Continuum Mechanics And Transport Phenomena

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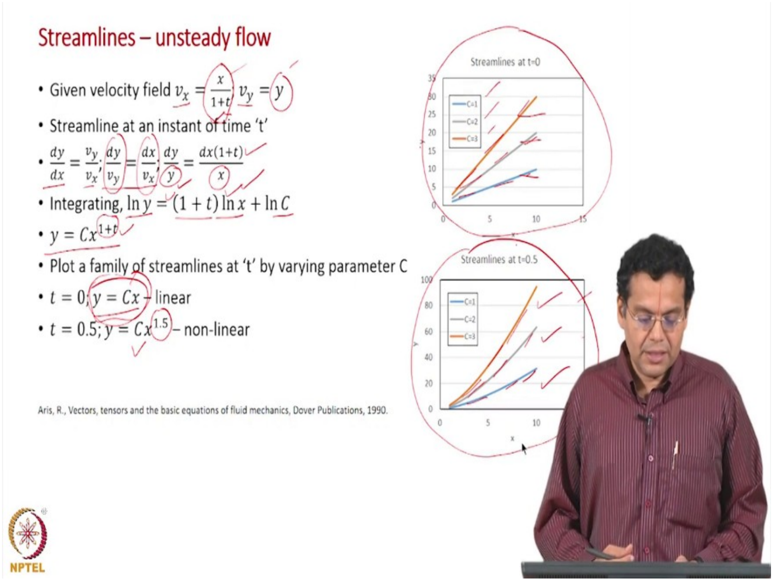
Lecture – 15
Streamline, Pathline, Streakline: Unsteady Flow Example

Example: (Refer Slide Time: 00:15)



The second example as I told you is calculating Streamlines, Pathlines and Streaklines. In the last example we are not discussed streaklines, in this example we will also discuss streaklines. Other objective is to see how to find out these lines and plot them for a unsteady flow and we have already discussed for unsteady flow these lines are not same. So, we will also, you will also see that these lines are not same they are not they do not coincide and then you will also see how to plot a streak line, those are the specific special features of this example, other things are a little bit common with the previous example.

We start with this velocity field; the velocity field is given by,

$$v_x = \frac{x}{1+t}; v_y = y$$

Because the x component is a function of time, it is a unsteady flow field, if it were not then it is just a steady flow field like in the previous case. Now, this example also illustrate, we have been talking about streamline at a particular instant there will also be emphasized here. Now, streamline as we have seen represents the flow at a particular time instant.

Now, let us start with the equation for the streamline which are using the previous example

$$\frac{dy}{dx}=\frac{v_y}{v_x}$$

And do a rearrangement, convenient for integration as

$$\frac{dy}{v_y}=\frac{dx}{v_x}$$

$$\frac{dy}{y}=\frac{dx(1+t)}{x}$$

Now, you will have to integrate the left hand side, integrate the right hand side, when you integrate remember, because streamlines at a particular instant t is just a constant given value to you, I want streamline at first second, 0 second. So, you integrate as usual with respect to y, as usual with respect to x keeping just t as a constant.

$$\ln y=(1+t)\,lnx+lnC$$

Now, we can rearrange

$$y=C\,x^{1+t}$$

Please pay attention that you have a time here; so this equation is for a particular time t. Based on the time t for example, which we will see now, we can plot a family of streamlines at time t by varying parameters. In the previous example there was no time you just vary C and got a family of streamlines. This case at every instant time of t you will get a family of streamlines, you will see that now.

Let us take t = 0 and we substitutes t = 0 you get

$$y=Cx$$

This is linear. So, these are the family of streamlines. What is it I have done? I have taken different values of C here and then plotted this equation that is, that is all I have done here (shown in the figure) the x axis y axis, taken C equal to 1, 2, 3 etcetera here and then plotted this linear relationship $y=x$, $y=2x$, $y=3x$ all these are straight lines.

Now, let us say some other instant it, I take as $t=0.5$.

$$y=Cx^{1.5}$$

Now, it becomes non-linear, just want to say that the streamlines differ. And now I get another family of streamlines by once again varying the value of C, $y=x^{1.5}$, $y=2x^{1.5}$ and $y=3x^{1.5}$ and I get another set of family of streamlines. This is what we have been emphasizing that this represents set of streamlines at that instant t = 0. The second equation represent another set of streamlines at time t = 0.5, that is why we say streamlines are instantaneous. Now, if you draw tangent to that; of course, they are all straight lines you will get the velocity over the entire region and now if you draw tangent; of course, these curves if you draw tangent along for all these curves at different locations you will get the velocity vector in the entire region. A very good example to illustrate that streamlines are instantaneous flow patterns and they change from time instant to time instant. This was not the attention of the earlier example, so we took a steady state example.

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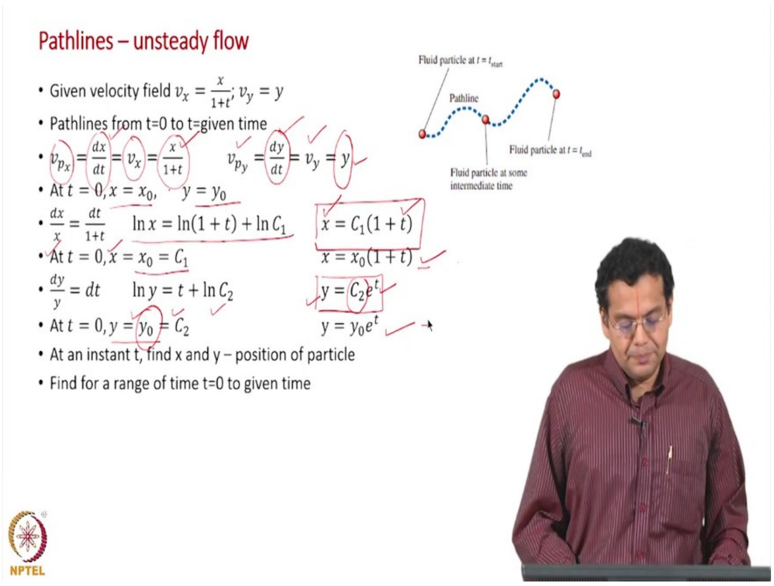

Path lines for unsteady flow. So, we start with the given velocity field

$$v_x=\frac{x}{1+t};v_y=y$$

Now path lines from $t=0$ to given time t which we have seen earlier. These steps are similar to the previous example,

the velocity of the particle = the rate of change of its x coordinate = the x compound of the velocity field

$$v_{p_x} = \frac{dx}{dt} = \frac{x}{1+t}; v_{p_y} = \frac{dy}{dt} = y$$

Now, this step is also we have seen earlier, I have a differential equation, no longer t is a constant remember, t varies along the path line, so we cannot take it as a constant as at an first streamline and $t$is an independent variable.

So, I have to integrate this two equation. We need initial conditions, so

$$At\, t=0; x=x_0, y=y_0$$

So, we integrate these two equations, rearrangement

$$\frac{dx}{x} = \frac{dt}{1+t}; lnx = \ln(1+t) + \ln C_1; x = C_1(1+t)$$

So, the equation for the x coordinate of the particle as a function of time is given by this expression. Now, we will have to evaluate the constant C_1. What is that we know about the particle? We know that the particle exponent is x_0 at t = 0. So, substitute

$$x = x_0 = C_1; x = x_0(1+t)$$

Please keep this in mind when we later on discuss streak lines there will be a difference the way in which we evaluate the constant, because the condition that we know is different, the condition what we know here is different. So, we know the initial condition t = 0, $x=x_0$, so we get this expression for the x coordinate.

Let us repeat this for the y direction

$$\frac{dy}{d} = dt; lny = lnt + \ln C_2; y = C_2 e^t$$

Now, evaluate the constant of integration as we have done along the x direction, we know the y coordinate, so substitute t = 0, $y=y_0$,

$$y = y_0 = C_2 ; y = y_0 e^t$$

So, the equation for the y coordinate of the particle as a function of time is given by this equation. So, these two equations together tells you how the x and y coordinates change as a function of time for the particular particle, at an instant find x and y portion of particle that is that is what I explained you. Now, you can find for a range of time $t=0$ to given time, the x and y positions.

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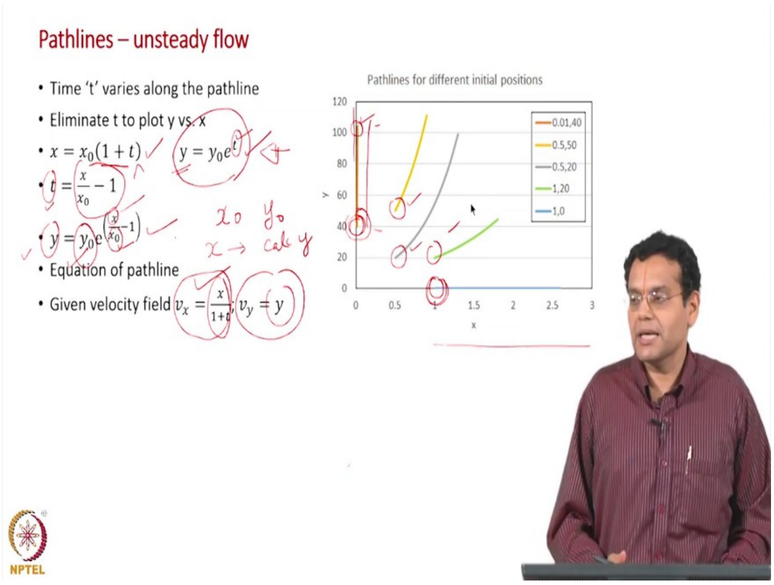

Now, as we are done in the earlier case I can eliminate t; t varies along the path line, so we can eliminate t, so that I can plot y versus x. Finally, the path line is a plot between y and x. So, we will have to eliminate t. So, how do we do that?

$$x = x_0 (1+t) ; y = y_0 e^t$$

So, we will take the first equation and express t in terms of x and x_0 and then substitute in the second equation for t which means we eliminated the time variable t.

$$t = \frac{x}{x_0} - 1$$

$$y = y_0 e^{\left(\frac{x}{x_0} - 1\right)}$$

This is the equation for the path line. These two are the equations to get the x and y coordinate as a function of time, if you are interested in getting only the final expression for the path line, eliminate t and this will be the equation for the path line which tells you how y varies as a function of x, which are the y and x coordinates the particles.

Remember for given initial portion x_0 and y_0 and that is what is shown here in this figure I have taken different initial positions x_0 and y_0 and then drawn the path line. How do you plot this path lines, how do you calculate them? Let us take one particular path line. The $x_0=0.1$, and $y_0=40$, so I know these values, I know x_0 and y_0. Then it is a matter of taking different x values and calculating y values. I will just repeat I know the value of x_0 and y_0 in this particular case 0.1 and then 40. Then what do I do? I vary x and then calculate y using this equation. So, and such plots are shown here for initial portion of point 0.1, 40.

This particular case if you see the, in this region which means we are closer to the y axis, the given velocity field is given by $v_x=\frac{x}{1+t}; v_y=y$ and because they are almost close to the y axis, the x component of velocity is very small, y component has a large value that is why the particles starting here almost travels vertically. Over the time period the distance travel along x direction is very less, but of course, travels a long distance over the y axis, because the y component is much higher than the x component.

Now, similarly if you take this other extreme where the particle lies on the x axis, the y velocity is 0 and you have only the x component of velocity and that is why it just travels along a straight line. So, one almost travel along a vertical line and one travels almost, just almost perfectly travels along the x axis. You can also see that from this equation also when $y_0=0$, y remains 0 and that is why this particle travels along the x axis. Of course, intermediate locations have different path lines based on the x_0 and y_0 values. We can also see that these path lines do not coincide with the streamlines which we have seen, it's a unsteady state flow field, we had streamlines change based on the instant and these path lines do not coincide with the that of those of the streamlines.

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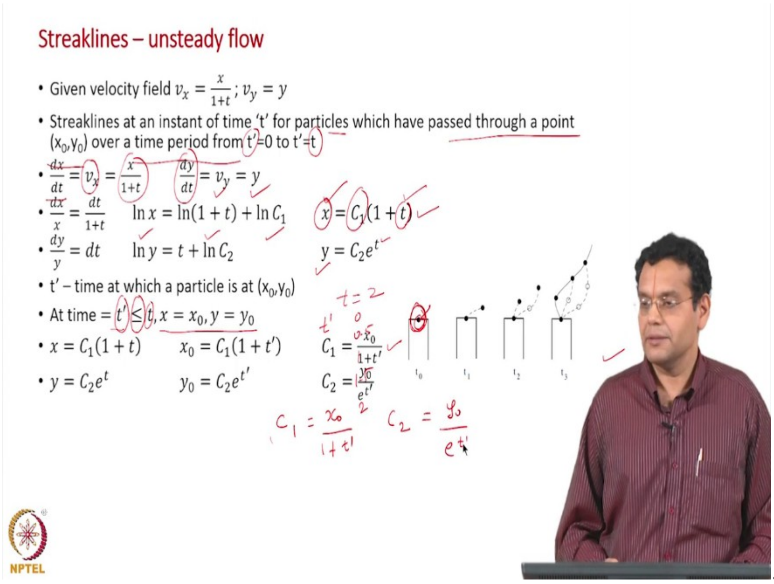

Streak lines; keep repeating this diagram which is very illustrative, as we go along we will also keep this figure in mind, so that we understand better ok. We are given the velocity field

$$v_x = \frac{x}{1+t}; v_y = y$$

That is a starting point remember, visualization of flow patterns. The given velocity field could be measured could be simulation, we are representing using different visualization ways.

Just to recall streak lines at an instant of time they are at an instant of time for particles which have passed through a point x naught y naught over a time period, because I need to introduce one more time I used nomenclature t'. What does what is a meaning of t'? t' tells you the time at which a particle enters or is at the point of interest or enters the domain. In this particular case the point at which a particle has this location is t' for us and that t' varies from 0 to t. What is t? t is the present time, the example which I have going to take, t is 2 we are going to plot the streak line at t = 2.

So, we will have to consider particles which have entered from 0 to 2, like 0.5, 1, 1.5, 2. So, streak lines at an instant of time for particles which have passed through a point x_0, y_0 over a period of time from $t'=0$ to $t'=t$.

The first two steps are same as that of the path line, what is that? Fine integrating the equations.

$$\frac{dx}{dt}=v_x=\frac{x}{1+t};\frac{dy}{y}=v_y=y$$

Now, we integrate this, this step is also same as that for the path line,

$$x=C_1(1+t)$$

Let us do this for the y direction as well

$$y=C_2e^t$$

So, until this the steps are same. Now, how are we evaluating the constant that is the difference between path line and streak line. In the case of path line we know that the what is known to us is a t = 0 that particle was at the position x_0 , y_0. What is known for streaklines is at some time *t* ' a particle is the initial position, some other next *t* ' some other particle is here.

So, the condition known to us is at time $t=t$ ' we know the coordinate, not a time t = 0. We know the condition at which the particle is entering, first particle let us say 0 then first, second, etcetera.

So, *t* ' time at which a particle is at x_0, y_0. I would define that to begin with itself, time at which a particle is at x_0, y_0. So, the conditions, the initial conditions which we know are

$$At\,time=t^{'}\leq t;x=x_0,y=y_0$$

For an example t, we are going to take as 2 seconds. So, $t^{'}$ we will take as 0, 0.5, 1, 1.5 up to 2. So, at time some $t^{'}$ less than or equal to t we know the position of the particle; $x=x_0,y=y_0$. So, to evaluate the constants I should substitute $t=t^{'}$ and then $x=x_0$. In the earlier case we just substitute t = 0 initial time and we evaluated the constant, but here the what we know about the, what is that we want to know the to evaluate the constant, we should know the position of the particle at some particular time. In the earlier case it was time t = 0, the present case it is at some time $t=t^{'}$.

So, we use this equation, substitute $t=t^{'}$ and of course, $x=x_0$ and evaluate the constant as

$$C_1 = \frac{x_0}{1+t'}$$

Now, similarly we do for the y direction,

$$C_2 = \frac{y_0}{e^{t'}}$$

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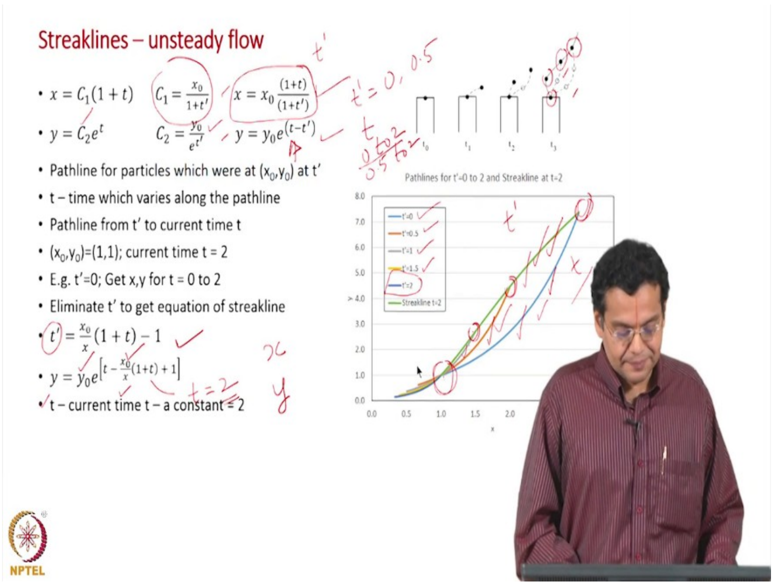

$$x = C_1(1+t); C_1 = \frac{x_0}{1+t'}; x = x_0\left(\frac{1+t}{1+t'}\right)$$

Similarly in the y direction

$$y = C_2 e^t; C_2 = \frac{y_0}{e^{t'}}; y = y_0 e^{t-t'}$$

Now, what is the significance of this equations? To do that let us go to the figure shown in the above slide image, these are equations for the path line of particles, different particles which were at x_0 , y_0 at t', and t is a time which varies along the path line. So, these are just path line equations, but for different fluid particles. Let us take the example here, let us say our point of injection is 1,1 so x_0 , y_0 is 1,1. Now, I consider the fluid particle which was at this location at $t'=0$, our current time is $t=2$.

So, I how do I use this equation? I substitute $t^{'}=0$ and vary t from 0 to 2 seconds and then get this curve which represents the path line of the particle which was at this location at the 0^{th} second. Right now the particle is here and this a path line which was followed by the fluid particle.

Now, let us take second particle which was at this location at $t^{'}=0.5$ seconds, what do I do? Now, I take $t^{'}=0.5$. Once again vary t now from 0.5 to 2, I get the path line of the fluid particle which was at the location (1,1) at the 0.5^{th} second. Similarly, this is the path line of fluid particle which was at (1,1) at the 1^{st} second this path line is not visible in this scale and t dash equal 2 the particle has just entering, so it is at (1,1) itself, it is not even just into the domain.

So, we have got several path lines for $t^{'}=¿$ 0, 0.5, 1 and then 1.5 and then 2. So, these two represents the x and y coordinates of the different fluid particles. The difference become comes because of the fact that t dash varies for different fluid particles. Now, so we drop path lines from $t^{'}$ to the current time t that is what we have done, x_0, y_0 is 1 in our case, the current time is 2 seconds.

An example as I have illustrated $t^{'}=¿$ 0 we get x and y by varying t = 0 to 2. Now, how do we get the streak line which of our interest? You can get, you can join all the endpoints of the path line. Please keep this figure in mind the streak line is obtained by connecting the endpoints of all the path line and exactly if you do the same procedure here and then connect by a smooth line you get the streak line.

Other way of doing that is; for the case of path line t varied along the path line, but for a streak line what varies is $t^{'}$, the time at which different particles enter at our location 1,1, that $t^{'}$ varies from 0 to 2. And, so in this case we eliminate $t^{'}$ to get equation of streak line, how do you do that?

Take the first equation for the x coordinate as a function of time and rearrange that for $t^{'}$.

$$t^{'}=\frac{x_0}{x}(1+t)-1$$

In the case of path line we rearrange for t; of course, there was no $t^{'}$ there and then substitute in the expression for y. In this present case we rearrange for $t^{'}$, because that is a variable to be eliminated, we get this expression for t' and then substitute in the second equation for y, so that we have an expression relating y and then the x which are the y and x coordinates along the streak line.

$$y = y_0 e^{\left[t - \frac{x_0}{x}(1+t)+1\right]}$$

How to use this equation? Remember we are interested in plotting streak line a particular time instant and that is in this case t = 2. And what is x_0 ,y_0? Our point of injection (1,1). So, I know constant value of t = 2 and x_0 ,y_0 is known to me, and then now I vary x and then find out y and then you straight away get the streak line equation or you can straight away plot the streak line. So, if you are interested in plotting the streak line alone using this method you can use this equation and get (y, x).

But if you are interested to show this kind of path lines also then use the earlier approach as we have discussed, where we plot all the individual path lines and then connect the endpoints of the path lines to get the streak line. This streak lines are what we experimentally measure, as I have told earlier by injecting a dye or injecting a smoke. If it is a gaseous medium inject a smoke, let us say colored smoke if it is a liquid medium we inject a dye and what we measure is this streak line.

If it is steady state then this streak line becomes same as path line and then stream line; otherwise it is not the same, but we should keep in mind what we measured is actually a streak line, because we continuously keep injecting particle and we take a photograph which is their position after some time from their point of injection, time of injection. As I told you t; of course, a current time a constant, which is in this case is 2.

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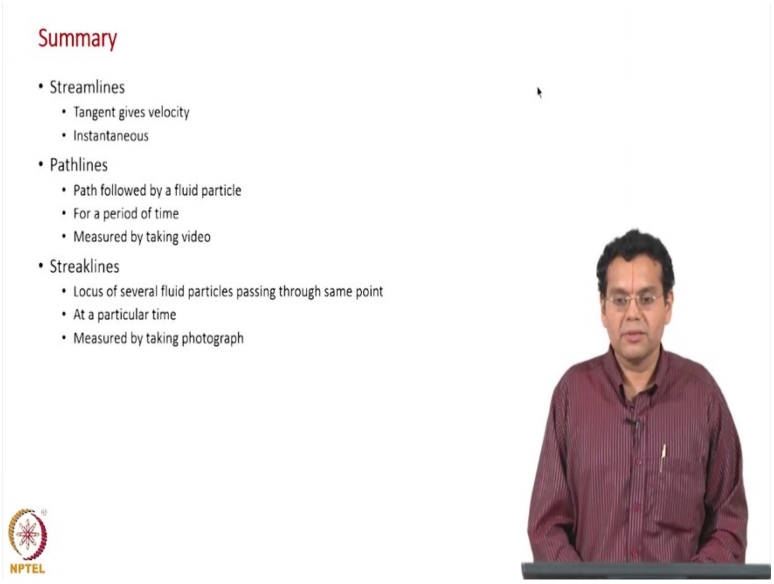

So, just to summarize what we have discussed; stream lines are a tangent to the stream lines gives us the velocity that instantaneous by the same should be clear. Path lines, path followed by a fluid particle for a period of time, it is for a period of time, measured by taking video ok, because it is over a period of time.

Streak lines; locus of several fluid particles passing through the same point at a particular time. We talked about streak line at a particular time and measured by taking photograph. Keywords are video and photograph, you will never forget, everybody likes camera, everybody likes video, everybody likes photograph. So, path lines video, streak lines photograph.