Continuum Mechanics And Transport Phenomena
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Lecture - 30
Integral linear momentum balance Part 2

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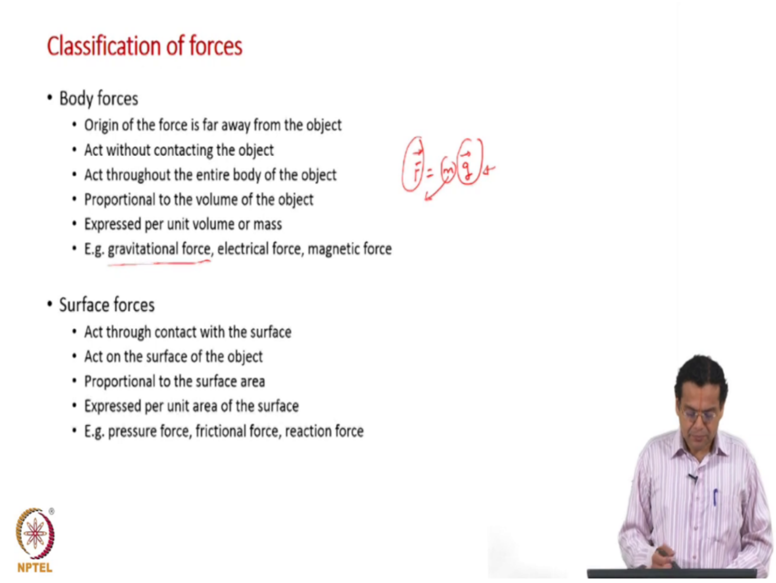

Now, let us look at the forces which are to be considered. For that first, we will discuss how do we classify forces, and we classify forces as

- Body forces and
- Surface forces.

We will discuss what are the characteristics of the forces with an example.

✔ The best example of body force is the gravitational force. We will keep that as an example and then see what are the characteristics of body forces. First, if you take an object the origin of the force away from the object. Let us say gravitational force acts on this control volume, it acts on its, but the origin is away from the object, and now it the body forces acting on the control volume without contacting the object. The gravitational force is nowhere in contact with the object, so the origin is away which of course, leads to the fact that force is not in touch or contact with the object. Now, it acts throughout the entire body of the object, the entire control volume is objected to the force, and of course, that is the way it is called a body force. So, acts throughout the entire body of the object and because it acts through the entire body object, it is proportional to the volume of the object. Then, if you say body force or gravitational force, for example, that is proportional to the volume of the object, hence we express body forces as per unit volume or per unit mass. For example, force is equal to the mass into the gravitational acceleration, and what is g; it is body force per unit mass. So, if you write $F = mg$; of course, we can see g is the acceleration due to gravity. What is the other way of saying; the body force F per unit mass will give you the expression for gravitation vector. So, body force is expressed in terms of per unit mass that is why you multiply by mass. Of course, for example, gravitational force, other body forces could be an electrical force, magnetic force. Within the scope of this course, the only body force which will consider is a gravitational force. So, to quickly summarize body forces, the origin is away from the object, they are not in contact, they act throughout the body and hence proportional to the volume and expresses per unit mass.

✔ Now, coming to surface forces. Surface forces act through contact, body forces there is no contact at all, but surface forces act through contact with the surface. So, let us take an example, pressure. Pressure force example for surface force. So, when you say pressure acts it is in contact with the surface. It is not like it is without contacting have a pressure force. The moment we say any surface force, for example, the pressure it acts through contact with the surface either perpendicular or tangential etcetera, in this case of pressure it is perpendicular. Obviously, it acts on the surface of the object. In the earlier case body force, it affects was felt throughout the body, but for the case of surface force, it acts on the surface of the object. Now, because it acts on the surface of the object becomes very obvious that it is proportional to the surface area on which it acts. Earlier it was proportional to the volume of the object because it was felt throughout for the case of body force. So, this also leads of fact that surface forces are expressed per unit area. Body forces were in terms of per unit mass; surface forces because they act in contact on the surface and it is proportional to the surface area, so expressed per unit area. Of course, they are very familiar with pressure is force per unit area that is how you get the unit of surface force as force per unit area. There are other surface forces namely frictional force, reaction force. We will come across reaction force. What is the reaction force; suppose if an object and you want to support it and whatever force acted by the support on the control volume is a reaction force. Of course, the frictional force is because of friction between the fluid flowing and the surrounding walls. So, we will come across all these kinds of forces in the applications when we discuss them.

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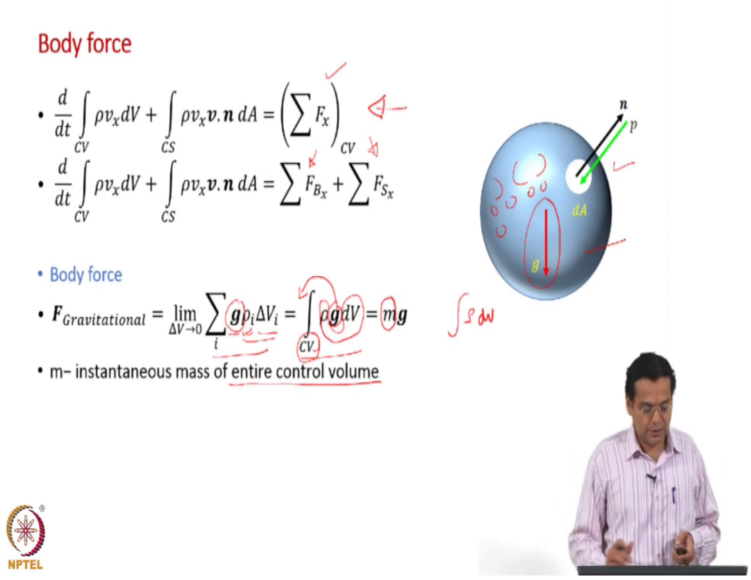

So, now, what do we do; we write the integral form of momentum balance, and on the right hand side now we have a little more information, earlier we kept telling that sum of all the forces acting on the control volume. Now, we express that in terms of the body forces and surface forces, a little more detail than we have done earlier.

$$\frac{d}{dt}\int_{CV} v_x \rho\, dV + \int_{CS} v_x \rho\, v.n\, dA = \left(\sum F_x\right)_{CV}$$

In this form of the integral linear momentum balance, the forces are returned more generically. Now, since we have classified them, expressed this in terms of body forces and in terms of surface forces.

$$\frac{d}{dt}\int_{CV} v_x \rho\, dV + \int_{CS} v_x \rho\, v.n\, dA = \sum F_{B_x} + \sum F_{S_x}$$

Now, in this slide, we will discuss about body force. As I told you the body force which we are going to discuss is a gravitational force and what is shown here is the control volume for convenience's sphere is shown and acted upon by gravity. This is for the surface force which we will discuss in the next slide.

Now, how do you represent the body force in terms of an expression? Ok; now, F gravitational force. So, I take the control volume divide it into smaller and smaller volumes, and then for each volume, you can have different densities, so which use a mass of each element, but now every we said body force acts throughout the object and in this case the because it is the gravitational force that is uniform for every smaller volume.

$$F_{gravatational} = \sum_{i} g\rho_i \Delta V_i = \int_{CV} \rho g \, dV = mg$$

If you are considering electric force and magnetic force it can vary based on this spatial location. So, because we are considering only gravitational force as the body force, I have not used i, for g, this i represents the volume of the ith element which can have which won't density rho, but I have not your subscript for i for g because the same gravitational force as throughout the control volume which means every smaller volume is objected to the same gravitational force.

So, as usual, we will not express this in terms of an integral, so we consider many such volumes so small that the summation can be represented in terms of an integral over the control volume. Look at the limits, the limits is over the control volume because the volumetric force or a force per unit mass, it is a volume integral. So, the gravitational forces mass into the g vector.

Now, what is this mass? This mass includes the mass of the control volume, which means the shell and mass of the contents of the control volume, both are included because suppose I have to support this, I have supported the weight of this container and also the liquid in it, that is why it says the mass of the entire control volume. Of course, instantaneous because the volume of liquid let us say that can vary as a function of time also. So, the mass of the container may not vary but mass of liquid in the container can vary. So, it says instantaneous mass of entire control volume.

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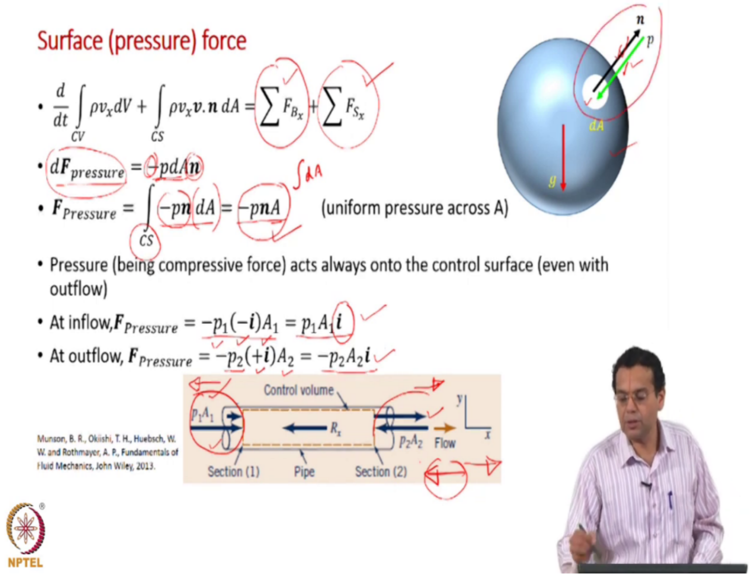

Now, let us discuss surface force, the surface force which are discussing is the pressure force.

$$\frac{d}{dt}\int_{CV} v_x \rho\, dV + \int_{CS} v_x \rho\, v.n\, dA = \sum F_{B_x} + \sum F_{S_x}$$

Now, we write the conservation equation, the linear momentum balance integral form of that and we have discussed the body force and now we are going to discuss the surface force.

Now, we express the force because of pressure as

$$dF_{pressure} =- p\, dA\, n$$

Let us look at this diagram. As I told you it is shown as a sphere for convenience, but could be any object and we are taking a small area there. So, let us take a small area of dA, and then this n vector represents the outward normal, always outward normal. So, taking a small area and the surface, and the outward normal can vary depending on where you take the area.

Now, I have shown pressure, pressure is the compressive force. What does it mean? It always acts in to the control volume or on to the control volume. Let us say we have an inlet and outlet, but still pressure acts in both the direction and always onto the control volume. It is a compressive force and that is what is shown here also (bottom figure in above slide image). What is shown here is a pipe and you have pressure acting into the control volume onto the control volume. Similarly, here also pressure acts on the control volume being a compressive force.

Remember our n vector is an outward normal, it is always pointing away from the control volume. If you have an inlet then n vector point away from the surface, for outlet also away from the control volume, exactly there directions are opposite. Pressure always acts on the control volume, n vector is always away from the control volume. So, n and p vectors are opposite to each other.

So, now, we are proceeding towards expressing the force due to pressure over a small area. So, the is dF represents the force for a small area and the magnitude is pressure into the small area dA and the direction is negative of outward normal, that is $-\ p\ dA\ n$.

Now, this is the force over a small area. We will integrate over the entire surface area. So, let us do that. The force due to pressure is

$$F_{pressure} = \int_{CS} -\ p\ dA\ n =-\ pnA$$

Remember the integral is a surface integral because the pressure is a surface force. It acts over the surface and so you are integrating over the surface area. To contrast the body force, body force acts over the entire volume and the integral was volume integral. So, pressure being a surface force, the integral is a surface integral. So, the body force as volume integral, the surface force as surface integral.

So, most of the time are all the cases that we are going to discuss the pressure is uniform across the area either inlet or outlet. So, this simple expression will suffice for us $-\ pnA$.

Now, pressure being a compressive force, acts always on to the control surface even with outflow, whether it is outflow or inflow, but always pressure acts on to the control surface. Let us see how does it apply for the case shown here.

Let us take the inflow boundary, the inlet, at inflow the force due to pressure is given by

$$F_{pressure} =-\ p_1(-\ i)A_1 = p_1A_1i$$

I am assuming pressure to be uniform and at location 1, the pressure is p_1. The normal is outward normal. It is along the negative x-axis, so $n\ =-\ i$. The area denotes as A_1. The force is $p_1 A_1$ that is the magnitude. The direction is towards the positive x-axis. That is obvious by intuition, the pressure acts into the control volume, so the direction is towards the positive x-axis.

Let us look at the outflow

$$F_{pressure} = -p_2(+i)A_2 = -p_2A_2i$$

The location is 2, so the pressure is p_2 and then, the outward normal is along the positive x axis. The force due to pressure acts along the negative x-axis, which is also correct by intuition at the outlet, pressure acts on to the control volume, so it should act along the negative x axis direction.

So, the force is p_1A_1i on the at the inlet, at the outlet the force is $-p_2A_2i$. So, that is the significance of the negative sign. That negative sign ensures that force due to pressure always acts into the control volume, and that is what we also took into account while writing the negative sign. So, this representation is important to remember.

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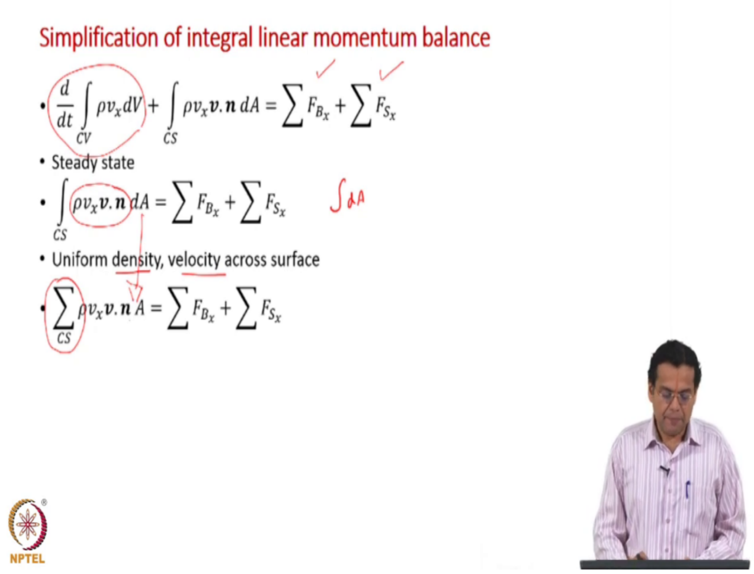

Now, as we have simplified the total mass balance we will simplify the integral linear momentum balance, but not so much. I will tell you the reason shortly.

$$\frac{d}{dt}\int_{CV} \rho v_x \, dV + \int_{CS} \rho v_x \, v.n \, dA = \sum F_{B_x} + \sum F_{S_x}$$

Now, we will start with the general form of the integral momentum balance. Of course, always we are working with the x component, and the right hand side we have now, we have body forces and surface forces, and the previous slides we have seen how to express body force as a volume integral, how to express surface force as a surface integral.

Now, if it is the steady state of course,

$$\frac{d}{dt}\int_{CV} \rho v_x \, dV = 0$$

So,

$$\int_{CS} \rho v_x \, v.n \, dA = \sum F_{B_x} + \sum F_{S_x}$$

So, in all our examples we will consider only steady state integral momentum balance, even if we look at books most of the examples are on steady state momentum balance. The transient because of momentum change usually negligible.

Now, as we have done for mass balance if we assume in the inlet, outlet etcetera, density, velocity is uniform then I can take out $\rho v_x \, v.n$ after the integral and then you have integral dA and that becomes A, so we can simplify that integral in terms of summation.

$$\sum_{CS} \rho v_x \, v.n \, A = \sum F_{B_x} + \sum F_{S_x}$$

This summation means overall the inlet, outlets whatever I have over the control volume. Whatever parts of the control volume or part of the control surface, wherever inflow outflow is there I sum over all those in surfaces through which there is inflow or outflow.

So, the assumption is the density and velocity or constant across the area, so that you can take out and write this way. So, as I told you we are not going to simplify further. In the case of total mass balance, we are a series of simplifications. Here we may not understand also if we simplify, but how do we simplify and apply will understand directly when you look at the applications.

So, that is the way I am not proceeding and showing you a lot of further simplifications. How do we apply? We will straightaway understand and understand better in fact, when we go to the applications. So, we will stop with simplification at this stage.

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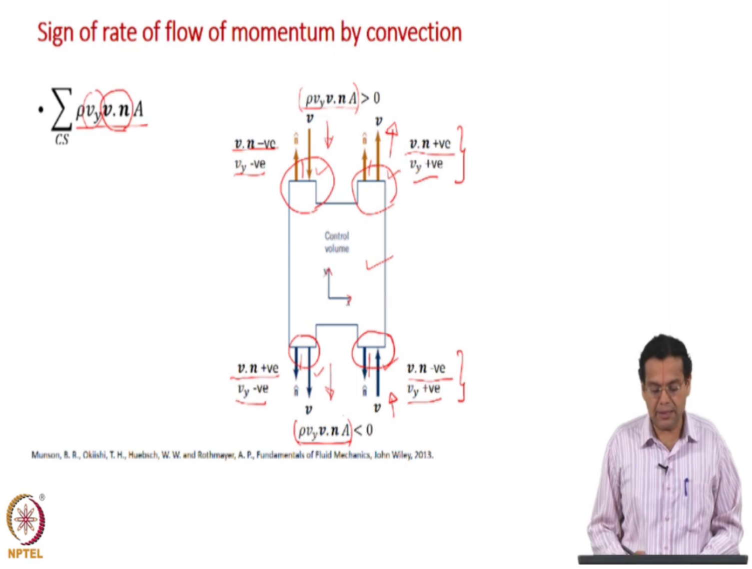

Now, we have this net rate of flow of momentum by convection term.

$$\sum_{CS} \rho v_y \, v.n \, A$$

If you take one for one particular surface, inlet, or outlet, this represents the rate of flow of momentum by convection. What we will do nowhere is, what is the sign of that particular term and first observation is that two terms contribute to the sign first is $v.n$. We have seen that $v.n$ is positive for outflow, negative for inflow, and then v_y can also be positive or negative depending on whether the y component of velocity is along y axis or against y axis.

So, we have four combinations of the sign of v_y and the sign of $v.n$, together determine the sign of this rate of momentum term, that is why we have to discuss very specifically. In the case of mass balance, it is very simple which is $v.n$ either positive or negative, but here four combinations are possible and that is shown using the control volume here a very nice representation from this book by Munson et al on Fundamentals of Fluid Mechanics.

So, there are four surfaces where there is an inflow or outflow, let us take one by one. The first observation is that n is always drawn as an outward normal. It is always pointing outside the control volume, so n is away from the control volume for all the inlets and outlets.

Now, let us take the top right surface through which there is an outflow, because there is an outflow we know that $v.n$ is positive Now, the direction of velocity is along the positive y axis. So, the direction of v_y is along the positive y axis. So, both are positive, through rate of momentum has a positive sign for this phase.

Now, let us look at the top left phase where you have inflow, because it is inflow $v.n$ is negative, and what is the direction of velocity? It is towards the negative y axis. So, v_y will be negative. The rate of momentum is positive because both are negative, so either both are positive or both are negative resulting in a positive sign for the rate of momentum through the whatever phase that the left hand side or the right hand side.

These are an individual rate of momentum for the respective phases. When you sum you tell us the net rate of momentum leaving. So, for both the cases, this rate of momentum term is greater than 0, so positive.

Now, let us analyze these phases at the bottom. The bottom right phase is the inlet phase and because it is inlet phase $v.n$ is negative. But what is the direction of velocity? It is along the positive y axis. So, v_y is positive. So, you get a negative sign for the rate of momentum. Now, on the left hand side for this case it is an outflow. So, $v.n$ is positive, but what is the direction of velocity that is towards the negative y axis, so v_y is negative. So, because one is positive other is negative this term the rate of momentum is negative. So, we should be careful in arriving at the correct sign for this rate of momentum term.

So, we will look at this of course, in the application but just to want to mention that in the case of mass balance it was very relatively easy, the sign depended on $v.n$ alone, but here it depends on $v.n$ and the direction of velocity as well, analogously for other directions etcetera.