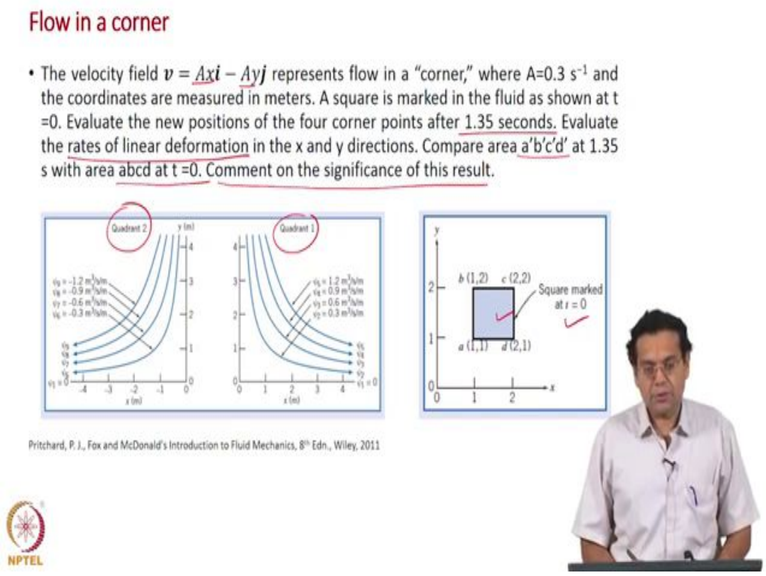
Continuum Mechanics And Transport Phenomena

Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 69
Strain Rate : Example 2

Example: (Refer Slide Time: 00:13)



Let us take another example. So, now, let us take an example, exactly which has the other component namely the normal strain rate, but does not have shear strain rate and rotation, that is a use of that is objective of this example. So, velocity field is given by

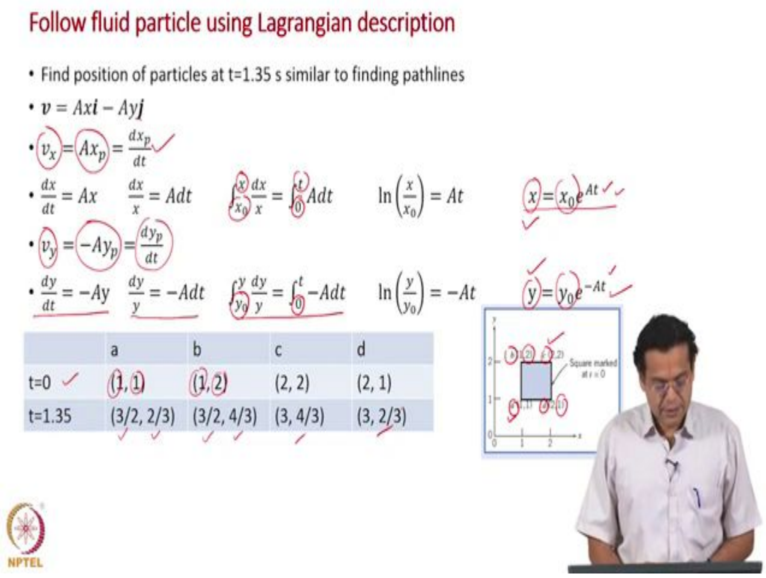
$$v = Axi - Ayj$$

Represents flow in a corner that is what we shown here in quadrant 1 and quadrant 2. Let us take quadrant 1 for easy understanding and A = 0.3 second inverse and the coordinates are measured in metres.

Now, just like you marked a two perpendicular lines in the earlier example, in this case we will mark a region a square is marked in the fluid as shown at t = 0. Evaluate the new positions of the four corner points, something like what we did in the earlier case after 1.35 seconds after some time interval.

Evaluate the rates of linear deformation in the x and y directions rates of linear defamation is nothing is another way of saying rate of normal strain or normal strain rate in the x and y directions. And then compare area a'b'c'd' at 1.35 seconds. And then we have to compare with that area with area a, b, c, d at t = 0; whether it will change or not we will see that. And then comment on the significance of this result. So, you have flow over a corner and then, we are in this region you are let us say marked some red colour dye this is square region and we are tracking that.

Solution: (Refer Slide Time: 02:49)



Now, tracking a particular point is something similar to what we have done earlier. If you track a particle is equivalent to finding the path line of that particular particle. So, find position of particles. So, we have four particles a, b, c and d; we will have to evaluate that position after some time interval which is nothing, but finding the path line. If you want to show the progress of these points then we draw the path line. Or if you want the only endpoint in the end point of the path line will give you the final position. So, we will find out the path line so that at any time t we will know the position of a, b, c and d.

So, let us find a general expression. So, we will have to recall what we discussed when we did examples on path lines. When we discussed the theory for path lines, so the velocity field

$$v = Axi - Ayj$$

We used the velocity of the fluid to find the path line; velocity of the fluid that is Eulerian description which is equal to the velocity of the particle given by rate of its displacement in the present case in the x direction.

$$v_x = Ax_p = \frac{dx_p}{dt}$$

And this we have used when we derived substantial derivative when we discussed path line same cost of concept is being used here. The Eulerian velocity field is equal to velocity of the particle which happens to at that which happens to be at that particular location, which is given by the rate of change of its displacement the present case x direction.

So, now, because our coordinates are x I am going back to x, but that is the physical principle ok. So,

$$\frac{dx}{dt} = Ax \rightarrow \frac{dx}{x} = Adt \rightarrow \int_{x_o}^{x} \frac{dx}{x} = \int_{0}^{t} Adt \rightarrow \ln ln \left(\frac{x}{x_0}\right) = At \rightarrow x = x_0 e^{At}$$

Now, let us do the same exercise for the y direction, we will equate the y component of velocity to the rate of change of y displacement of the particle.

$$v_y = - Ay_p = \frac{dy_p}{dt}$$

And,

$$\frac{dy}{dt} = - Ay \rightarrow \frac{dy}{y} = - Adt \rightarrow \int_{y_o}^{y} \frac{dy}{y} = \int_{0}^{t} - Adt \rightarrow \ln ln \left(\frac{y}{y_0}\right) = - At \rightarrow y = y_0 e^{-At}$$

So, integration rearrangement gives you this equation relating the y coordinate of the particle as a function of time. So, at any time t you can calculate the x co ordinate and the y coordinate and in this particular case we are asked to calculate the coordinates. If you want to track this region as you progress then you can substitute for different values of time and get the x and y coordinates. This example what we are asked to do is to find the coordinates at t equal to 1.35 seconds.

	a	b	c	d
t = 0	(1,1)	(1,2)	(2,2)	(2,1)

t = 1.35	(3/2, 2/3)	(3/2, 4/3)	(3, 4/3)	(3, 2/3)

So, in this table the first row shows the coordinates at time t = 0. Second row shows the coordinates how do I obtain I just substitute at t = 1.35.

So, the second row gives x and y coordinate of a, b, c, d after time interval of 1.35 seconds. Now, see the concept of path line were discussed sometime back and we have now used it and interpreting in a different way. What we have actually calculated is only path line but now, we are going to analyse in terms of what happens to this region and what is the implication of the velocity field on this particular region.

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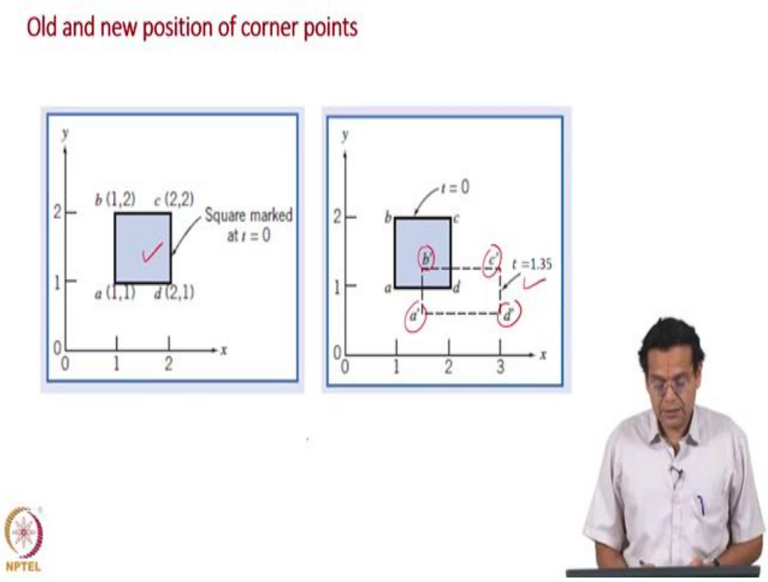

So, let us show the old and new position of the corner points. So, the this is the square region marked at time t = 0 and it has become a; it has become a rectangle at t = 1.35 seconds. These are the coordinates of a’, b’, c’ and d’ and it has become a rectangle. So, let us see how do we understand this.

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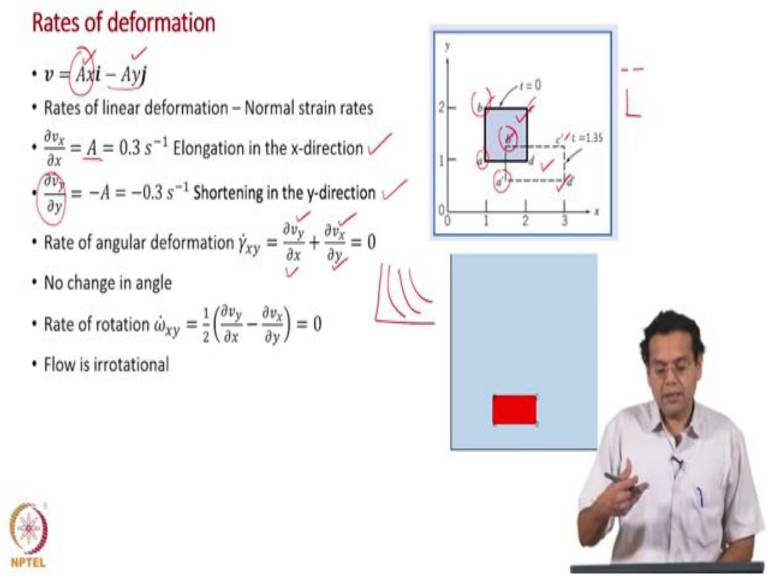

Now, the velocity field was given by

$$v = Axi - Ayj$$

So, rates of linear deformation let us calculate them the normal strain rates

$$\frac{\partial v_x}{\partial x} = A = 0.3\ s^{-1}$$

The value is 0.3 second inverse, which means that there is elongation in the x direction. Now, let us evaluate the normal strain rate in the other direction which is

$$\frac{\partial v_y}{\partial y} =- A =- 0.3\ s^{-1}$$

This means that there is shortening in the y direction. And that is what we have seen if you see compare b, c and b'c', there is elongation which is for line segment along x direction. And if you compare a, b and then a'b' there is a shortening in the line segment which was along the y direction.

So, for line segment along x direction there is elongation as shown by the value of 0.3 second inverse line segment along y direction there is decrease in that length and given by the -0.3 second inverse. So, keep in mind that as we have discussed earlier, this tells about rate of elongation and also rate of change of length per unit length.

Now, the rate of angular deformation is

$$\dot{\gamma}_{xy} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0$$

So, in this case there is no change in angle; when I say no change in angle, if you take 2 line segments perpendicular to each other they remain as such that is what you see here also. The angle b a d is 90 degrees the angle b'a'd' also 90 degrees there is no change in angle at all. That is why I said earlier example we had angular deformation and of course, rotation in this example angular deformation is 0 we will see rotation is also 0. But, we have normal strain rates.

So, rate of rotation

$$\dot{\omega}_{xy} = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) = 0$$

Such a flow is called irrotational where, you do not have any rotation such a flow is called irrotational and that is what you see this animation. Now, for showing this animation I calculate the coordinates at every instant of time this a'b'c'd' tells you the coordinates after sometime t equal to 1.35 seconds. But now, for this animation I can find out the path line equations substitute t equal to 0.1, 0.2, 0.3 whatever time interval and then calculate the position and plot them and the region covered by a, b, c, d is shown as red so that we see it as a region moving.

And as you see it also represents the flow around the corner, if we have seen the streamlines where we had flow around the corner and that is also represent there is also represented by this region which we are tracking ok. And we can see that the square becoming a rectangle how the square gradually becomes rectangle this was one initial square. And then let us say a final rectangle a t equal to 1.35 here you see how the square evolves gradually to the rectangle. And of course, remember it is continuously deforms that will always should be kept in mind we said fluid continuously deforms.

So, as long as the flow as long as you track this will continue to deform of course, for the sake of simulation we have stopped and replaying it again, but otherwise continuously deforms ok. So, in the earlier example no normal strain rates only angular deformation rotation were present in this particular in this example normal strain rates are present angular deformation and rotation are absent. This is relatively simpler case compared to the earlier case, when you have angular deformation rotation it becomes little difficult to analyse in terms of the what you see.

And what we analyse remember when we said in the earlier example what we see is a summation of all the effects. Here because no angular deformation and rotation whatever you see is representing what are we calculated in terms of normal strain rates. Earlier it was representing both the combined effect of both the deformation and the rotation.

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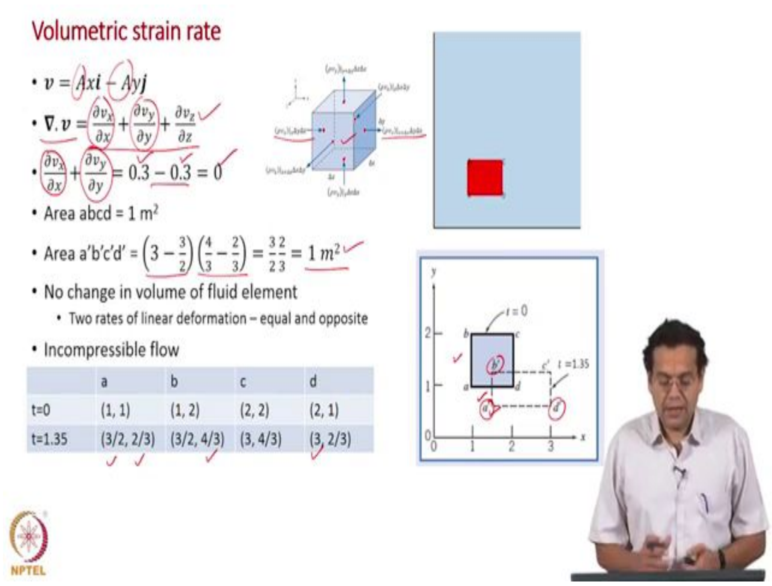

Now, let us calculate the volumetric strain rate. So, the volumetric strain rate we start with the velocity field, which is

$$v = Axi - Ayj$$

We have seen how to express volumetric strain rate which is that fractional rate of change of volume. And we have seen that can be expressed in terms of

$$\nabla . v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = A - A = 0.3 - 0.3 = 0$$

What does it tell you, there is no change in volume, rate of change of volume, or fractional rate of change of volume is 0; the volume of the element does not change with respect to time.

So, in this present case it is not volume it is area because we are considering 2D. So, let us verify that numerically, the area of the initial square both the sides are 1 unit so area is 1 meters. So, the final area should also be 1 metre square.

Let us calculate that what is the final area a'b'c'd'; if you consider points a'd' the length of the line a'd' which is difference in the x coordinate of a'd' which is $\left(3-\frac{3}{2}\right)$. Now, the difference in the y coordinate of, let us say a'b'. So, it is $\left(\frac{4}{3}-\frac{2}{3}\right)$ so,

$$Area\ of\ a'b'c'd' = \left(3-\frac{3}{2}\right)\left(\frac{4}{3}-\frac{2}{3}\right) = 1\ m^2$$

So, we have theoretically proved that the volume or area cannot change based on our calculation also we have proved that the area does not change. So, no change in volume of fluid element; why is it happening? Two rates of linear deformation are equal and opposite the normal strain rate in the x direction is positive because length is increasing. And normal strain rate in the y direction is negative because, the length is decreasing they are equal and opposite hence there is no change in the area. Whatever increase in area because of the increase in length along x direction is compensated by the decrease in area because of the decrease in length along the y direction. Of course, such a flow is incompressible that is what we have seen.

I like to discuss another view point remember $\nabla . v$ we discuss the significance in two different ways; the present way is that fractional rate of change of volume. Now, what is the earlier way it was relating to a Eulerian view point, present view point is Lagrangian the earlier view point is Eulerian. And remember these terms where derived as part of the continuity equation where you took as control volume like this. Let us say if you take $\frac{\partial v_x}{\partial x}$ how did it come? We took the mass flow leaving through the right face mass flow entering the left face and then we took the difference and so, $\frac{\partial v_x}{\partial x}$ represents.

So, let us say we have rho there it represents net mass flow leaving the x direction per unit volume for the moment leave per unit volume. So, just represent something like net mass flow leaving in the x direction. Because there is no rho it tells net rate at which volume leaves in the x direction and what about $\frac{\partial v_y}{\partial y}$ that tells you net rate at which volume leaves in the y direction of course, per unit volume.

Now so, if you look from a Eulerian viewpoint if you take a small location what happens is that because, $\frac{\partial v_x}{\partial x}$ is positive which means that there is net flow leaving in the x direction. And then because $\frac{\partial v_y}{\partial y}$ is negative, which means that there is net inflow in the y direction they compensate each other and hence the net rate of flow volume through all the control surfaces is equal to 0.

So, that is two different view points for $\nabla.v$, this example does not require the second view point, but because we have discussed the 2 viewpoints for $\nabla.v$. And numerically we can see in this example that there is net outflow in one direction the x direction net inflow in the y direction and hence if you sum up all the through all the directions is 0 other way of looking at it.

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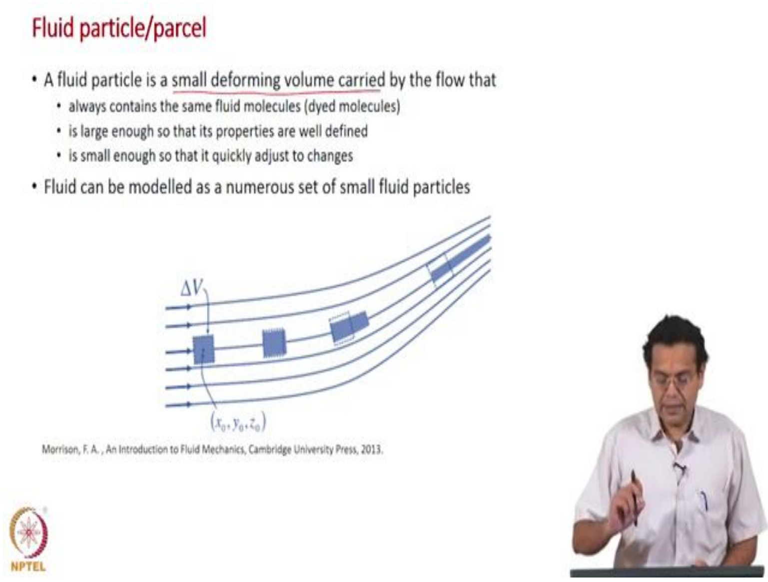

When we almost began the course. We introduced what a fluid particle is and we defined a fluid particle as a small deforming volume and then, we took this example and showed that the fluid particle should represent whatever is happening to the flow field. And then we said along the flow field the particle elongates becomes smaller etcetera in the direction etcetera.

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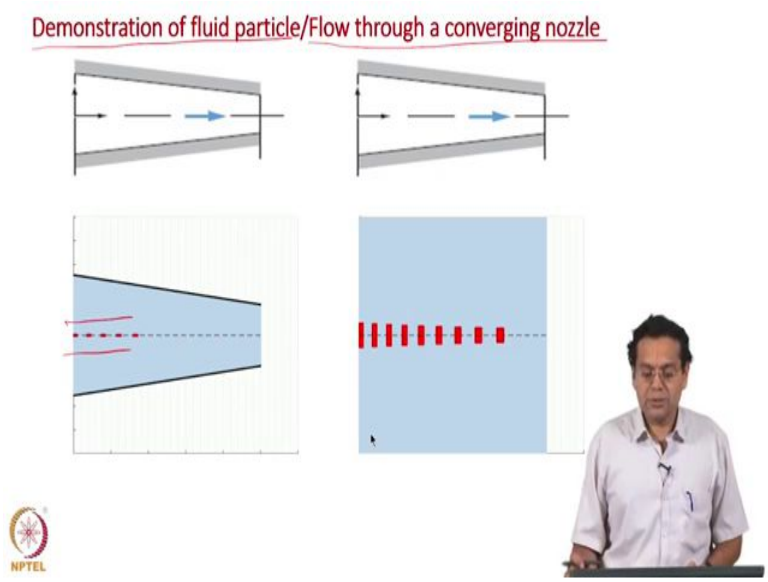

And we also showed this animation where we said the earlier title was demonstration of fluid particle is same example now, I say call it as a flow through a converging nozzle. And then we had simulations in 2 scales one when I just ran we were tracking the fluid particle. And then we said should be very small and then we said for the same time interval it travels longer distance as it approaches to as it travels along the length of the channel.

Because, it is a converging channel the velocity increases along the direction of flow then what we discussed was we zoomed this region small this region. And then that is what we saw in there right animation, where we can clearly see that the fluid for the fluid element the length of the fluid element increases and then the height decreases. And then the statement which I made at that point of time was that the length increases because, the velocity increases along the flow direction. And I said correspondingly there is a decrease in the height of the fluid element. Now, they need not be accepted as such.

Now, we can by theory based our discussion we can explain those statements, I will just repeat earlier. We have looked at this element fluid element or fluid particle and said that its length increases along the flow direction and height decreases along the flow direction. We explain saying that the length increases because the velocity increases along the flow direction. Correspondingly, I said the height decreases. Now, we can physically reason out these two statements that is what we will do in the next slide.

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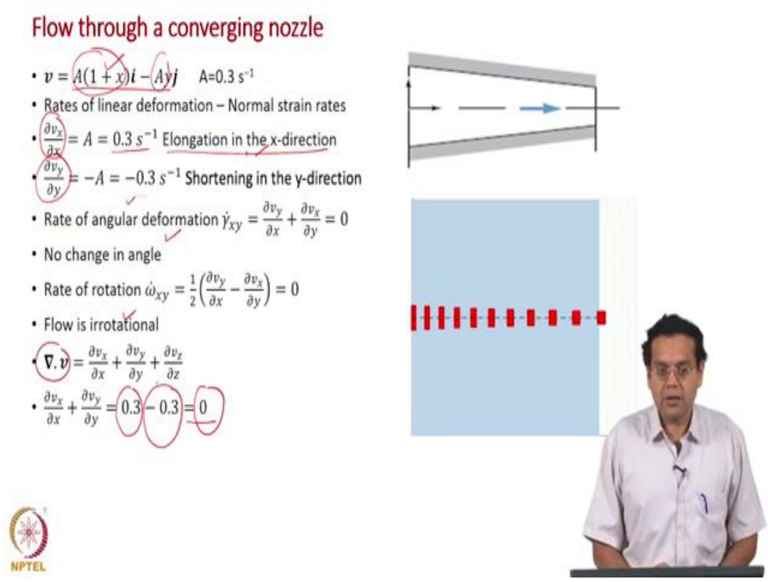

Now, what is the velocity field for a converging nozzle is given by

$$v = A\,(1 + x)\,i - Ayj; \qquad A = 0.3\ s^{-1}$$

But, the way in which we should view is that x velocity is a function of only x and the y velocity is a function of only y. That is how we should view for our purpose for calculating rate of normal strain shear strain the x component of velocity depends on x. Then as we have calculate in the previous example, we have a nonzero value of $\frac{\partial v_x}{\partial x}$ and that is positive why? The x component increases along the x direction. That is also is line with the physics of the geometry because of the reduction area the x component should increase. Now, what about the y direction? It is $-$A and we have a -0.3 second inverse and there is reduction in the height of the fluid element.

So, because of this only when we run the simulation because of this only this tells you that there is elongation in the x direction. Now, this clearly justifies our earlier statement that because the velocity increases along the x direction there is elongation of the fluid element earlier we made as a qualitative statement. Now, we can quantitatively establish in this statement.

If you look at the normal strain rate in the y direction it is negative and that is why you see a decrease in the height of the fluid element. Now, we made a statement that accordingly or correspondingly there is a decrease in the height of the fluid element how do you explain that. Before that like in the previous case there is no angular deformation there is no rotation because, v_x depends on x only, v_y depends on y only.

If you calculate $\nabla.v$ like in the earlier case you get a value of 0; that this explains y accordingly or correspondingly because it is incompressible flow $\nabla.v$ should be 0. And so, when there is increase in length along x direction it should be compensated by decrease in length along the y direction. That is why we said increase in length along x direction because of increase in velocity correspondingly there is a decrease in length along the y direction why their correspondingly? Because, $\nabla.v = 0$ for incompressible flow.

So, when one direction there is rate of increase in length, there is a decrease in rate of length in the other direction, we are kind of connected we have seen this in the previous example, in terms of concepts just want to connect this example with the simulation which have seen earlier where we introduced the concept of particle.

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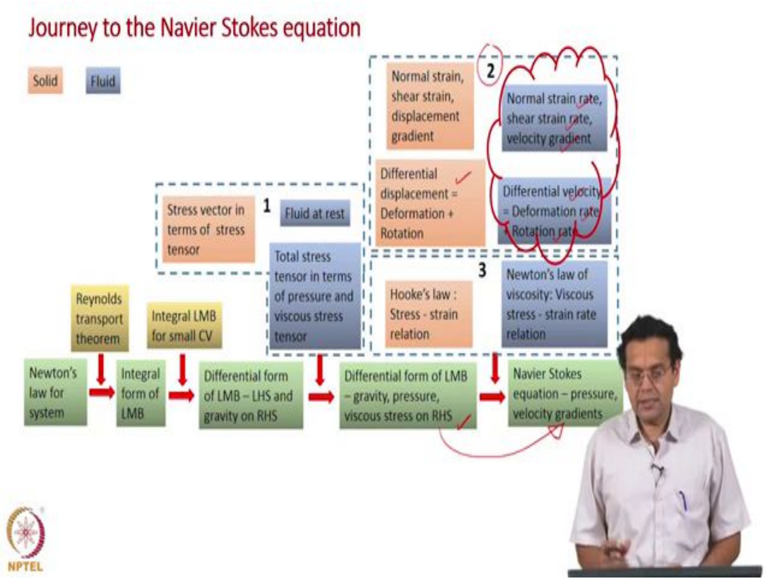

So I think that brings us to the close of our this lecture, where we I have discussed fluids strain rate in fluids analogous to strain and solids. So, in terms of a journey to the Navier Stokes, we have derived the differential form of linear momentum balance with a viscous stresses in the right hand side we said we need to express that in terms of the velocity gradients to understand velocity gradients.

We took a diversion to solid mechanics understood normal strain, shear strain, displacement gradient rotation. You also understood how to decompose or split differential displacement in terms of deformation and rotation. And now, we have analogously explained for the case of fluids, but everywhere we had rate.

So, everywhere we had rate so, normal strain rate shear strain rate and velocity gradient and then the differential velocity is sum of deformation rate less rotation rate. Superficially we say just addition of rate, but we discuss that based on the difference between the behaviour of solids and fluids ok. And of course now, we will go back once again to solid mechanics to relate stress and then strain.

We need to relate viscous stress and velocity gradients and we said instead of velocity gradients, we will relate only to the rate of defamation. But, before doing that we will go to the solid mechanics once again to understand and relate stress and strain. So, that when we come to fluid mechanics we can easily extend and relate the viscous stress and strain rate of course, this we will understand as we go along.

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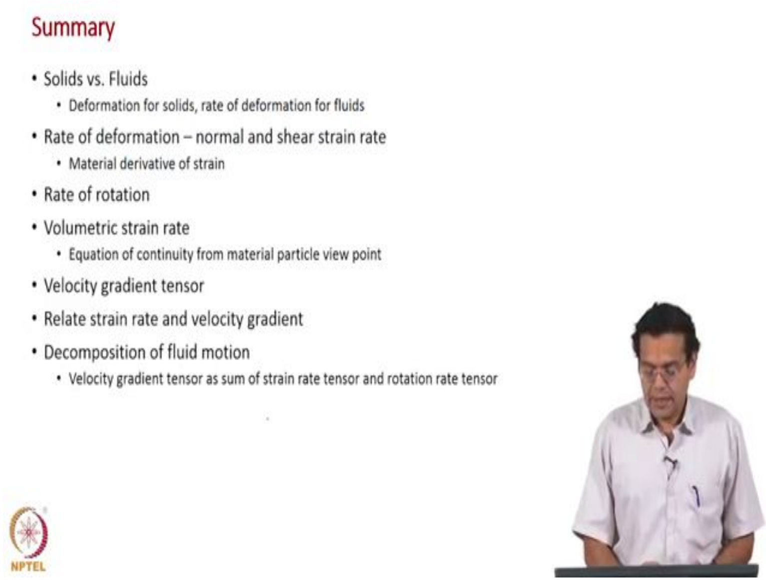

To summarise, we started with the distinction between solids and fluids in terms of response to a tangential force. And we understood say deformation for solids is rate of deformation for fluids and rate of deformation could be normal and shear strain rate one is rate of change of length other is rate of change of angle. And we also discussed that they are material derivatives of strain ok, we also discussed about rate of rotation, we discussed about volumetric strain rate and the volumetric strain rate is represented by the divergence of velocity field.

And so, we understood two ways or physical significance of the divergence of velocity in two different ways and based on that we express the equation of continuity from the material particle view point. We discussed the velocity gradient tensor analogous to the displacement gradient tensor for solids. And then we related the strain rate to the velocity gradients and we also discussed the decomposition of fluid motion which is expressing the velocity gradient tensor as sum of strain rate tensor and rotation rate tensor.