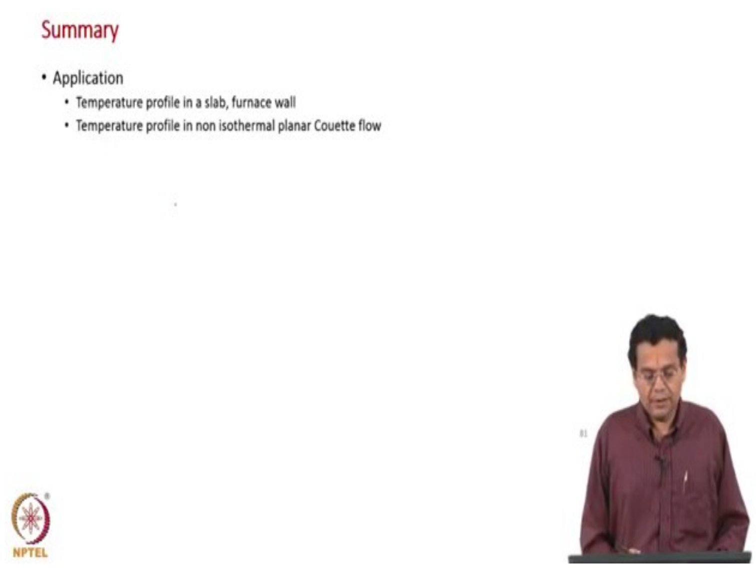

So, let us summarize we have looked at the applications of the differential energy balance equation. I would say very very simple application; one to find out temperature profile in a slab extended that to a furnace wall also found out the heat flux. Next is to find out temperature profile in non-isothermal planar couette flow. So, far we have been looking at planar Couette flow, because it is a two different temperatures we call it as the non-isothermal planar Couette flow.

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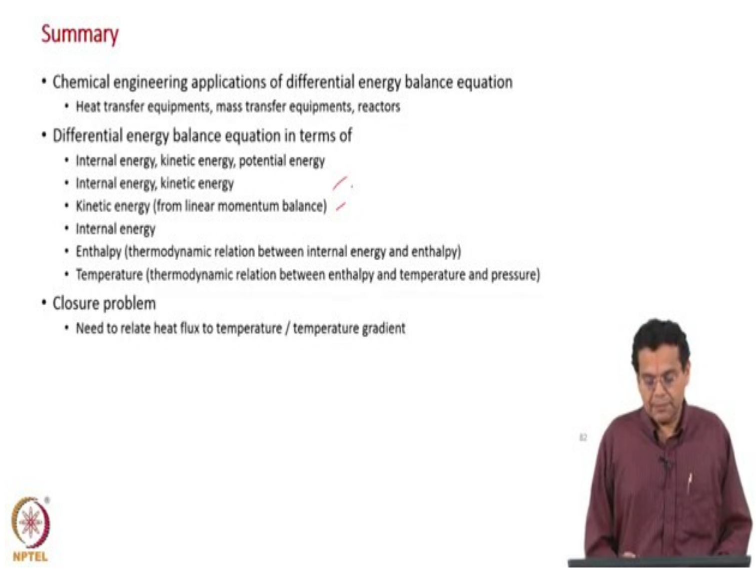

Let us summarize what we have discussed under differential energy balance equation. First looked at the chemical engineering applications of differential balance equation with respect to heat transfer equipment, mass transfer equipment and reactors. Then we derived the differential energy balance equation in terms of the total energy internal kinetic and potential and from that we got the equation in terms of internal and kinetic energy.

Separately derived an equation for kinetic energy from linear momentum balance equation, from these two equations we got the differential energy balance equation in terms of internal energy. Then based on thermodynamic relations, we expressed the equation in terms of enthalpy and in terms of temperature as well. We discussed the closure problem where we realize that the heat flux term has to be expressed in terms of temperature more precisely temperature gradient.

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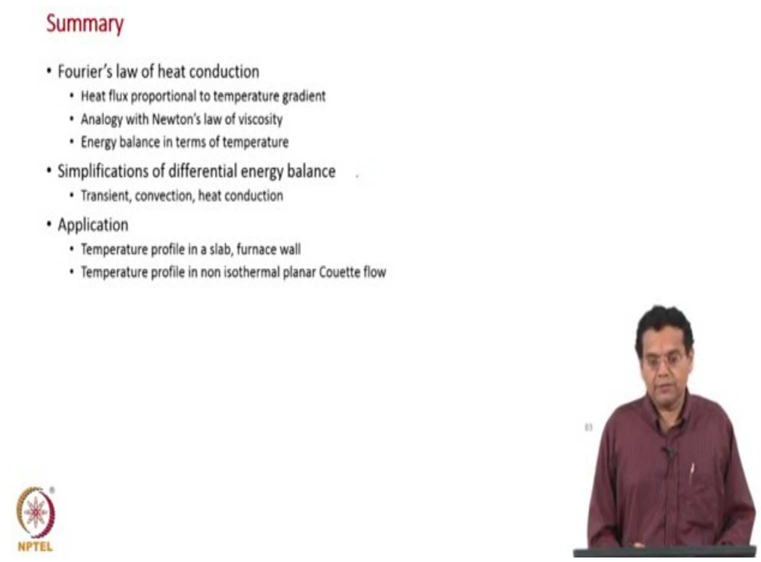

To close the energy balance equation, we discussed the constitutive equation namely the Fourier's law of heat conduction, which says the heat flux is proportional to the temperature gradient including a negative sign. We discussed the analogy with Newton's law of viscosity. Using the Fourier's law of heat conduction we expressed the energy balance equation completely in terms of temperature.

We also simplified the differential energy balance equation to forms which we usually use and finally, it act transient convection heat conduction terms. Looked at applications to find out the temperature profile in a slab, furnace wall and then non-isothermal planar Couette flow.