Continuum Mechanics And Transport Phenomena
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Lecture – 42
Properties of stress tensor – Part 1

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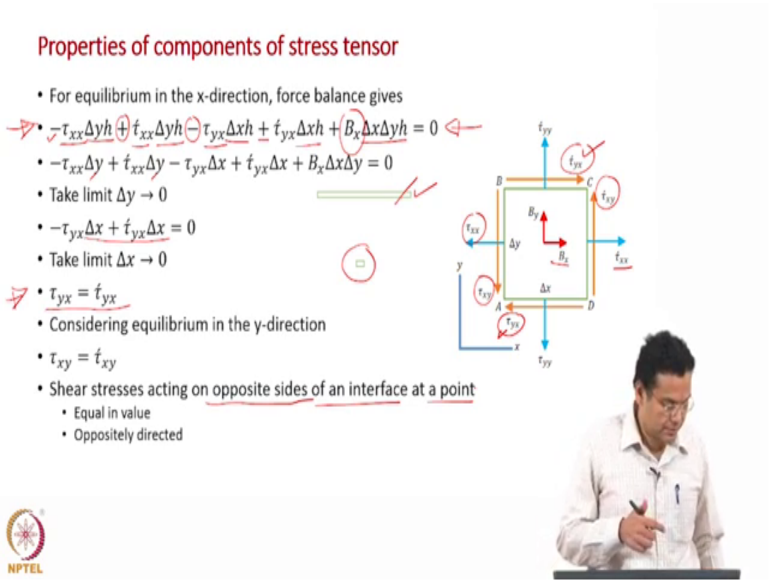

Now, we will discuss the properties of the components of stress tensor and how does it help us in terms of simplification. The two dimensional plane element is shown here. So, how do you imagine two ways of imagining, every kind of imagine is a plate like this or a character of imagining is the small two dimensional region inside a solid object as usual. We said tetrahedron inside a solid object now this two dimensional element inside the solid object.

And then it has a thickness we are going to proof for two dimensional case then we can extend for three dimensional case. Now, so we have a plate and it has a thickness and we are given the dimensions Δx and Δy and then let us say thickness is h.

And the stresses acting on the different surfaces the normal and shear stresses are shown, let us go one by one. We have $\dot{\tau}_{xx}$ on the right face, remember all the forces are shown in the positive sense. Which means that we are using our sin convention on a positive face force should be along positive direction negative face force should be along negative direction, that is what I mean by positive sense.

So, the right side face called as $\dot{\tau}_{xx}$ and it is along the positive x axis, the left side face we have τ_{xx}, so along with negative x axis. Now, take the y direction we have $\dot{\tau}_{yy}$ on the positive y face, τ_{yy} on the negative y face, so these are the normal stresses. We have specifically taken τ_{xx} on the left hand side and some other value $\dot{\tau}_{xx}$ on the right hand side, similarly in the y direction also to begin with later on we will see what happens.

Now, let us go to the shear stresses on the right face we have $\dot{\tau}_{xy}$, xy because the plane is x and direction is y; shown in the positive sense because, normal is along with positive axis force along positive axis. So now, coming to the left face I have shown as the τ_{xy}, it is a x face hence tau x second subscript is y because force along y direction, negative face so force along negative axis.

Now, let us take the top and bottom face and the top face it is $\dot{\tau}_{yx}$, normal is along y direction force along x direction both are positive and then we have τ_{yx} normal is y forces along x direction and of course both are negative and then of course, what is shown are also the body forces B_x along x direction, B_y along y direction ok. That is the terminology for this diagram we will use this 2D element to arrive at properties of the components of stress tensor.

Now, as we have taken earlier we will take this two dimension element under equilibrium. So, let us say for easy understanding plate is also under equilibrium which means that we can do the force balance along x direction and we can do along y direction as well. Let us write the force balance along x direction taking this element to be under equilibrium. Now, for writing the force we need the direction, we need the stress and we need the area. So, three are required let us do one by one, let us take the left face

$$-\tau_{xx}\Delta yh + \dot{\tau}_{xx}\Delta yh - \tau_{yx}\Delta xh + \dot{\tau}_{yx}\Delta xh + B_x\Delta x\Delta yh = 0$$

Remember we should take components of the stress acting along x direction just like written for the tetrahedron. So, τ_{xx} the force is along negative x axis so negative sign.

So, now $\dot{\tau}_{xx}$ on the right side it is along positive axis so, this is positive. And now coming to the shear stresses τ_{yx} that is along negative x axis. This equation represents the force balance along x direction.

So, now let us cancel out h it is common, other way of explaining is just two dimensional case h is common to all the terms.

$$-\tau_{xx}\Delta y + \dot{\tau}_{xx}\Delta y - \tau_{yx}\Delta x + \dot{\tau}_{yx}\Delta x + B_x\Delta x\Delta y = 0$$

Now, we as usual even for this properties we are interested in getting properties violated at a point always keep that in mind. We should shrink this plate this let say rectangle to a point we do it in two different ways.

What we do now is take limit $\Delta y \rightarrow 0$, it will become a line. So, we have a plate Δy Δx we are shrinking $\Delta y \rightarrow 0$ it becomes a line like a horizontal line. If you include this face alone will be available the entire plate will become just this face alone. Just for visibility I have shown this as a small rectangle either you can say it becomes a line if I attach a surface becomes a one thin plane and that is what we see now.

$$-\tau_{yx}\Delta x + \dot{\tau}_{yx}\Delta x = 0$$

So, I have this line or face now I will make $\Delta x \rightarrow 0$; it becomes a point.

$$\tau_{yx} = \dot{\tau}_{yx}$$

Now, if we take a rectangle it became a line then it became a point or if you take it as a plate with the surface it became a surface alone then became a point.

So, eventually what is that we have done we have proved this relationship $\tau_{yx} = \dot{\tau}_{yx}$

I will explains what it means, for a point on a surface. We have a surface at every point on the surface this relationship is valid. What is the physical significance of that? So, we have a let us assume this is the plane on opposite sides of the plane the shear stresses are equal and opposite that is all it says. On a surface the shear stresses acting on opposite side of a interface of a surface are equal and opposite in direction.

Now once again remember when we derived the expression for a control volume tetrahedron etcetera, we have always start with average values similarly applies here also at this stage they are all average values. When you take limit finally, this limit in two successive stages they become point values, all our relationship are to be derived at a point.

So, we have not distinguished in terms of nomenclature, but at first stage they are average values the last equation they are point values. So, we have seen that $\tau_{yx} = \dot{\tau}_{yx}$.

If you consider equilibrium along y direction then do the force balance along y direction, and do the similar steps you will see is

$$\tau_{xy} = \dot{\tau}_{xy}$$

Here also, same physical significance but along other direction. If we have to face the shear stress acting on opposite sides are equal and opposite in direction. So, these two are same. So, shear stresses acting on opposite sides of an interface at a point, look at the sentence it says opposite sides of an interface at a point. So, at every point on the surface this is valid, what is it valid; their shear stresses equal in value of course and then oppositely directed. So, if you take a face you have shear stress here shear stress here and they are opposite direction equal in magnitude, either it be in x direction or x face y face or whatever face it is.

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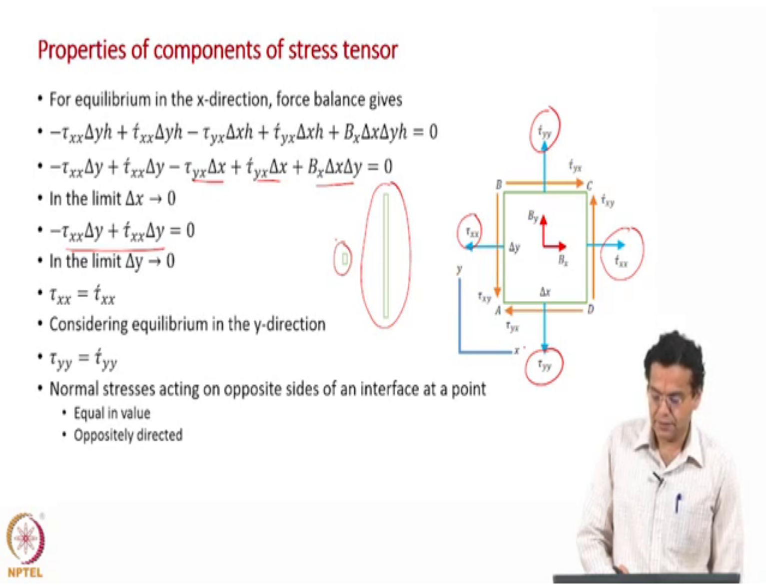

Now, what we will do? We will now start once again with the force balance in the x direction, that step is same no change in it.

$$-\tau_{xx}\Delta yh + \dot{\tau}_{xx}\Delta yh - \tau_{yx}\Delta xh + \dot{\tau}_{yx}\Delta xh + B_x\Delta x\Delta yh = 0$$

This step is same as last slide then divided by h as we done in the previous slide.

$$-\tau_{xx}\Delta y + \dot{\tau}_{xx}\Delta y - \tau_{yx}\Delta x + \dot{\tau}_{yx}\Delta x + B_x\Delta x\Delta y = 0$$

Now, the difference appears, earlier we first took $\Delta y \rightarrow 0$ and then took $\Delta x \rightarrow 0$. So, what happened? This rectangle became this face and then became a point, now what I am doing I am taking $\Delta x \rightarrow 0$. So, first it becomes a vertical line. So, terms with Δx will now vanish only terms with Δy will remains.

$$-\tau_{xx}\Delta y + \dot{\tau}_{xx}\Delta y = 0$$

Now, I take $\Delta y \rightarrow 0$ exactly opposite of what we did in the last case. So now, what happens the vertical line just become a point and this goes to a point.

$$\tau_{xx} = \dot{\tau}_{xx}$$

Now, this relationship is valid at every point in the face. So, if you have an interface and what is significance, if you have a plane opposite sides of the plane the normal stresses acting on opposite sides of the plane are equal in terms of magnitude opposite in direction. Now, repeat the same exercise along the y direction you will prove that these normal stresses are equal in magnitude opposite in direction. So, normal stresses acting on opposite sides of an interface at a point equal in value oppositely directed. So, if you want to combine the previous slide inference and this slide conclusion. If we have interface normal stresses are also equal and opposite, shear stresses are also equal and opposite that is the conclusion from these two slides.

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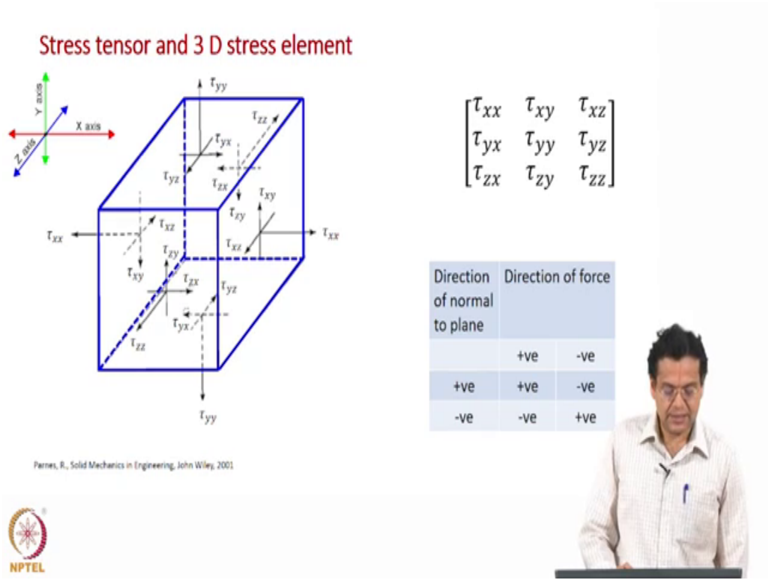

Now, what we have discussed now, in fact, already used it though I did not explicitly explained. Where do we use remember this is the stress element 3D stress element when I marked and then we said nine components are there one more set nine components are there in opposite direction, they both are equal and when I marked look at the way I represented, right hand side it is τ_{xx} also left hand side I noted τ_{xx} I have not shown different value. Which means that remember these are two different representations same surface on one side τ_{xx} other side τ_{xx} opposite direction.

So, in even in this nomenclature this has been used, though of course that not the point discussed. Now, no we have to know how what was the concept used in representing τ_{xx} of this side on τ_{xx} on the other side. Similarly I have used τ_{xy} in this direction and τ_{xy} in the opposite direction, similarly τ_{xz} in this direction and τ_{xz} in the opposite directions.

So. In fact, we have used on opposite side of this in surface, the normal stresses are opposite and equal similarly shear stress are equal and opposite. So, it has been used, but we have now proved theoretically and justifying the nomenclature for the stress element.