Continuum Mechanics And Transport Phenomena
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Lecture - 16
System and Control Volume

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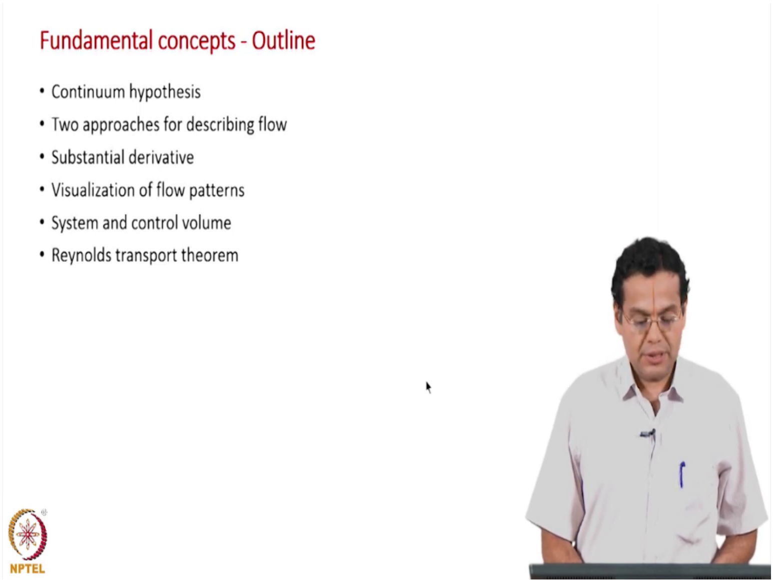

We are in the fundamental concepts, the beginning part of the course, where discussed continuum hypothesis, the Lagrangian and Eulerian approaches for describing flow, the substantial derivative following the fluid motion and then in the last lecture we discussed about different ways of visualizing the flow patterns in terms of streamlines, pathlines, and streak lines.

Now, we are at the end of the fundamental concepts, but two important topics namely system and control volume are left out. We will define them and distinguish them clearly. Then the Reynolds transport theorem which is going to form the basis for the derivation of all the conservation equations.

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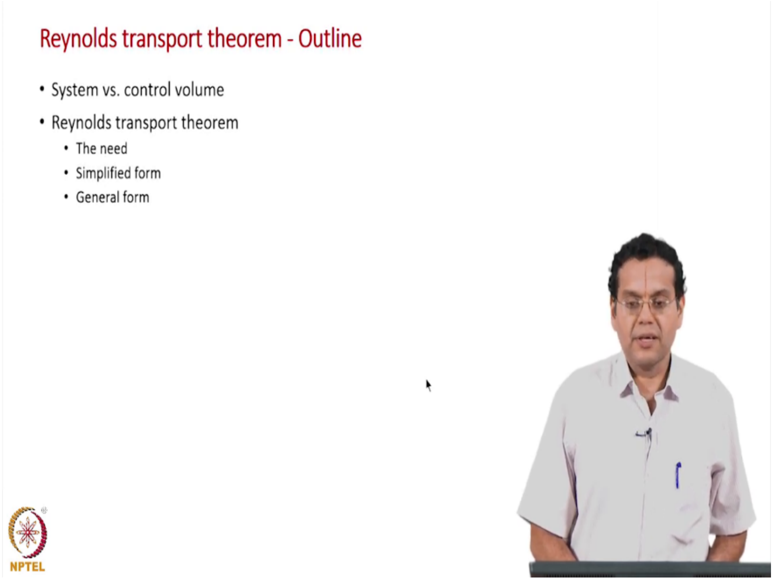

Of course, we start with the distinguishing system and control volume. The system is the same meaning as in a thermodynamics course. Control volume could be any equipment could be a heat exchanger, a distillation column, or reactor. Then Reynolds transport theorem, we will do it in steps. First, of course, we discuss the need for the Reynolds transport theorem. Next, we do the derivation in two steps. First, we derive a simplified form of the Reynolds transport theorem; we do a lot of assumptions, and then get a simple expression. Once we understand that we extend that to a more general form. The straightaway derives the general form would be a little difficult, so we do it in steps.

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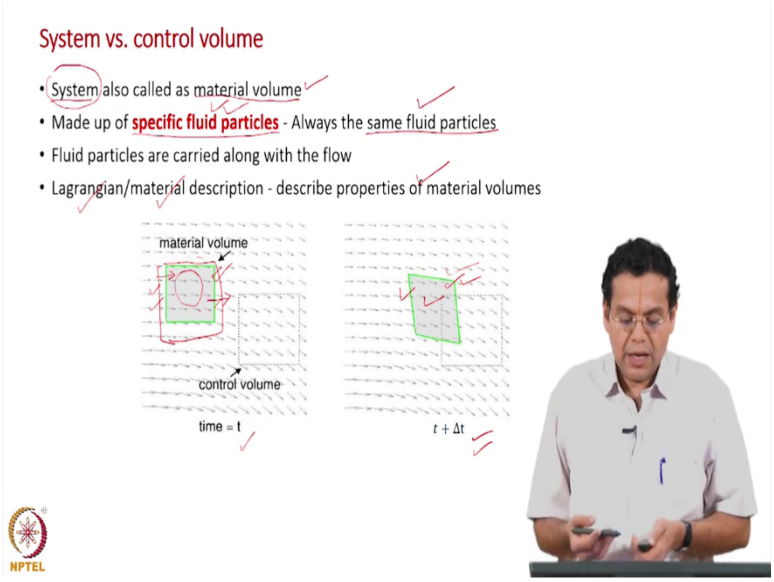

So, let us start with defining and explaining what a system is, what a control volume is. In terms of nomenclature, the system is also called as material volume. You will shortly see why is it called a material volume. But in terms of nomenclature, in terms of books, it is called a system or a material volume, both are equivalent.

Now, how do you define a system, how do you imagine a system? As we have imagined a particle, how do we imagine a particle, we said particle consists of a large number of molecules, and then we color it with a die. Analogously here, we identify a set of fluid particles; there we identified a large number of molecules, the molecules constituted the fluid particle. Now, we are going to identify a lot of fluid particles and that is why you have a region (square box in the referred slide). At some time t, I inject a dye and color a portion of the fluid which I call as a system. Then I keep tracking what happens to these same set of fluid particles that is why I made it red and bold as well that is the most distinguishing feature or that is what defines a material volume.

At time t, I color a set of fluid particle let say 100 fluid particles, and then at some later time t + Δt, let say 1 second, or 1.1 seconds, the different geometry is shown (right-hand side in referred slide). This material volume has moved from an initial position to final (t + Δt), what is common between these two is the particles, which are constituting the material volume or the system is same. Based on the fluid flow, it has undergone some change in shape etcetera. To repeat again made up of specific fluid particles, you color a set of fluid particles and keep tracking them, and what happens to those set of fluid particles. As an entire region that may change in shape etcetera, but we always keep focusing those let us say 100 particles always a same fluid particles.

Now, of course, as I told you these fluid particles are carried along with the flow and that is why there can be some change in shape etcetera. Now, the way in which I describe this system or material volume almost has the Lagrangian or material description. What do we do there? There we identified a fluid particle made up of a large number of molecules, and then we follow that fluid particle. In this case, it is a large set of fluid particles and then I keep tracking those set of fluid particles and that is why it is a Lagrangian description or also called as material description.

We said Lagrangian or material description because we are following a material fluid particle and that is why the system is also called as material volume because we are following a volume made up of fluid particles which are material fluid particles. So, can be familiar with both the terminology system and material volume, most of the fluid mechanics books will use the terminology system. And of course, we are describing properties of a material volume.

So, to just one significant definition or keyword from this slide is specific fluid particles - always the same fluid particles. We do not allow any fluid particle to enter, any fluid particle to leave. You always keep tracking the same set of fluid particles, no fluid particle enters, no fluid particle leaves and which means that this system moves along with the fluid flow that is why there is no particle entering, no particle leaving. So, the velocity of this material volume is the same as the local fluid velocity.

If it were different, what will happen, some fluid particles will enter, some fluid particles will leave, and that is why the way in which we are defined a material volume is all the local velocities should be the same as the local fluid velocity. So, it just flows along with the fluid flow, and whatever happens to the fluid flow also happens to the material volume.

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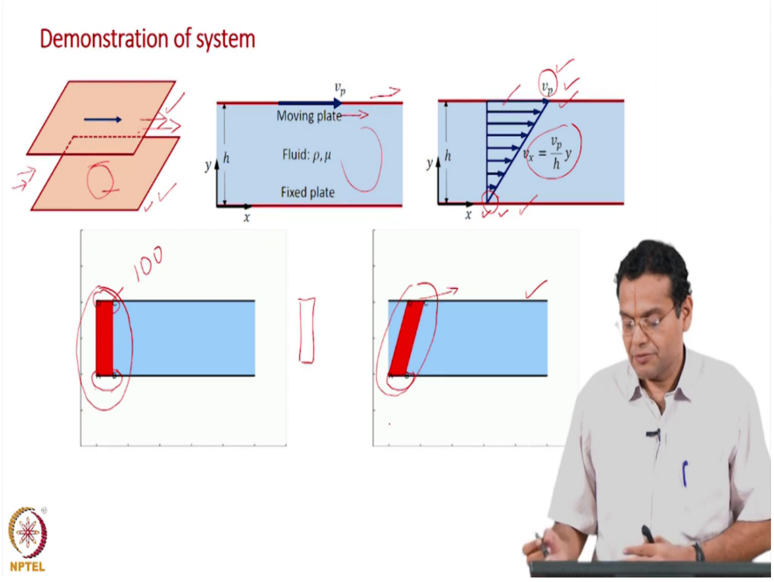

So, let us demonstrate that this geometry is looking similar to our usual flow between two parallel plates, but with the difference. Here again, we have two parallel plates, but in the earlier case both the plates were stationary, and a pump or external device made the fluid flow between the two parallel plates, but in this case, bottom plate is stationary. Now, the top plate is given a constant velocity and this movement of the top plate causes the movement of fluid in the region between the two plates. So, the way in which we cause the motion of the fluid is different both the case. In the earlier case, some external pump made the fluid flow between the two plates. In this case, we impart momentum or impart motion to the fluid in the region because of the moment of the top plate. So, the top plate is set in motion that tracks the lower layer, and so the entire fluid in this region is set in motion.

Now, how does the velocity profile look like, the fluid at the bottom plate, let say water clings to the surface. When it clings to the surface, it has the same velocity of the bottom plate which is 0. Now, similarly, the layer of fluid at the top, clings to the top surface, and the top plate moves at the constant velocity because it clings to that the fluid layer also moves at the same velocity at other plates.

So, the velocity at the bottom is 0 and the velocity at top is v_p. Then at several lectures several classes we will derive this velocity profile or velocity field, but right now we will take it as a linear profile, it is a velocity that varies linearly from the bottom plate to the top plate. Of course, this velocity profile is required to understand what I am going to show now. So, typically summarizes geometry, we have two plates. The bottom plate is stationary; the top plate is as given a constant velocity, and that sets the fluid between the two plates in motion, and the velocity profile is linear.

Now, what I do at a particular instant of time. I select a region and then let us say coloring with the red dye, and that is what is shown (refer video for visualization). So whatever you color defines your system. So, in this case, this region rectangular region defines by A, B, C, D is your system (red-colored region in the referred slide). Now, because of the fluid being set in motion, let us see what happens to this system.

Now, let me stop the simulation video (referee lecture video). What has happened now? The first point, we said the system is made up of a specific set of fluid particles. So, we have to focus on a let us say some 100 particles are there in this shaded region, we have to focus on those 100 particles always. Now, those 100 particles are occupying different shapes, let us say it is a parallelogram after some time, but it was a rectangle initially. Now, why is this happening? The fluid particles, which are in contact with the bottom plate have the same velocity of the plate. So, they do not have any velocity at all they are just stationary and that is why they were never moved as time progressed as well.

What happened to the fluid particles which were in contact with the top plate, they have the same velocity as that of the top plate. So, they are also moving along with the top plate and that is why the system which had a rectangular configuration, to begin with has become a parallelogram after some time and something like my foot is held stationary as if my hand is alone being stretch, and of course, I completely deform. The point of emphasis here is once again the system is made up of the same fluid particles, and we track what is happening to those 100 fluid particles as time progresses.

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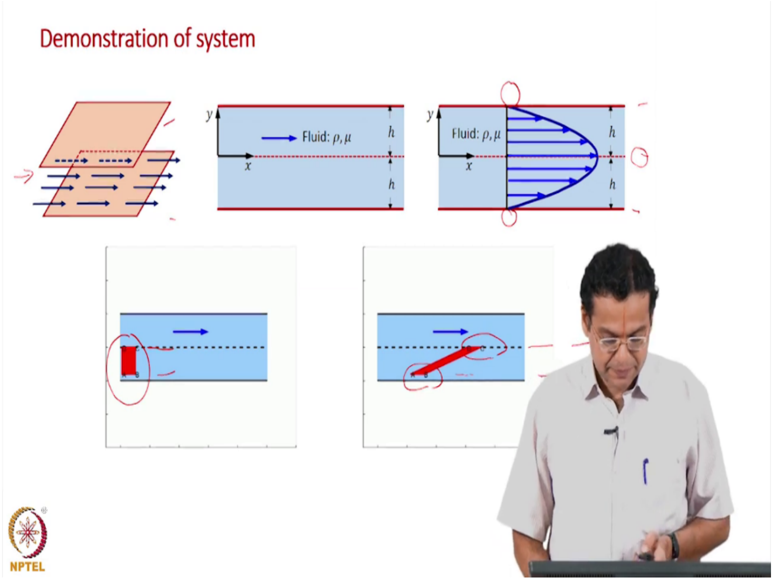

Let us take another example. This configuration is very well known to us just to quickly recap. We have two plates that are stationary and then fluid is made to flow through this using a pump. Then the velocity is 0 at the bottom plate; the velocity is 0 at the top plate, and it is maximum at the central line. So, what should we know, the velocity increases from the bottom plate to the central line, of course from top plate also it increases.

Now, what is that shown just like in the previous case (red-colored resign in the referred slide), I have colored a portion of the fluid at some time t with a red dye and that defines my system. And in this case, as the fluid flows, let us see what happens to the system. Once again we focus on those 100 fluid particles only.

Now, we are following those 100 fluid particles. What has happened now? These fluid particles which are nearer the wall have a lower velocity. These fluid particles which are near the center have a higher velocity. So, unlike the earlier case, in the earlier case, the particles along with AB were just stationary; and then along with CD were alone moving because of the contact of the plate.

Of course, when I say those particles, all the particles in the system, they accordingly correspondingly move. But particles near the bottom plate move at a slower velocity and particles near the center move at a faster rate at a larger velocity and hence once again to begin with we have a rectangle that has deformed to a parallelogram. The difference between the earlier and this cases both the layers keep moving, and but at different velocities; earlier it was zero and nonzero velocity. But here two nonzero velocities one is a lower velocity and another is a higher velocity. But the connecting point for all for both the demonstration is that at a particular time, we color a set of fluid particles a larger region, and then we tracked and focus on what is happening to those set of 100 fluid particles and the same fluid particles.

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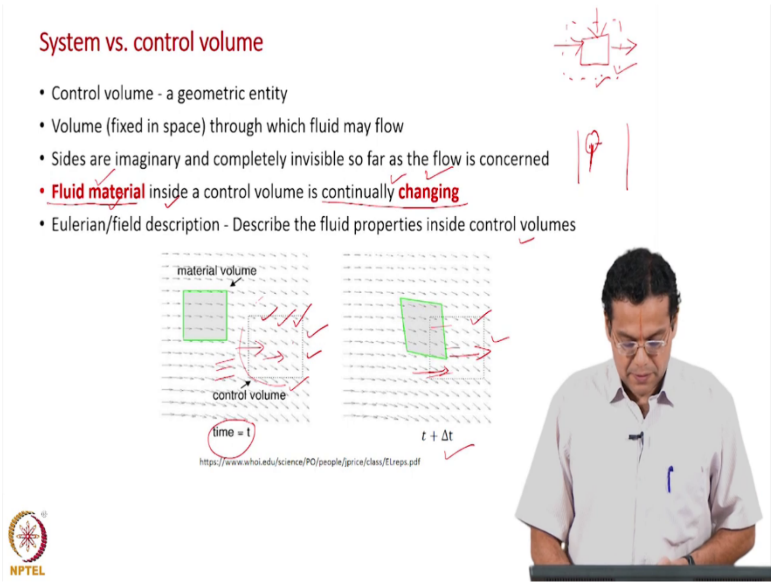

Now, let us see what is a control volume. The first control volume is a geometric entity. Why is a geometric entity? Control volume something you would have used very frequently in your process calculation course. For example, you would have taken a let us say a mixing tank, and then two streams are mixed, and then you get a product stream. Then you would have drawn a dotted line around this mixing tank, the dotted line nothing but a control volume that is why it is a geometric entity.

What is shown is a control volume in terms of a dotted line and then usually the control volume is stationary, it can be moving. We will look at examples and then fluid flows through this control volume. So, let us look at that a volume fixed in space through which fluid may flow, because sometimes, it may be a batch condition and fluid may not flow etcetera that is why it says may flow. What I have shown is a control volume the geometric entity because we have drawn a control volume to our convenience and then fluid flows through this control volume.

Now, as far as the fluid is concerned, the sides are imaginary and completely invisible so far as the fluid flow is concerned. For our convenience, we have drawn a control volume inside the region, inside the fluid flow region and then here we have a drawn a control volume around the mixing tank. So, it is for our convenience over which we write conservation equations. As for the fluid flows, it is just imaginary; it is completely invisible. Just like we had a key point for the system what is the key point for control volume fluid material inside a control volume is continually changing.

What does it mean? We have fixed the control volume. At this point of time t, some set of 100 particles, the some set of particles are flowing through the control volume. Some other time $t + \Delta t$, some other set of fluid particles are flowing through the control volume. That is why the fluid material inside the control volume is continually changing, incompletely contrast to the system where the fluid particles which constitute the system were always the same that is a big and clear distinction between system and control volume.

Fluid particles that make up a system are the same, fluid particles that are inside a control volume keep changing. Some set of particles at this time; some set of fluid particles at some other later time. Now, the way in which we are described you can easily recall that this description is the Eulerian or field description.

Just want to recall, remember we had a chimney example. What did I say, I put a sensor and then measure the temperature there. And then in terms of fluid particles, different fluid particles are being sensed by the temperature probe. So, we were at a particular point and different fluid particles passed through that particular point. Similarly, here also instead of a point we have a region here and different fluid particles pass through that region, of course, it is along with the Eulerian or field description, describe the fluid particles inside the control volumes.

So, to summarize this slide, the key point is fluid material inside the control volume keeps changing that is the key definition of the control volume. The control volume is drawn or for our convenience, it could be drawn in a region, or it could be drawn over entire equipment. A region over we want to describe the conservation, we draw a boundary. Of course, now it should be clear why it is a geometric entity. The geometric entity to distinguish from a material entity which is the material volume that is why we distinguish that is a material volume; this is a geometric entity.

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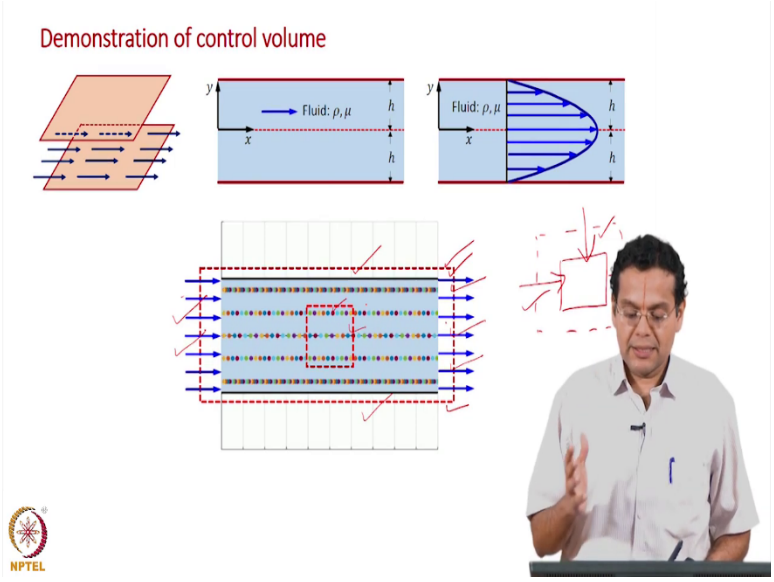

Let us demonstrate the control volume. The configuration fluid flow between the two plates is known to us. Now, let us straight go to the video (referee lecture video for visualization). Different fluid enters, and then crosses this region.

So, now let us focus. I have shown two control volumes. One control volume is shown inside the region between the two fixed plates. Different fluid particles passing through the region, let us say at a time, some set of fluid particles were flowing through that control volume and then at some other time t + Δt, some other time later, some other sets of fluid particles are flowing through the control volume.

Now, control volume can be over the entire equipment or an entire region also. So, another choice for control volume is covering the two plates together. Now, fluid crosses this control volume and of course, enters the control volume and leaves the control volume. Now, a terminology the surface that surrounds the control volume is called the control surface. So, to be more precise, I should say that fluid enters the control volume through this control surface, and then fluid leaves the control volume through this control surface. Of course, fluid does not cross the upper controls surface, it does not cross a lower control surface.

If you recall back near the process calculation course, you would have had some mixing equipment. Then you would have drawn a dotted boundary which represents control volume. And then when you do a mass balance energy balance, you would consider streams with cross the boundary exactly here again. When I take this (inside) as my control volume, I consider only flows which cross the boundary, and what is happening is not considered.

When I take bigger control volume I focus on what is happening to the fluid flow in this control volume which is the region inside the between the two plates. So, different forms of control volume are possible. Once again what is to be understood is that fluid particles that are in the control volume keep changing with the time that is the bottom line.

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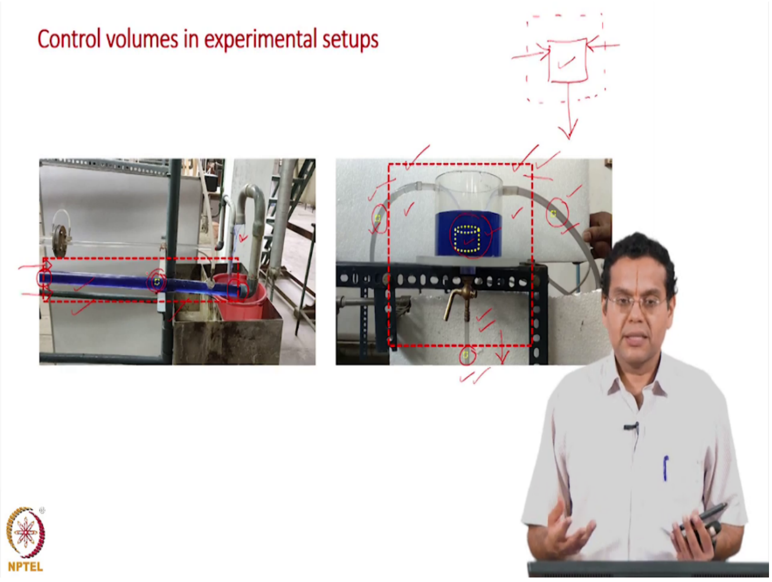

Let us look at control volumes in the experimental setups. The right-hand side setup is something familiar to us that is where we start the whole course in fact, and the left-hand side setup is a new experimental setup. So, let us run the tank system (referee lecture video). Just to recall, it is a small tank. It has two inlets and then one outlet. Water enters through both the inlets and leaves through the outlet.

Now, I have shown different control volumes here. First, let us take this larger control volume. This larger control volume is around the entire tank. The inflow crosses the boundary at left, another inflow crosses the boundary at right, the outflow crosses the boundary at the bottom. This is exactly the same as what you would have done in your process calculation course. Let me draw something similar to this. Two inflows and then one outflow and then you would have taken a boundary and for writing a mass balance and energy balance. The only difference is that that is a schematic, and what you are seeing an actual experimental setup that is a difference between these two.

In fact, remember when we start the course we said we will measure the two inlet flow rates and then calculate the exit flow rate by writing a mass balance, that mass balance was written around this control volume. Of course, I did not discuss about control volume etcetera it was too early to discuss. But what is that indirectly we are done we are taken this control volume and written a mass balance taking mass flows which cross these control surfaces.

Now, I have shown a few other control volumes. What is that? I can take a control volume inside the tank and analyze what is a fluid flowing in, flowing throughout etcetera. So, my control volume need not be over the entire equipment, and it can be inside the equipment. Remember, I just mentioned about integral balance and differential balances. So, integral balances, of course, I will repeat this later again. For integral balances, we take control volume over the entire equipment for differential balances we take control volume inside the equipment.

Now, I have shown a few other control volumes. I can also analyze what is a fluid flow in the pipe which takes in water to the tank. Similarly, I have shown another control volume through which I can analyze the fluid flow in this pipe. Similarly, I have shown another control volume in the exit pipe. So, my control volume could be around the entire tank; it could be within the tank; it could be even in the tubes which lead the water to the tank or which takes water out of the tank. So, control volumes, the selection of control volume is based on our requirement, sometimes based on convenience as well, mostly based on the requirement.

Now, the setup is shown (left-hand side in the referred slide) is a simple flow through a pipe. Once again colored for better visualization. What is shown here is a pipe, and then it just water rises and then falls here. Now, my control volume just like in this case can be the entire region the bigger rectangle. And I can analyze the fluid flowing through that is in this region, what is a fluid flow entering, and then what is the fluid flow living and crossing the control surfaces are as I have shown here I can take a small control volume inside the pipe and analyze the fluid flowing in out etcetera. So, control volumes could be over the entire equipment or it could be inside the equipment as well. So, in the earlier case, it was a demonstration of a control volume. This slide shows control volume in actual experimental setups. Of course, you can extend this to equipment and industry, and so on.

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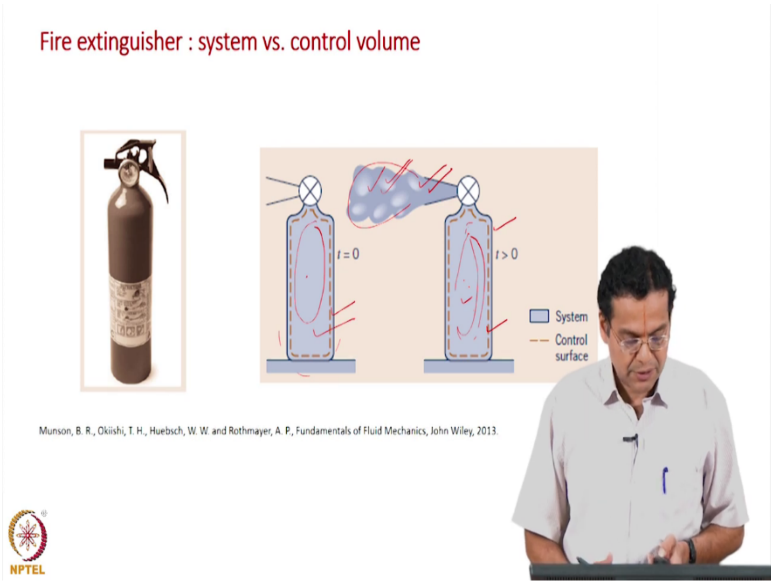

Now, let us compare this system and control volume. Taking an example, a very good nice example from this book Munson et al., Fundamentals of Fluid Mechanics. What is shown here is a fire extinguisher and filled with let say CO_2 gas. Now, the fire extinguisher valve is completely closed. Then at time t = 0, I color the entire gas inside the cylinder with some let say red color or whatever color and define that as my system.

What is my control volume? The cylinder is my control volume and the dotted line represents the control surface. So, the control surface is a boundary surrounding the control volume. And what is that we are done at time t = 0, we identified the contents of the control volume as the system, and this the entire cylinder as my control volume and the control surface shows the boundary.

Now, let say a time t = 0, I open the valve of the cylinder. Now, what happens to my system part of the gas has escaped CO_2 gas is escaped. So, now, my system is still whatever the gas inside the cylinder plus whatever gas that has escaped, why is that we said the moment we identify a system, and it is constituted of a set of 1000 particles we should always keep tracking those 1000 particles.

Let us say about 800 particles are inside a cylinder, and 200 particles escaped. So, my system still constitute constitutes whatever gas inside the let say remaining gas inside the cylinder and whatever gas has escaped. Now, what is a control volume? In this case, the control volume remains the same, the same control surface the same control volume. So, this is a good example to distinguish what is the system, what is a control volume. We identified a time t = 0 a set of particles as a system. And we always keep tracking even though they have moved out of the control volume, but still, my system includes whatever particles here and whatever particles here. In this case, the cylinder is my control volume defined by the control surface.

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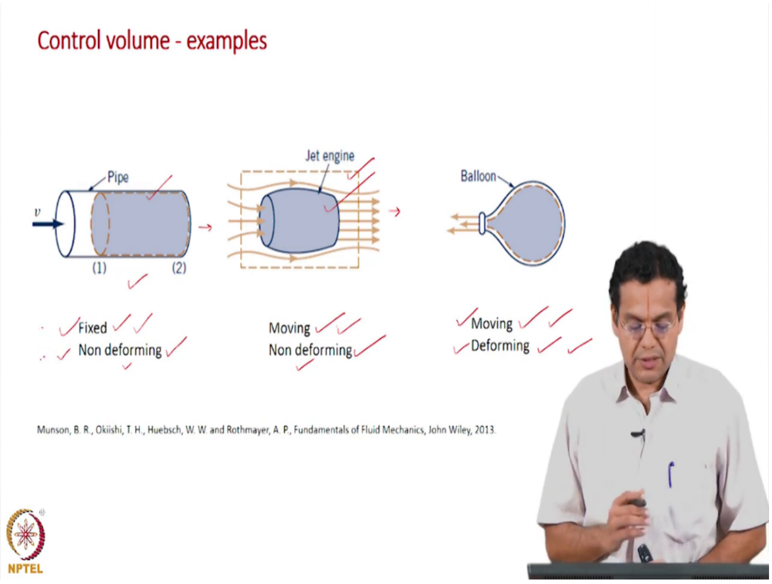

Now, what are the different possibilities for control volume? Shown three different possibilities. The first case what we have is a flow through a pipe, and the dashed line represents the control surface. Now, fluid flows inflows out etcetera. This control volume is fixed. The pipe usually does not move. You have usually had a fixed pipe. So, the control volume is fixed. And the surface of the pipe does not deform it is rigid and that is why it is a non-deforming and then fixed control volume, because it is stationary. It is non-deforming because the surface does not deform, deform meaning change and shape etcetera.

Now, let us take the second case, where our control volume is an engine an aircraft jet engine, and then we would not analyze the fluid flow through this engine let's say around the engine etcetera. Now, this is my control surface which defines a control volume. In this case, my control volume is moving, because the jet aircraft is moving jet engine is moving. So, my control volume also you cannot keep the control volume stationary, because otherwise, we keep it stationary your region of interest would not be there at all it will be moving. So, you will have to use a moving control volume. What happens to the deformation, of course, engines are non-deforming, it is very rigid. So, it is a moving and then non-deforming control volume. Here the control volume is fixed; in this case, the control volume moves.

Let us take the more general case. Example shown here is a balloon, a deflating balloon which means that the air inside is escaping. Because air escapes you know the surface of the balloon cannot be rigid it will start collapsing and that is why it is a deforming control volume. And then if the balloon is deflating it will go here and there which means that the balloon also moves. We have a balloon completely filled let say we just start releasing air, it will start to move here and there, and the surface will also start collapsing that is why is a deforming control volume, it also moves.

So, the most general form of the control volume is a moving control volume with a deforming surface. Of course, it’s a most general case which becomes difficult for us to analyze. So, what is our scope? Our scope is the simplest case namely fixed control volume and non-deforming control volume. So, the entire course all our control volumes will be fixed and will be non-deforming as well, which is the simplest case of the control volume.