Continuum Mechanics And Transport Phenomena
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Lecture – 87
Bernoulli Equation: Example 1

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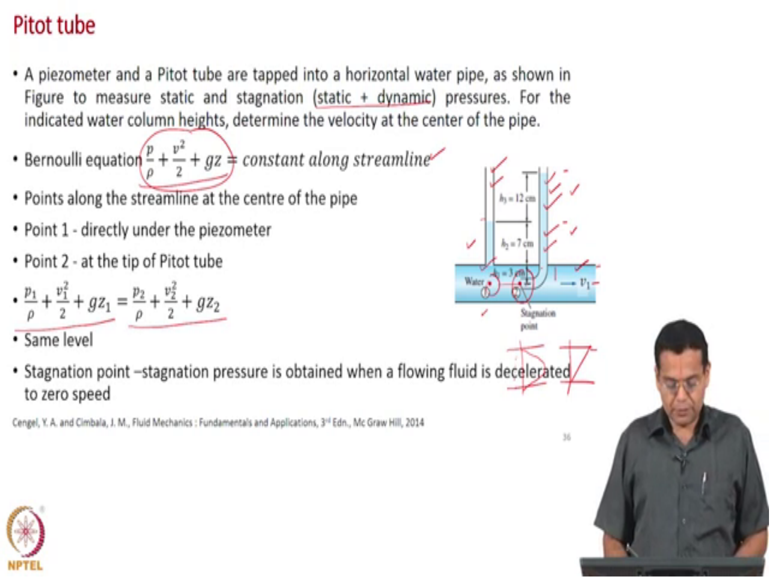

Let us look at a very simple application of this Bernoulli's Equation. We will have many such examples in Fluid Mechanics book. We will just discuss one of them and many will be representative of this, some terms will be given, some terms will be asked to find out. What is it we are interested in? Remember, the next level of application is going to be finding velocity profiles that is we are going to predict velocity profile. For example, in our flow between the parallel plates we are going to predict the velocity profile where one plate is set in motion and then when both the plates are fixed we are going to predict the velocity profile to be parabolic etcetera.

Now, suppose if you want to measure the velocity profile in between the plates or in a pipe how do you measure and that is the arrangement shown in the figure. This arrangement is used to find out velocity at different points in a pipe.

Now, what is the arrangement? What is shown here is a pipe carrying water and then you make a hole and then insert a tube in this way on the wall of the pipe. Now, water rises based on the static pressure at that particular location and that static pressure is balanced by the level to which water rises. So, you make a hole and attach a small tube, water will rise and that will represents the static pressure at that particular point and the static pressure is represented by the height to which the water level rises.

Now, you make another tube of L shape. Why L shape? This end should face the flow and once again water rises in this tube as well. What happens now let us say water has some velocity at point 1 and when it reaches point 2 it gets decelerated the velocity becomes 0 at point 2. So, it loses out all the kinetic energy and so, that results in a increased pressure and that is why you observe a higher level here that increased pressure is represented by the water level in this tube. So, the first tube represents the static pressure, the second tube reflects the increased pressure because of the conversion of kinetic energy.

Now, let us read the exam question a piezometer; when I say piezometer this simple tube is called a piezometer and a pitot tube are tapped into horizontal water pipe as shown in figure to measure static and stagnation pressure. What the piezometer measures is a static pressure and the second pressure is called the stagnation pressure. And, why is it stagnation pressure? Because the velocity becomes 0 at that particular point, for the indicated water column heights.

What are the water column heights indicated? So, in the first tube the water rises to 7 centimeter, and in second tube the water rises to 7 + 12 = 19 centimeter. Of course, you have a 3 centimeter between the point, in this case we are interested in measuring the velocity along the central line. So, the point of measurement and the wall there is a 3 centimeter height.

So, now, let us write the Bernoulli's equation

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = constant$$

Now, we will choose a streamline along the center line of the pipe and point 1 is directly under the piezometer and point 2 is at the tip of the pitot tube, that is what is shown here. The point 1 is below the piezometer, 2 is at the tip of the pitot tube. Now, because points 1 and 2 are along the streamline this sum should be same at point 1 and point 2 and that is what is written here

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2$$

This should be same because we are considering points along the streamline that is what we have discussed. The sum of these three terms should be same. Now, points 1 and 2 are at the same level. Both the point are at this along the center line of the pipe and then this we have already discussed 2 is at the stagnation point and we have just now discussed stagnation pressure is obtained when a flowing fluid is a decelerated to zero speed.

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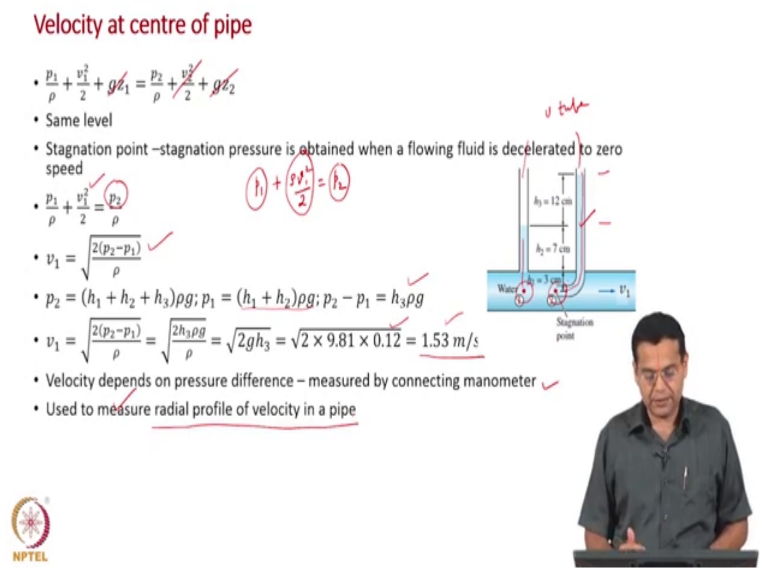

Now, let us write down the Bernoulli's equation which means the sum of energy at the two points are same.

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2$$

And, because the levels are same so, z_1 and z_2 are same, z is the level from some datum some reference and we have seen the stagnation point the velocity is zero which means that $v_2 = 0$. So, that is a simplification which are going to apply for the Bernoulli's equation. So,

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho}$$

Now, this p_2 is called the stagnation pressure. So, the stagnation pressure represents the sum of static pressure and the dynamic pressure and that is why you have a higher level of rise of water in this particular pitot tube compared to the piezometer.

We are interested in finding out the velocity which is in the dynamic pressure term. So, the dynamic pressure is the difference between the stagnation pressure and the static pressure. So, rearrange this equation for velocity

$$v_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho}}$$

Remember, the difference between the two pressures represents the dynamic pressure; p_2 is stagnation pressure, p_1 is static pressure.

So, now, let us evaluate what is the pressure at point p_2

$$p_2 = (h_1 + h_2 + h_3)\rho g; \qquad p_1 = (h_1 + h_2)\rho g$$

So,

$$p_2 - p_1 = h_3 \rho g$$

Now, let us substitute that in the equation,

$$v_1 = \sqrt{\frac{2h_3\rho g}{\rho}} = \sqrt{2gh_3} = \sqrt{2(9.81)(0.12)} = 1.53\frac{m}{s}$$

What we usually do is because it depends on the level difference. We connect a U-tube nanometer here and directly measure the pressure difference.

We need not measure separately the static pressure and then the stagnation pressure. Just connect a U-tube nanometer that will give us the pressure difference and that is what is written here velocity depends on pressure difference measured by connecting a manometer and remember the velocity is high remember we should be at high velocity because we are considering the flow to be inviscid and we are also in this case quite away from the wall and we said inviscid flow prevails in the region away from the wall where net viscous was a negligible. So, that condition is also satisfied. As it I told you, this method is used to measure the radial profile of velocity in a pipe or it could be even velocity between the two plates.