Continuum Mechanics and Transport Phenomena
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Lecture – 70
Stress Strain Relation : Introduction

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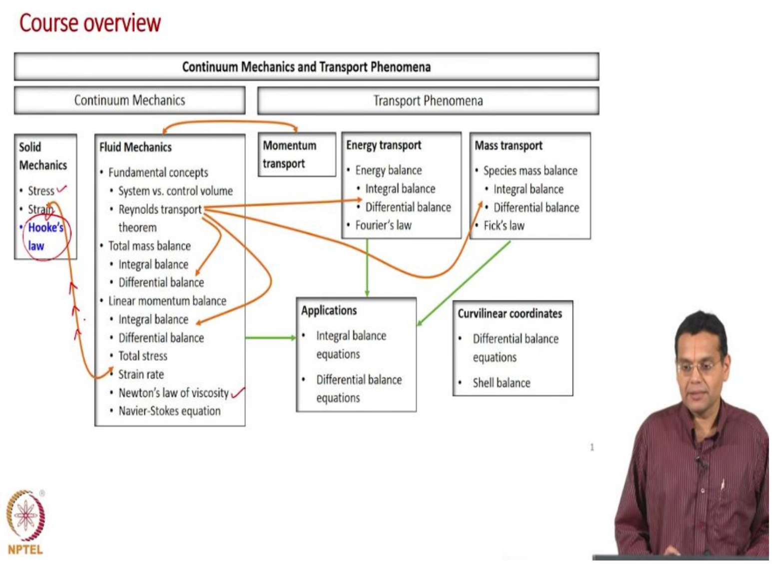

So, we will get started. We derived the differential form of linear momentum balance and we are proceeding towards the Navier-Stokes equations. And during that path to understand total stress and fluids, we took the first diversion to solid mechanics. The first time we travel in this through is arrow mark, and then discussed stress in solid mechanics, came back to fluid mechanics and understood total stress. Then we need to understand strain rate.

So, once again we took a diversion to solid mechanics, understood strain and extended that to strain rate in fluids. Now, we are going to take once again diversion to solid mechanics, the third and the last time. What is the reason? We need to relate viscous stresses and the strain rate or the velocity gradients through the Newton's law of viscosity. But before doing that we will relate stress and strain through Hooke's law by going to solid mechanics for the third and last time. So, objective of this lecture is to discuss about Hooke's law. We are going take a diversion to solid mechanics, discuss about Hooke's law and then of course, followed later on by coming back to fluid mechanics.

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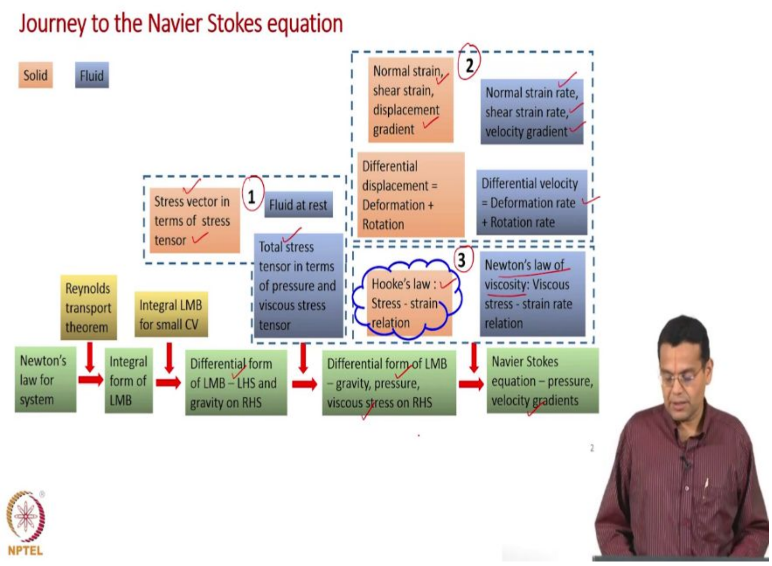

So, in terms of the journey to the Navier-Stokes, we derived the differential form of linear momentum balance, we had surface forces on the right hand side, understand that we took the first visit to solid mechanics, understood stress vector, stress tensor and came to fluid mechanics, understood total stress in terms of pressure and viscous stresses. Then, we could complete the differential form of linear momentum balance including the right hand side with the viscous stresses, with the viscous stress terms.

Now, those viscous stress terms had to be expressed in terms of velocity gradients, that is where we discussed closure. To understand velocity gradients need to understand about displacement gradient, so we made a second visit to solid mechanics, understood strain, displacement gradient and then the strain tensors. Then extended that knowledge to fluid mechanics, understood normal strain rate, shear strain rate, velocity gradient and the strain rate tensor or the deformation rate.

Now, we need to look at the actual relationship between the viscous stress and the velocity gradients which is what going do here in Newton's law of viscosity. But before that like in the previous cases we take a third visit to solid mechanics, discuss this block which says Hooke's law which is the relationship between stress and strain relationship. Once we understand that the analogous relationship of fluid is stress and then strain rate that is the overall idea.

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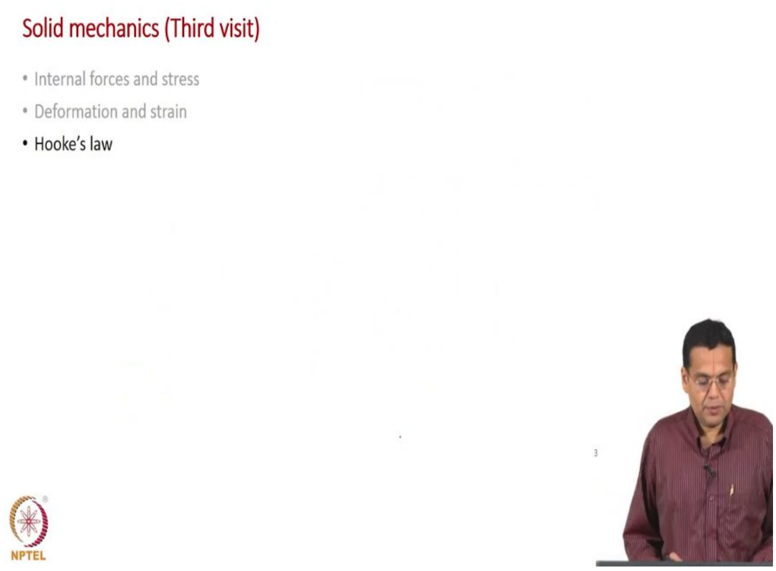

So, third visit to solid mechanics. In terms of titles first we discussed about internal forces and stress; second visit, we discussed about deformation and strain and third, in this visit we are going to discuss about Hooke's law which relates stress and strain.

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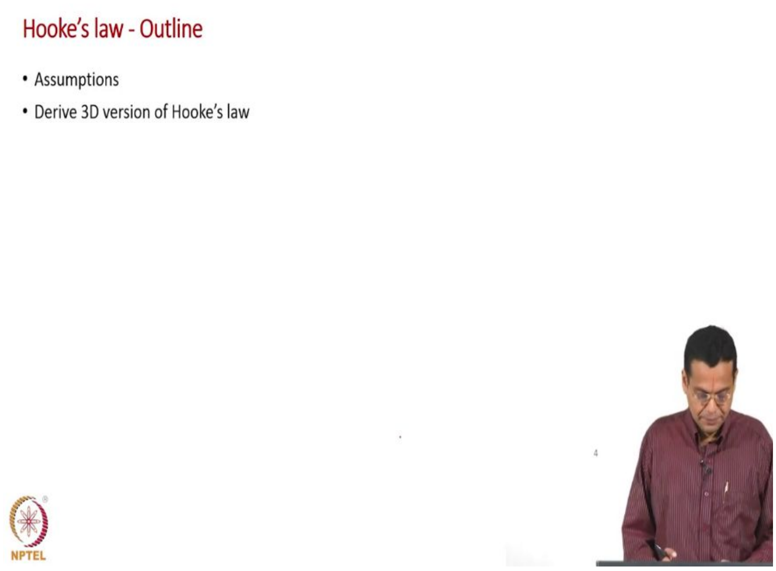

What is a outline? First we discuss about assumptions and then derive the 3D version of Hooke's law. Hooke's law in a simpler form should be very well known to most of us. We will derive the 3D version of Hooke's law

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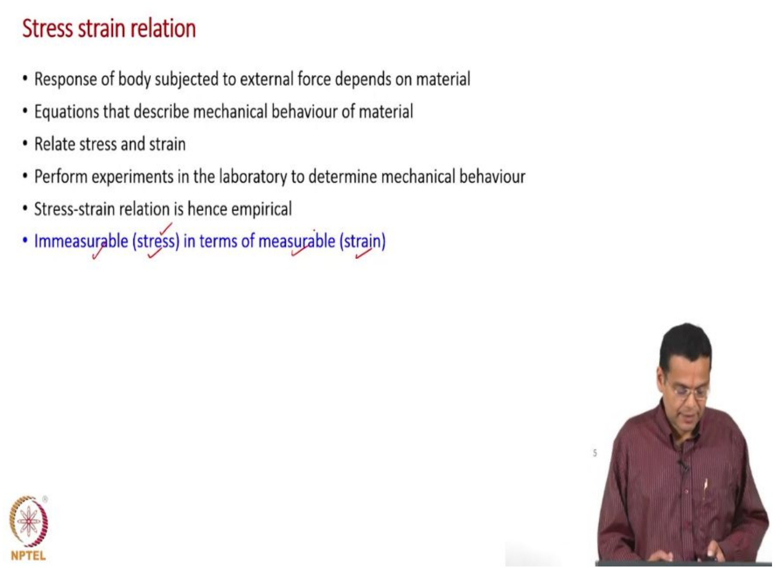

Now, what is the overall objective? I have a solid body as subjected to some force and I am interested in seeing the response of that body, which is called the mechanical behavior of the solid object. I need equations that describe the mechanical behavior of the material, and moment we talk about a material we need to bring in properties of the material. When we first came to solid mechanics, we discussed about surface forces, stress vector, stress vector in terms of stress tensor, and through the discussion we never focused on what material we are discussing about. We just discuss it could be a plane inside a solid object, and then we characterize that intensity of force in terms of stress.

Then we came to strain where we discussed about change in length, change in the angle which we called as deformation, quantify that in terms of strain, there again we did not discuss about the material at all. We discussed in terms of quantification of the deformation. And now, we have to relate this stress and strain, and when they are going to relate the stress and strain then the property of the material has to be brought in. So, while stress and strain can be discussed without reference to any material, the stress-strain relationship becomes material dependent, and that is why we are discussing about the assumptions. When we say assumptions the stress-strain relationship, which are going to discuss is limited to that particular set of assumptions.

If you recall back about stress and strain independently we hardly had any assumptions throughout our discussion they were all exact relationship. But when we discussed stress and strain it is with reference to a particular material and that is where assumptions are brought in.

Now, we can derive the form of the expression, but when you derive there are some constants which we called as material properties. One material property which is known to you is Young’s modulus. But that Young’s modulus has to be determined by experiments. So, the stress strain relationship that with the actual values the parameters is determined by performing experiments in the laboratory. And hence, the stress strain relationship is empirical; when I say empirical it is based on experimental observation because was an experimental evidence.

And what is that more importantly we are doing? We are relating immeasurable quantity with a measurable quantity and that is the key, that is why highlighted here. When we discussed about the stress it is certainly a very physically meaningful quantity, no doubt about it, they had a surface, what is the force acting on the surface, stress vectors, stress tensor etcetera. Perfectly very much meaningful, but we cannot measure components of stress.

But remember, when we came to strain we said we have a plate, there are some coordinates, difference in coordinates and then we analytically derived the expression for different normal strain, shear strain, which means that you can measure strain and then quantify strain based on measurements. Even the questions if you recall back some coordinators were given initial coordinates, final coordinates are given, which means they are measurable value, so strain is measurable. So, strain is measurable and stress is immeasurable by finding a relationship between stress and strain, we are expressing an immeasurable quantity in terms of a measurable quantity and that is the key.

Stress is not directly measurable by expressing in terms of strain it becomes indirectly measurable. You can determine stress, you can calculate stress, but not directly measure stress, and that is the key here. So, to summarize this while stress and strain discussion is independent of the material, the relationship between stress and strain depends on the material. Because the response depends on whether for example, if we take a solid, if it is made of steel it has my response, if it is nylon some other response. Let us say aluminum, it has some other response. So, properties of the material have to be brought in and that is where we discuss assumptions.

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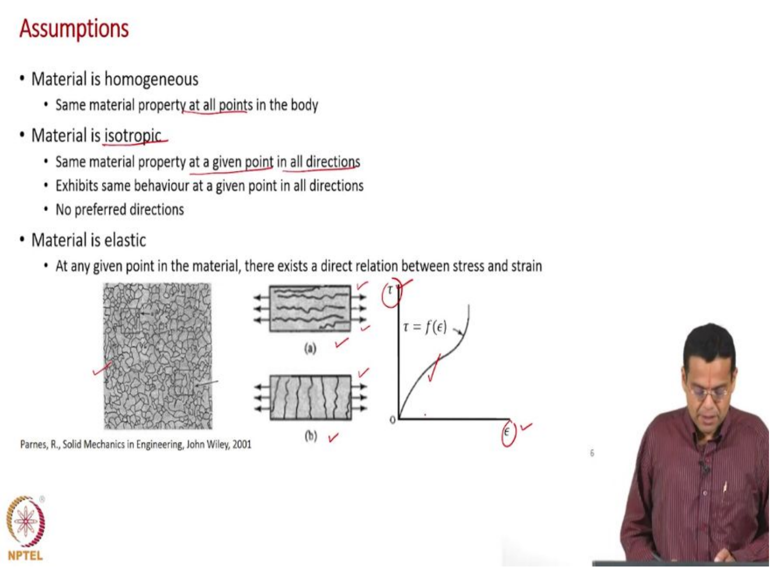

So, what are the assumptions? When I say assumptions, the stress strain relationship is valid only under this set of assumptions. We never had this restriction when we discussed about stress separately and strain separately. Of course, for strain we said it is infinitesimal strain that was a small assumption when we discussed for solids and when we went to fluids even we said that is not even assumption.

- The first assumption is that the material is homogeneous

What does that mean? If you take a let us say a solid object, these the properties which are going to experimental determine. When I say properties I mean material properties. What are the material properties? We will come across few material properties one which is very well known to us is the Young's modulus. So, if you take every point in the body, the value of the material property for example, Young's modulus is same. So, look at the definition of a homogeneous material, same material property at all points in the body. So, any point you take in the body has the same value of material property. What is shown in this diagram is just magnified view of steel. So, if we zoom in and zoom in of course, there can be non-homogeneities. But on a macroscopic scale, if you look at steel assets then you say that the properties are same at every point. So, that is still level we are looking at. We are not magnifying; magnifying, and saying going to say that there can be non uniformities from point to point. We are not going to go to that level of magnification. If you are given let us say steel block like this and you say at every point it has a material property. So, material is homogeneous means same material property at all points in the body. So, that is a first assumption. So, the Hooke's law which I going to derive is for a homogeneous material.

- Second assumption, material is isotropic.

What is isotropic? Let us read the definition. Same material property at a given point in all directions, the key is at a given point in all direction. When we discussed about homogeneity we said same property at different points, at all points in the body. When you discuss about isotropic assumption the same material property you are at a given point, but in let us say x-direction, y-direction, z-direction, all the directions you have the same material property for a for example, Young's modulus.

So, independent of the direction, the material is assumed to have the same material property. And which means that exhibits the same behavior at a given point in all directions. Why do we say this? The property determines the behavior, so when the property is same in all directions, exhibit same behavior and got a given point in all directions.

And the other way of putting this is no preferred direction. When I say that at a point the property is same in all directions, there is no question of talking about direction at all. When do we have a preferred direction? When the property is different along different directions then there is a question of talking about a preferred direction, but because the property is independent of directions there is no preferred direction.

What is shown here is an example for anisotropic material; the one of the examples for an isotropic material is wood. Now, what is shown here are fibers and the wood let us say a block of wood is subjected to tensile force. Now, the mechanical behavior of wood will depend whether the tensile forces are aligned along the fibers or they are perpendicular to the fibers.

Even if you look at let us say just make a simple Google search if you say Young's modulus for steel, you will be given one value of Young's modulus. But if you let us say search for Young's modulus of wood, you would get Young's modulus of wood along direction, tangential direction, longitudinal direction which means that the Young's modulus depends on the direction. Such kind of materials are called an isotropic materials.

The Hooke's law which we are going derive is not applicable for an isotopic materials. But anyway we are safe that, remember we will see shortly that isotopism is a very convenient assumptions. If you do not do that we will have really lot of difficulties. So, but in general we can for our scope we are most of the cases we can assume the material to be isotropic.

So, just to summarize, homogeneous refers to same material property at all points throughout the, its means throughout the body. Isotropism refers to direction, you are in the same point the material property same in the all directions. That is why the terminology says isotropism, no preferred direction.

- The third assumption is that the material is elastic.

What is the meaning of that? At any given point in the material, there exists the direct relationship between stress and strain and that is what is shown here. You have a some relationship it could be non-linear or linear. What is shown here is a stress on the y-axis, strain on the x-axis and you have some functional relationship τ as a function of ε. So, at any given point in the material there exists a direct relationship between stress and strain, which means that τ is a function of ε.