Continuum Mechanics And Transport Phenomena
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Lecture - 22
Integral Total Mass Balance

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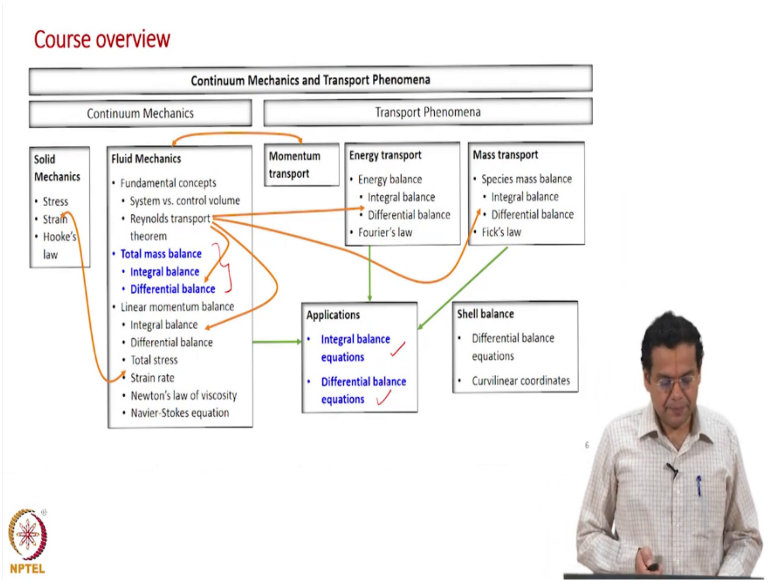

So with that introduction for integral and differential balance equation, macroscopic and microscopic balances. We will begin with deriving the conservation equation for total mass. So, we are in the part of the course which says total mass balance, conservation equation terms of integral balance and differential balance. And, then we will also look at applications of the integral balance and the differential balance that is the integral total mass balance and differential total mass balance.[]\7

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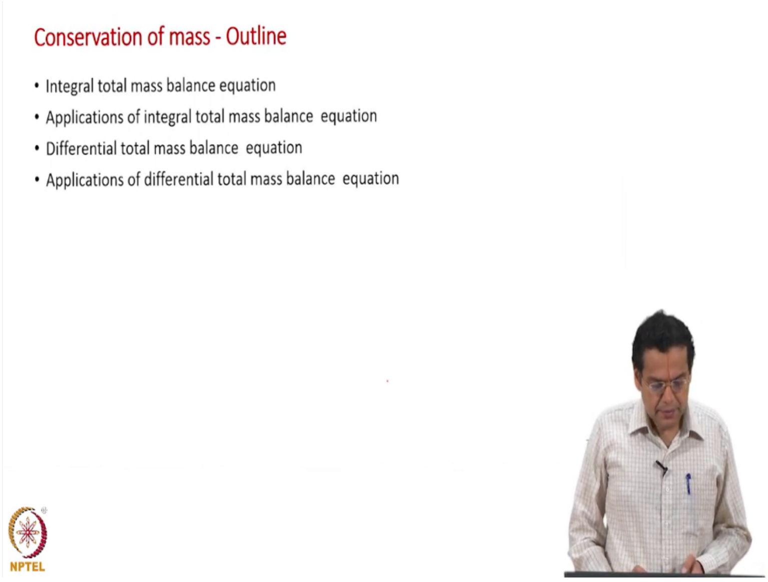

So, in terms of outline, we derive the integral total mass balance equation starting with the law of physics and look at applications of integral total mass balance equation and then derive the differential total mass balance equation and look at applications of differential total mass balance equation. So, that is the outline under the conservation of mass title.

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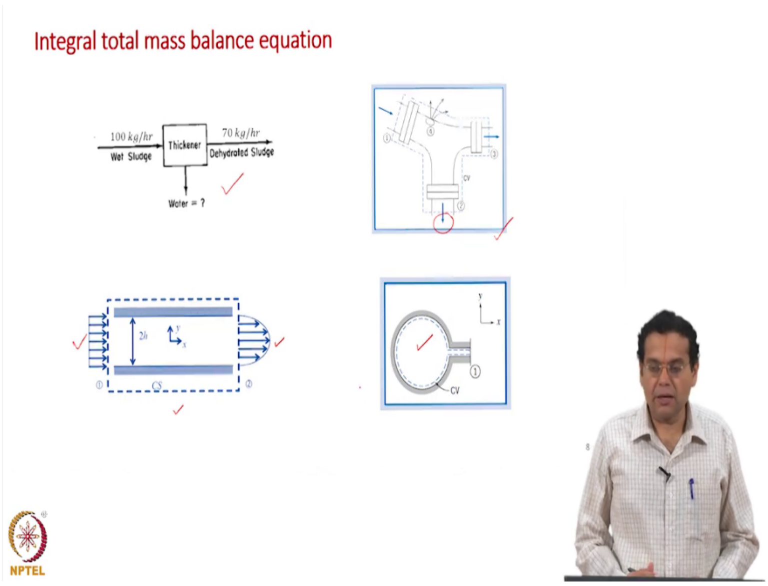

After deriving the integral total mass balance equation, we would be taking a few applications and it is volume what are they just give an idea what is that is in store for us. We will be able to apply the integral total mass balance equation to solve a typical process calculation problem likes mass balance over a thickener. We will be able to apply the integral mass balance equation for flow over a pipe junction and find an outlet say one of the velocity is one of the flows and, then we will also be able to apply for flow between the parallel plates and find out how the maximum velocity is related to the inlet velocity and, then under a transient condition we will be able to find out how does a density vary as a function of time in a tank through which the air escapes. These are the kind of applications which would be able to solve at the end of this integral total mass balance equation.

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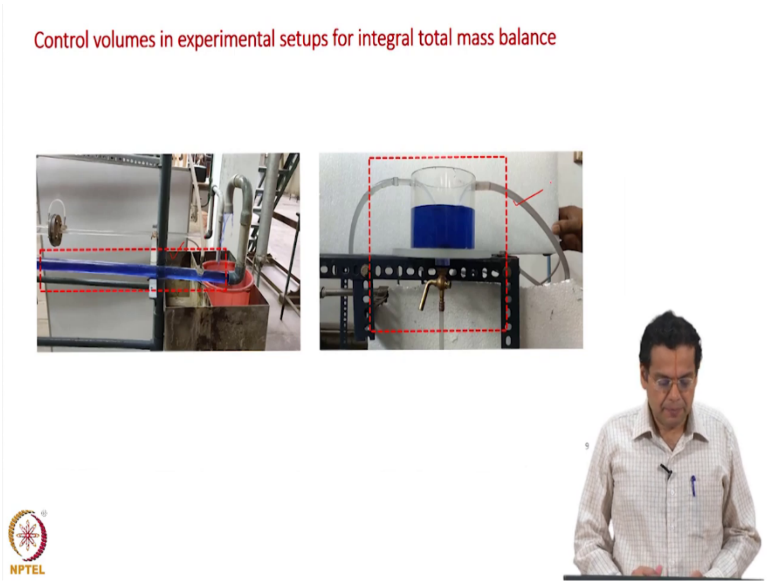

So, the control volumes are now around the entire pipe and entire tank as we have seen in the previous slide.

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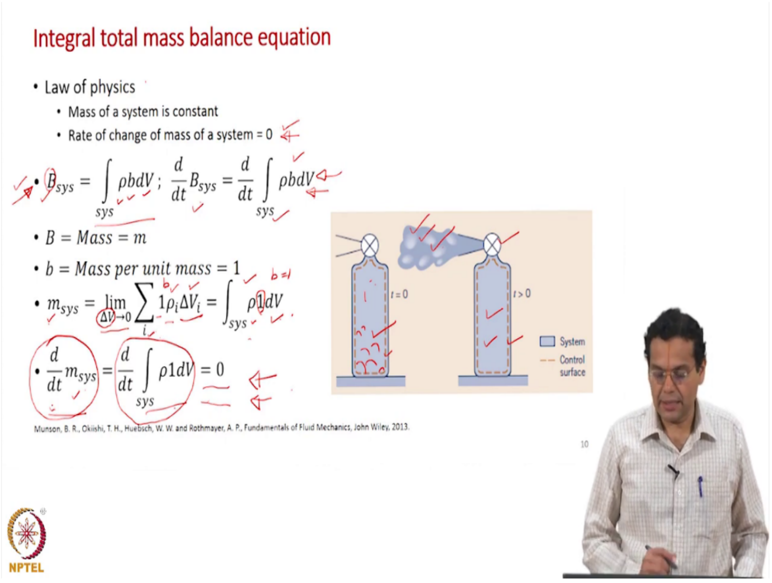

So, let us start deriving the integral total mass balance equation, the starting point for any derivation is the law of physics and we said that is one of the major objectives of this course that relate the law of physics to the conservation equation. The very first statement is the law of physics. What is a law of physics now? law of physics for a system which states that mass of a system is constant, just to recollect this example which I have discussed few classes earlier, we had a fire extinguisher and then a time t = 0, let us say carbon dioxide (CO_2) gas is inside the cylinder. I identify the gas inside the cylinder a time t = 0 as my system.

Sometimes later if I open the wall and part of the gas escapes the cylinder still, my system is constituted by whatever gas outside and whatever remaining gas inside. The control volume still remains as my cylinder, but the system has partly moved out of this control volume. So, we said the system is made of specific fluid particles and so by this definition the way in which are defined as a system. Let us say initially you have a mass of 10 kg, let say 2 kg mass has escaped and remaining mass is 8 kg but still a mass of the system is 10 kg so a mass of a system is constant.

Now, we like to write this in terms of a rate of change. So, the rate of change of mass of a system equal, and mass is constant. So,

The rate of change of mass with respect to time = 0. Now, these are the statements let see how do we go ahead deriving the equation from this.

Now, while deriving the Reynolds transport theorem we introduced the extensive property B introduced intensive property b.

$$B_{sys} = \int_{sys} \rho\, b\, dV \quad ; \quad \frac{d}{dt} B_{sys} = \frac{d}{dt} \int_{sys} \rho\, b\, dV$$

We related the B for a system in terms of this integral and the rate of change of the property can be expressed in terms of the second expression. So, this we have come across when we derived the Reynolds transport theorem. Now for the present case

B = Mass = m

b = Mass per unit mass = 1

So, just to repeat this is the general equation which is for any property B. Right now we are focusing on the total mass, so B for us right now is total mass and b is mass per unit mass hence becomes unity. Let see how do we write for the mass of a system.

$$m_{sys} = \sum_i 1\, \rho_i\, \Delta V_i = \int_{sys} \rho\, 1\, dV$$

We take our system split into smaller and smaller volumes of ΔV_i and each volume has it is own density ρ_i, earlier we had here b now which was property per unit mass present it is just unity that is why I have used 1 here and now I sum over all the smaller volumes and now I want to represent this as an integral. So, I consider very small volumes really infinite of them and as the number of volumes becomes larger and larger, the volume of each element becomes smaller tends to 0. So, I evaluate the sum in the limit of each volume tending to 0, then you can represent the sum as this integral. So,

Mass of the system has been represented as = $\int_{sys} \rho\ dV$.

So, when you look at $\rho\ dV$ the way as you interpret is in terms of this splitting into smaller elements and then summing up all the masses of each of the elements.

Now, let us write this expression for the case of the mass of a system

$$\frac{d}{dt} m_{sys} = \frac{d}{dt} \int_{sys} \rho\, 1\, dV = 0$$

We are writing this equation for mass of a system. Rate of change of mass of a system m system has been written in terms of this integral expression $\rho\ dV$.

Now, from the law of physics, we know that rate of change of mass of a system is equal to 0, so this equal to 0 comes from the law of physics. The left hand side just tells you rate of change of mass of a system and the right hand side tells you the same thing, but in terms of the integral expression for m system. Now, when you equate this to 0 you are invoking the law of physics that the rate of change of mass of a system is equal to 0.

So, here we have written the law of physics let say an English statement, this is a mathematical representation of the English statement made in the first bullet. We say rate of change of mass of system equal to 0 which has been represented in terms of the equation in the last line.

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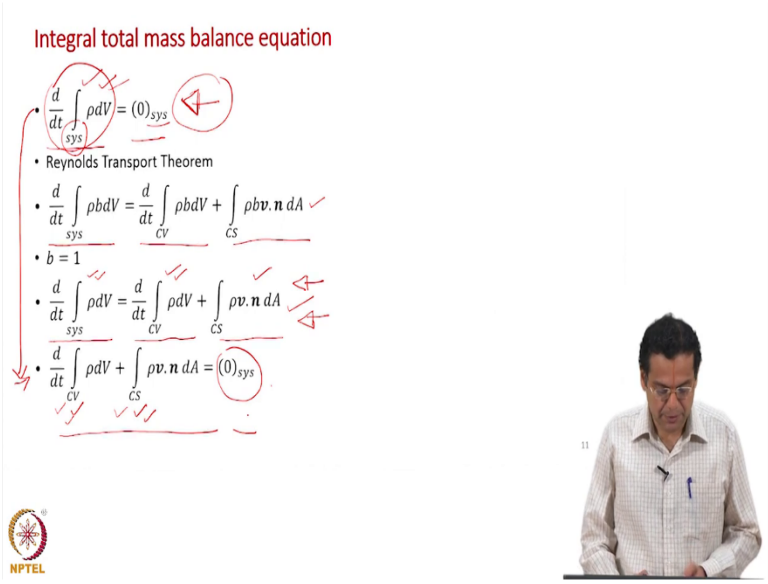

So, we have brought written the earlier step again rate of change of mass of a system is equal to 0,

$$\frac{d}{dt} \int_{sys} \rho\, dV = (0)_{sys}$$

Now, we more precise we have given a subscript saying for a system. Now what we will do now is use the Reynolds transport theorem because Reynolds transport theorem relates the rate of change of property for a system to the rate of change of property for a control volume.

$$\frac{d}{dt}\int_{sys} \rho\, b\, dV = \frac{d}{dt}\int_{CV} \rho\, b\, dV + \int_{CS} \rho b\, v.n\, dA$$

So, let us right the general form the Reynolds transport theorem for applicable for any property, just to recall this first term on the left hand side term represents rate of change of property for the system, right hand side represents the rate of change of property for the control volume and then this last term second term on the right hand side tells you the net rate at which the property is leaving the control surface.

Now, as we are discussing the intensive property is b is mass per unit mass which is 1 and so we substitute b = 1 in the Reynolds transport theorem and then get a Reynolds transport theorem applicable for mass.

$$\frac{d}{dt}\int_{sys} \rho\, dV = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, v.n\, dA$$

So, how do you read this equation, now rate of change of mass for the system that is the left hand side term, right hand side first term rate of change of mass for the control volume. Then the second term on the right hand side tells you net rate at which mass leaves the control surface.

Now, becomes a little more meaningful earlier we are using the word property now that property is mass, so we can read out this equation in terms of mass. So, of course, the Reynolds transport theorem relates rate of change of property for the system to the rate of change of property for the control volume. In this case, mass so the rate of change of mass for the system to the rate of change of mass for the control volume and net rate which mass leaves a control surface.

$$\frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, v.n\, dA = (0)_{sys}$$

Now, what we do, this is the equation that came from the law of physics. The left hand side can now be represented using the Reynolds transport theorem and that is what I have done. I have taken this equation taken this equation on the left hand side of this equation as substituted the Reynolds transport theorem and completed the expression for this equation.

So, just want to emphasize that this first equation is the law of physics, the left hand side in terms of system that has been represented in terms of the control volume and control surface that is a idea of using Reynolds transport theorem. In the law of physics in the left hand side I use the Reynolds transport theorem and then so when I do that the left hand side terms are in terms of control volume control surface of course, get the right hand side term still in terms of 0 for system.

But what is the main thing you achieved here that the integral which was in terms of system has been represented in terms of integral over control volume and integral over control surface using the Reynolds transport theorem. Remember Reynolds transport theorem tells you how to replace the system integration in terms of control volume control surface, this equation does not come from Reynolds transport theorem that statement comes from the law of physics. Using Reynolds transport theorem you are able to relate integral over control volume control surface, right now it is 0 for the system let us see how do we represent that also for a control volume. Unless all the integrals are in terms of control volume control surface we will not be able to apply for solving a mass balance problem.

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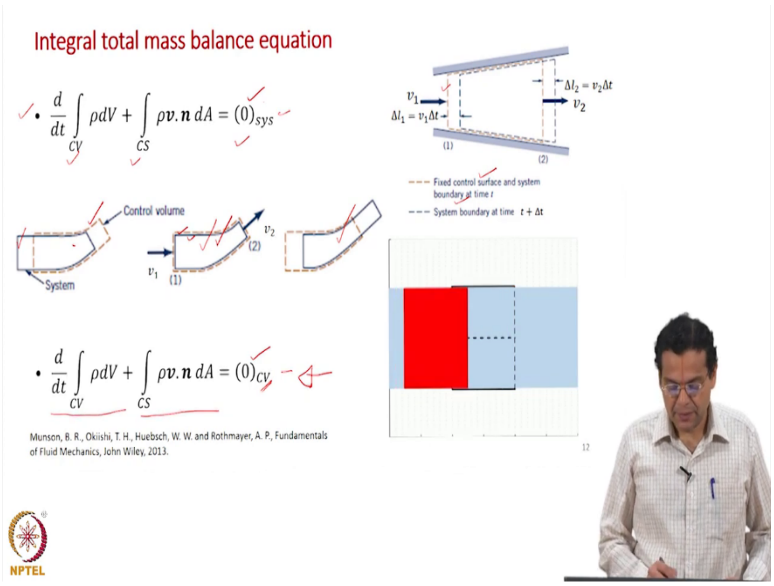

Let us see how do we do that. So, the stage we are in is that we have written the law of physics and use a Reynolds transport theorem, and obtained this equation.

$$\frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, v.n\, dA = (0)_{sys}$$

In this equation, the left hand side expressions are what we wanted meaning there in terms of control volume control surface. But, the right hand side is says that 0 for the system has to be expressed in terms of the control volume.

Now, to recall back when we drive the Reynolds transport theorem both for the special case and general case at any time t we took the control volume and the system to be coincident at any time t. Fixed control surface and system boundary at time t, we wrote

$$B_{sys}(t) = B_{CV}(t)$$

So now, that is what is shown here (above slide image) this is our control volume, we could choose a system that is partly outside and partly entering. We could choose a system that is just coinciding with the control volume or which is just leaving the control volume, three possibilities are there in several possibilities. But what is the possibility is most convenient was the possibility where the system in control volume coincides. So, that the boundary of the system in the boundary of the control volume both coinciding with each other. So, the system gets much more well defined. So, in terms of our the animation which have seen earlier. So, these are different possibilities for the system we consider the system which is coinciding at time t, we consider the system to be coincide with the control volume at time t. What is the implication of that this 0 for system can be replaced with 0 for the control volume, left hand side is what we want.

$$\frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, v.n\, dA = (0)_{CV}$$

On right hand side, we replace 0 with control volume of course, later on when we start writing usually do not write this control volume this implied that is for control volume. But at this stage when you go from system to control volume we should understand how do we really go from 0 for system to 0 for control volume. This comes from the law of physics and that the rate of change of mass is 0 for the system and now both are coinciding. So, I replace 0 for system with 0 for control volume.

So, now this equation is the integral form of the total mass balance equation, where all the terms are in terms of control volume and control surfaces. Now of course, in terms of significance the first term on the left hand side represents rate of change of mass within the control volume, the second term tells you the net rate of mass leaving the control surface of course right hand side is 0.

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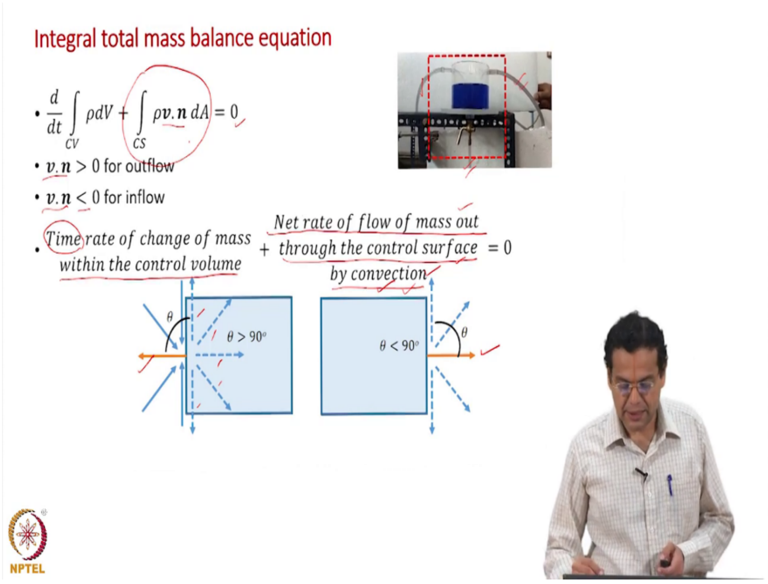

So, as I told you we write the conservation integral form of mass balance equation, we do not specify control volume in the right hand side it is understood.

$$\frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\, v.n\, dA = 0$$

Now we have seen that $v.n$ automatically takes care of outflow and then inflow and $v.n$ is positive for outflow just to recall and the same diagram is shown here (above slide image) and the n represents the outward normal. So, always the normal is drawn outside of the control volume. So, let us say with respect to this physical geometry if some are inflows and some are outflows, for one phase the outward normal is drawn in the direction normal to surface and for the other inflow the outward normal drawn in the direction normal to surface.

So in both the cases both out outlet and inlet the normal always points away from the control volumes. So, this is the outward normal and in terms of the angle between the velocity vector and the n vector for outflow, that angle is less than 90 degrees, and then the $v.n$is positive for outflow. And, for the case of inflow, the outward normal is shown and these are the extended inflow velocity vectors the angle between the velocity vector normal vector is greater than 90 degrees and hence $v.n$ is less than 0 for inflow.

So, because of this integral automatically takes care of surfaces where there is outflow and inflow because $v.n$ is positive for outflow the term tells you the net rate at which mass leaves the control surface. So, let us put this all this very formally in terms of the significance of this equation.

- The first term represents a rate of change of mass, to be more specific we say time rate of change of mass, and then it is within the control volume this tells what is happening to the mass as a function of time within the control volume.
- The second term tells you as we are discussed now, the net rate of flow of mass out through the control surface remember all these are whatever flows in and out etcetera through the control surface all these are parts of the control surface through with there is inflow or outflow. Of course, we already discuss that wherever there is no inflow outflow the velocity is 0 and this integral will not contribute. So, that is why it says net rate of flow of mass out through the control surface when I say through the control surface wherever you have inflow and outflow all other parts one contribute.
- Now, we have a term which is going to play a major role as we go along by convection by convection, what do a mean by convection? you have water entering through one tube, similarly water entering through the second tube and water leaving through the third tube. Now, as we have discussed earlier this stream entering carries mass with it. All these wherever there is the transport of mass by flow by bulk flow we call it as convection.

  So, this term we will use several times throughout the course right now we are using the context of total mass, we will say later on extend these two let us say momentum, energy, species mass etcetera. But all other same physical significance that whatever flow entering let us say through inflow and outflow with them they carry these properties namely mass etcetera. So, that is why it says net rate because accounts for positive and negative in terms of outflow and inflow net rate of flow mass out through the control surface and by convection. So, every term has a significance there and of course, right hand side is 0.