Continuum Mechanics And Transport Phenomena
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Lecture - 21
Integral and differential balances

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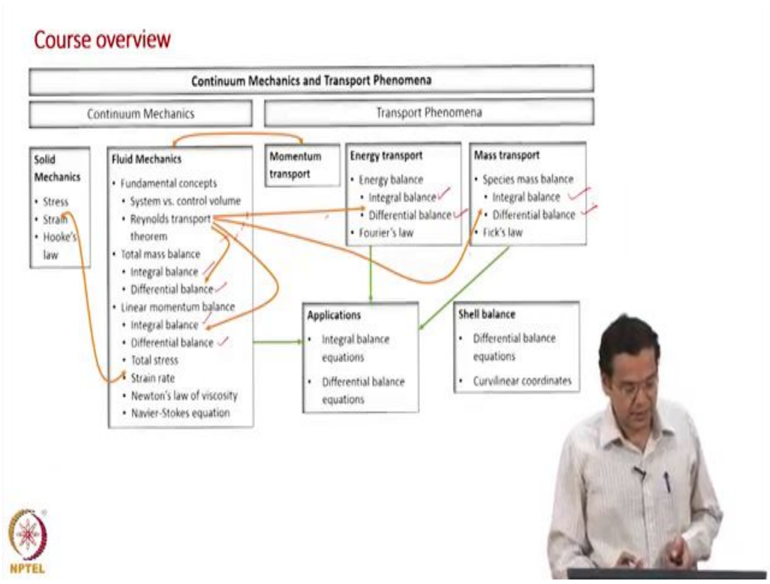

So, let us get started, we have completed the fundamental concepts required for this course in particular we discussed the Reynolds transport theorem, which relates the laws of physics to the conservation equations. Now, we are ready to derive the conservation equations using the Reynolds transport theorem and that is what is indicated by these arrow marks (above image). Start from the Reynolds transport theorem and goes to the total mass balance, goes to the linear momentum balance, goes to energy balance, and goes to the species mass balance.

So, we start with the laws of physics use, Reynolds transport theorem, and derive these conservation equations and within each of these conservation equations, we derive the integral balance and the differential balance. So, if you look at the course overview we have an integral differential balance and a total mass balance, similarly, we have integral and differential balance for linear momentum balance similarly integral and differential balance for energy balance, and similarly for species balance ok. Of course, we are going to take several lectures to discuss all these and derive all these, but we are into the derivation of these conservation equations. So, just an introduction to the conservation equations as such.

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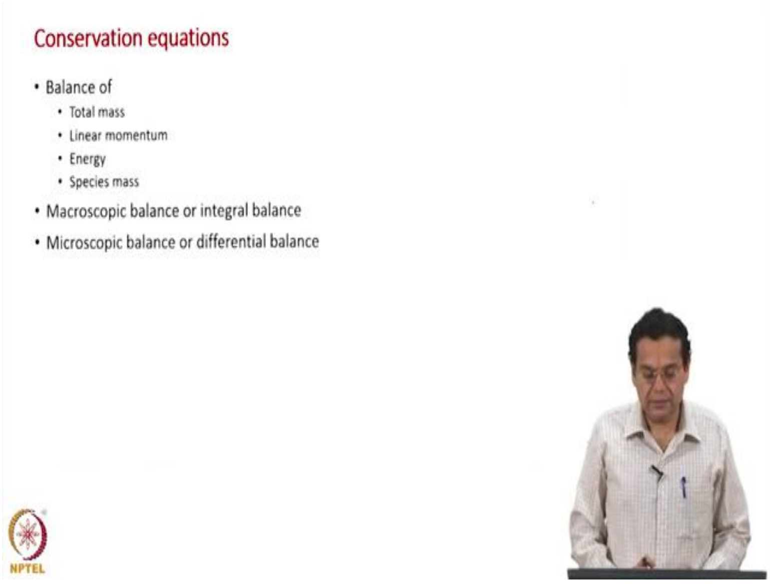

So, as we have seen in the last slide we are going to discuss and derive the balance of total mass, linear momentum, energy, and then species mass. When we say mass we clearly distinguish total mass in the species mass, species mass comes much later in the course, so until then mass and total mass are equivalent. We said we are going to discuss both the macroscopic balance also called as integral balance and the microscopic balance or the differential balance.

So, we are going to derive conservation equations which state the balance of total mass, linear momentum, energy species mass, both in the macroscopic form and the microscopic form alternate names are integral and differential.

- Macroscopic tells you it is over the entire equipment and the form the equation what you get is integral that is why it is integral balance,
- Microscopic balance is a small region inside the equipment and the equation that you get is the differential equation that is why differential balance over the differential element.

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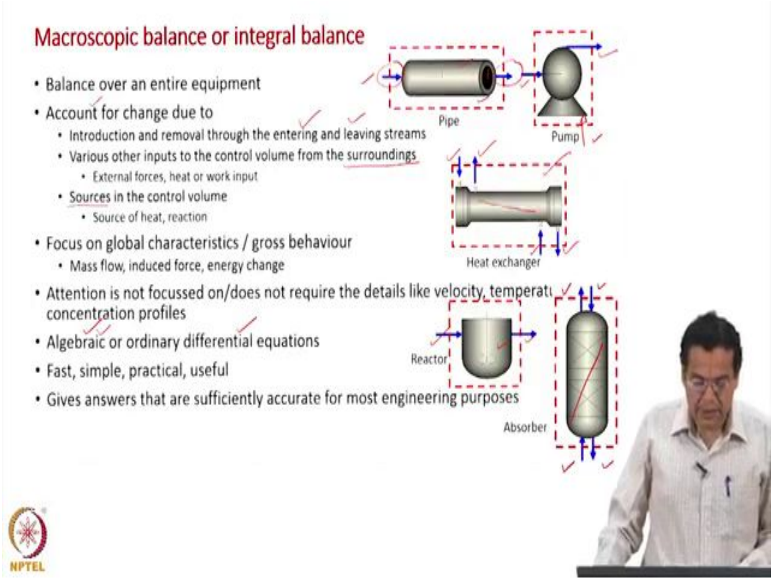

So, a small introduction about Macroscopic balance and the microscopic balance before we start actually deriving the conservation equations. So, as I told you macroscopic balances are written over the entire equipment and few equipment's are shown here. For example, we have a pipe here (top right hand side image) and the dashed red boundary indicates the control volume. The control volume now surrounds the entire pipe and it takes into account what is the fluid flowing in and then the fluid flowing out.

Similarly, for the pump a boundary is drawn around the pump and then it accounts for what is flowing in and flowing out addition to that you will also supply some power to the pump so that the fluid is pumped. Of course, the power is not a material stream what is shown are the fluid entering and fluid leaving streams, in addition to that power is also supplied in terms of let us say work interaction to the surroundings.

Then a heat exchanger is shown (middle image), once again the boundary is drawn around the entire heat exchanger, and streams entering the heat exchanger and leaving the heat exchanger cross the boundary. So, we do a macroscopic balance or integral balance around the heat exchanger accounting for energy that is carried by a stream into and out of the control volume.

Similarly for the absorber absorption column, whereas where again we draw a control volume over the entire equipment over the entire absorber and account for the material which is brought in, brought out, and then similarly taken in and taken out. Similarly, for the reactor, we have the boundary drawn over the entire reactor and we account for the components which are entering the reactor and leaving the reactor.

In all these cases we do not focus on what is inside the equipment, the boundary is drawn around the entire equipment. So, the balance of entire equipment that is seen several examples of that, what are the changes we account for interaction and removal through the entering and leaving streams. That is very obvious from all these figures that whatever a material which is brought in, brought out that is accounted for introduction and removal through the entering and leaving streams not alone material. It could be energy as well when you do energy balance we account for energy as well. So, when I say account for the change to interaction removal through the entering and leaving streams, it accounts for total mass could be species mass it could be momentum it could be energy. A flowing stream contains any of these, so depending on the balance we do we account for any of these properties.

Now, other than the property which is being brought in and brought taken out by the streams, you have other inputs to the control volume in the surroundings. They are like external forces, for example, the pipe has to be supported some force is required what is that force and then as I told you sometime back we will have to supply work to this. So, work input in some places there could be heat input, heat output, heat exchange the surroundings, let us say so all those are interaction with the surroundings inputs and outputs are from the surroundings.

Other than that there could be sources in the control volume itself. For example, there could be a source of heat you could have a small heating element or reaction is a source of a component ok, it is not a source of total mass but a source of component. So, all these are accounted for when we do a macroscopic balance, whatever carried by the entering and leaving streams and then other interactions with the surroundings in terms of forces heat work and then other sources in the control volume in terms of heat or reaction. Based on what we have discussed should be obvious that we focus on the global characteristics of gross behavior.

We do not look at the local behavior we look at the global characteristics and gross behavior of the equipment of the control volume. So, what are the global characteristics, for example, we interest in the mass flow you may find out the unknowns would be finding out mass flow, induced force, energy change. For example, you may find out what is the power to be supplied and then what is the mass fluid flow entering leaving that could be our unknowns and then similarly what is the induced force. So, though those are the unknowns that will solve for when you do a macroscopic balance. And then attention is not focused on the details like velocity, temperature, concentration profiles, we do not look at the velocity profile in the pipe or the let us say the at temperature profile in the heat exchanger or the concentration profile in the absorber.

So, we do not focus attention on the profiles and the other way to put that is we do not require details on these profiles for this kind of analysis and because we write macroscopic balance when you simplify what you get is an algebraic equation if it is a steady state process. If it is a transient process you get an ordinary differential equation the reason is we do not consider the spatial variation. So, all the spatial coordinates are gone so only the independent variable could be timed if it is a transient process, if that is also the steady state process time is also not an independent variable.

So, will result in a simple algebraic expression, which is the advantage of doing a macroscopic balance. So, that becomes easy to solve algebraic equations or even ordinary differential equations are easy to solve. So, it becomes fast simple, and very practically useful method. And so far gives the answer that is sufficiently accurate for most engineering purposes. If you are looking at the engineering level of accuracy then you get answers which are sufficiently accurate.

So, to summarize the macroscopic balance we do balance over the entire equipment then account for changes because of the streams interactions surroundings, and sources within the volume focus on global characteristics gross behavior. In terms of mass flow, the force then energy change, we do not focus on the profiles we result in algebraic or ordinary differential equations and a simple practical useful method of engineering accuracy.

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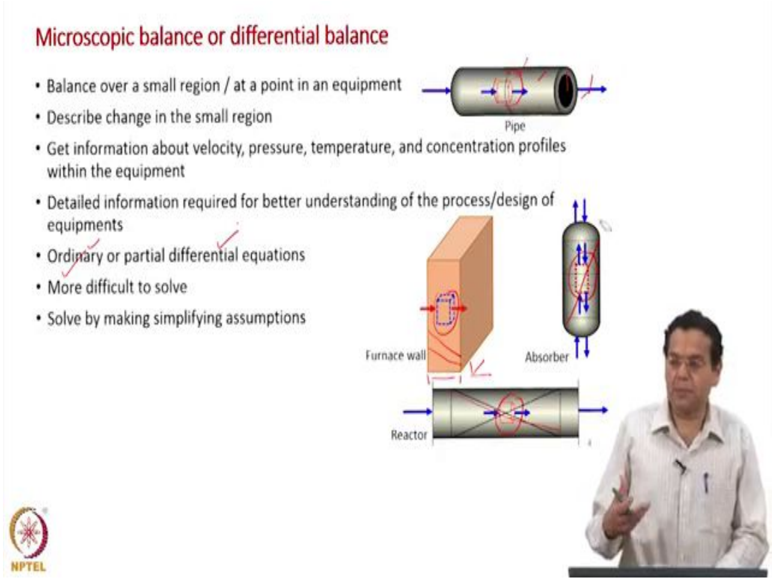

Now let us move on to the microscopic balance the differential balance. Now as the name indicates microscopic balance we take control volumes are within the equipment which are very small and in fact, every point is considered as a control volume.

Few examples are shown here (above slide image) a pipe is shown where we take a control volume which is inside the pipe and then account for changes for this small control volume make it very small it becomes just a point. And, then similarly for the case of furnace wall is shown and where we have shown a control volume within the furnace wall and then we account for changes of what is happening inside this small control volume differential control volume. And, similarly, in the absorber, a small control volume is shown and then we do balance for the small control volume.

Similarly, the reactor control volume is shown within the equipment and we do balance equations conservation equations for this small control volume. So, as we have seen we write balance over a small region and if you make the region smaller and smaller it just becomes at a point in equipment, that is why in this case we are able to get more and more details able to get profiles of velocity, temperature etcetera.

So, describe the change in the small region, as we have seen we described the changes in a small region and it becomes a small point as well. And because we are focusing on a very small region, we will be able to get information on the velocity, pressure, temperature, concentration profiles within the equipment.

So for example, in the pipe when you write the microscopic balance and then solve you will be able to get what is the velocity profile across the diameter, in the furnace case the temperature profile across the thickness of the wall and the concentration profile along the length of the absorber and similarly the concentration of let us say the reactant or product along the length of the reactor.

Because we write differential balances over a small region when we solve we get information about profiles of all these variables which means we are getting much more detailed information compared to what we got in the macroscopic balance and once you get detailed information, we have a better understanding of the process and then better design of the equipment.

Now space is a variable, the way in which I have seen we said we are going to get profiles which means the special variables. For example, x-axis, y-axis, z-axis, and x, y, z coordinates are one variable. So, the least what you get is an ordinary differential equation, if it is steady state process and then only one special variation. Let us say in the pipe the there is variation along the radius it is all steady state. So, we will have an ordinary differential equation in terms of let us say r. Let us say in this case along with the thickness of the furnace wall you are interested in finding the temperature profile. So, the access the variation so you have one independent variable descending in a differential equation. So, likewise, there is at least one special variable resulting in an ordinary differential equation, of course, the more general case would be variation along with the different directions. Let us say in this particular case we are interested in the variation along the x-axis, y-axis, z-axis all the three directions, similarly the r, θ, z directions which means that you will result in partial differential equations. If time is a variable time also adds and then you result in a once again of course a partial differential equation.

So, you either result in ordinary or partial differential equations depending on the variable, but what is important is that these equations are going to be more difficult to solve. Earlier we had just either algebraic equations or if it is the unsteady state we have the ordinary differential equation with time as a variable. Now least is an ordinary differential equation without time with only one spatial variable, otherwise, many times result in the partial differential equation so more difficult to solve certainly. Usually what we do is we make simply make assumptions and solve the resulting simplified equations.

These two slides are to give an overview as we derive the macroscopic and microscopic balances for the different properties, namely mass, momentum, temperature, and species you will understand what is contained in these slides much better.

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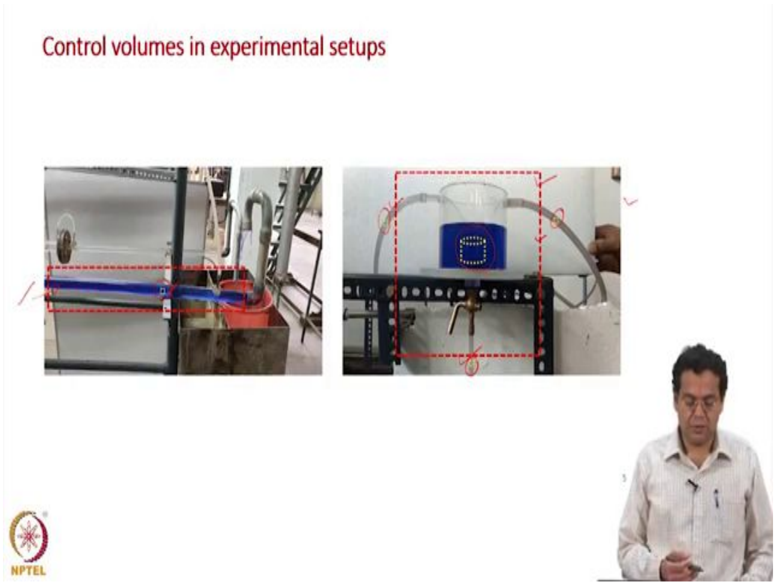

This slide shows the control volumes in the experimental setups which we have come across and discussed earlier. On the left hand side is the experimental setup for flow through a pipe and the right hand side shows the setup for the flow through a tank. Now, what is shown here are the control volumes for integral balance and differential balance. On the left hand side, the red dashed boundary represents the boundary for integral balance, where the control volume is drawn over the entire pipe through which water flows in and water flows out.

Similarly, on the setup on the right side the red dotted line is the control volume and through which water flows in and through two inlets and water flows out through one outlet. Once again the control volume is over the entire equipment through which there is flow in and flow out. Now, these streams carry with them the mass as such the total mass, and then they also carry with them the momentum they carry with them energy and then they also if there are multiple species are.

So, when you do an integral mass balance we account for these properties are carried by the streams into and out of the control volume. This slide also shows the control volume for differential balance those are shown by this yellow color boundary dashed boundary. In the setup of the pipe the control volume is shown within the pipe and in this second setup on the right hand side there are different smaller control volumes possible, one control volume could be inside the tank or the small control volumes could be inside the inlet pipes or in the outlet pipe as well.

So, these are control volumes within the equipment and we chose such control volumes for doing a differential balance, and then for the small control volume, we analyze the flow of total mass, momentum, energy, and species mass. And, because we consider these control volumes within the equipment and become smaller and smaller that is become a point, we are able to analyze the profiles of velocity, temperature, concentration by doing a differential balance by choosing such small control volumes.