**Continuum Mechanics And Transport Phenomena**
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**Lecture - 55**
**Strain Displacement Gradient Relation: Example**

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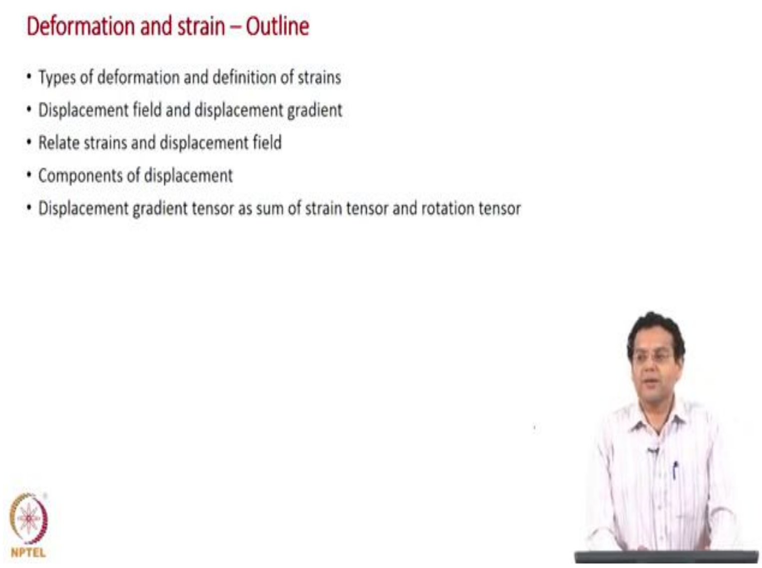


We are discussing Solid Mechanics and we have come to solid mechanics for second time to understand displacement gradient that was our objective and the title for our second visit is deformation and strain in that we have discussed the types of deformation namely change in length, change in angle and then we quantified them in terms of strains namely normal strain shear strain. Then we said the strains are related to the displacement of the particles or points. So, then we discussed what was the displacement field, what was the displacement gradient slowly moving from 1D, 2D to 3D.

Now, we have strain on one side and displacement field another side and then we will have to relate these two and that is the objective of this lecture. Later on we will see what is components of displacement and then the last bullet later on.

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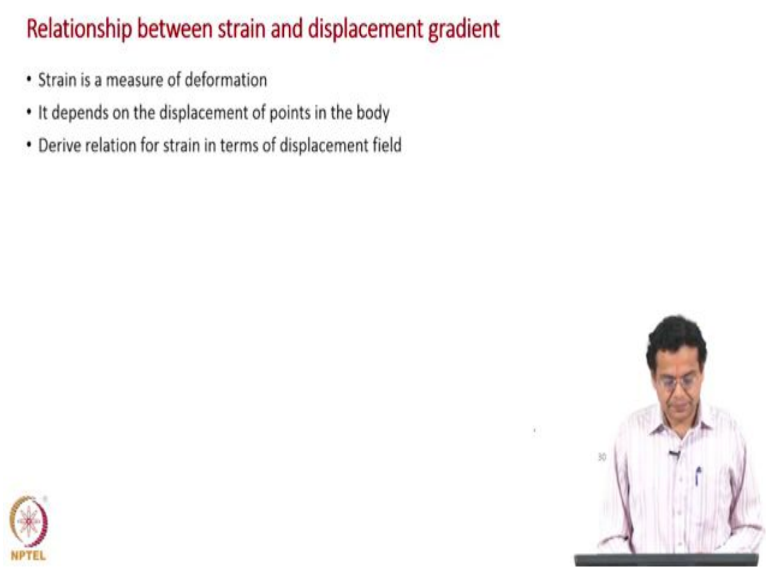


The one question which I going to answer throughout this lecture is that, we are given displacement field how do I calculate strain that is what we are going to do now.

We introduced deformation which are change in length, change in angle and strain is a measure of defamation which means that, you first introduced change in length, change in angle, but we need to quantify it and quantification was through strain namely normal strain shear strain that is the meaning of the statements strain is a measure of deformation.

Now, when we explained even normal strain, shear strain we said take some points in the object along one direction, and then the point got displaced they move to some other location which means that the strain deformation are all related to the displacement of points or particles in the body. We are going to derive a relationship for strain in terms of displacement field that is the objective.

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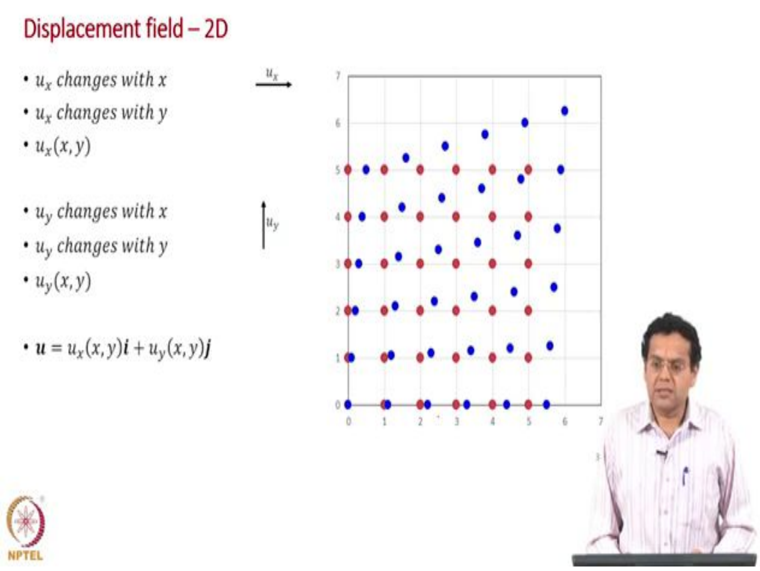


Our discussion is going to be in 2D. So, just to recall that the discussion earlier what was the displacement field; what is shown here; the red dots represent some points identified or particles identified in a plate or 2D plate like this.

In the initial state undeformed state, you apply a force it deforms go to final state or deformed state, the blue dots represent the same points in the deformed state, and then the we said $u_x$ represents the horizontal displacement which is nothing, but the difference in the x coordinates of the points and if you focus the difference in x coordinate which is x displacement keeps changing in the x direction also in the y direction.

And now if you look at the y displacement which is the difference in the y coordinate that also changes if you go along the x axis and the y axis that is what we summarized and that is what we discussed in the slide, $u_x$ changes with x and $u_x$ changes with y as well. So, $u_x$ is a function of x comma y.

Similarly $u_y$ changes with the x, $u_y$ changes with the y so, $u_y$ is a function of x comma y and we have represented the displacement field in terms of a vector,

$$u = u_x(x,y)\,i + u_y(x,y)\,j$$

We are given this displacement field we need to find out the strains that is the idea.

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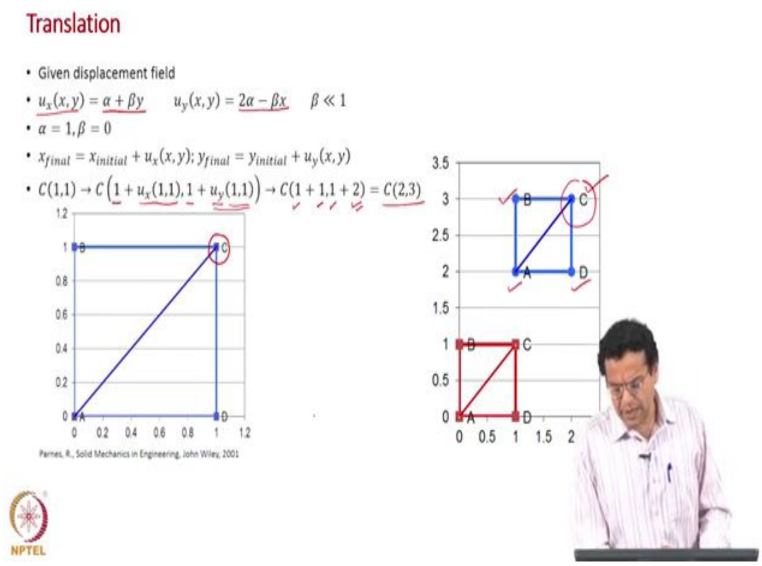


Now, we are proceeding towards finding a quantitative relationship between the displacement field and the strain. Before that what we will do now is qualitatively we will see how we can explain strain if you are given displacement field. We are going to derive quantitative relationship later before that to get understanding. What we are going to do is qualitatively can I say whether we have strain or not etcetera.

So, let us start we are given a displacement field what is a given displacement field; 2 dimensional case; so, we are given $u_x$ as a function of x comma y, $u_y$ as a function of x comma y

$$u_x(x,y) = \alpha + \beta y; \quad u_y(x,y) = 2\alpha - \beta x \qquad \beta \ll 1$$

So, as I told you $u_x$ depends on y only and $u_y$ depends on x only. Now alpha and beta constants and we have a condition that beta is very very small than 1. Now how do you understand this? We have a plate as you have seen earlier and we are given the displacement field for this plate.

Now, in all these diagrams what you will see is, the red boundary represents the red figure represents the initial configuration un-deformed configuration and the blue figure represent the same plate after deformation the final state. So, red is initial state blue is final state.

Now for the moment for the first case we will take

$$\alpha = 1,\ \beta = 0$$

Now when I say what happens to the plate what I mean is given the coordinates ABCD I want to find out the new coordinates ABCD, initial coordinates are given final coordinates are to be found out.

How do you find out? You take the coordinate, add the x displacement, add the y displacement get the final coordinate now, but the displacement depends on x and y. So, accordingly you should calculate the displacement. Let us see one example

$$x_{final} = x_{initial} + u_x(x, y)$$

$$y_{final} = y_{initial} + u_y(x, y)$$

So, that is what I said final coordinates are equal to initial coordinates plus the displacement in the respective directions. Let us take an example coordinate C let us see what is initial coordinate. Initial coordinate both in this diagram both are super imposed. So, the initial coordinate of C is (1, 1). You will see the final coordinate shown here which is (2, 3) let us see how do you find out that.

$$u_x(x, y) = \alpha + \beta y; \quad u_y(x, y) = 2\alpha - \beta x$$

$$\text{For,} \quad \alpha = 1,\ \beta = 0$$

$$u_x(1, 1) = 1; \quad u_y(1, 1) = 2$$

$$C(1, 1) \rightarrow C(1 + u_x(1, 1), 1 + u_y(1, 1)) \rightarrow C(1 + 1, 1 + 2) = C(2, 3)$$

So, likewise we can calculate for A B and D and you can draw the square here connecting ABCD which shows the new coordinates are ABCD.

So, what is that we are done? We are given displacement field in terms of $u_x$ and $u_y$ in this particular case we are taking $\alpha = 1$, $\beta = 0$ we are given the initial coordinates using the displacement we found out the final coordinates now let us see what has happened to the plate.

The plate if you see here has just moved from one location to the other it has just got displaced. If you look at the length of all the sides there is no change it means there is no normal strain if you look at the angle there is no change there is no shear strain.

So, it has just moved from one place to the other place the technical terminology for this is translation that is where the title says translation. You applied a force it just moved the plate that is all has happened. Remember first slide we had change in length, change in angle we had also said plate is also moving, we ignored that point of time now we are discussing that by applying the force nothing has happened to the plate in terms of shape or length that angle nothing has happened; it just moved that is why we called as translation.

To be more specific we call this as rigid body translation, why; just like a rigid body it just moved from one place to another place and you do not see any change in length, any change in shape, it means no normal strain on shear strain.

And now we will also introduce one more terminology called rotation there is no rotation also the plate is just moved on one way to assess rotation is based on the diagonal. So, if you look at the diagonal it there is no change at all.

So, to conclude this case $\alpha = 1$, $\beta = 0$ the plate remember there is displacement because all the coordinates have changed but the displacement is because of translation and there is no normal strain no shear strain, no rotation the simplest case.

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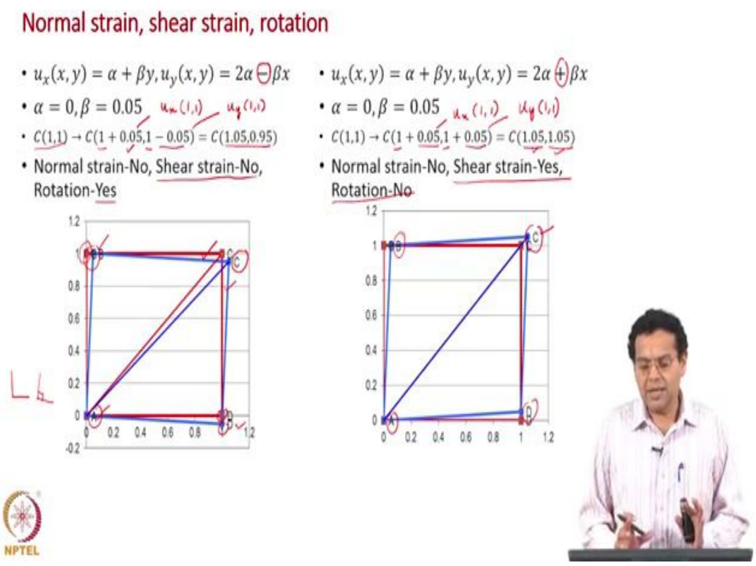


Now, let us take a more realistic case where there may or may not be normal strain shear strain and rotation.

$$u_x(x,y) = \alpha + \beta y; \quad u_y(x,y) = 2\alpha - \beta x$$

Now, same displacement field; now we will consider two cases. The slide split into the two columns let us discuss the left hand side column. Now I take

$$\alpha = 0; \quad \beta = 0.05$$

Remember $\beta$ has extremely small values order of $10^{-6}$. So, you have seen that if you apply any force in a solid object, almost rigid a small deformation happens. So, slightly deformable. So, $\beta$ is very small, if I take beta $10^{-6}$ hardly we can see anything that is why we are taken $\beta$ as 0.05 but remember $\beta = 0.05$ is a very large value to show clearly we are considering $\beta = 0.05$.

Now, let us proceed. As we are seen earlier the red figure represents the initial configuration, the blue figure represents the final configuration. As we have done earlier, to know the final configuration a final state I need to calculate the coordinates let us see how do we calculate

$$u_x(x,y) = 0.05y; \quad u_y(x,y) =- 0.05x$$

So,

$$u_x(1,1) = 0.05; \quad u_y(1,1) =- 0.05$$

So,

$$x_{final} = x_{initial} + u_x(x,y)$$

$$y_{final} = y_{initial} + u_y(x, y)$$

$$C(1,1) \rightarrow C(1+0.05,\ 1-0.05) = C(1.05, 0.95)$$

So, you find the coordinates of C (1.05, 0.95). Similarly find the coordinates of B, A and D you can draw the figure in blue which is the final state. So, red represents the initial configuration blue represents the final configuration.

Now, let us analyze what has happened to the shape. If you look at the length there is no change in length what do you mean that by that? Focus on the initial AB you have A here and then this point and the length of this blue are almost the same or they are same.

Here because I have taken 0.05 it may look like there is a change, but remember β is extremely small value. So, there is no change in that length which means that there is no normal strain. Similarly if you do if you take A and D and then AD once again there is no change in length. So, there is no normal strain along x axis and along y axis.

Now, what about shear strain? Look at the angle initially the angle was 90 degrees, now in final state both lines have slightly moved this way, but their angle is still 90 degrees. Look at the angle but in red lines they are 90 angle between the blue lines are also once again 90 degrees. So, which means that there is no shear strain also.

But look at the plate the plate has undergone a rotation how do you understand as I told you look at the diagonal the red diagonal versus the blue diagonal it has rotated clockwise. So, there is rotation. So, the final configuration is rotated with reference to the original configuration, final state is rotated with respect to initial state.

So, to conclude there is no change in length and no normal strain and then there is no change in angle still 90 degrees no shear strain, but the plate has undergone rotation. So, there is rotation.

Now, consider another case the right hand side a slightly different displacement field why is it slightly you will understand. The displacement fields are given as

$$u_x(x,y) = \alpha + \beta y; \quad u_y(x,y) = 2\alpha + \beta x$$

Now let us see what is implication of this. Once again take

$$\alpha = 0; \quad \beta = 0.05$$

So,

$$u_x(x,y) = 0.05y; \quad u_y(x,y) = 0.05x$$

Now as we have done earlier for the left hand side we will find out the new coordinates of point C let us find out this coordinate. Same like last time (1, 1) is the old coordinate to that you add the displacement,

$$u_x(1,1) = 0.05; \quad u_y(1,1) = 0.05$$

So,

$$C(1,1) \rightarrow C(1+0.05,\ 1+0.05) = C(1.05, 1.05)$$

Now, similarly you can calculate for A B and then D. Now let us see what happens in terms of strain. Look at the length there is no change in length, if you compare red AB and blue AB no change in length. Consider let us say red AD and blue AD once again no change. So, there is no normal strain now look at the angle. Look at the angle; look at the angle BAD the initial state 90 degrees in the deformed state look at the angle BAD say smaller angle, angle has reduced which means that there is shear strain in this particular case we said shear strain exist when there is a change in angle.

So, in this particular case the because angle is reduced there is shear strain now what about rotation? How to analyze rotation how to view rotation? Focus on the diagonal and the red diagonal, blue diagonal are just superimposing on each other which means there is no rotation.

So, the second case there is no normal strain just like the first case; first case there is no shear strain it just turned like this; the second case it has become like this closer to each other slightly very slightly because there is shear strain. First case just rotated, second case was symmetric remain symmetric see it is 1.05, 1.05 along the same diagonal. So, there is no rotation. Now going back to objective what did we say given a displacement field we should be able to tell about normal stain, shear stain strain and rotation that is what we are done talked about translation also.

Now, what we have done this qualitatively we are told by demonstration finding out coordinates, looking at the picture, looking at the figure etcetera we are going to do this quantitatively that is what our objective is. So, very nice example from the book Rayman Barnes illustrates our objective very clearly before proceeding to quantitative evaluation of the expression.

We are going to know find expressions for normal strain shear strain that is all, but these cases really illustrate what is that we are going to do? In fact, after deriving the relationship we will come back to this example and then say here we have set rotation; yes, but what is the value we will be able to quantify it.

Right hand inside we said shear strain is existing there is shear strain but later on after deriving the relationship you will be able to quantify it, find a x find a value for that.