

Artificial Lift

Prof. Abdus Samad

Department of Ocean Engineering

Indian Institute of Technology Madras, Chennai

Lecture-10 Reservoir Deliverability-Single Phase flow

In this lecture, we will discuss reservoir deliverability. When talking about reservoir deliverability, we refer to the inflow performance relationship. First, you need to understand transient, steady-state, and pseudo-steady-state flow. There are several formulas, and I will provide the formulas. While remembering large formulas might be challenging during the exam, you must understand how to use them, grasp the units, convert units, and comprehend the purpose of these formulas. Mere mechanical learning isn't sufficient; it's essential to understand why you are learning these concepts, particularly regarding IPR (Inflow Performance Relationship) and flow regimes concerning artificial lift.

When selecting or designing an artificial lift system, you should be knowledgeable about fluid behavior, wellbore properties, how fluids flow into and out of the wellbore, how pressure and temperature changes affect fluid properties, whether single-phase or multiphase flow is present, and how factors like sand, gas, or twists impact the system. This understanding is crucial before choosing or designing any artificial lifting system. You should also understand transient and steady-state flow because if you choose an artificial lift today, conditions change tomorrow, such as pressure, temperature, or increased water influx because it is a water drive reservoir, your production rates can be affected. Water coning is one consequence: pressure forces oil and gas into the wellbore. If too much oil and gas are extracted, water might enter the wellbore instead, altering your production and water cut. Water cut represents the percentage of water in your production fluid. For instance, some wellbores might have a 20% water cut, while others have as high as 90%. In cases with a high water cut, you need to assess whether it's economically viable to continue production. If not, you must find ways to minimize water production to maximize oil and gas recovery.

It's important because water production not only doesn't generate revenue but also incurs costs for separating hydrocarbons from the water in surface separation systems. Your primary goal is to profit from oil and gas production, and dealing with excess water can be financially and environmentally challenging if not appropriately managed. When producing, you must consider the flow rate necessary to prevent water or excess gas from entering the wellbore. In an oil wellbore, you want to avoid excessive liquid ingress; in a gas wellbore, you aim to prevent fluid blockages. Therefore, understanding the different flow regimes, including transient flow, steady flow, and pseudo-steady flow, is crucial. Based on this understanding, you can decide the type of artificial lift to use and the level of productivity to achieve.

Steady-state flow,

$$q = \frac{kh(p_e - p_{wf})}{141.2B_o\mu_o \left(\ln \frac{r_e}{r_w} + S \right)}$$

Transient flow,

$$p_{wf} = p_i - \frac{162.6qB_o\mu_o}{kh} \times \left(\log t + \log \frac{k}{\phi\mu_o c_t r_w^2} - 3.23 + 0.87S \right)$$

$$q = \frac{kh(p_i - p_{wf})}{162.6B_o\mu_o \left(\log t + \log \frac{k}{\phi\mu_o c_t r_w^2} - 3.23 + 0.87S \right)}$$

Pseudo-steady-state flow,

$$q = \frac{kh(p_e - p_{wf})}{141.2B_o\mu_o \left(\ln \frac{r_e}{r_w} - \frac{1}{2} + S \right)}$$

$$t_{pss} = 1,200 \frac{\phi\mu_o c_t r_e^2}{k}$$

$$q = \frac{kh(\bar{p} - p_{wf})}{141.2B_o\mu_o \left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right)}$$

$$q = \frac{kh(\bar{p} - p_{wf})}{141.2B_o\mu_o \left(\frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)}$$

Now, let's consider a reservoir as a circular shape. It looks like this, with a wellbore situated within. The reservoir has a pressure marked as reservoir pressure, and the flow rate is denoted as Q . You also have parameters like horizontal permeability (K), flowing pressure (P_{wf}), oil viscosity (μ), formation volume factor (B_o), wellbore radius (R_w), reservoir radius (R_e), and pay zone thickness (h). Speaking of the pay zone, I've already explained that it represents where you can economically extract oil and gas. Although your reservoir may be large, not every part is conducive to oil and gas extraction. The pay zone is where production is efficient, profitable, and contributes to earnings.

If we take a top view of the situation, it appears like this: R_w represents the radius of the wellbore, and fluid flows in the direction indicated. In this complete view, we have parameters such as permeability (K), oil viscosity (μ), oil formation volume factor (B_o), and reservoir pressure (P), which I previously explained. Let's assume this is our reservoir with a wellbore at its center.

Initially, when you commence production, the reservoir pressure is very high, and the reservoir boundary is denoted as P_e . As production starts, pressure ripples from the wellbore towards the reservoir boundary. The reservoir may extend over a significant area, but production begins in your wellbore. Due to production, the pressure gradually decreases and propagates outward. For example, the pressure may reach this point after a certain time, and after some more time, it might reach here, and so on. Consequently, the pressure gradually decreases over time.

In some cases, we consider a constant flowing pressure (P_w), while in others, we assume a constant flow rate. Here, I'm guessing a continuous pressure (P_w), but it could be a constant flow rate in different situations. Now, let's define the phases: When the pressure pulse has not yet reached the reservoir boundary, we call it transient flow. However, when the pressure pulse has reached the reservoir's end, we refer to it as steady-state flow, signifying that a steady state has been achieved.

So, if it's a water-drive reservoir, as I mentioned earlier, whether it's a water-drive reservoir, gas-cap reservoir, or solution gas-drive reservoir, the reservoir boundary pressure will be primarily maintained by water pressure. In such cases, a steady-state condition can be kept

longer. However, if the pressure isn't sustained due to factors like solution gas or other issues, your pressure will decrease, leading to pseudo-steady-state flow.

In this scenario, we go through transient flow, followed by a steady state, and eventually reach a pseudo-steady state, where the pressure declines constantly. Reservoir flow depends on several factors: reservoir pay zone thickness, reservoir radius, wellbore radius, permeability, viscosity, reservoir type, and distance. For example, whether it's a gas-cap or water-driven reservoir, these factors influence how fluid flows into the reservoir.

For example, a fluid with very high viscosity will move slowly, resulting in a lower flow rate. Conversely, if the pay zone is extensive, denoted by 'h' being very high, you can have more perforations and achieve higher production rates. The reservoir's permeability ('k') is also crucial for determining flow rates, as is the total area of the reservoir. If the transition to a steady state and pseudo-steady state occurs rapidly, the decline rate can be high, potentially leading to a shorter duration of high production. On the other hand, a large and wide reservoir can sustain higher production rates over a more extended period of time. Now, let's delve into the transient flow. These equations have been sourced from Gu et al.'s 2007 book, 'Petroleum Production Engineering,' which contains these mathematical formulations.

So, the flowing pressure (P_w) at time 't' equals the initial reservoir pressure (P_i) plus a constant term. This equation might seem complex, and while you may not need to memorize it, it's essential to understand how to use it. 'q' represents the flow rate, ' B_o ' stands for the formation volume factor, ' μ_o ' denotes the viscosity of oil, 'k' is the permeability, 'h' is the reservoir thickness, and because it's transient, it's a function of time (in seconds). ' Φ ' represents porosity, ' μ_o ' is again the oil viscosity, ' C_i ' is the drainage area compressibility factor, represented as 'psi inverse,' and 'S' stands for skin.

What is skin? When drilling a reservoir and creating a wellbore, damage can occur due to rocks breaking in a certain way, which may hinder sufficient production. In such cases, we use the 'skin factor' or simply 'skin.' A completely undamaged wellbore would have a skin factor of 0. However, a specific value for the skin factor needs to be applied for damaged wellbores. When I provide a problem, I will specify the skin factor value.

If you utilize 'S' to calculate the flowing pressure 'P_w' from the wellbore, the same equation can be modified into this form. Remembering constants can be challenging, but by rearranging this equation, you can express it in terms of the formation volume factor 'Bo,' the viscosity of oil 'μ_o,' permeability 'k' in milliDarcy, reservoir thickness 'h' in feet, initial reservoir pressure 'Pi,' time 'T,' permeability again in milliDarcy, the viscosity of oil as 'Cp,' compressibility factor 'Ct' (represented as psi inverse), wellbore radius 'R_w' in feet, and the skin value (which is non-dimensional, without units).

When solving problems based on these equations, it's essential to remember the units. 'Pi' minus 'psi' equals 'psi,' where 'psi' represents the skin factor. In steady-state flow, if you observe the flow rate 'q,' it equals 'Kh' times 'P' minus 'P_{wf}.' Here, 'K' represents permeability in milliDarcy, 'h' is in feet, and 'P' is the reservoir thickness, denoted as 'h.' Additionally, 'P_{wf}' stands for flowing pressure. In the formula, 'Bo' refers to the formation volume factor (previously defined in another class), 'μ' represents the viscosity of oil in Cp, 'Re' is the reservoir radius, 'rw' is the wellbore radius, 'S' signifies the skin factor, and 'ln' represents the natural logarithm, with 'E' as the base.

Next is pseudo-steady-state flow, where I mentioned that the pressure pulse has already reached the boundary and gradually declines with time. In pseudo-steady-state flow, the formula 'q' equals 'Kh' times 'P' minus 'P_{wf},' with units as follows: permeability in milliDarcy ('K'), thickness in feet ('h'), 'P' in Psi, formation volume factor ('Bo'), viscosity of oil (μ_o), reservoir radius ('Re,' representing the pay area), wellbore radius ('R_w'), and skin factor ('S'). Since this flow type varies with time, it includes a time-dependent function denoted as 'Tp_{ss}.' The 'Tp_{ss}' formula is 'a' multiplied by '1200' and divided by the product of 'phi,' 'μ_o,' 'Cts,' 'Re,' and 'K.' All these terms have been previously defined: 'phi' for porosity, 'μ_o' for oil viscosity, 'Ct' for compressibility factor, 'R' for reservoir radius, and 'K' for permeability in milliDarcy.

Assuming the reservoir pressure is the average, the equation changes slightly. Instead of 'half,' it becomes '3,' while the other parameters remain mostly similar. The following equation also varies based on the provided area of the reservoir. Now, let's delve into the Inflow Performance Relationship (IPR), where 'q' represents the flow rate, 'Pe' and 'P_{wf}'

denote the pressure in psi. Here, 'P' stands for reservoir pressure, and 'P_{wf}' represents the flowing pressure.

$$J = \frac{q}{(p_e - p_{wf})}$$

Consider a scenario where the reservoir pressure is 'P' (or 'P_e'), and 'P_{wf}' is zero. In this case, 'P_{wf}' equals 0 implies that 'q' equals 'J' times 'P_e' minus 'P_{wf}', which simplifies to 'J P_e.' Here, 'J' signifies the slope of the curve. If you know the slope ('tan theta') and the reservoir's 'Q' (flow rate), you can determine that the inverse of the slope, denoted as '1/J,' equals 'P_e' minus 'P_{wf}' divided by 'Q,' resulting in 'Q' equaling 'J' times 'P_e' minus 'P_{wf}.'

$$J^* = \frac{q}{(p_e - p_{wf})} = \frac{kh}{141.2B_o\mu_o \left(\ln \frac{r_e}{r_w} + S \right)}$$

The inverse of the slope is referred to as the productivity index. It's crucial to consistently check units, as any errors in unit conversion can lead to different results, potentially impacting your performance and grades.

You can achieve the maximum production rate if we have a very low P_{wf}. The maximum production rate, denoted as 'q max,' is obtained when P_{wf} tends to 0, nearly 0, resulting in the minimum 'q,' which equals 0 when P_{wf} equals the reservoir pressure, or P_e (here, 'P' is written). However, this is applicable for single-phase liquid flow. The curve will appear differently in the case of a two-phase flow, such as in a solution-driven gas reservoir. It may take various forms, such as curved, straight, or completely smooth curves, each requiring separate mathematical formulations. We will explore these formulations later.

Now, let's consider a simple problem to find the productivity index for a reservoir with an average pressure (P_e or P average) of 2500 psi and a liquid flow rate of 500 barrels per day (BPD or BD). You must be familiar with the various terms used in field units, such as BBL per day (BBL/D) or barrels of oil per day (BOPD). Sometimes, they use 'stock tank barrel' (STB) for dead oil or oil under surface conditions. In this case, the wellbore flowing pressure, P_{wf}, is 2000 psi. With a pressure difference (ΔP) of 500 psi (ΔP = P_{wf} - P_e), fluid will flow. To calculate P_i (productivity index or J), you can use the formula J = Q / ΔP. In

this scenario, it becomes $500 / 500$, which equals 1. The unit for this will be 'barrel per day per psi.

Problem:01

- Find productivity index, IPR for reservoir average $P=2500$ psi.

Liquid flow rate is 500 bbl/day,
wellbore flowing pressure is 2000psi.

Assume a single phase fluid. Find PI.

$$PI = q / (a_v \cdot P - P_{wf})$$

$$= 500 / (2500 - 2000) = 500 / 500 = 1$$

bbl/day/psi

$$Q_{max} = PI \cdot 2500 = 2500 \text{ bbl/day}$$

If you look at the productivity curve, as it is for a single phase, it is already mentioned there. It shows the relationship between P_{wf} and Q , representing the Inflow Performance Relationship (IPR). However, while you have determined the productivity index (IP) from this curve, you still don't know the maximum flow rate (Q_{max}). To calculate Q_{max} , consider the value of θ , P_e , 2500 psi, and other data, such as 500 barrels per day (BPD) in this case, and P_{wf} , which is 2000 psi.

Calculating Q_{max} is straightforward: Q_{max} equals J multiplied by 2500. Since J equals 1, Q_{max} equals 2500 barrels per day (BPD). The relationship between J and P is expressed as Q equals J multiplied by $(P_e - P_{wf})$. When P_{wf} is 0 ($wf = 0$), Q_{max} equals J multiplied by P , simplifying the calculation. It's a straightforward problem. You first need to find J to find the productivity index or maximum flow rate. Once you have J , multiplying it by the reservoir pressure (P) will give you Q_{max} . Based on this, you can construct the productivity curve. So, Q_{max} is 2500, understood.

At the point where Q equals 0, it occurs when the P_{wf} becomes equal to the reservoir pressure. For example, when the flow rate is stopped in certain conditions, reservoir pressure will enter the wellbore and equal the wellbore pressure. When both pressures are equal, there will be no flow, resulting in $Q = 0$. However, if you open the valve to start producing at a high rate, the flowing pressure will be almost 0, while the reservoir pressure

will remain very high, creating a significant pressure difference between the flowing pressure and the reservoir pressure. With such a high-pressure difference, a large volume of fluid will enter. Typically, reservoir engineers or production engineers aim to avoid situations where an exceptionally high flow rate occurs. Such situations can lead to gas coning or water coning, where water enters instead of the desired oil or gas. Reservoir and production engineers work to prevent these scenarios by operating within a safe region on the P-I curve.

This slide describes single-phase liquid flow. In the case of single-phase liquid flow, the IPR curve is a straight line. The most critical criterion for single-phase flow is that the flowing pressure (P_{wf}) should be higher than the bubble point pressure (P_b). This is a crucial factor to consider when discussing single-phase flow. In contrast, if we were dealing with a two-phase flow, the flowing pressure would be lower than the bubble point pressure, which makes the situation more complex. However, for now, we assume single-phase flow, where the bubble open pressure is lower than the flowing pressure, ensuring that no bubbles are formed when the fluid enters the wellbore.

Single phase IPR,

$P_{wf} > P_b$

$$J^* = \frac{q}{(p_i - p_{wf})} = \frac{kh}{162.6 B_o \mu_o \left(\log t + \log \frac{k}{\phi \mu_o c_t r_w^2} - 3.23 + 0.87S \right)}$$

Steady state

$$J^* = \frac{q}{(p_e - p_{wf})} = \frac{kh}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} + S \right)}$$

pseudo-steady-state flow around a vertical well

$$J^* = \frac{q}{(p - p_{wf})} = \frac{kh}{141.2 B_o \mu_o \left(\frac{1}{2} \ln \frac{4A}{\gamma c_t r_w^2} + S \right)}$$

So, here again, we have the J-Stoickel productivity index, denoted as $Q P_i$ divided by P_{wf} . P_i represents reservoir pressure, P_{wf} stands for flowing pressure, B_o is the formation volume factor, μ_o is the oil viscosity, ϕ is porosity, c_t is the compressibility factor,

R_w is wellbore radius in milliliters, K is permeability, h is the reservoir thickness or pay zone thickness (not the whole reservoir), K is again in milliliters, ϕ , and S is your skin. If you have these parameters, you can calculate J . You obtain this formula derived from the previous reservoir formula for steady-state flow. Similarly, you get the formula for pseudo-steady-state flow. Based on these equations, we must plot IPR curves. If given a specific data set, such as B_o , μ_o , T , K , ϕ , C_t , R_w , S , you can calculate J . From J , you can derive your IPR curve. Ultimately, you need to obtain the IPR curve to determine your desired flow rate and how it correlates with your overall inflow performance.

So, inflow performance linked with outflow performance ensures stable production. Your artificial lifting system, surface separator system, tubing, valves, and everything else will work smoothly without any disturbances. Now, let's move on to another problem. Consider single-phase flow and draw an IPR curve for steady flow. I've made a mistake in my previous statement. It should be 'single-phase liquid flow,' and you must draw IPR curves for steady, pseudo-steady, and transient flow. In this explanation, I'll solve it for a steady flow, and you can practice the other two.

Problem:02

- Single phase flow, draw IPR curves for Draw IPR curve for steady flow, pseudo steady state, transient flow.
- Data: $P_{\text{average}}=5651$ psia
- $P_b=50$ psia, porosity =0.19, $S=0, k=8.2$ md
- $h=53$ ft, $\text{visc}=1.7$ cP, $C_t=0.00000129$ psi⁻¹
- $r_e=2980$ ft, $r_w=0.328$
- $B_o=1.1$

The provided data is as follows: The average pressure is 5651 PSI, and for this case, we assume P_e , P_i , and P_r to be in psi. The bubble point pressure is also given as 50 psi. The porosity, denoted as ϕ , is provided as 0.19. Porosity is a non-dimensional number calculated as the volume divided by the total volume. The skin value is given as 0.

If your skin is not provided as 0, then you should use the given value for calculations. However, in this case, we are assuming an undamaged wellbore. Sometimes, instead of stating S equals 0 in an exam, I might write 'undamaged wellbore.' In such cases, you should assume S is 0 for the calculations. If you don't take S as 0, you won't be able to solve the problem because the S value won't be provided. The permeability, denoted as K, is 8.2 mD, where 'D' stands for Darcy. Please note that you should not use a capital 'M' for milli. Use 'm' for milli, and avoid using 'M' for mega. Use the correct units and notations for psi, Newton meters, centipoise (cP), and other measurements.

Solution of Problem:02

$$J = \frac{q}{(P_i - P_{wf})} = \frac{kh}{141.2 B_o \mu_o \left(\ln \frac{r_e}{r_w} + S \right)}$$

$\bar{P} = 5651 \text{ psia}$, $P_b = 50 \text{ psia}$
 $\phi = 0.19$, $S = 0$, $k = 8.2 \text{ md}$
 $C_t = 0.000129 \text{ psi}^{-1}$
 $B_o = 1.1$

$$J = \frac{8.2 \times 53}{141.2 \times 1.1 \times 1.7 \times \left(\ln \frac{12980}{0.328} \right)} = 0.1806 \text{ stb/d-psi}$$

$J = \frac{q}{(P_i - P_{wf})} \Rightarrow q = J(P_i - P_{wf})$

P_{wf}	q
0	10.11
5651	0

$J = \frac{q}{P_i - P_{wf}} = \frac{10.11}{5651 - 0} = 0.001789$
 $J = 0.1806$
 $0.1806 = \frac{10.11}{5651 - P_{wf}}$
 $10.11 = 0.1806(5651 - P_{wf})$
 $10.11 = 1020.2 - 0.1806 P_{wf}$
 $0.1806 P_{wf} = 1020.2 - 10.11$
 $0.1806 P_{wf} = 1010.09$
 $P_{wf} = \frac{1010.09}{0.1806} = 5593.5 \text{ psia}$

So, you have to write exactly what I need. If you are writing something wrong, you might lose marks. Now, let's proceed with the problem. The bubble point pressure (P_b) is 50 psi, porosity is 0.919, and skin (S) equals 0. The reservoir thickness (H) is 53 feet. It will be helpful to draw the wellbore first to understand the situation better. The values you have are as follows: K (permeability) is given, H (reservoir thickness) is provided, P_i (initial reservoir pressure) is given, and P_b (bubble point pressure) is 50 psi. The reservoir pressure

(P_e) is 5651 psi, C_t (compressibility factor) is $1234 \times 10^{-9} \text{ psi}^{-1}$, the reservoir drainage radius is 2980, and the wellbore radius is 0.328.

Additionally, the formation volume factor (B) is given as 1.1. Now, let's solve this problem. It would help if you used the formula to find J , the productivity index. Calculate J using the provided values, and you already have the flow rate given. This part is unnecessary; you should proceed with these values to solve the problem. So, you can use the equation with K as 8 for J .

Now, let's calculate J . Using the formula, you can calculate J as follows: $J = (2 * H * K) / (141.2 * B * \mu * \log(R_e / (0.328 + 0)))$. We already have the H value of 53 feet, the formation volume factor (B) is 1.1, and the μ value is 1.7 cP. The logarithmic R_e value is 2980 ($R_e = 2980$), with no skin factor ($S = 0$).

After calculating these values, I obtained $J = 0.1806$. It's important to note that J is not unitless; it has units, specifically STB (Stock Tank Barrels) divided by d psi.

Now, let's calculate the other points to create the curve. Since we assume single-phase flow, we will have a straight-line curve. But what are the values? To determine that, we proceed as follows:

$$J = Q * (P_e - P_{wf}) \text{ implies } Q = J * (P_e - P_{wf}).$$

If I create a table with P_{wf} and Q , starting with $P_{wf} = 0$, we can calculate the corresponding values. When we put $P_{wf} = 0$, Q becomes $J * P_e$, which is $0.1806 * 5651 = 1011$. Then, we put $P_{wf} = 5651$, and Q equals 0.

This gives us the curve with P_{wf} on the x-axis and Q on the y-axis, with Q_{max} being 1011 barrels per day. The reservoir pressure (P_e) is 5651 psi A. This is your IPR curve; if you encounter a similar problem, you can follow these steps to solve it.