Continuum Mechanics And Transport Phenomena
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Lecture - 98
Viscous Stress vs. Molecular Momentum Flux Part 2

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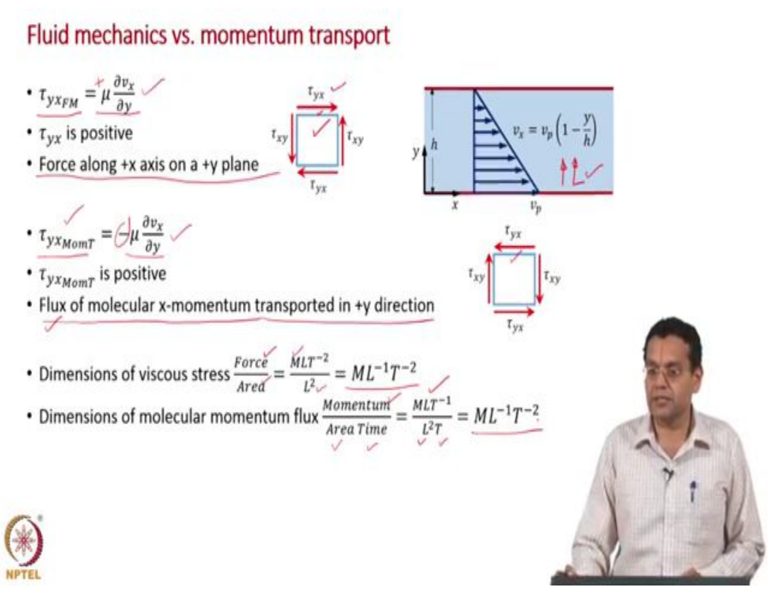

Now, what we will do is compare our discussion under fluid mechanics and the present discussion under momentum transport. Let us do that. Let us first discuss what we have discussed in fluid mechanics.

$$\tau_{yx,FM} = \mu \frac{\partial v_x}{\partial y}$$

To be more specific, there was a positive sign because we have now negative sign, so let us be more specific.

And suppose if we say τ_{yx} is positive what does it mean? Force along positive x axis on a positive y plane that is what is shown here, positive y plane and force along positive x axis. That is why for this case because τ_{yx} is negative for this case on a positive phase, you know positive y phase we have shown the force along negative x axis.

$$\tau_{yx,MomT} =- \mu \frac{\partial v_x}{\partial y}$$

Now, we have discussed τ_{yx} momentum transport. So, far throughout the course, throughout fluid mechanics we have discussed τ_{yx} as $\mu\frac{\partial v_x}{\partial y}$. Now, when we came to momentum transport because of our sign convention we are relating τ_{yx} to $-\mu\frac{\partial v_x}{\partial y}$.

Now, if you say $\tau_{yx,MomT}$ is positive, what is it, what does it mean? As we are discussed in the previous slide it says that flux of molecular x momentum transported in positive y direction that is what we have, flux of x momentum transported in the positive y direction.

So, in one case we related $\tau_{yx,FM}$ as $\mu\frac{\partial v_x}{\partial y}$, the second case they have related $\tau_{yx,MomT}$ to $-\mu\frac{\partial v_x}{\partial y}$. Also liked to emphasize which we will discuss in the next slide also, here I say force along positive x axis on a positive y plane. Here what do I say? Flux or flow of something following flux of molecular x momentum transport in positive y direction. This we will discuss next slide as well.

But now; so, fluid mechanics represents $\tau_{yx,FM}$ as force of course, force per unit area and momentum in momentum transport we interpret $\tau_{yx,MomT}$ as flux of molecular x momentum. That is the basic difference in the viewpoint itself. Both are τ_{yx} only, in fluid mechanics it is a mechanical view point or mechanics viewpoint. So, we represent interpret τ_{yx} as force and of course, per unit area.

Now, how do we interpret? We interpret as flux of molecular x momentum. Now, let us compare the dimensions of the viscous stress and the molecular momentum flux. Now,

$$Dimensions\ of\ viscous\ stress = \frac{Force}{Area} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$Dimensions\ of\ molecular\ momentum\ flux = \frac{Momentum}{Area\ x\ time} = \frac{MLT^{-1}}{L^2T} = ML^{-1}T^{-2}$$

Now, if you see the dimensions are molecular momentum flux, we have seen that moment is a flux it is that quantity per unit area per time, we have discussed volumetric flux mass flux, now we are discussing momentum flux, molecular momentum flux. And remember, when we defined the τ_{yx}, the first instance we said x momentum is transported in the y direction, this rate of transport of x momentum per unit area is the momentum flux.

So, once again get the same dimensions. Certainly they have a same dimensions, physical interpretations different.

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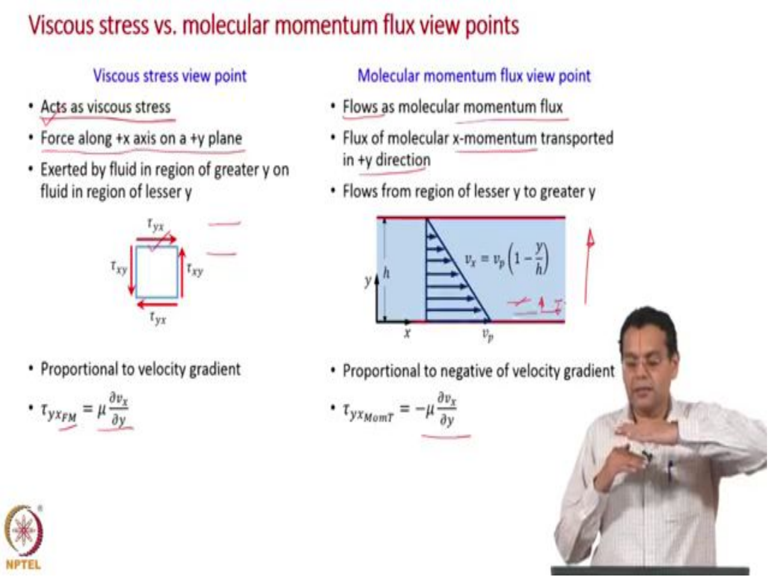

So, let us compare these two viewpoints. Look at the title, viscous stress view point in the molecular momentum flux viewpoint. Two viewpoints of the same quantity τ_{yx}, let us see. So, left hand side we have the viscous stress viewpoint which have been following so long throughout our fluid mechanics. When we came transfer phenomena when we are discussing now momentum transport, the way in which have viewing τ_{yx} has changed, it is molecular momentum flux viewpoint.

- It acts as viscous stress in the viscous stress view point because τ_{yx} is a force, we say it is acting as a viscous stress. And τ_{yx} acts as flows as molecular momentum flux in the molecular momentum view point.

Look at the difference in interpretation, one means interpret as a force and one we interpret as a molecular momentum.

- And for a positive τ_{yx}, it is a force acting the positive x axis on a positive y plane that is what is shown in the left side figure in the viscous stress view point. It is flux of molecular x momentum transported in positive y direction in the second view point.
- And in terms of regions, exerted by fluid in region of greater y on fluid in region of lesser y in the first view point. And Flows from region of lesser y to region of greater y in the second view point.

- In the fluid mechanics case proportional to velocity gradient and now, proportional to negative of velocity gradient.
- So, $\tau_{yx,FM} = \mu\frac{\partial v_x}{\partial y}$; and $\tau_{yx,MomT} =- \mu\frac{\partial v_x}{\partial y}$.
- This two viewpoints also can be explained from a molecular viewpoint.

Now, for the molecular momentum flux what we said was we have molecules at a higher velocity, and then and when they go to a layer above them, they carry with them higher x momentum and hence transport x momentum to the layer above them. Now, other way of looking at it is, the molecules near bottom plate at a higher velocity when they reach at top they have to be decelerated because that layer is it a lower velocity.

Molecules here are lower velocity and when they come here then it get accelerated. So, when you have deceleration acceleration you should have an associated force. So, it is like on you have a layer of fluid and molecules from lower to upper layer when they reach they need to be decelerated, and molecules from the upper layer to the lower layer they should get accelerated.

And of course, this is deceleration acceleration of their own forces, so it is like a layer and two equal and opposite forces acting on this layer and which we call as the shear force, if we divide the area you get the shear stress. And that is a viewpoint which have which helps us to view this molecular motion as viscous stress the first viewpoint is the molecular momentum flux.

So, one way we say that x momentum is transported from one layer to the other layer, other way is as if layer because of acceleration deceleration associated forces you have two forces of equal magnitude acting in opposite direction which is a shear force divided by area you get shear stress. So, that is a two interpretation of the same phenomena. Both this can be related through the Newton's second law of motion at this level of the molecular level.

So, Newton's law you have rate of change of momentum and then right hand side forces let us say divide each by area. So, left hand side is something similar to the molecular momentum flux, right hand side is force per area which is our viscous stress view point. More formal derivation possible we are not discussing that.

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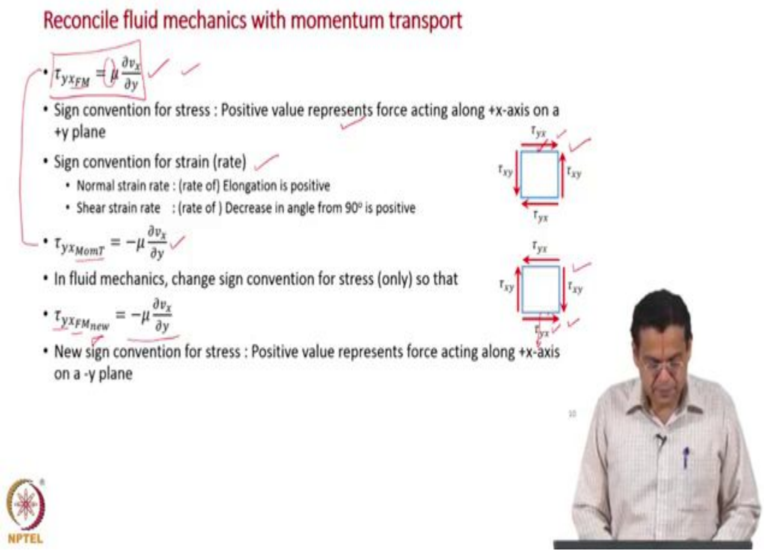


Now, one case we have plus, one case we have minus. How do you reconcile both of them? That is the slide about. Reconcile fluid mechanics with momentum transport. Fluid mechanics has positive sign momentum transport has negative sign, but can we reconcile both of them? We can do that let us see how do we do that.

$$\tau_{yx,FM} = \mu \frac{\partial v_x}{\partial y}$$

What is the sign convention for stress which are being using so far? A positive value represents force along positive axis and positive y plane that is what is shown here we come across several times.

Now, what is the sign convention for strain rate? Because this is the one dimensional Newton's law of viscosity which relates stress to strain rate. Now, so let us see what is the sign convention for strain rate which we have already discussed for normal strain elongation it is positive. So, our fluids for normal strain rate, rate of elongation is positive, and for shear strain rate shear strain is positive when the angle between the two line segments decreases.

So, here if the rate of decrease in angle from 90 degree is positive then it represents positive strain rate. So, we adopted one sign convention for stress and then a another sign convention for strain rate. Now,

$$\tau_{yx,MomT} = -\mu \frac{\partial v_x}{\partial y}$$

How do we introduce the negative sign here? That is a question we have to answer. How do we do that? As we have discussed now, one sign convention for stress, another sign convention for strain rate suppose if you change both the sign conventions, nothing will happen here. So, We keep one sign convention same and change only one of the sign conventions, so we change the sign convention for stress.

I will repeat again this relationship is based on two sign conventions, one for stress one for strain rate. We introduce a negative sign there. So, we will change one of the sign conventions namely for stress.

So, why are we doing that? So, that we get the same expression as we have obtained for momentum transport. So, that the expression in fluid mechanics and expression momentum transport both will look alike. So, what do we do in fluid mechanics? Change sign convention of stress only.

$$\tau_{yx,FM_{new}} =- \mu\frac{\partial v_x}{\partial y}$$

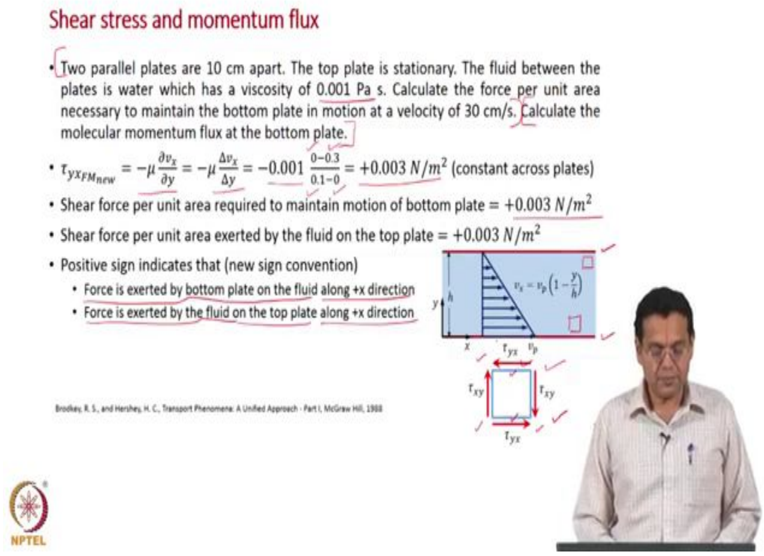
So, far we have been τ_{yx} because we are discussing fluid mechanics in momentum transport we introduce a FM and then momentum transport, but within FM we have an old sign convention and a new sign convention that is why it is new. Nomenclature is very clear.

In this case, in the new sign convention what do you mean by a positive τ_{yx}. What does it mean? Positive value represents force acting along positive x axis on a negative y plane and that is what is shown here in the bottom figure. What is that? This is a negative y plane and the force is shown along a positive x axis. So, the bottom image represent the fluid mechanics new sign convention, and top image represent the fluid mechanics old sign convention.

I will repeat again. We will keep a positive force, easy to discuss. So, positive force acts on positive y phase here positive force acts on a negative y phase that is a difference because we are change the sign convention. So, once again quickly summarize this slide. We are trying to reconcile fluid mechanics and momentum transfer.

What do I mean by that? The expressions which we have discussed for τ_{yx} for fluid mechanics and momentum transport differ by a negative sign. We want to introduce that negative sign in fluid mechanics and that is we have done that by changing the sign convention for stress. In the old sign convention positive force acts on a positive y plane the new sign convention positive force acts on a negative y plane.

Example: (Refer Slide Time: 15:07)



Just to understand this two different viewpoints, that is a very simple small numerical example from this book Brodkey and Hershey, some of some concepts are explained really nicely in this book. Of course, very old book, but physical explanations are good in this book.

Two parallel plates are 10 centimeter apart, the top plate is stationary. The fluid between the plates is water which has a viscosity of 0.001 Pa.s. Calculate the force per unit area necessary to maintain the bottom plate in motion at a velocity of 30 cm/s. Calculate the molecular momentum flux at the bottom plate. It is a typical fluid mechanics problem because now we are going to compare both the viewpoints and additional question is added. Calculate the momentum molecular momentum flux at the bottom plate.

So, let us use the new sign convention

$$\tau_{yx,FM_{new}} = -\mu \frac{\partial v_x}{\partial y}$$

Let us write in terms of a differences, so

$$\tau_{yx,FM_{new}} = -\mu \frac{\partial v_x}{\partial y} = -\mu \frac{\Delta v_x}{\Delta y} = -0.001 \frac{0-0.3}{0.1-0} = +0.003 \frac{N}{m^2}$$

Of course, it is a constant across the plates. It does not depend along the y axis. So, shear force per unit area required to maintain motion of bottom plate is equal to 0.003 Newton per metre squared, that is very simple as well.

Now, what is the shear force per unit area? Exerted by the fluid on the top plate once again 0.003 that is also along the positive axis. So, let us try to interpret this in the new sign convention, that is the idea of this question as well. Positive sign, we have got a positive value what does it indicate, according to the new sign convention. Please keep that in mind. And the new sign convention is shown in the figure. Why is it new sign convention? Negative phase, positive force; positive phase, negative force, that why it is new sign convention.

Now, let us say this imagine the stress element and this positive force indicates that the force exerted by the plate on the fluid is towards the positive x axis. Force exerted by the bottom plate on the fluid is along the positive x direction that is understandable. Now, just imagine the stress element, look at this force, how do you interpret that? Force exerted by the plate on the fluid is towards the negative x axis. So, force exerted by the fluid on the plate is towards the positive x axis; so very simple. All these are known to us what extra we are doing here is interpret based on the new sign convention that is all you are doing.

So, let us read this force exerted by the fluid on the top plate is along the positive x direction. Once again I want to emphasize, all these are forces acting on the fluid, right from beginning, whenever you say viscous stresses or even in solids stress is all acting on the solid acting on the fluid. So, all these are force acting on the fluid. So, bottom plate force on the fluid is along positive x axis. Here force acting on the fluid is along the negative axis.

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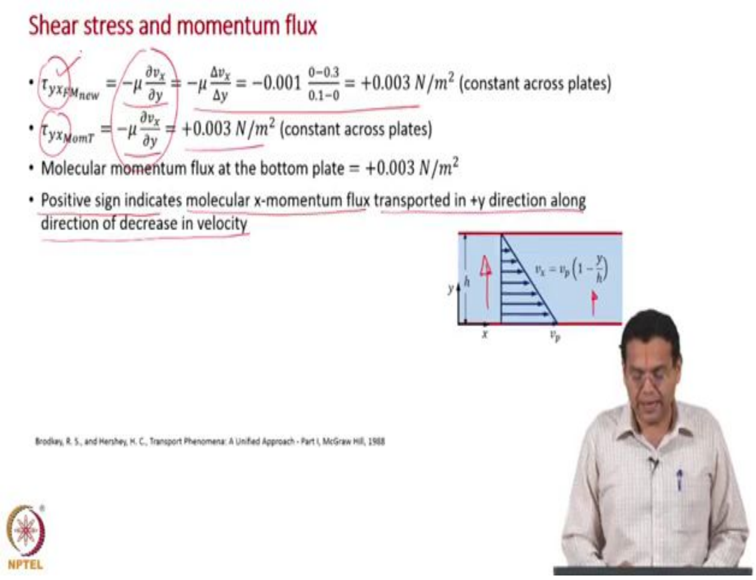

Now, we will calculate the momentum flux.

$$\tau_{yx,FM_{new}} = -\mu\frac{\partial v_x}{\partial y} = -\mu\frac{\Delta v_x}{\Delta y} = -0.001\frac{0-0.3}{0.1-0} = +0.003\frac{N}{m^2}$$

This is what we have done in the previous slide based on the new sign convention for fluid mechanics. And now, for momentum transport

$$\tau_{yx,MomT} = -\mu\frac{\partial v_x}{\partial y} = -\mu\frac{\Delta v_x}{\Delta y} = +0.003\frac{N}{m^2}$$

The τ_{yx} momentum transport has the same expression. Why is it same? Because the earlier expression has been written based on new sign convention that is why both the expressions are same. τ_{yx} fluid mechanics new, and τ_{yx} momentum transport have the same expression. So, same steps will give us 0.003 Newton per metre square. Once again it is constant across the plates. Numerically they are same, how are we going to interpret this; that is different.

How are we going to interpret? It is a positive value. What does it mean? Molecular momentum flux at the bottom plate is equal to plus 0.003. What is the correct interpretation? If you have a positive sign, indicates that molecular x momentum flux transport in the positive y direction along direction of decreasing velocity, very complete description. Molecular x momentum transport into the positive y direction. Why is it along positive y direction? Because the flux is positive and along direction of decreasing velocity.

So, now, in the first case we interpreted this as shear stress or shear force acting per unit area. The second case we are interesting that has molecular x momentum flux transported in y direction. In the first case, it was shear force acting in x direction on a y plane. In this case, it is x momentum flux transported in the y direction. See that, the direction of momentum is x, direction of transport is y. Like in the other case direction of force along the x axis, the plane on which axis y. Here similarly the direction of momentum is x momentum, the direction along which it flows as y axis. So, that they are analogous. We will see more about this later as well.

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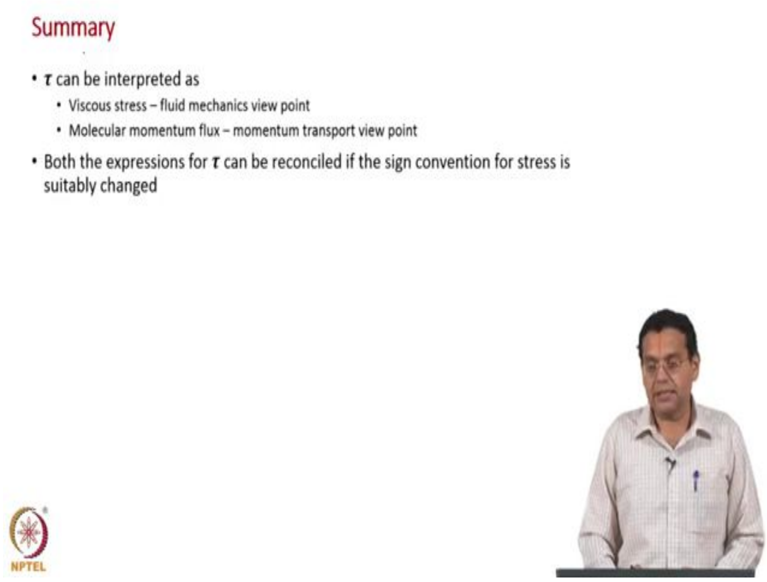

So, let us summarize what we are discussed so far, τ can be interpreted as viscous stress, the fluid mechanics viewpoint and molecular momentum flux, the momentum transport viewpoint. Both the expressions for tau can be reconciled if the sign convention for stress is suitably changed. That is what we have discussed so far.

We took our usual planar couette flow, took the other case where the bottom plate is moving, gave a molecular interpretation to τ , but that resulted in a negative sign. So, we want to reconcile that with the expression which we have discussed so far in fluid mechanics that could be done by changing sign convention for stress.