Continuum Mechanics And Transport Phenomena
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Lecture – 06
Lagrangian and Eulerian Descriptions – Part I

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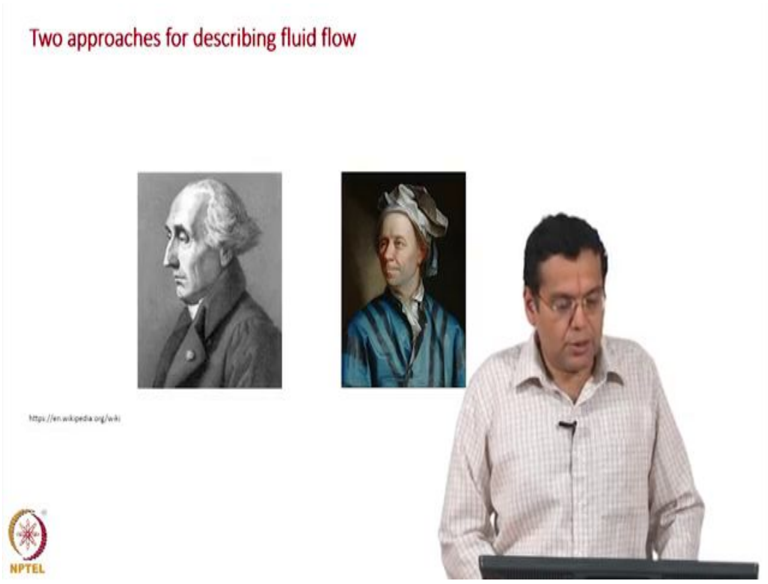

Now, move on to the next concept of explaining Two Approaches for describing Fluid Flow. There are two approaches for describing fluid flow one is called the Lagrangian description and other is Eulerian description. What we will do now is see what do we mean by Lagrangian description and what we mean by Eulerian description.

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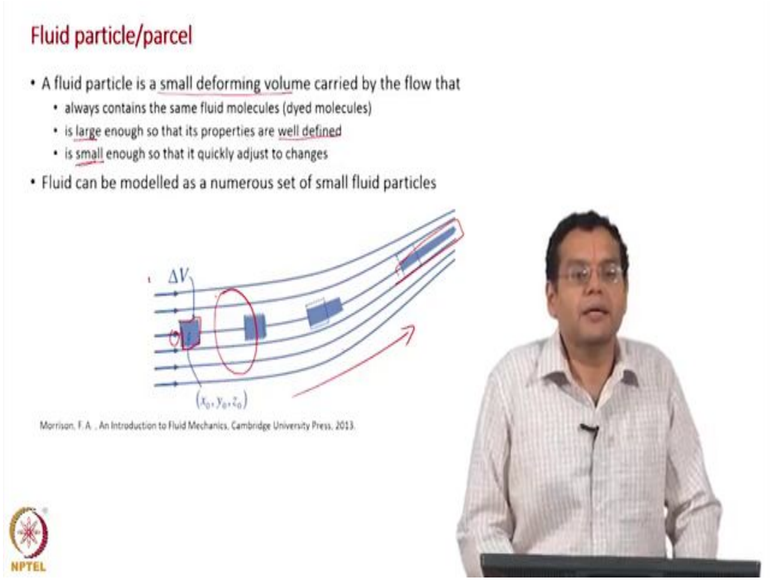

Now, before even discussing that we should know what do we mean by fluid particle, how do we imagine in a fluid particle. If we refer books frequently refer a term called fluid particle, fluid parcel. We will spend some time seeing how do we imagine a fluid particle how do we visualize a fluid particle.

So, let us look at the definition first. Says that a fluid particle is a small deforming volume carried by the flow that it has some characteristics. Let me list and then explain to you. Always contains the same fluid molecules and then is large enough, so that its properties are well defined and then it is small enough, so that it quickly adjust to changes.

Now, first characters always contain the same fluid molecules; how do you imagine. What is shown here (above refer slide) is a flow and then the cross section of the flow keeps changing along the flow direction. Now, to imagine a fluid particle you should imagine as if you dye a small portion of the fluid with some colored dye, and then see what is happening to the dyed fluid molecules. So, you are always keeping your attention on the molecules which are dyed to begin with. So, always contains the same fluid molecules. So, in our case the dyed molecules to begin with.

Now, it is large enough, so that it’s properties are well defined that is what we have seen sometimes back. Our volume cannot be extremely small, cannot be extremely large, so our volume should be small enough, but should contain enough number of molecules that is what we mean here by is large enough so that is properties are well defined and should be small enough, so that the spatial variation is considered that is what it says here as, so that quickly adjust to changes. So, this large and small or same as what we had discussed sometime back. When I say well defined enough number of molecules in our case more than 10^6 molecules and small enough, so that you consider. You cannot consider the particle which is so large, you lose out on the spatial variation, you consider a small fluid particle though that you can account for the variation within the flow direction in any other direction.

So, you can imagine this fluid particle as made up of large number of molecule fluid molecules. So, then you can imagine fluid to be model as numerous set of small fluid particles. Now, so initially we said a fluid molecules lot of fluid molecules put together form a fluid particle or parcel, lot of fluid particles or parcels made makeup a fluid. So, that is the hierarchy. Fluid molecules as enough number of fluid molecules make up a fluid particle or a parcel and then you can imagine the fluid to be made up of numerous fluid particles. So, in terms of imagination that is how we visualize.

Now, there is one small characteristic which I have not discussed that small deforming volume. We have discussed what small volume is deforming volume. What do we mean by deforming volume? And that is what shown here. This is the group of molecule which have been initially dyed, and then as it flows through the channel the cross section reduces, and the whatever happens to the flow also has happened to the fluid particle, it has got an elongated, and then the height has decreased, the length has increased.

So, whatever has happened to the fluid also happens the fluid particle. The fluid particle should mimic what is happening to the flow. if you put it to the other way, a fluid particle is small deforming volume carried by the flow. Why deforms? When the fluid undergoes deformation the fluid particle also undergoes deformation, it mimics what has happened to the fluid flow.

Always contained the same fluid molecules, and large enough, so its properties are well defined. Small enough, so that quickly adjust to changes, and fluid can be modelled as a numerous set of small fluid particles.

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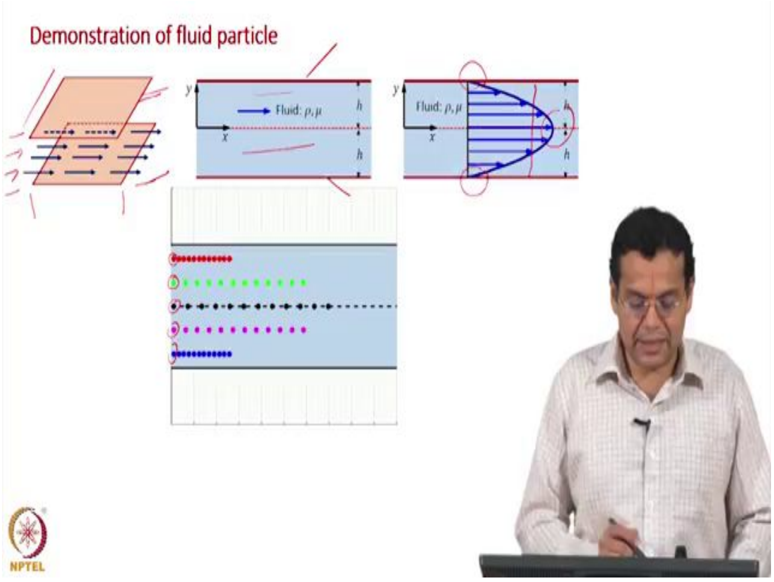

Now, what we will do is demonstrate this fluid particle. The example which are considered here is the flow between two parallel plates and this example which we will come across several times throughout this course. You have two plates, and the length and height between the two plates and the width are shown in the slide. You have flow taking place between these two plates in the right direction, and that is shown is here schematically (above refer slide). Let us see the front view of that. These two lines represent the plates and this region represents the region through which fluid flow takes place.

Now, if you measure the velocity or predict the velocity profile, after several lectures, several classes we will get this velocity variation along this height for the moment we see that the liquid clings to the upper surface and so its velocity 0. Similarly, it also clings to the lower surface the velocity 0, and shows a maximum velocity in the center. So, this is the velocity varies along the distance between the two plates.

Now, we will use this example to demonstrate what fluid particles are. Have a small demonstration here; let us run it (see in the video). We said fluid particle is one where we dye a enough number of molecules and keep tracking that, and that is what happens here. Let us say I take a group of a large number of molecules, large enough number of molecules, color it with red dye and keep tracking that fluid particle. and as I said, the fluid particle contains the same set of molecules which have identified to begin with. Similar, I make and consider another fluid particle here dye with green color and keep tracking what is happening to that fluid particle. Similarly, I have shown several fluid particles.

What is that I am trying to show here (see the video)? We are identifying a group of molecules with some identification. For easy identification we can just color it by imagination or color it in this way, and then the molecules are dyed are being followed over the a region of interest and here we have seen several fluid particles, a color with different dyes and then they always contain the same fluid molecules that is of importance and then we are a tracking them.

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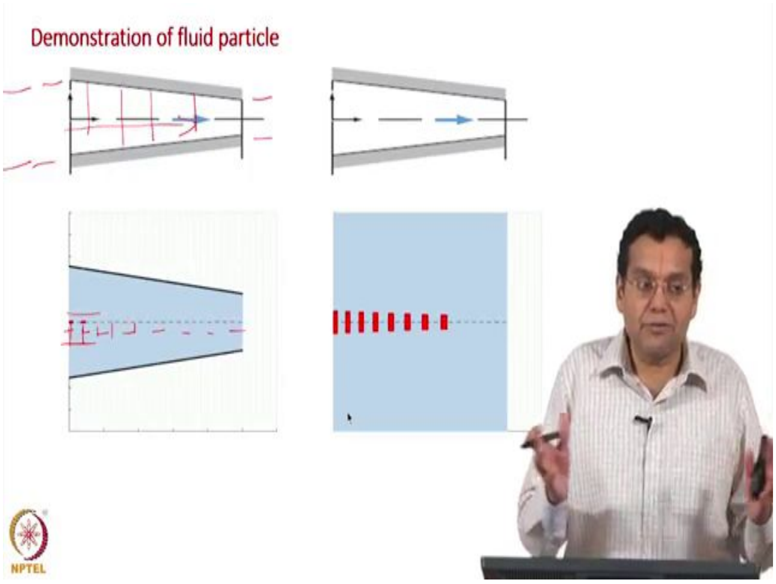

Now, we said another characteristic of fluid particle is that which I have seen the earlier figure is that whatever happens to the flow also happens the fluid particle. It should mimic what is happened into the fluid flow. This demonstration show you that the example I chosen here is the flow between two plates, but in this case the distance between the plate decreases, which we call as a converging nozzle, the reason for choosing this example is that because of the reduction in the area reduction in the height for the fluid flow, the velocity increases along the length of the fluid flow direction. So, in the earlier case there are two parallel plates and the distance was constant throughout.

In this case once again we have two plates, but in this case the distance between them reduces, the area for flow over which a flow happens reduces in this direction, so there is increase in velocity along the flow direction. That is the reason we have chosen this particular example. Once again this is the example will consider the later as well.

So, now, let us run this simulation (see the video to understand better). Let us see what happens. What I have shown here is once again large number of molecules are being dyed with red and then I am tracking that fluid particle along the length of this channel. Now, this is what is seen earlier also. We identified a group of molecules and tracked them along the region of interest. Now, if you observe this carefully, the locations of the particle shown here are at constant times intervals, for the same time instant the distance traveled is larger as it goes towards the end of the channel, along the length of the channel because of the increase in velocity it travels larger distances for the same time interval.

Now, if you zoom this region alone (see in above referred slide), what we have shown you on the right hand side is that just zoomed portion of this. Now, if you see, if you run this video what you see here is that there is a change in the dimensions of the fluid particle, the length of the particle keeps increasing as it travels along the channel, the height of the fluid particles keeps decreasing as it travels along the length of the channel.

The increase in length is because of the increase in velocity along the channel direction and correspondingly there is a decrease in height. So, this is what we meant by saying that fluid particle, whatever happens to the flow also happens to the fluid particle. So, now, these two simulations kind of summarizes in terms of video the sentences which are described earlier. What is that we said? A fluid particle is a small deforming volume that is what we are seeing here.

It is small so that we capture what is happening in the fluid domain. It has enough number of molecules, there are no molecular uncertainties. And of course, small as we have seen, so that we can consider many fluid particles and describe what is happening in the entire region and then by considering several fluid particles, we can describe the entire fluid in terms of fluid particles.

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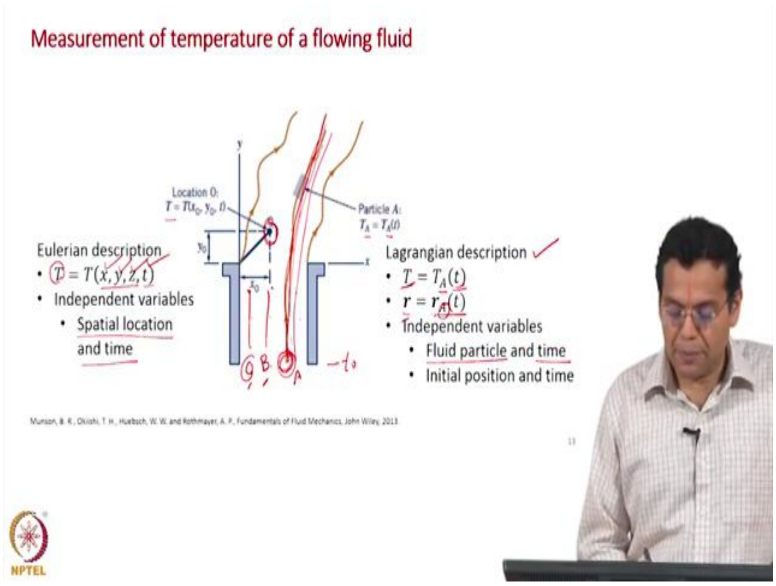

Now, we can distinguish between the two approaches a fluid flow description namely the Eulerian description and the Lagrangian description. And that is what we are going to do now.

Now, what is shown here is a chimney (see in above referred slide), through which there is a flow of flue gas. Now, our objective is to describe the temperature in this region. Our objective is the same, the way in which we are going to do this in is in two different ways. Our objective is to describe the temperature in this region somewhere near let us say top of a chimney.

Now, how do we measure the temperature here? Two different ways. First one, I will take a temperature sensor and then place at a particular location. Let us say because of flue gas the temperature can fluctuate can vary as a function of time. So, at this particular location I measure the temperature as a function of time. Likewise, I keep the temperature sensor let us say thermocouple or simple thermometer amount be applicable for simple imagination understanding can imagine a thermometer. Keep at different locations in this region and then measure the temperature at different spatial locations. So, the position of the measurement is fixed and then you are measuring the temperature as a function of spatial location and time as well.

Now, the temperature is the dependent variable here, this is a variable of interest to us. Now, what are the independent variables? The spatial locations x, y, z, and time are the independent variables. So, this method of measurement is called the Eulerian approach of measurement or Eulerian description. You are describing the temperature in this region by making measurements at one spatial location and of course, at different spatial locations to get the entire temperature, entire region and that is called Eulerian description. We represent the dependent variable temperature in terms of the spatial locations x, y, z and the time as well. So, you should keep note, make note that the independent variables are the spatial location and time.

Now, let us look at another method of describing the temperature here, in this region and that is why we discussed about fluid particle. For this discussion the concept of a fluid particle is not required, almost at least straight away it is not required and this is what you would naturally do as well. If I asked you to measure the temperature in this region, you will ask me for a sensor, you will keep at a particular location, you will measure, similarly you will place a different locations and then measure the temperature and describe the temperature here. Now, what is the alternative method? Which requires the imagination of a particle.

Let us consider a particle here and let us say the fluid particle based on whatever discussion we had so far and that fluid particle passes through this region. Now, just imagine once again that you attach the temperature probe to that particular particle. So, what happens as the particle flows through this region it is measuring the temperature through the region of interest as well.

So, in the second case we consider fluid particle and then we imagine a temperature sensor attached to that. So, as the fluid particle flows through this region it measures the temperature along the fluid particle.

Now, this method of describing the temperatures in this region is called the Lagrangian description. Why is that? Because we are following the fluid particle and describing the temperature for a fluid particle. In this particular case the fluid particle is denoted as capital A. So, if I track this particular path I will get the temperature of particle A as a function of time, $T_A(t)$. Suppose, if I want another particle let us say I have a particle B here, particle C here, remember in the earlier case where different particles dyed a differently to different colors. Similarly, you have particle A, particle B, and particle C. So, now, if you track all these particles as a function of time then you will be able to describe the temperature in this region.

So, in the second case you track different fluid particles and they sense the temperature as they travel along the path and you will be able to describe temperatures in this particular region that is the Lagrangian description. Let us see how do we represent that. T, now, this is for a particular particle called A. So, $T_A(t)$. Because we are following the fluid particle; obviously, we will also get the position of the fluid particle as a function of time.

So, I call that as r vector and that r vector is for a particular particle A and as a function of time, $r_A(t)$. So, we get two kinds of information. Number one, first the variable which are interested to measure, that temperature as a function of time for that particular particle that is why we say $T_A(t)$ and because we are following the particle; obviously, we get the position of the particle as a function of time which I call as $r_A(t)$.

Now, what are the independent variables in this case? Dependent variable remember is still temperature, additionally you get the position of the particle also. So, those two are the dependent variables. What are the independent variables? For example, if I describe in terms of A and time then the fluid particle A B C D, they are the independent variables and time.

Now, it is little inconvenient to say particles A B C as independent variables, you cannot even quantify how many particles. So, you need another way of specifying independent variables. So, what do we say, instead of saying particle A B C, I will tell that at a particular time let us say some T_0 or 0 a position occupied by the particle. So, I specify a particle instead of numbering as A B C or lettering as A B C. I will tell that I have particle A which was at this position at time t equal to T_0 or time $t = 0$. I have particle which was at this position, so which I call as r_0. So, instead of A, B, C, D, I replace it with r_0 which is the initial position and time.

So, as eventually the independent variables in a Lagrangian approach are the fluid particle and time, but fluid particle is inconvenient to be described, so we use initial position and time as the independent variables in the Lagrangian approach. So, it should be kept very clearly in mind that the dependent variable is same, the independent variables are different. In the Eulerian description the spatial coordinates whichever usual x y z and time are the independent variables. Time is anyways common to both of them. In the case of Lagrangian approach, because you are following the fluid particle those fluid particles are the in independent variables, but because it is inconvenient identify particles by numbers or letters, we identify them by position at a particular time let us say initial time, so the independent variable becomes the initial position and time.

So, what we have seen here is take an example of flue gas flowing through a chimney. We have described the temperature in this region using two different approaches Eulerian description and Lagrangian description. One is at a particular location, other is fluid following a fluid particle.

One other way of explains Eulerian description is that you are at that particular point if you want to attach meaning of fluid particle to that it sensors different fluid particles, is that right. So, in this case we are following one fluid particle, in the case of Eulerian because your sensor was placed at one particular location it senses the temperature of different fluid particles which cross through that point. So, that should be kept in mind. So, one is in terms of independent variables other in terms of the particles which are being considered. Lagrangian is one particle, Eulerian is several fluid particles which cross that particular portion of interest.

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Having discussed this in detail we will extend this to two other examples. This photograph shows a very busy road in Chennai and this road is just outside our institute IIT, Madras. One of the busiest roads I would say and a lot of traffic here. And our interest is not the traffic right now, our interest is the air pollution caused by this traffic here. And let us say we are interested in measuring that. Air pollution could be in CO_2 concentration particulate matter, any other impurities as well.

Now, analogous to the description what we had earlier. I can describe the pollutant level in this region, two different ways. What is the first way, I would keep sensors at several spatial locations in the road and then make them measure the pollutant constant let us take CO_2, I measure the CO_2 concentrate, I put CO_2 sensors all along the road and I take readings of CO_2 concentration as a function of time, then I will be able to describe the CO_2 concentration in the entire region which is our Eulerian approach. At a fixed location the CO_2 sensor measures the concentration as a function of time put several sensors and then you can measure.

Now, what is the Lagrangian method? We follow a fluid particle. Now, our fluid particles are very large here, they are nothing, but our vehicles, so attach one sensor to all of them. As they travel they will give us the pollutant concentration. Not so precise in telling fluid particle, but easy to see our particle here all our vehicles are a particle which is the Lagrangian description. So, first fix sensors measure the concentration, second fix sensors to this so called fluid particles vehicles and measure the CO_2 concentration in that region.

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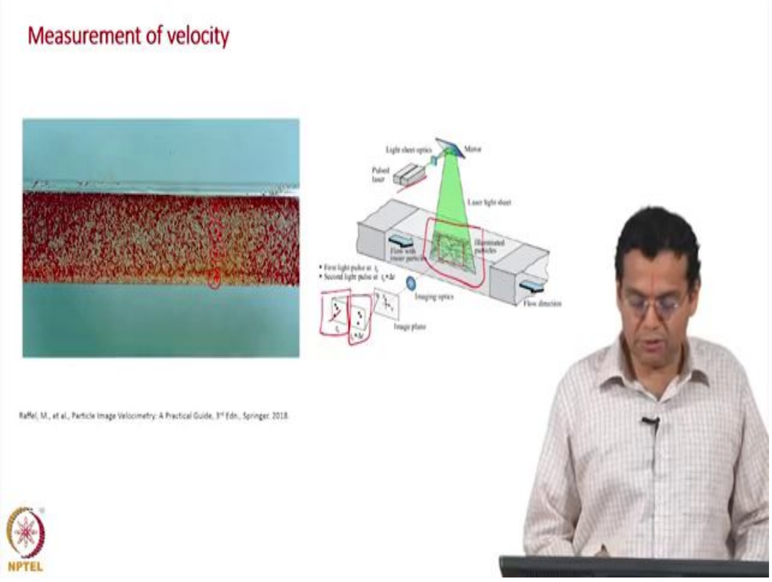

Now, third example. And you would be wondering why temperature first concentration second, then only velocity. In our usual flow of topics its velocity first, and then temperature second and then concentration. For obvious reasons temperature most easily imaginable variable for all of us, easily we can sense a temperature, temperature of course, outside is hot, it is cold inside and of course, heat transfer is the most favorite subject for all the students also. So, that way also and easy to explain in terms of temperature. We start with velocity it is become little difficult to imagine that is why temperature. Similarly, concentration also, you can easily have a quick imagination for concentration measurement, and that is why velocity comes last.

So, what is that we are interested in? Very nice example. You have flow through a pipe and of or any region of interest and I want to describe the velocity in that particular region. Now, once again two different ways, first is to use a instrument to measure the velocity to different spatial location. So, I will put a sensor here, I will put a sensors at different locations. We will later on learn in a separate fluid mechanics course that these kind of measurements can be done using pitot tubes, pitot tubes instrument to measure the local velocity across this region. So, you use a velocity measuring device. So, place that at several locations and measure the velocity. So, this is our Eulerian description.

Alternatively, how do you measure the velocity is in the Lagrangian approach? Little more explanation is required for that. What we do is we seed the flow with what are called neutrally buoyant solid particles. First let me say what is seeding. Seeding is just adding along with the fluid, add these neutrally buoyant solid particles. Now, what are these neutrally buoyant solid particles. The solid particles they have two characteristics, number one very small size or (Refer Time: 25:13) say 10 microns, 20 microns, extremely small size or even of the nanometer range, they could be even 10 nanometers, 20 nanometers range. So, they have two characteristic as I said, one is very small let us say 10 nanometer, 20 nanometer size; second is that as the name indicates they are neutrally buoyant.

What does a neutrally buoyant mean? Their densities are almost same as that of the fluid which we are studying. For example, if we are studying velocity of water, the density these of these solid particles should be around let us say 999 plus or 1000, around 1000, around 999 almost close. So, why do we have these characteristics? We want the solid particles to follow the fluid motion.

So, there should not be any difference at all between the fluid motion and the solid motion. So, if the solid particle is almost like a fluid particle what does it mean? Very small size and then density is also same as fluid. We cannot have exactly like that, so our size is something of are of nanometers and then densities almost close to water. These are called neutrally buoyant solid particles.

So, we seed them to the flow, and then we pass laser sheet on that, that is what is shown here you have a source of laser. And using some laser optics you pass a laser light sheet on the region of interest and then using a high speed camera you take two successive images at very short time intervals. Just like taking a snapshot something like our burst. So, you take the image using the camera, a high speed cameras. Let us say you take two measures differing a few microseconds or milliseconds difference. So, what is that we have done? We have taken the Lagrangian position of the particles. So, using these two frames you will be able to get the velocity of the particles, Lagrangian velocity, then you can of course, convert to Eulerian etcetera.

So, Lagrangian measurement of particles involves seeding of the fluid, and then passing a laser light sheet over it, and then using a high speed camera to capture a image, and then images to be taken a quick time instants, very small time difference and using the position of the images you can find out the Lagrangian velocity of the particles, and the velocity in the region.