Continuum Mechanics And Transport Phenomena

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Lecture - 64
Normal and Shear Strain Rate

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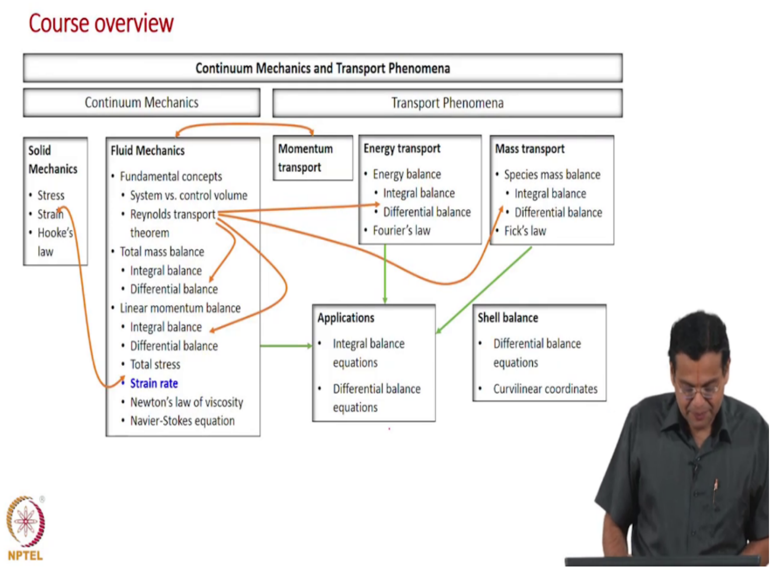

We have derived the linear momentum balance and then, we have proceeding towards the Navier Stokes equation. To understand the surface forces on the right hand side, we going to solid mechanics understood stress came back to fluid mechanics understood total stress and then we had completed the differential linear momentum balance. And then we realize that the viscous stress tensor components are unknowns they have to be related to velocity gradients.

To understand velocity gradients, we need to understand displacement gradients. So, we went to solid mechanics discussed about strain and then now we are coming back to fluid mechanics to discuss about strain rate that is where we are.

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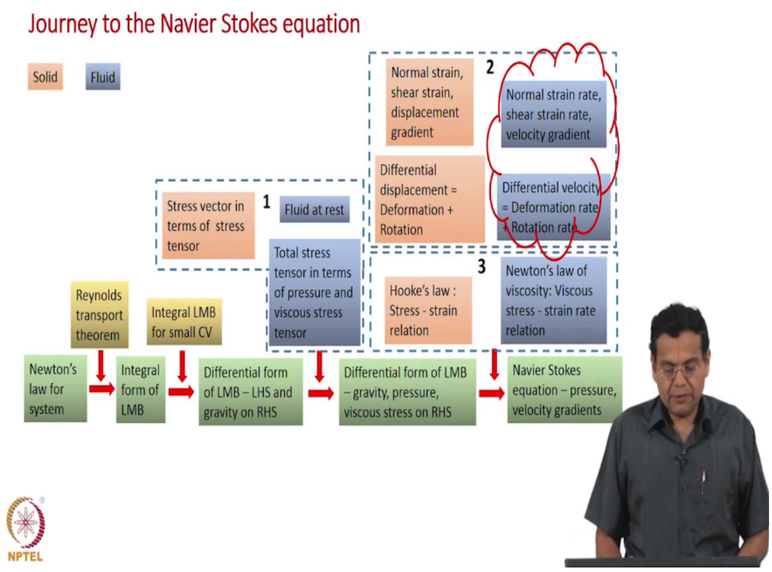

In terms of our journey to the Navier Stokes, we derived the differential form of the linear momentum balance with the viscous stresses in the right hand side. They need to be related to the velocity gradients. To understand velocity gradients we first understood displacement gradient by taking a diversion to solid mechanics and then we discussed strain, displacement gradient, strain tensor, rotation tensor etcetera. Now, we are going to analogously discuss these two blocks for fluid mechanics, the highlighted two blocks are going to be discussed now.

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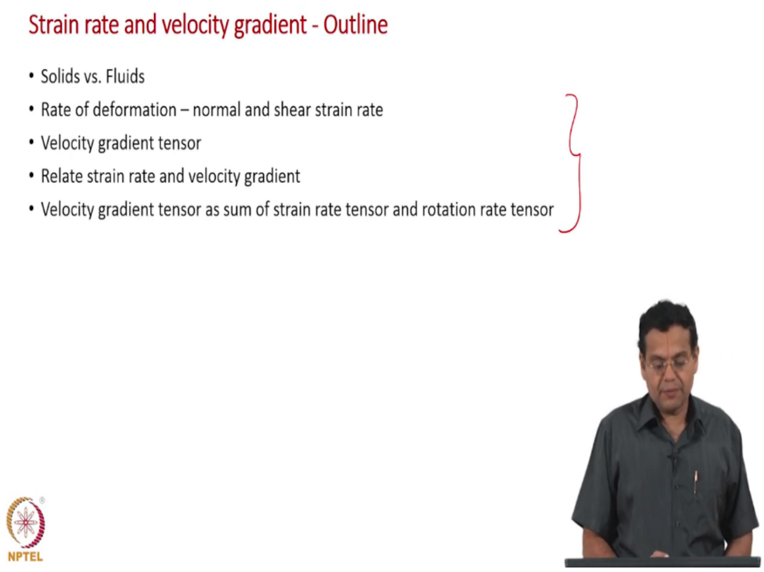

We have discussed difference between solids and fluids after our first visit to solid mechanics. When we came back to fluid mechanics after the first visit to solid mechanics, we discuss difference between solids and fluids. So, once again now, we are coming back to fluid mechanics. So, we will discuss the difference between solids and fluids it will be a revision, but we will highlight one particular aspect there. The topics which are going to follow are analogous to what we discussed under strain when we discussed solid mechanics.

- There we discussed deformation, normal and shear strain, here we are going to discuss rate of deformation, normal and shear strain rate.
- There we discussed displacement gradient tensor; here we are going to discuss velocity gradient tensor,
- Then we related the strain and displacement gradient; here we are going to relate strain rate and velocity gradient and
- There we expressed the displacement gradient tensor as sum of strain tensor and rotation tensor. Analogously here, we are going to express the velocity gradient tensor as sum of strain rate tensor and rotation rate tensor.

So, these four topics are almost analogous to what we discussed for strain in solid mechanics.

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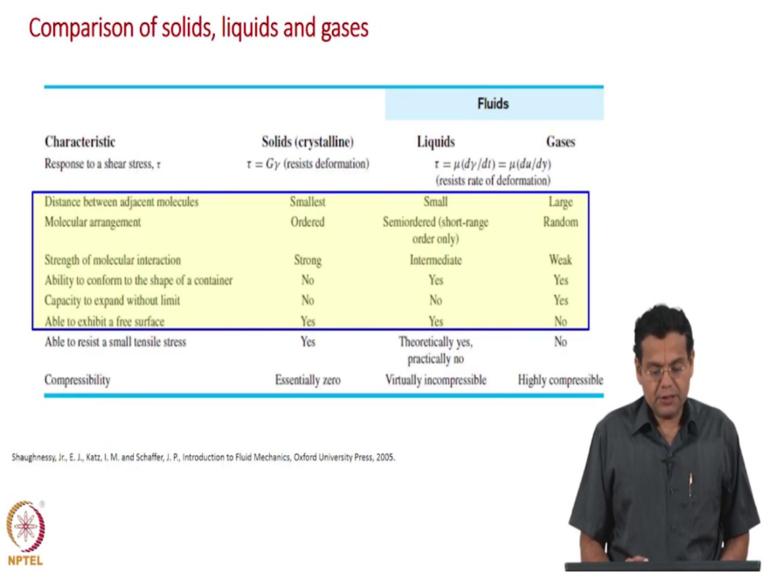

Let us start with the comparison of solids and fluids; we discussed the difference between solids and fluids under two categories. First set of characteristics for which we know the difference between solids and fluids that is what is shown here. We discussed it in terms of distance between adjacent molecules, molecular arrangement, strength of molecular interaction, ability to conform to shape of container, capacity to expand without limit and able to exhibit a free surface.

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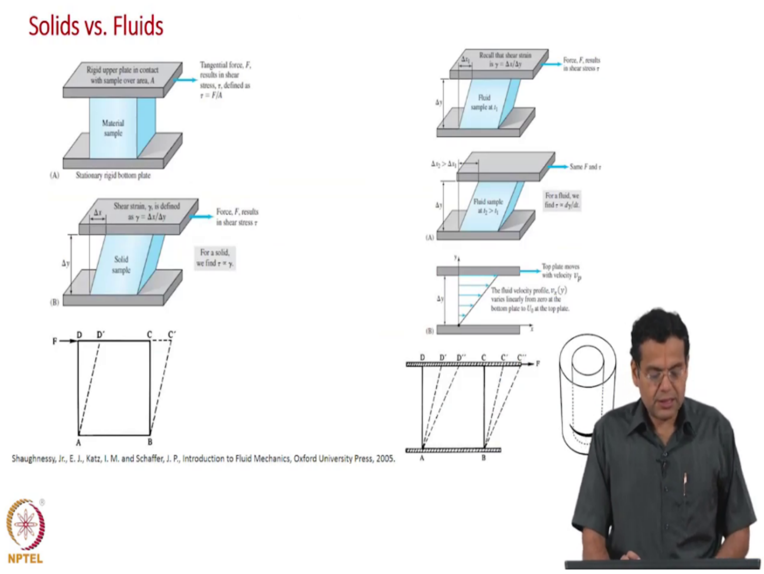

So, these differences all known to us, then we went on to discuss the difference between solids and fluids which is from a mechanics point of view and that is this slide about. So, when we apply a tangential force to a solid, it starts deforming and deform until the internal stresses balance the external apply tangential force and then it stops deforming and that is what is shown here, an undeformed state and a deformed state.

So, it goes from one equilibrium state to another equilibrium state and then it stops deforming and so, solids can resist shear stress under static condition and that is what is shown here D C initial state D‘ C’, the final state. And if you remove the tangential force, the solid goes back to its original state.

Now, if you apply a tangential force for a fluid as shown in the right hand side, it starts deforming, but now it continues to deform as long as you apply the tangential force and that is what is shown here D C are the points at some time t. As long as you apply, D moves to D’ or D’’ C moves to C’ or C’’. So, it continuous to deform as long as you apply the tangential force and now if you take out the tangential force, it just retains the final shape as such ok. So, that is why we say fluids flow.

So, now, what is the implication? because of this fluids cannot resist shear stress under static condition. Even if apply a very small tangential force, they start moving. So, fluids cannot resist shear stress; however, small it is under static condition, they immediately start flowing. Now, we also discussed the difference between solids and fluids using this coaxial cylinder arrangement.

If you have a solid between the two cylinders and then try to rotate the inner cylinder, then the force required to rotate depends on how far the solid is deformed from an initial state. So, there is some initial state and more you deform from the initial state, more the force is required that is what happen in the case of solids. But in the case of fluid what happens? If you fill a fluid between the two cylinder; now the force required to rotate in the cylinder will now depend on how fast you rotate the cylinder so, depends on the rate of deformation.

So, in the first case in the case of solids it depended on an how far it is deform which means depends on deformation, the force depends on deformation. In the case of fluids it depends on how fast you rotate the cylinder and other words the force required to rotate the in the cylinder depends on the rate of deformation that is the key difference between solids and fluids which are going to take as a basis for further discussion.

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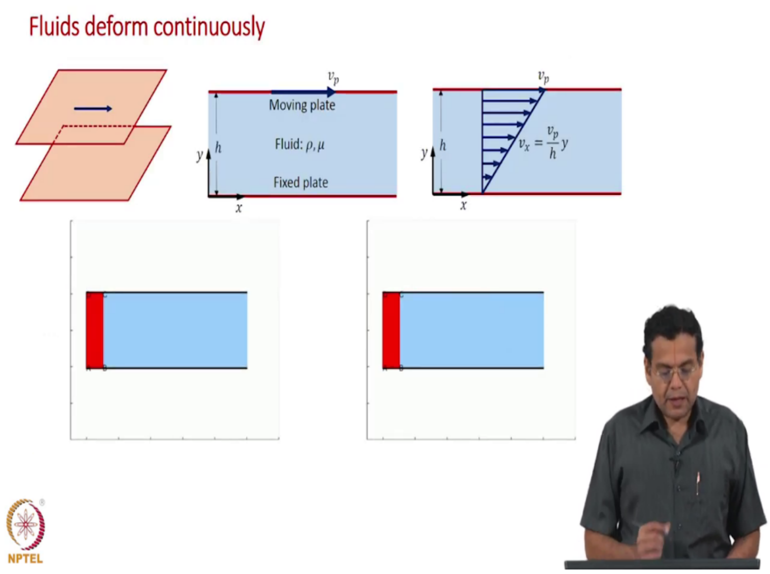

We also shown animation to show that fluids deform continuously, there is a flow between two parallel plates; the bottom plate is fixed the top plate is set to motion at a constant velocity v_p and the velocity profile is linear we have come across the several times. We identify a region of fluid and then if you focus on that region of fluid, we can easily see that it continuously deforms as long as the plate is moving; this region continuously deforms and that is what we say that fluids deform continuously. So, the animation shows that this region continuously deforms of course, it replace back, but otherwise it deforms continuously.

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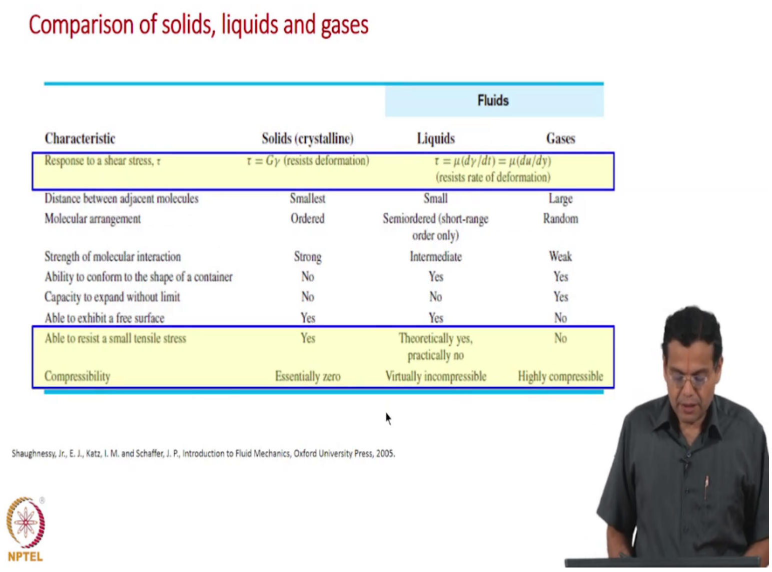

Now, this table highlights the differences from a mechanics point of view which is of present interest to us what are the differences response to shear stress. The key difference is that solids resist deformation, fluids resist rate of deformation as you have seen the cylinder example is the best example to understand this. On one case force depends on deformation; other case the force depends on rate of deformation and we have already discuss this earlier ability to resist small tensile stress and compressibility.

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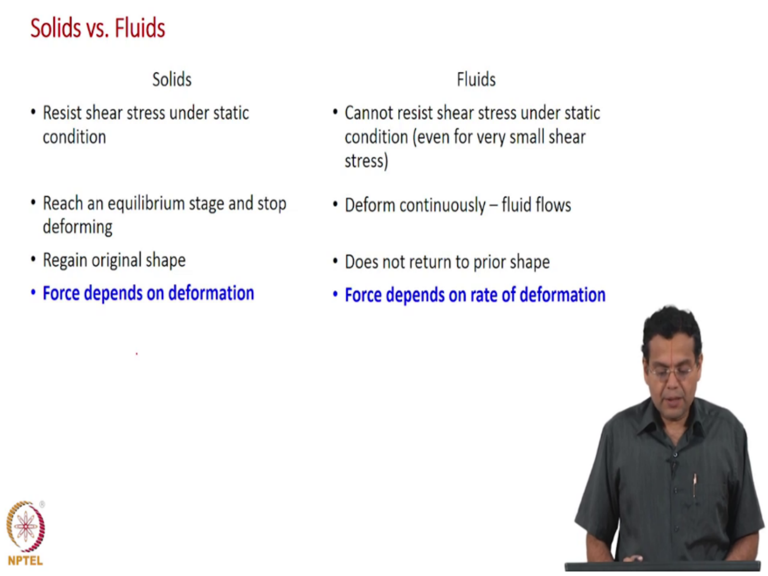

So, now let us summarize all the differences between solids and fluids.

- Solids resists shear stress under static condition, fluids cannot resist shear stress under static condition even for a very very small shear stress. They cannot resist shear stress under static condition.
- Solids reach an equilibrium stage and stop deforming, fluids deform continuously and so, we say fluid flows.
- When the tangential force is removed, solids regain original shape. Fluids do not return to their prior shape.
- Now, the major difference which is of importance to us for the present discussion is that the case of solids force depends on deformation, with the case of fluids force depends on rate of deformation.

When we discussed stress for a solids and then when we came back to fluids to discuss about total stress we took lead from the first difference and then we said fluids under static condition have some stress when they flow their additional stress and so on.

Now, the difference which is of relevance to us is the last difference and that is why this has been highlighted in the earlier difference the first difference was highlighted now, the last difference highlighted and we are going to take lead from this difference. It says force depends on deformation for solids force depends on rate of deformation for fluids.

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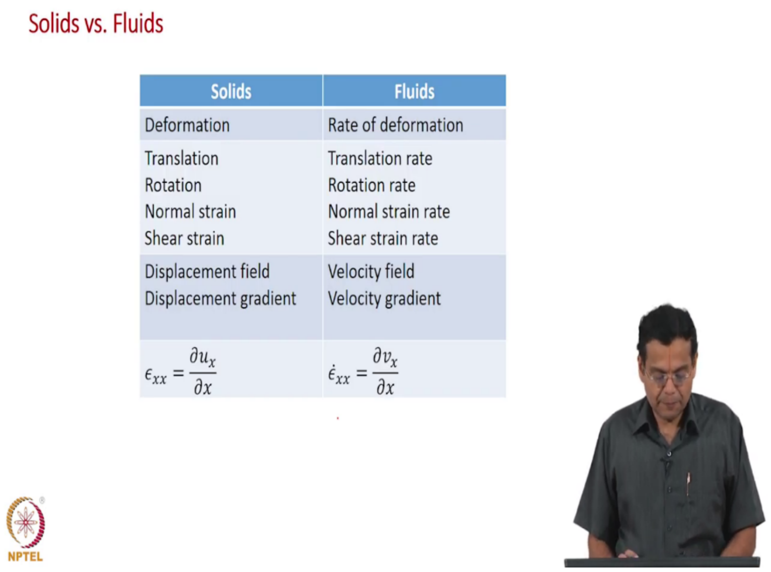

Now, based on this difference, we can intuitively write this table.

Solids	Fluids
Deformation	Rate of deformation
Translation, Rotation, Normal strain, Shear strain	Translation rate, Rotation rate, Normal strain rate, Shear strain rate
Displacement field, Displacement gradient	Velocity field, Velocity gradient
$\epsilon_{xx} = \frac{\partial u_x}{\partial x}$	$\dot{\epsilon}_{xx} = \frac{\partial v_x}{\partial x}$

Now, we are going to relate the normal strain rate to the velocity gradient. So, of course, we are going discuss further slides in detail about all this, but now based on the fact that force depends on deformation for solids and force depends on rate of deformation for fluids, we can come up with this table.

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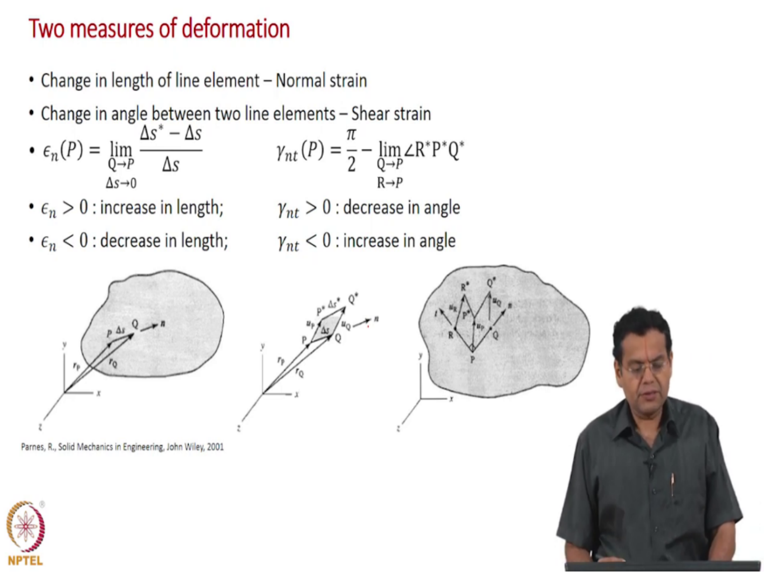

Now, the way in which we are going to discuss the order of topics, they are all going to be analogous to what we discussed for strain under solid mechanics. So, some will be recall slides, then other will be new slides for fluids. So, this is the recall slide where we defined normal strain and then shear strain.

$$\epsilon_n(P) = \frac{\Delta S^* - \Delta S}{\Delta S}$$

$$\epsilon_n > 0: \ \ increase\ in\ length$$

$$\epsilon_n < 0: decrease\ in\ length$$

For normal strain, it is change in length by original length at a point we take a small line element along direction n and the change in length by original length gives a normal strain.

$$\gamma_{nt}(P) = \frac{\pi}{2} - < R^* P^* Q^*$$

$$\gamma_{nt} > 0: \ \ decrease\ in\ angle$$

$$\gamma_{nt} < 0: increase\ in\ angle$$

For the case of shear strain, we take two perpendicular line elements and then look at the change in angle between the initial state.

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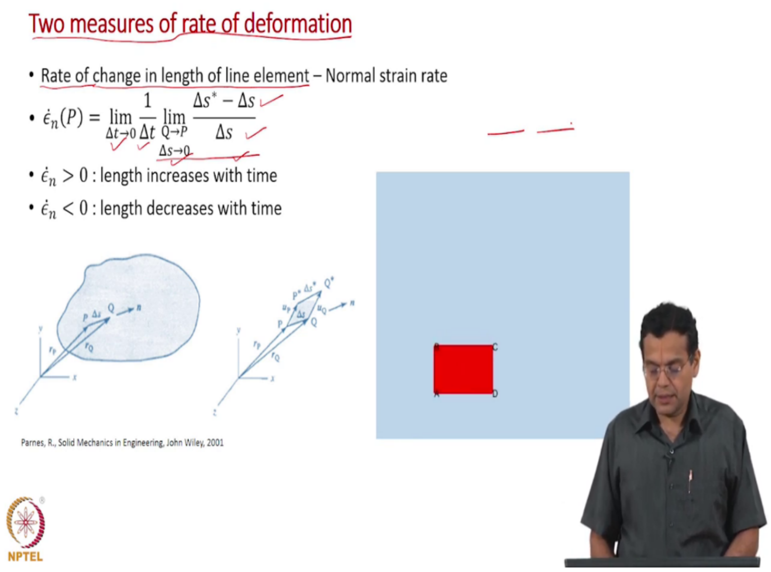

Now, we will analogously introduce two measures of rate of deformation earlier it was two measures of deformation. When we discuss solid mechanics we said there could be change in length, change in angle. They are quantified in terms of normal strain, shear strain. Similarly here for the case of fluids, we are going to discuss normal strain rate and shear strain rate and this slide is for normal strain rate.

So, how do we define normal strain rate? Earlier we have seen strain to be defined as change in length of line element, now it is rate of change in length of line element. Of course, normalized by the length. So, rate of change in length of line element is a normal strain rate, you express this formally as

$$\dot{\epsilon}_n(P) = \frac{\Delta S^* - \Delta S}{\Delta S}$$

$$\dot{\epsilon}_n > 0: \ increase\ in\ length$$

$$\dot{\epsilon}_n < 0: decrease\ in\ length$$

Now, the last part of the definition is same as normal strain which tells about change in length by original length. Now we need to express in terms of rate of change of length. So, numerator tells change in length divide by Δt gives rate of change in length of course, per original length. And now we have two limits the first limit is as we have seen already. At a point we consider a small line element that is why $\Delta s \to 0$ and now we are divided by Δt and the rate is defined as the limit of $\Delta t \to 0$.

So, that is why you have another limit here which tells $\Delta t \to 0$. So, one limit makes the definition of normal strain rate where at a point that is why $\Delta s \to 0$, other limit makes the definition of normal strain rate instantaneous.

This animation shows rectangle element. Let us see what happens. If you focus on the line element A D, its length keeps increasing with respect to time; if you focus on the line element A B, its length keeps decreasing with time. So, this is what we mean by normal strain rate. In one case of course, it is greater than 0 which means the length increases with time that is a that is happening for line element along x axis. If you focus on line element along y axis $\dot{\epsilon}_n$ its length decreases with time and hence the normal strain rate is less than 0. So, this is a simple animation.

Earlier when we were discussing solid mechanics, we were discussing yeah line element, initial length and then final length, but now we are interested in what is the rate of change of that length; of course, normalize, but normalize by the instantaneous length. So, same significance, but now it is respect with respect to time and that is what you see here the length of line segment AD keeps increasing, length of line segment AB keeps decreasing.

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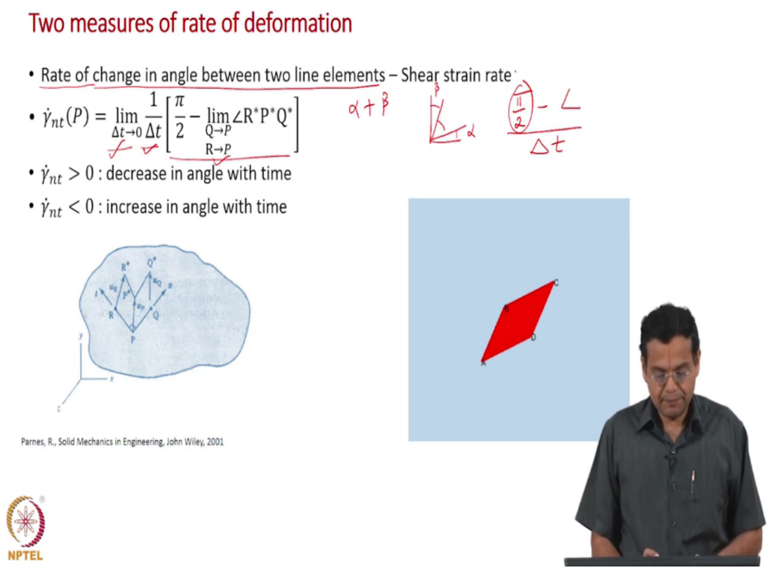

So, now let us define shear strain rate ok. How do you define? Earlier we said change in angle between two line elements; now it is rate of change in angle between two line elements. So, let us see how do you formally define the shear strain rate.

$$\dot{\gamma}_{nt}(P) = \frac{1}{\Delta t}\ [\frac{\pi}{2} - < \overset{*}{R}\overset{*}{P}\overset{*}{Q}]$$

$$\dot{\gamma}_{nt} > 0: \ decrease\ in\ angle$$

$$\dot{\gamma}_{nt} < 0: increase\ in\ angle$$

So, the terms within bracket are the same what we have seen earlier for the definition of shear strain and now we are interested in the rate of change of that. So, divide by Δt and then $\Delta t \rightarrow 0$ and once again, we have two limits here. The first limit tells that at a point we consider two line elements which are perpendicular to each other. So, that makes the definition of the shear strain rate valid at a particular point and we have another limit $\Delta t \rightarrow 0$. So, that makes the definition of shear strain rate valid at every instant of time.

In the case of solid mechanics, we need a definition which is valid at every point, but now we have a definition which is valid at every point and every instant of time. And let us look at this animation where the initially the angle BAD was ninety degrees and now the sides the angle between AB and AD decreases, they come closer to each other and the rate at which this angle changes is what we mean by shear strain rate.

Like to mention that the way in which you understand shear strain rate is that we consider two line elements. Let us say sometime t and some other time t + Δt. Let us say the two line elements become like this, they come closer to each other and now the rate of change of angle is the shear strain rate.

And the rate at which this change happens in the limit of $\Delta t \rightarrow 0$ is the shear strain rate. Then rate at which $\alpha + \beta$ changes with time is also shear strain rate either way of looking at it and remember always it is the change in angle per unit time and the angle at time t is always $\frac{\pi}{2}$. So, in this case the shear strain rate strictly applies only to the initial time instant not as it progresses.

Of course, if the shear strain rate is greater than 0 which means that the two lines have come closer towards each other; there is a decrease in angle between them with respect to time.

And if the shear strain rate is less than 0, then the two lines have gone away from each other which means there is increase in angle between them with respect to time. Whatever we discussed for solids under static condition, now we are discussing as a rate.