Continuum Mechanics And Transport Phenomena
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Lecture – 31
Integral linear momentum balance: Examples - Part 1

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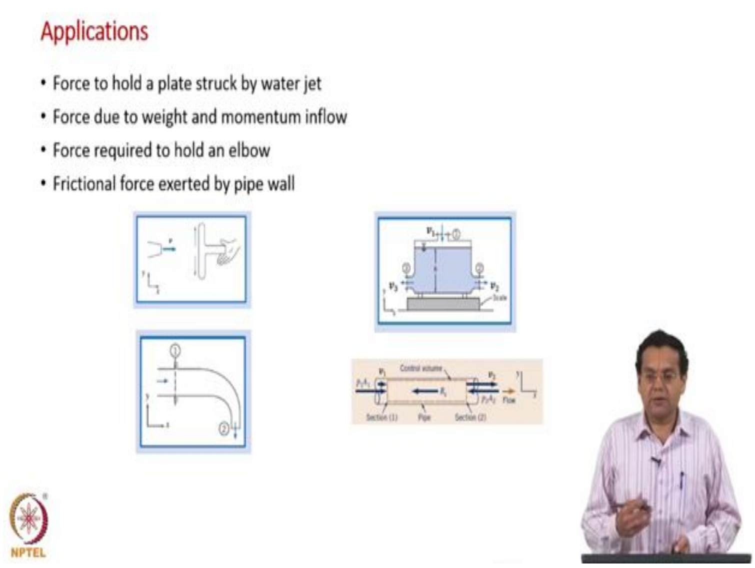


We have derived the integral form of momentum balance and looked at the significance of the terms there. We also looked at how do you get the sign for the rate of momentum term, linear momentum term leaving. Let us look at applications of the integral momentum balance.

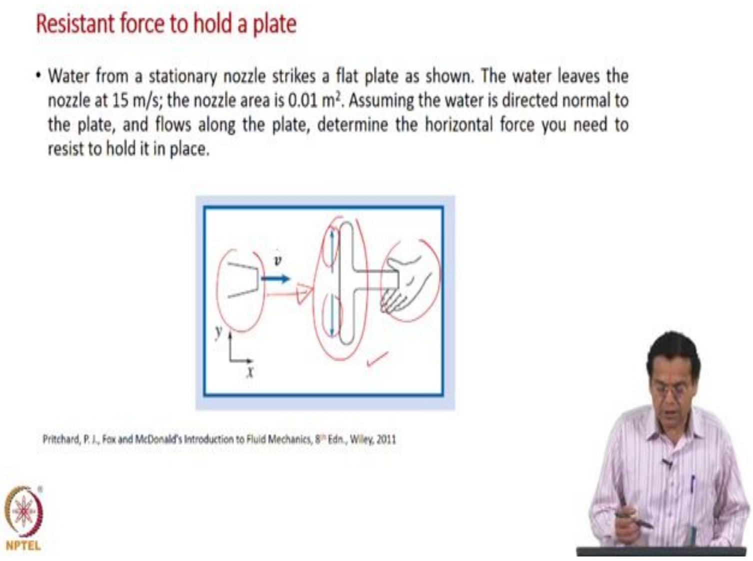
Now, one common term or common variable which I going to find out in all these examples is force. So, we keep talking about force in all the examples that is something that is common to all the examples. So,

- The first example is, we have a water jet and it impinges on a plate and you are holding this plate and you have to give some force so that you can hold this plate. So, we are going to find out what is that force. So, forced to hold a plate stuck by a water jet. Even without doing an example, we can say that this force required is to just balance a momentum. So, all these examples will deal with the balance of force and momentum. So, that should be kept in mind.

- Now, the second example is once again a simple and nice example. We have a tank and then there is water, but it just not water; water is entering and then leaving maybe we will take it this way. So, water is entering and then leaving and this is on a weighing balance and I like to know what is the reading shown by the balance and the reading shown will include the weight of the water in it the tank also and of course, the momentum of the fluid and that is what we are going to see now.
- The third example is the force required to hold an elbow. You have an elbow here which is the pipe joint, we use an elbow to connect two pipes either the same diameter or different diameter. In this case, it is a reducing elbow why because the diameter is changing and there is turn off direction as well. So, because of momentum, pressure etcetera, this elbow will start moving if you do not support it. So, you need to support it. So, what is the force required to hold an elbow and
- The last example is an example to illustrate the frictional force. We have a pipe flow through a pipe and then we measure the pressure drop, temperature, pressure etcetera and find out what is the frictional force, from the pressure drop; we find out what is the frictional force.

Of course, other force which we came across was the pressure which are going to come across the boundaries and of course, some example which will also it will body force. So, these examples will include the momentum flow term and the forces namely body force and surface forces.

Example: (Refer Slide Time: 03:29)

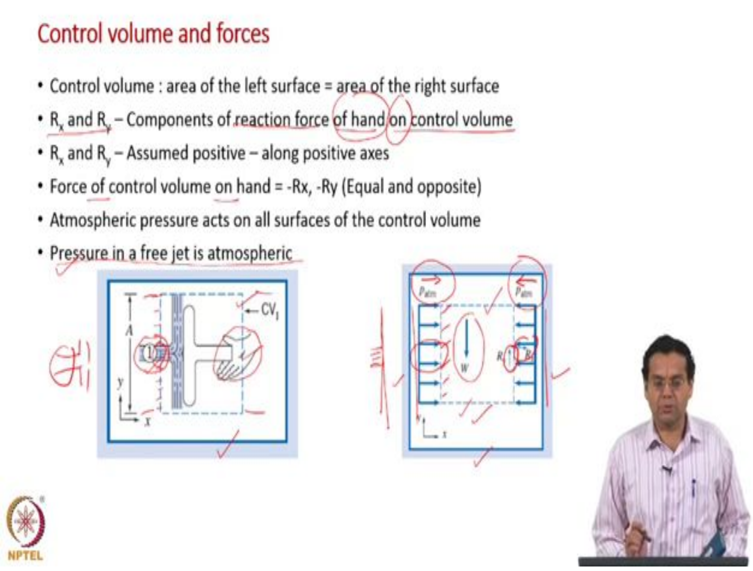


Let us take the first example and let us read the example, water from a stationary nozzle strikes a flat plate. The water leaves a nozzle at 15 m/s, it is a high velocity. The nozzle area is given assuming the water is directed normal to the plate. The reason is that if it is an angle in reality, it may be at an angle, but then you will have to take the component of them, but now if you say it is exactly normal, then only that component along with the normal acts. Suppose, if it is inclined; a part of it will act on the plate makes some more problems more complicated that is why we say water is directed normal to the plate, and once again after it hits, it may get splashed. It may go at some other angle. We do not take all that we say water nicely rises parallel to the plate. All these are simplifications done so that the analysis becomes simple.

And then, determine the horizontal force you need to resist to hold it in place. If you do not do this suppose if you let us say water splash strikes this and go somewhere here and here, then if you take some component of that along the x-direction, but if I say water just goes parallel to the plate falls down, parallel rise up parallel; then, and there is no component of this along the x-direction. This makes a problem much more simpler.

So, we are asked to find out the horizontal force which means we are interested in the momentum in the x-direction, force in the x-direction etcetera ok. So, this one whatever it described is shown schematically here, the plate here and then our hand to support that the nozzle, the water coming out of the nozzle and the direction in which water flows after hitting the plate.

Solution: (Refer Slide Time: 05:27)



Now, the first step is to select the control volume and represent the forces. Now, the control volume that we have selected is shown here (left side image in above slide). When you look at the control volume, there is a momentum flow that crosses the control volume and then there is a force of our hand on the control volume which acts as the part of the control surface.

So, when you choose a control volume you should make sure that whatever we are interested in acts or crosses the control volume or control surface. In this case, the flow crosses the control surface and the force acts on the control surface. Now, the control volume is so chosen that the areas are same. I am focusing only on the x-direction. So, let us look at it.

Now, all the forces are shown here, let us look at what are the forces.

- The first force is atmospheric pressure or force exerted at atmospheric pressure and
- We have discussed that pressure is always compressive and acts into the control volume; that is why the direction of pressure is shown to the right towards the positive x-axis and opposite phase it is shown towards the left towards the negative x-axis.

And we have shown uniform pressure across the entire area and just the effect of pressure will just cancel out each other. The force due to the pressure is $P_{atmosphere}A$ on both the side that is why we said area of the left surface is equal to the area of the right surface. That is the implication of that statement. If we do not do that, the force due to pressure will not cancel each other. The force magnitudes are the same, the directions are opposite. They just cancel each other because of atmospheric pressure.

- Now, the other forces shown is, of course, the weight is shown which is not of significance to us because we are going to do a balance in the x-direction. The force due to weight acts in the along the y-direction but anyway shown here for completeness.
- Now, when we have this plate and apply force, force is exerted by the hand on the control volume. I think that is very important because that changes the sign and this force is called a reaction force. Components of the reaction force of hand on control volume. So, the force exerted by our hand on the control volume. Now, it could be at any angle that is why we split it into two components R_x and R_y. The x component is R_x, the y component R_y. Now, to begin with, we will not know what is the correct direction.

So, by convention, we take R_x and R_y to be positive along the x and y-axis. Eventually, we may find out there it is negative which then we will say that we have to exert force in the opposite direction. But to begin with and analyzing any example, we take the direction to be positive along the x-axis and y-axis.

So, R_x and R_y are components of reaction force of hand on control volume and then, R_x and R_y are assumed positive along the positive axes, the end result may not be same, it may be negative, in which we will have to interpret differently.

- So, the force of control volume on hand is, we use Newton's third law and it so $-R_x$ and $-R_y$; that is why I said reaction force of our hand on the control volume that terminology is very important because if it is the force of control volume on hand whatever control volume exerts on our hand is negative of what we are exerting. So, it is $-R_x$ and $-R_y$ and which we have discussed atmospheric pressure acts on all surface of the control volume. I mentioned about the effect of atmospheric pressure on the left and right surface and of course, pressure acts on these two top and bottom surfaces, but we are not interested in that because that is in the y-direction.

So, what we have seen here is the selection of control volume how do you select and what we have seen in the right side diagram what are the forces acting. This picture is very clear. It will become easy for us to write the integral balance equation.

Now, if you look at the diagram on the right I have shown uniform pressure throughout. Now, of course, the pressure here everywhere is atmospheric that is very obvious. Now, what is pressure across the cross section, across the surface area where there is flow. For that, we should use the physical fact that pressure in a free jet is atmospheric. What is the free jet? You have a nozzle and water just flows out of the nozzle is a free jet.

So, the pressure inside the fluid region is atmospheric, if it is a free jet. If you are not stating this, then I cannot show atmospheric pressure for this part of the surface because it will be some other pressure. Only if you assume that the pressure in this area is also atmospheric, only then I can show uniform pressure throughout. If it were not atmospheric, here on let us say slightly higher pressure, the pressure will be different. That is only based on that assumption I can draw a uniform atmospheric pressure throughout.

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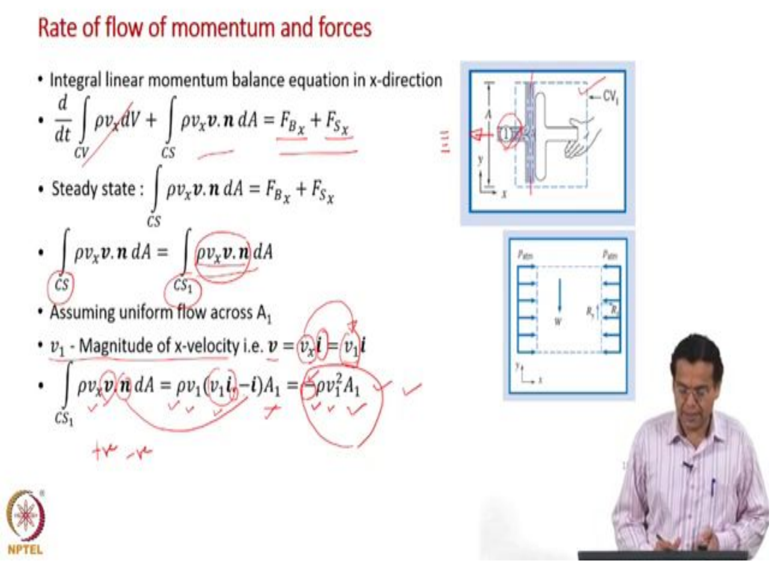

Let us write the integral linear momentum balance equation in the x-direction.

$$\frac{d}{dt}\int_{CV} \rho v_x \, dV + \int_{CS} \rho v_x \, v.n \, dA = F_{B_x} + F_{S_x}$$

In this example, we are doing a balance along the x-direction. Now, of course, steady state problem. So, the first term goes off leaving with the second term and the forces on the right side.

$$\int_{CS} \rho v_x \, v.n \, dA = F_{B_x} + F_{S_x}$$

Now, let us first take the rate of momentum term. Now, in this control surface only at section 1, there is inflow. No other part of the control surface, there is flow in the x-direction. So, you do have the rate of momentum leaving in etcetera, but that is in y-direction because we are doing we are writing a balance along the x-direction. The only part of the control surface, where you have flow in x-direction is only part 1 of the control surface.

$$\int_{CS} \rho v_x \, v.n \, dA = \int_{CS_1} \rho v_x \, v.n \, dA$$

So, that is why I replace the control surface with control surface 1, that part of the control surface, where the free jet enters. Now, let us simplify this integral expression. Now, we assume the uniform flow what does it mean as usual there is no change in velocity in a direction perpendicular to the flow. We have a uniform velocity which means that I can take out the entire term outside. Of course, I also assume density is uniform.

$$v_1 = Magnitude\ of\ x - velocity; \quad i.e\ v = v_x i = v_1 i$$

$$\int_{CS_1} \rho v_x \, v.n \, dA = \rho v_1 \, (v_1 i \, x - i) A_1 = - \rho v_1^2 A_1$$

So, we have to be a little careful in arriving at the correct expression especially the sign of this particular term. If you recall back our earlier discussion on the combination of two terms resulting in a sign, we can quickly check this sign. v_x is positive, $v.n$ is negative because it is inflow resulting in a negative sign ok. We said these two signs together constitute give the sign for the entire term. So, in this case, one is positive, other is negative resulting in a negative sign.

So, this and that is why we did not discuss much on simplification of integral balance earlier. No way we can discuss all this in a general way. So, we discussed for a specific problem.

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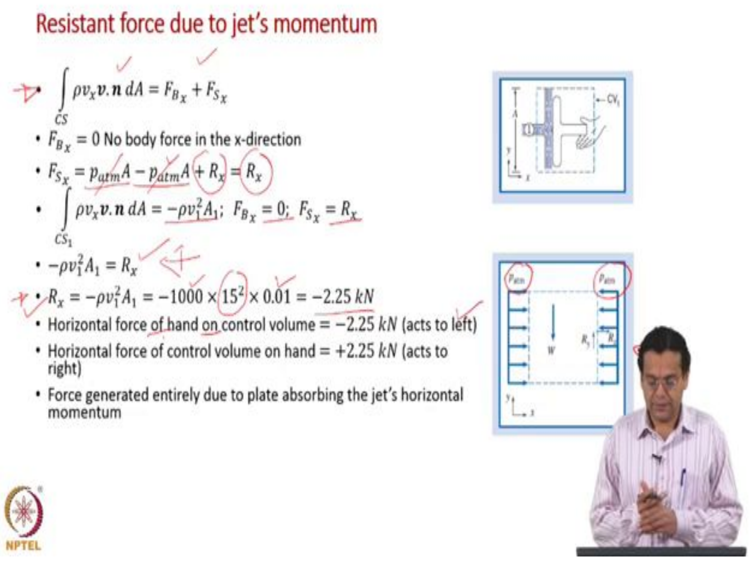

$$\int_{CS_1} \rho v_x \, v.n \, dA = F_{B_x} + F_{S_x}$$

So, we have written the integral form of the momentum balance, the net flow momentum rate of momentum flow term, and the forces. So, far we have discussed only the rate of momentum term.

$$\int_{CS_1} \rho v_x \, v.n \, dA = -\rho v_1^2 A_1$$

Now we will discuss the forces on the right hand side.

$$F_{B_x} = 0, \;\; No\ body\ forces\ in\ x - direction$$

Since we are doing balance along the x-direction, momentum balance along the x-direction, there is no body force. The only body force we are considering is the gravity that does not act along the x-direction; it acts along the y-direction only.

$$F_{S_x} = P_{atm}A - P_{atm}A + R_x = R_x$$

Now, the surface forces due to atmospheric pressure of course it cancel out, and due to our hand, the reaction force because of our hand. Now, focus on this diagram, where we have shown all the forces. So, the force due to atmospheric pressure is $P_{atm}A$ and remember it is the into the control volume that is why it is $+P_{atm}A$ and on the right hand side, it is once again $P_{atm}A$ but it is against the positive x-axis, it is along the negative x-axis because it is into the control volume and that is why we have $-P_{atm}A$. When I say force, it is positive when it is acting along the positive x-axis and this force acting along the negative x-axis and that is why we have $-P_{atm}A$.

Now, the direction of our reaction force is assumed to be positive along the positive x-axis that is why this also $+R_x$. So, as far the forces are concerned, there is no body force. The only surface that was remaining is that due to the x component of the reaction force exerted by our hand on the control volume.

Now, let us substitute all the terms in the integral balance equation.

$$\int_{CS_1} \rho v_x \, v.n \, dA = -\rho v_1^2 A_1; \qquad F_{B_x} = 0; \qquad F_{S_x} = R_x$$

So, let us substitute in the integral form of the momentum balance equation,

$$\int_{CS_1} \rho v_x \, v.n \, dA = F_{B_x} + F_{S_x}$$

$$-\rho v_1^2 A_1 = 0 + R_x$$

$$-\rho v_1^2 A_1 = R_x$$

The way in which I substitute it is that the rate of convection term, let us use our convection term. The convection term is on the left inside and the force term is on the right hand side. So, by convention; the convection term is always on the left hand side, the unsteady state term is also on the left hand side, but we do not have an unsteady state term. So, by convention, we have written the convection term on the left hand side, the force on the right hand side but now as I told you the key physical variable of interest in all the integral balance equation is the force.

So, I take left hand side R_x and bring the convection term to the right side.

$$R_x = -\rho v_1^2 A_1 = -1000 \, x \, 15^2 x 0.01 = -2.25 \, kN$$

To begin with, we said the x component of force acts along the positive x-axis, but now we have a plate even by intuition we can fill, we have water jet hitting on it and we are applying a reaction force. So, our force should be directed towards the negative x-axis; that is a result we have obtained. We apply a force, our force on the control volume should be towards the negative x-axis but we what did we say R_x is towards the positive x-axis that is why we have got a negative sign for R_x that is what the negative sign implies.

So, horizontal force or to be more precise, the horizontal component of the force of our hand on the control volume every word here is important, of our hand on the control volume is equal to -2.25 which means that it acts towards the left. Every time it is also good to check with a physical intuition also. Sometimes we may get confused, the negative or positive sign.

A horizontal force of hand on control volume = $-2.25\ kN$ (acts to left)

A horizontal force of control volume on hand = $+2.25\ kN$ (acts to right)

So, control volume exerts a force on the hand to the right and we have to exert a force to balance that towards the left. So, the force generated is entirely due to the momentum; we have to balance the momentum with our force in the x component and of course, horizontal momentum.