Continuum Mechanics And Transport Phenomena
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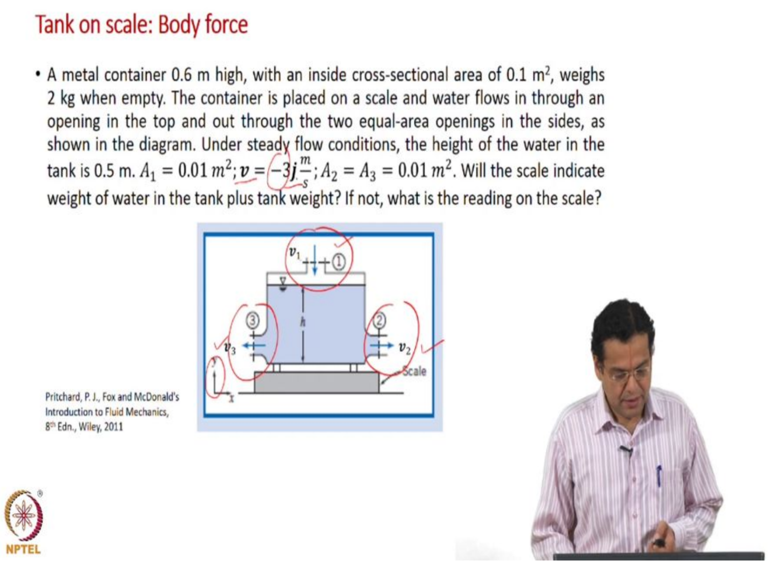
Lecture – 32
Integral linear momentum balance: Examples – Part 2

Example: (Refer Slide Time: 00:13)

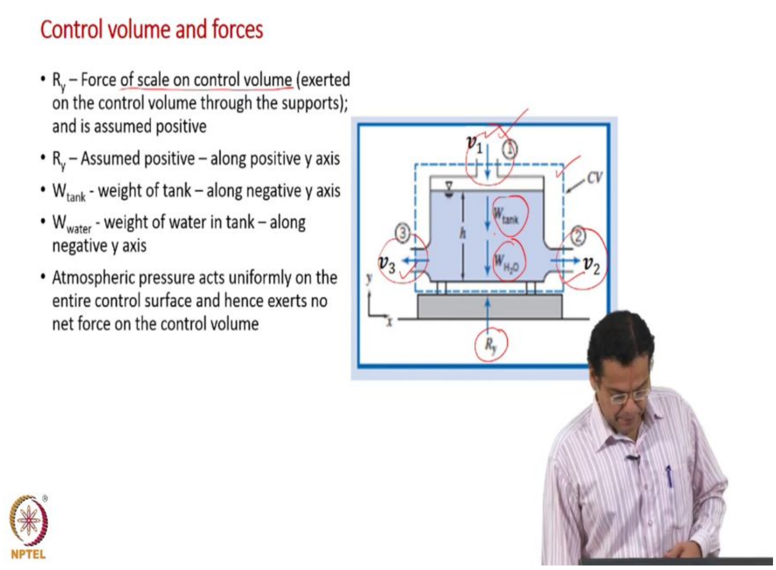


In the second example, we have a container and then, for example, you would measure any sample in your lab, you first put the container and then it will show the mass, and then add some liquid it will show the total mass. Let us say, the balance now shows the weight then it first shows the weight of the container, and then the weight of the container and liquid. Now, we have an opening on the top and then the liquid flows down into the container and leaves through the sides. Now, the reading shown by the balance will be different it will account for the momentum of the liquid entering. So, that is what you are going to see, now in this example more formally.

Let us say read the example, a metal container 0.6 meters high with an inside cross-sectional area of 0.1 m^2, weighs 2 kg when empty. So, this something like putting your beaker and then weighing and the container is placed on a scale and water flows in through an opening in the top and, out through the two equal-area openings in the sides as shown in the diagram (upper referred slide image). Under steady flow conditions, the height of the water in the tank is 0.5 meters. So, that indirectly gives us the volume of water in the tank, you are given the area of the inlet, and the two outlets us all of them as the same area.

We are also given the velocity, the velocity vector is given, it is flowing downwards. So, it has only one component v_y and because it is towards the negative y-axis it is -3, and then j. So, the magnitude of velocity is 3 and along the negative y-direction, it is -3 j. Will the scale indicate the weight of water in the tank plus tank weight? If not, what is the reading on the scale?

Solution: (Refer Slide Time: 02:30)



Now, the control volume is shown (above slide image) are surround by the control surface and, mass enters and leaves through the control surface. In terms of momentum, section 1 flow contributes towards the y-direction and section 2 and 3 flows contribute towards the x-direction.

Since we are interested in the reading shown by the balance we are interested only in the y momentum balance, this means only section 1 flow will be contributing. Now, we denote

- R_y as a force of the scale on the control volume. As in the previous example, we should be careful of the direction of force, the force exerted by the scale on the control volume, and, of course, through the supports and is assumed positive along the y axis,

- We are taken R_y to be directed along the positive y-axis, assumed positive along the positive y-axis.
- Now, W_{tank} denotes the weight of the tank and it is towards the negative y-axis, which you already know. Similarly,
- W_{water} represents the weight of water in the tank that is also along the negative y-axis, and
- Atmospheric pressure acts uniformly on the entire control surface and hence exerts no net force on the control volume. If you have any object and then it is subject to atmospheric pressure, there is no net force acting on the object or the control volume. Be it any shape, this given particular shape, or even if it is the regular shape like a cuboid. Then, if it is just subject to atmospheric pressure there is no net force acting on it. That also we can intuitively understand.

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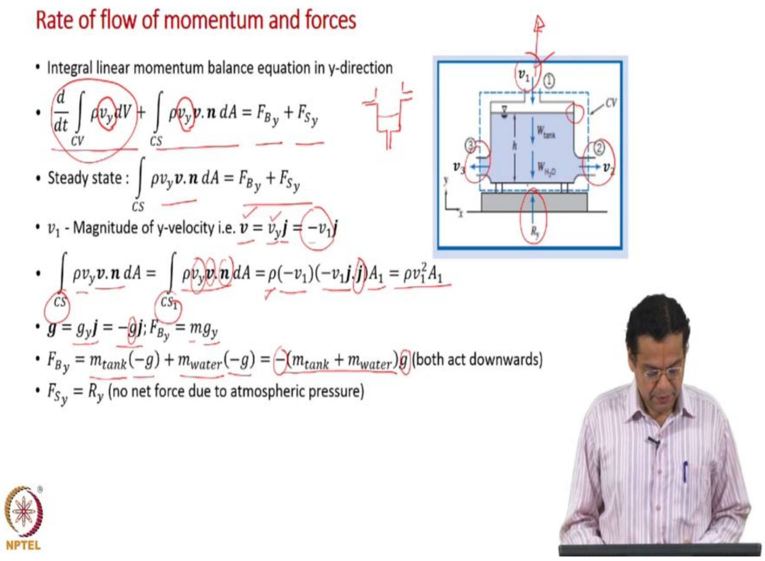

So, with all this information will proceed further will start at the integral momentum balance in the y-direction, because we are interested in the force of the balance on the control volume.

$$\frac{d}{dt}\int_{CV} \rho v_y \, dV + \int_{CS} \rho v_y \, v.n \, dA = F_{B_y} + F_{S_y}$$

So, that is the integral balance equation in the y-direction that is why we have used the y component of velocity in the expression. So, the first term tells you the rate of change of y momentum in the control volume. The second term tells you the net rate at which y momentum leaves a control volume and right-hand side we have body forces and surface forces, acting along the y-direction.

Now, we will consider the steady-state condition,

$$\int_{CS} \rho v_y \ v.n \ dA = F_{B_y} + F_{S_y}$$

The level in the tank does not change with time and hence we say it is under steady-state condition. Remember, if you recall the experiment which you have done we had 2 inlets in the experimental setup and then one outlet, and, here again, when we took that data, we make sure that the level in the tank remained constant. If it is changing then it is under transient condition, but we make sure that, the inlet flow is equal to the sum of the two outlet flows and the level does not change with time.

Now, let us call v_1 is a magnitude of y velocity and so, the v vector has only one component

$$v = v_y j = - v_1 j$$

Now, let us compute this convective momentum term, and as I told you the momentum flow, which contributes along the y-direction is only the section 1 flow. The other 2 flows are in the horizontal direction. So, they do not contribute to momentum in the y-direction.

$$\int_{CS} \rho v_y \ v.n \ dA = \int_{CS_1} \rho v_y \ v.n \ dA = \rho(-v_1)(-v_1 j.j) A_1 = \rho v_1^2 A_1$$

So, that is why the control surface is replaced with CS_1. If it is for mass balance then you have to consider CS_2, CS_3 control surfaces 2 and 3, but because it is momentum balance we consider only the control surface 1, flow-through that alone contributes the y momentum.

Now, coming to the gravity vector; the gravity vector has only one component here once again, the component of gravity along y-direction is

$$g = g_y j = - g j = - 9.81 j$$

If, you write in terms of magnitude g is 9.81 just a value and because it is along the negative y-axis it is -9.81 j.

Now, let us write the body force along the y-direction, the body force along the y-direction is mass into the component of gravity along the y-direction.

$$F_{B_y} = mg_y$$

$$F_{B_y} = m_{tank}(-g) + m_{water}(-g) = -(m_{tank} + m_{water})g$$

So, if you write; there are two components to it; one is the mass of the tank multiply by the $-$g, and second you have the mass of water in the tank multiply by $-$g.

So, if we simplify you get sum of both the masses into the g; remember g here is just 9.81, we have taken care of the negative sign here, because it acts towards the negative y-axis and, the surface force is

$$F_{S_y} = R_y$$

R_y is along the positive y-axis and you already discuss that there is no net force to atmospheric pressure.

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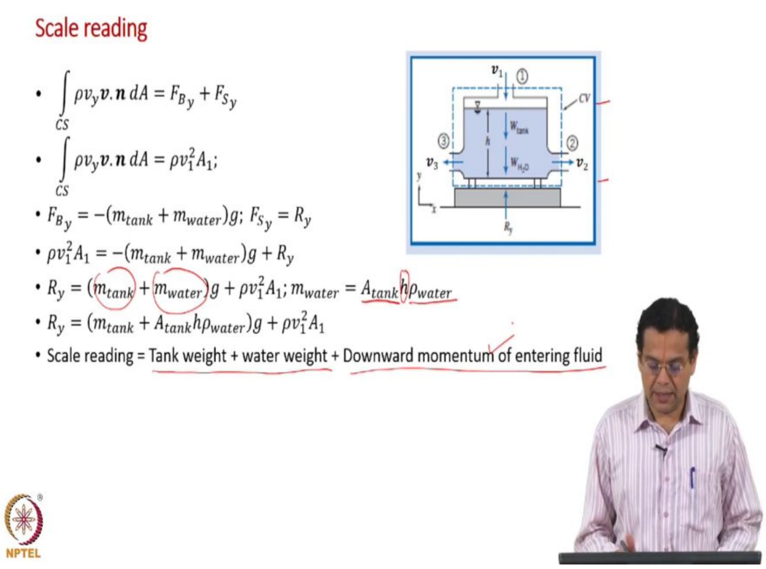

Now, let us substitute all the expressions in the integral momentum balance along the y-direction.

$$\int_{CS} \rho v_y\, v.n\, dA = F_{B_y} + F_{S_y}$$

We found out

$$\int_{CS} \rho v_y \, v.n \, dA = \rho v_1^2 A_1$$

$$F_{B_y} = -(m_{tank} + m_{water})g$$

And,

$$F_{S_y} = R_y$$

So, let us substitute as usual with the convective momentum on the left-hand side, the forces on the right-hand side.

$$\rho v_1^2 A_1 = -(m_{tank} + m_{water})\, g + R_y$$

And, then rearrange for R_y;

$$R_y = \rho v_1^2 A_1 + (m_{tank} + m_{water})\, g$$

R_y is the force of the balance on the control volume or the reading in terms of the force of the balance.

Now, the mass of the water in the tank is not given directly to us.

$$m_{water} = A_{tank} h \rho_{water}$$

It is given in terms of the level of water, in the tank multiply by the cross-sectional area of the tank which gives a volume multiply by the density of water gives you the mass of water in the tank. So, that is how you get the mass of the water in the tank, the mass of the tank is given to you directly. So,

$$R_y = \rho v_1^2 A_1 + (m_{tank} + A_{tank} h \rho_{water})\, g$$

So,

$$Scale\ reading = Tank\ weight + Water\ weight + Downward\ momentum\ of\ entering\ fluid$$

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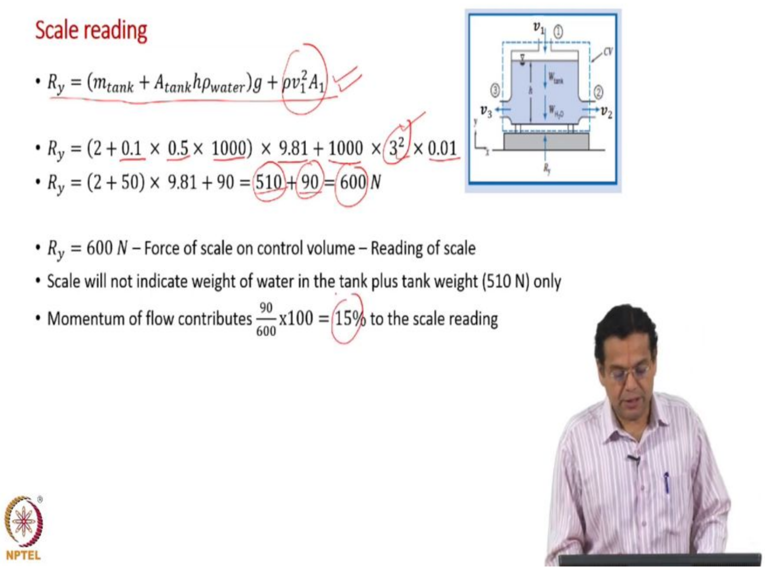


Let us substitute the values given on the problem. So,

$$R_y = \rho v_1^2 A_1 + (m_{tank} + A_{tank} h \rho_{water})\, g$$

$$m_{tank} = 2\ kg; A_{tank} = 0.1\ m^2;\ \ h = 0.5\ m;\ \rho_{water} = 1000 \frac{kg}{m^3}; g = 9.81 \frac{m^2}{s};$$

$$v_1 = 3 \frac{m}{s}; A_1 = 0.01\ m^2$$

So,

$$R_y = (2 + 0.1\ x\ 0.5\ x\ 100)\ x\ 9.81 + 1000\ x\ 3^2\ x\ 0.01$$

$$R_y = 510 + 90 = 600\ N$$

So, it has two components to it that is why specifically the steps are shown in terms of components; one component is because of the weight of the tank and water, other components because of the momentum.

So, if you are neglecting momentum then we would conclude I would say wrongly as 510 Newton, but the balance would show 600 Newton's taking into account the momentum of entering fluid. The force of scale on the control volume is the reading of the scale.

So, to answer the question scale will not indicate the weight of water in the tank plus tank weight. So, it will not indicate 510 Newton only.

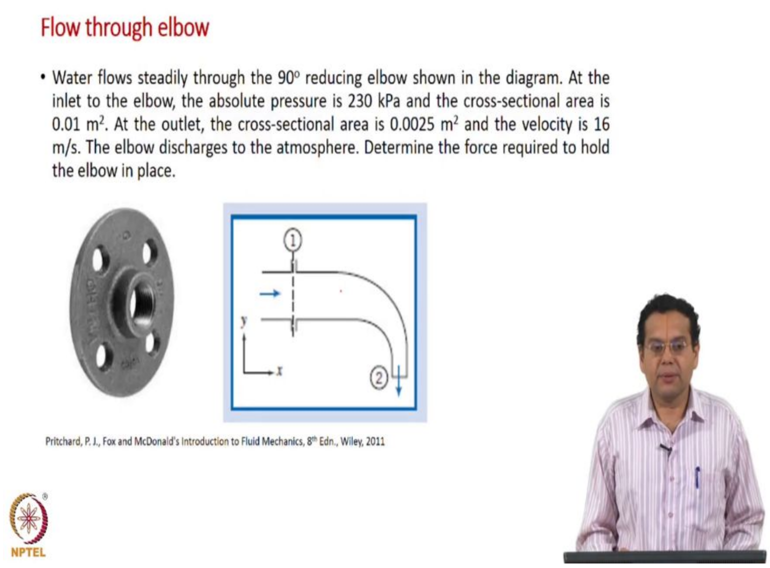
$$Momentum\ of\ flow\ contributes = \frac{90}{600} x100 = 15\ \%\ to\ the\ scale\ reading$$

That is why in this particular example the velocity is chosen to be very high value, for 3 m/s for water is a very high value. To emphasize the point that momentum can contribute high number is chosen. If, you choose the low velocity then, of course, this contribution becomes lower and lower. Just to emphasize that it contributes you have to 15 % a really large velocity has been chosen.

So, the only takeaway from this example is that the momentum of water entering also contributes to the scale reading. And, it is also a good problem solving practice that till the last step we work in terms of variables. And, then substitute in the last step, this awards numerical mistakes apart from that we have a very general expression which is valid for this any value of the variables. If, we start substituting the beginning that becomes very specific of that example.

So, any configuration of this expression is valid. For example, we are discussing that if the velocity becomes smaller and smaller, you can see that this contribution becomes lower and lower. So, for example, we can try with 1 m/s second or 0.5 meters per second this becomes smaller and smaller, that also it is squared.

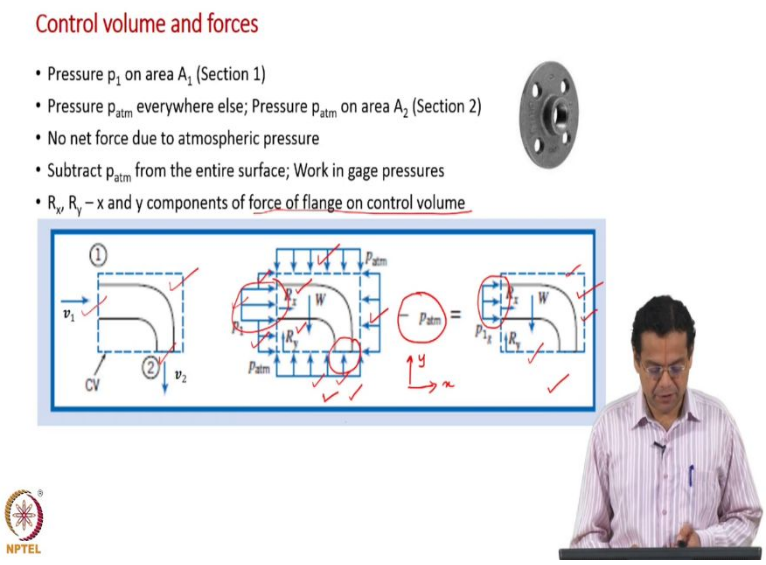
Example: (Refer Slide Time: 15:23)



The next example is the flow through an elbow which is the usual pipe joint for connecting 2 pipes and it is a reducing elbow because we have a larger cross-section at the inlet and a smaller cross-section at the outlet. And, usually, we attach this elbow to a flange. So, the elbow is attached here another way of looking at this something like a tap; almost looks like a tap. So, another way of looking at it is a tap through which water flows. Now, if you are not attached to this elbow for leaving this elbow alone and allowing water to flow it will just keep moving. So, we have to fix it; so some force has to be exerted on this control volume to hold in place and we are going to determine that.

So, let us read that water flows steadily through the 90 degrees reducing elbow (90 degrees tells the angle; reducing tells the reduction cross-sectional area) shown in the diagram, or the inlet to the elbow the absolute pressure is 230 kilopascal. So, which are different pressure not atmospheric pressure. The cross-sectional area is 0.01 m^2 at the inlet and the cross-sectional area is much smaller is 0.0025 m^2 at the outlet and the velocity is 16 m/s, so high velocity there. The elbow discharges to the atmosphere so, the pressure at the outlet is atmospheric we are asked to determine the force required to hold the elbow in place.

Solution: (Refer Slide Time: 17:03)



Now, what is shown here is the control volume along with the control surface and we have x-direction flow and the y-direction flow here. Now, in this particular example in addition to atmospheric pressure, we have one different pressure at the inlet and that is what is shown in the second diagram here (middle diagram in the above slide). The pressure force acting on the control surface. Everywhere we have atmospheric pressure. At the inlet, we have in addition to atmospheric pressure some extra pressure. Now, we have seen the last example or we can intuitively understand that atmospheric pressure does not contribute to any net force. So, we can just subtract out the atmospheric and we can work out in terms of gauge pressure. So, in this diagram total pressure is shown, in the last diagram the gauge pressure is shown.

The advantage of this will be; everywhere the value will be 0 at all these surfaces, only a reason of inlet or outlet you will have to have a non-zero value, in terms of problem solving Let us summarize this so,

$$p_1 g = \ gauge\ pressure$$

The pressure p_1 and area A_1 and pressure p atmospheric everywhere else and it is given that here also the pressure atmospheric. Suppose, if it is leaving a different pressure then here also the arrow marks will be different of magnitude. And, remember all the pressures are shown as compressive. All are to acting on the control surface, with no net force due to atmospheric pressure. So, we subtract p atmospheric from the entire surface work in gage pressures.

And, then we are interested in this problem on the force of the flange on the control volume, it can have 2 components. In fact, going to solve for both the components in this example, in the earlier example where only one of the components, in the case of the plate it was only R_x in the case of balance it was only R_y. So, a slightly proceeding little involved example.

So, we have both the components in this example so, R_x, R_y are the components of force of the flange on the control volume. And, like in the previous cases, we are taken positive along the x-axis and the y axis. So, the R_x and R_y are taken positive along the axis, then based on the sign we can interpret the physical significance.

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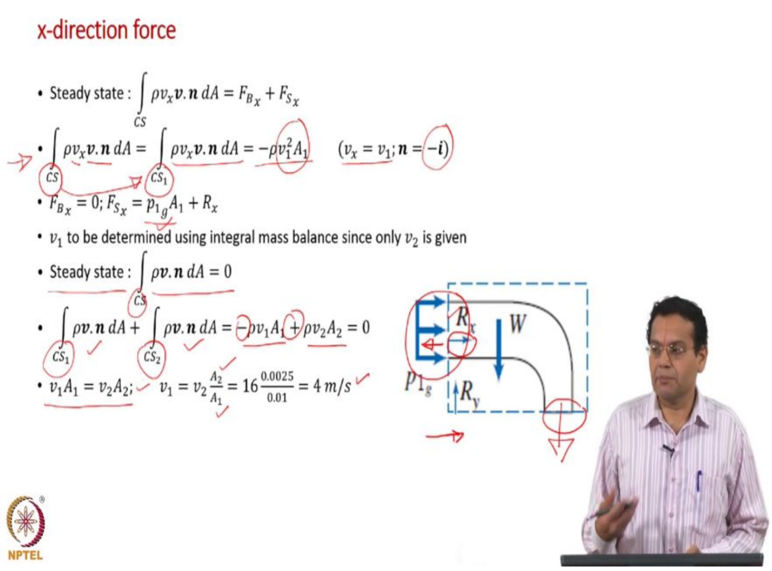

Now, let us find out the x-direction force R_x, which means that we should write the momentum balance in the x-direction, we consider the straightaway the steady-state momentum balance with a convective term on the left-hand side and the forces on the right-hand side.

$$\int_{CS} \rho v_x\, v.n\, dA = F_{B_x} + F_{S_x}$$

Now, for the x momentum in flow-through section 1 alone contributes, some replacing CS control surface with control surface 1,

$$\int_{CS} \rho v_x\, v.n\, dA = \int_{CS_1} \rho v_x\, v.n\, dA =- \rho v_1^2 A_1$$

Now, let us look at the other forces there is no body force along the x-axis.

$$F_{B_x} = 0$$

Surface force;

$$F_{S_x} = p_1 g A_1 + R_x$$

We have written in terms of the gauge pressure based on the discussion in the last slide into the area of this opening.

Now, we have this convective momentum in terms of v_1, but what we are given in the problem is a velocity $v_2 = 16\ m/s$. So, we do a mass balance, integral mass balance to find v_1. So, this example, in fact, involves both integral mass balance and momentum balance.

$$\int_{CS} \rho\, v.n\, dA = 0$$

So, the control surface split into control surface 1 and control surface 2.

$$\int_{CS} \rho\, v.n\, dA = \int_{CS_1} \rho\, v.n\, dA + \int_{CS_2} \rho\, v.n\, dA = 0$$

$$-\rho v_1 A_1 + \rho v_2 A_2 = 0$$

So, if we simplify we get

$$\rho v_1 A_1 = \rho v_2 A_2$$

So, now, we can find out the velocity v_1 we were given the area of the outlet and inlet and this gives us

$$v_1 = v_2 \frac{A_2}{A_1} = 16 \frac{0.0025}{0.01} = 4 \frac{m}{s}$$

Obviously, the areas reducing at the living at 16 m/s at the inlet and it is 4 m/s, we would have straightaway written this expression, but just to emphasize that that comes from an integral mass balance, we proceed a systematically and arrive at this expression.

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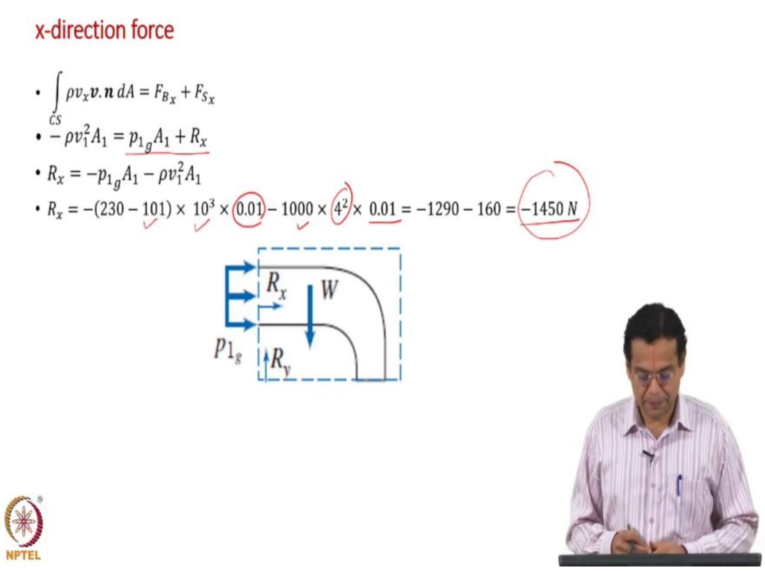

So, now we already with all the expressions, let us substitute in the x-direction momentum balance. So,

$$\int_{CS} \rho v_x\ v.n\ dA = F_{B_x} + F_{S_x}$$

We found out that

$$\int_{CS} \rho v_x\ v.n\ dA =- \rho v_1^2 A_1$$

$$F_{B_x} = 0$$

And,

$$F_{S_x} = p_1 g A_1 + R_x$$

So,

$$-\rho v_1^2 A_1 = p_1 g A_1 + R_x$$

So, let us rewrite for the reaction force, the x component of reaction force,

$$R_x = p_1 g A_1 + \rho v_1^2 A_1$$

So, we substitute and get

$$R_x = -(230 - 101)\ x\ 10^3\ x\ 0.01 - 1000\ x\ 4^2\ x\ 0.01 = -1290 - 160 = -1450\ N$$

So, the first observation is that the reaction force has two components, one due to the pressure and one due to the momentum of the incoming fluid, and you have the elbow, and fluid is entering, and then pressure. So, your force has to be in the opposite direction, not our force meaning the force exerted by the flange. So, the flange has to exert a force along the negative x-direction that is why we get a negative sign here minus 1450.

So, the force due to pressure and momentum are all towards the positive axis. So, the reaction force to hold the elbow should be along the negative x-axis. So, the force exerted by the flange because your flange is here on the control volume is along the negative x-axis, otherwise, it will just keep moving towards the right.

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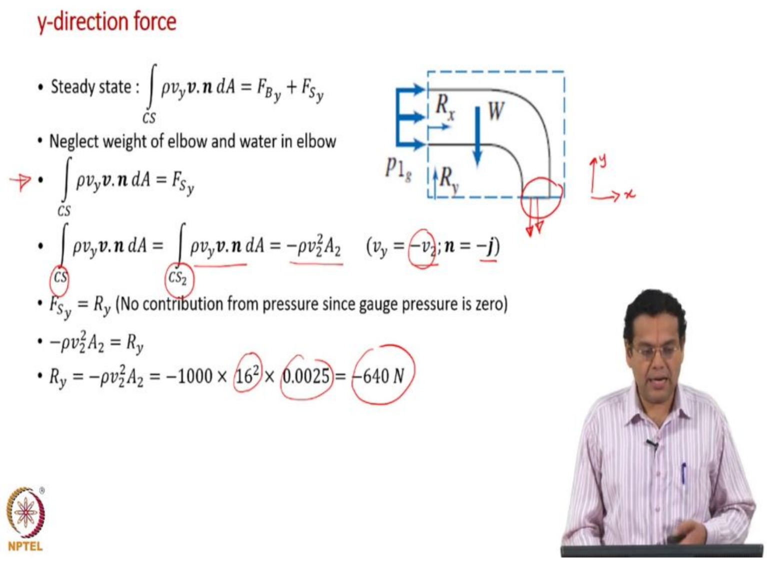

So, now coming to the y-direction force, we write the integral balance of linear momentum along the y-direction.

$$\int_{CS} \rho v_y\ v.n\ dA = F_{B_y} + F_{S_y}$$

And, we neglect the weight, if you are given the mass of the elbow, the mass of water in the elbow, then we can include in this particular example it is not given ah sometimes may be negligible will also; so we neglect that.

$$\int_{CS} \rho v_y \; v.n \; dA = F_{S_y}$$

So, we have only convective momentum and the surface force on the right-hand side. Now, the convective momentum flows through the outlet alone which is along the y-direction, that alone contributes. So, I am replacing CS with CS_2 and $v_y = -v_2$,

$$\int_{CS} \rho v_y \; v.n \; dA = \int_{CS_2} \rho v_y \; v.n \; dA = -\rho v_2^2 A_2$$

Now, the surface force is only because of the y component of reaction force, there is no component due to pressure.

$$F_{S_y} = R_y$$

Now, let us substitute in the integral balance and write it for the y component of the reaction force,

$$R_y = -\rho v_2^2 A_2$$

Now, let substitute the values

$$R_y = -1000 \; x \; 16^2 \; x \; 0.0025 = \; -640 \; N$$

Now, the first observation the y component of reaction is only because of momentum. Now, how do you interpret this negative force? You have water flowing down and because of that, there is upward thrust, because water is flowing down it exerts the upward thrust on the control volume or the elbow, which means that you should apply a force from the top so, that you hold the elbow now so, supply a force opposite the thrust.

So, that something like the launching of a rocket, because of the gases, of course, high velocity and so on there is the thrust on the rocket in the upward direction. And, similarly here because of water flowing outside the tap was this elbow there is an upward force on the tap. So, the y component of the force exerted by the flange should be pointing downwards. Otherwise, we have some movement in the upward direction, that is the interpretation for this negative sign. So, in this example, we have seen that the force finally apply will have some direction based on this Rx value and Ry value, that both the components. Accordingly, you will have a direction for the net resultant force.