

## Artificial Lift

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### Lecture-11 Reservoir Deliverability-Two Phase Flow

Dealing with single-phase fluid flow is easier because it results in a straight-line curve. However, you cannot use the same formula when you have a two-phase flow, such as in a solution gas drive reservoir. Instead, it would help if you employed an exponential formula, essentially a quadratic equation developed by a scientist named Vogel.

Two-phase equation,

$$\text{Vogel's eq: } q = q_{\max} \left[ 1 - 0.2 \left( \frac{p_{wf}}{\bar{p}} \right) - 0.8 \left( \frac{p_{wf}}{\bar{p}} \right)^2 \right]$$

$$q_{\max} = \frac{J^* \bar{p}}{1.8}$$

$$q = q_{\max} \left[ 1 - \left( \frac{p_{wf}}{\bar{p}} \right)^2 \right]^n$$

In the case of single-phase flow, we have seen that J equals Q divided by  $(P_{\text{bar}} - P_{\text{wf}})$ . If we set  $P_{\text{wf}}$  to 0, J becomes Q divided by  $P_{\text{bar}}$  because we have  $(P_{\text{bar}} - 0)$ .

$$\begin{aligned} J &= \frac{Q}{(\bar{p} - P_{wf})} \quad \text{--- (1)} \\ P_{wf} &= 0 \\ J &= \frac{Q_{\max}}{\bar{p} - 0} \quad \text{--- (2)} \\ \frac{Q_{\max}}{\bar{p}} &= \frac{Q}{\bar{p} - P_{wf}} \\ \frac{Q}{Q_{\max}} &= \left( \frac{\bar{p} - P_{wf}}{\bar{p}} \right) \\ &= \left( 1 - \frac{P_{wf}}{\bar{p}} \right) \\ &= 1 - \left( \frac{P_{wf}}{\bar{p}} \right) \end{aligned}$$

If we divide both sides by  $Q_{\max}$ , it becomes  $Q / Q_{\max}$  equals  $(P_{\text{bar}} - P_{\text{wf}}) / P_{\text{bar}}$ , simplifying to  $1 - (P_{\text{wf}} / P_{\text{bar}})$  for single-phase flow.

But for two-phase discharge, Vogel introduced an adjustment to this equation. He inserted specific parameters and introduced a square term as well. Thus, it becomes an equation like this:  $Q / Q_{max} = 1 - (P_{wf} / P_{bar}) + 0.2 * (P_{wf} / P_{bar})^2 + 0.8 * (P_{wf} / P_{bar})^2$ . This equation is known as Vogel's equation.

$$\frac{q}{q_{max}} = 1 - \left(\frac{P_{wf}}{P}\right) + 0.2 \left(\frac{P_{wf}}{P}\right)^2 - 0.8 \left(\frac{P_{wf}}{P}\right)^2$$

$$q = q_{max} \left[ 1 - 0.2 \left(\frac{P_{wf}}{P}\right) - 0.8 \left(\frac{P_{wf}}{P}\right)^2 \right]$$

Fetkovich eq

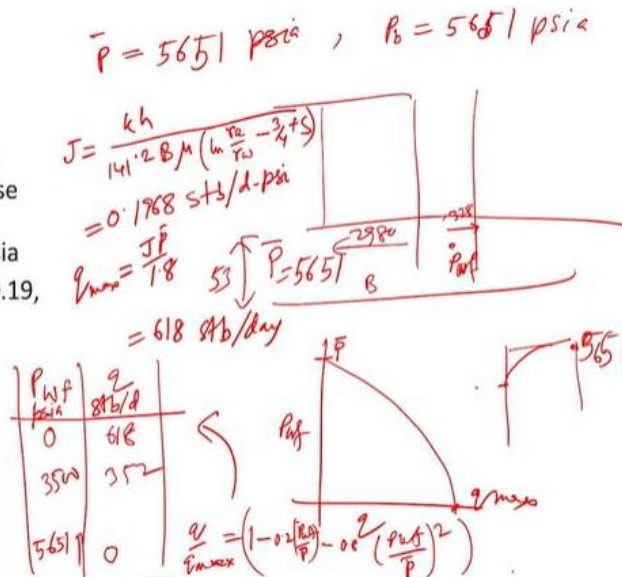
$$q = q_{max} \left( 1 - \frac{P_{wf}}{P} \right)^2$$

or  $q = c (P_{bar} - P_{wf})^2$

So, the final form is as follows:  $Q$  equals  $Q_{max} * (0.2 * P_{wf} / P_{bar} - 0.8 * P_{wf} / P_{bar})$ . This is the standard equation you should remember. There's another form of the equation called the Fickovich equation. It looks like this:  $Q$  equals  $Q_{max} * (1 - (P_{wf} / P_{bar})^n)$  or  $Q$  equals  $c * (P_{bar} - P_{wf})^2 * (P_{bar} - P_{wf})^n$ , where  $c$  typically ranges from 0.5 to 1.

### Problem:

- Construct IPR for verticle wellbore, 2phase flow. Use Vogel's eq
- Data:  $P_{average}=5651$  psia
- $P_b=5651$ psia, porosity =0.19,  $S=0$ ,  $k=8.2$ md
- $h=53$  ft,  $visc=1.7$ cP,  $ct=0.00000129$ psi<sup>-1</sup>
- $re=2980$ ft,  $rw=0.328$
- $Bo=1.1$
- $q/q_{max}=1-0.2(p_{wf}/Pr)-0.8(p_{wf}/Pr)^2$



Now, let's tackle a problem related to this equation. The problem is constructing an IPR curve for a vertical wellbore with a two-phase flow. We previously solved it for single-phase flow, but now it's a two-phase flow situation, and we have to use Vogel's equation.

$$q_b = J^* (\bar{p} - p_b)$$

$$\Delta q = q_v \left[ 1 - 0.2 \left( \frac{p_{wf}}{p_b} \right) - 0.8 \left( \frac{p_{wf}}{p_b} \right)^2 \right]$$

$$q = q_b + q_v \left[ 1 - 0.2 \left( \frac{p_{wf}}{p_b} \right) - 0.8 \left( \frac{p_{wf}}{p_b} \right)^2 \right]$$

$$q_v = \frac{J^* p_b}{1.8}$$

$$q = J^* (\bar{p} - p_b) + \frac{J^* p_b}{1.8} \times \left[ 1 - 0.2 \left( \frac{p_{wf}}{p_b} \right) - 0.8 \left( \frac{p_{wf}}{p_b} \right)^2 \right]$$

$$\text{Single phase, Unsaturated oil reservoir: } J^* = \frac{q_1}{(\bar{p} - p_{wf1})}$$

$$P_{wf} > P_b: J^* = \frac{q_1}{(\bar{p} - p_{wf1})}$$

$$P_{wf} < P_b: J^* = \frac{q_1}{\left( (\bar{p} - p_b) + \frac{p_b}{1.8} \left[ 1 - 0.2 \left( \frac{p_{wf1}}{p_b} \right) - 0.8 \left( \frac{p_{wf1}}{p_b} \right)^2 \right] \right)}$$

The data provided is similar to the previous question, with  $P_{\text{bar}}$  (or  $P$ ) equal to 5651, and the bubble point pressure is also given because it's a two-phase flow scenario. In this case, the reservoir and bubble point pressure are the same. Visualize it like this: draw a wellbore with the reservoir at  $P$  bar 5651 and the bubble  $P_{wf}$ . The  $P_{wf}$  will certainly be lower than your reservoir pressure. So, the flowing pressure is lower when the reservoir pressure is 5651 and  $P_b$  is 5651.

- An oil well (under saturated reservoir) has this data:
- $Q=108\text{b/d}$
- $P_b=1825\text{psia}$ ,  $p_{wf}=1980\text{psia}$
- $T=195^\circ\text{F}$
- $P_i=3625\text{psia}$

Above bubble pt,  
 $J=Q/(P_i-P_{wf})=0.066 \text{ stb/day/psi}$

When  $P_b=P_{wf}$ ,  $Q_{ob}=J(P_i-P_{wf})=118.5\text{b/d}$

So, that means it will create a two-phase flow when it enters the wellbore because you're making a pressure funnel. I already mentioned that a pressure funnel looks like this: with a reservoir pressure of 5651 and a flowing pressure that may be lower than this one and even lower than that. The pressure isn't maintained at the same level as 5651 inside the reservoir, so it becomes less than the bubble point pressure, automatically creating a two-phase flow.

Problem  
 IPR, use generalized vogels eq, Fetkovich eq  
 Data:  $P_{\text{average}}=3000$  psia  
 $P_{wf1}=2000$ psia,  $q_1=500$  stb/d  
 $P_{wf2}=1000$ psia,  $q_2=800$ stb/d

$\bar{P} = 3000$  psia,  $q_1 = 500$  stb/d  
 $P_{wf1} = 2000$  psia,  $q_2 = 800$  stb/d  
 $P_{wf2} = 1000$  psia,  $q_2 = 800$  stb/d

$$E_{\text{vgs}} = \frac{q}{1 - (0.2) \left( \frac{P_{wf}}{\bar{P}} \right) - (0.8) \left( \frac{P_{wf}}{\bar{P}} \right)^2}$$

$$= \frac{500}{1 - 0.2 \left( \frac{2000}{3000} \right) - 0.8 \left( \frac{2000}{3000} \right)^2}$$

$$= 778 \text{ stb/day}$$

$$C = \frac{q_1}{\bar{P} - P_{wf1}} = \frac{500}{3000 - 2000} = 0.0005 \text{ stb/d/psi}$$

$$E = \frac{q_2}{\bar{P} - P_{wf2}} = \frac{800}{3000 - 1000} = 0.0008 \text{ stb/d/psi}$$

$P_{wf}$	$q$
0	900
200	500
1000	0

$E = 0.0008 \left( \frac{q}{\bar{P} - P_{wf}} \right)$

The data is almost identical to the previous question: a reservoir thickness of 53 feet, a wellbore radius of 0.328, a reservoir radius of 2980, a formation volume factor of 1.1, viscosity of 1.7 Cp, undamaged wellbore skin, K of 8.2 milli Darcy, and porosity of 0.19. We need to calculate the IPR (Inflow Performance Relationship) data using this data.

First, calculate J using the formula:  $J = KH / (141.2 * B * \mu * \ln(\text{Re} * R_w^{(-3/4)} + S))$ . Plugging in all the values for K and H, we obtain 0.1968. The result is in stock tank barrels per day per psi, denoted as Stb/d/psi.

So, from there, you can calculate  $Q_{\text{max}}$ , which will be J times P bar divided by 1.8 as given in the formula. Plugging in the values for J, P bar, and 1.8, we finally obtain 618 STB per day, which stands for stock tank barrels per day or simply barrels per day (BOPD). Using consistent units within the same problem, avoid switching between different units.

Now, you can draw the curve again, which will look like this. The curve represents Pwf on the x-axis and Q on the y-axis, measured in STB per day per psi A. To calculate the points on the turn, start with Pwf at 0 to determine the initial flow rate. Then, try other Pwf values,

like 3500 and 5651, to create a non-linear curve. Unlike a straight line, you need to calculate more points on the curve for it to take shape because this curve isn't linear and may not follow a simple mathematical pattern.

So, from there, you can calculate  $Q_{max}$ , which is determined by  $J$  times  $P$  bar divided by 1.8 as per the given formula. Plugging in the values for  $J$ ,  $P$  bar, and 1.8, we finally obtain 618 STB per day, which stands for stock tank barrels per day or simply barrels per day (BOPD). You can use any appropriate unit, but it's crucial to maintain uniformity within the same problem and not mix multiple units.

Now, you can draw the curve again, which will look like this. The curve represents  $P_{wf}$  on the x-axis and  $Q$  on the y-axis, measured in STB per day per psi A. To calculate the points on the curve, start with  $P_{wf}$  at 0 to determine the initial flow rate. Then, try other  $P_{wf}$  values, such as 3500 and 5651, to create a non-linear curve. When you have  $Q_{max}$ , you can apply it to Vogel's equation, where  $Q$  by  $Q_{max}$  equals  $1 - 0.2 P_{wf} \text{ by } P \text{ bar} - 0$ .

So, from there, if you put  $P_{wf}$ , you will get the  $Q$  value. So, one by one, if you put 3 or 4 values slowly, the curve will be formed. Start by setting  $P_{wf}$  to 0, and you'll determine the initial flow rate. Then, try values like 3500 and 5651 and add more data points because this curve is not a straight line. When dealing with a curve, you can't rely on just 2 endpoint data; you need several points to create a smooth curve. Calculate from this equation, and you'll find that with a flowing pressure of 0, you get 618. You'll have different flow rates at approximately 35, 352, and 5651. This is how you should approach solving the problem.

Now, let's consider a situation involving a 2-phase oil reservoir.

So, you get two phases in the reservoir before it reaches your flowing pressure in the wellbore, transitioning from the initially single phase to two phases. It's like this: under certain conditions, here's  $P_{wf}$ . As it goes from here to here, it may start as single-phase and then become two-phase. It forms a curve when it crosses the sand phase boundary, which signifies the sand control system and other components entering the flowing pressure region or the wellbore region.

The graph consists of two sections,  $P_{wf}$  and  $Q$ , with  $Q_{max}$  and  $P_b$  representing the bubble point pressure. At the bubble point, you get this formula:  $J$  equals  $P$  bar minus  $P_b$ . However, the formula will be as follows when you have a two-phase flow. You've seen this  $Q_v$ ; the IPR curve shows  $P$  bar,  $P$  bubble point pressure,  $Q_b$ , and this flow rate indicates  $Q_v$  after the formation of bubbles. Then, you get some liquid along with some free gas.

So, that is called  $Q_v$ . Vogel's equation changes like this: instead of  $Q_{max}$ , you use  $Q_v$ . Up to  $Q_b$ , you get a certain flow rate, and after that, your system changes. You get two combined curves: a straight curve and a decline curve or exponential curve, and all the terms are already defined. For example,  $P_w$  is flowing water pressure, and  $P_b$  is bubble point pressure.

Using this equation, we will solve some problems. Constructing IPR curves using test points involves testing a wellbore and obtaining different flow rates and pressures. Based on that data, you construct the IPR curve. For that, we have a formula: if you have flowing pressure greater than bubble point pressure, you follow it; if it's the opposite, then you use it.

Now, using this formula, we have to solve a problem. So, one of the simplest problems involves an oil well in a saturation reservoir with this data:  $Q$  is 108 barrels per day. Again, note that multiple notations are used for barrels, such as BBL per day or BPD, so don't be confused.  $P$  is the pressure at the bubble point, which is 1825 psi. The flowing pressure is 1980 psi, almost similar to  $P$ . Temperature is also at 195 degrees Fahrenheit, and the reservoir pressure is 3625 psi. Since it is above the bubble point pressure, the formula is like this:  $J$  equals  $Q$  divided by  $P_i$  minus  $P_{wf}$ . You are getting 0.6066 STB per day when  $P_b$  and  $P_{wf}$  are equal. So, you have  $J$  equals  $P_i$  minus  $P_{wf}$ .

So, 118. Here's another problem: the average reservoir pressure is given as  $P$  average, which you can write as  $P_r$ , or simply  $P$ .  $P$  average is 3000 psi. The flowing pressure for the first test was 2000 psi, denoted as  $P_{wf1}$ , and at that time, they recorded a flow rate of 500 STB per day, or 500 barrels per day, as you prefer to say. In the second test, conducted in the same wellbore with  $P_{wf1}$  equal to 1000 psi, they achieved  $Q_2$  equal to 800 STB per day. They conducted two tests and obtained different values. To address this, you need to draw an IPR curve using Vogel's and Fedcovitch's equations. So, let's start with Vogel's equation.

$Q_{max}$  gives Vogel's equation for  $Q_{max}$  equals  $Q$  divided by  $(1 - 0.2 * P_{wf} \text{ divided by } P \text{ bar})$  minus  $(0.8 * P_{wf} \text{ by } P \text{ bar squared})$ . Now, when we plug in the data, for the first set of data, we get  $Q_{max}$  equals  $500 * (1 - 0.2 * 2000 \text{ divided by } 3000)$  minus  $0.8 \text{ times } 2000 \text{ divided by } 3000 \text{ squared}$ . So, finally, you are getting 978 stock tank barrels or STB per day. Please ensure you use uniform units when solving.

Now, create a table with  $P_{wf}$  and  $Q$  values. Equation 1 is as follows:  $Q$  equals  $Q_{max}$  times  $[1 \text{ minus } (0.2 * P_{wf} \text{ by } P \text{ bar}) \text{ minus } (0.8 * P_{wf} \text{ by } P \text{ bar squared})]$ . Now, if I plug in  $P_{wf}$  values, starting with 0, then 3000, and continue with some more values, you can calculate  $Q$ . When you plug in 0, you get 978; when you use 3000, you get 0; for example, if you input 1500, you get 685. Continue with more values, and you will eventually obtain a curve like the one shown – it won't be a straight line; instead, it will exhibit more curvature.

Now, you have to use the same data for the Fetkovich formula. I will represent it like this:  $N$  equals logarithmic  $Q_1$  divided by  $Q_2 \text{ times } P \text{ bar squared minus } P_{wf} \text{ squared}$ , divided by  $P \text{ bar squared minus } P_{wf} \text{ squared times } P_{wf1} \text{ times } W_2$ , which is also a logarithmic function. This equation yields a value of 1 when you input the  $Q_1$ ,  $Q_2$ ,  $P$ , and  $P_{wf}$  values, so  $N$  becomes 1. Now,  $C$  equals  $Q_1$  divided by  $(P \text{ bar squared minus } P_{wf} \text{ squared})$  to the power of  $N$ . Given  $Q_1$  as 500,  $P \text{ bar}$  as 3000, and  $N$  as 1, it becomes  $0.001 \text{ STB per day } \psi^{(-2)}$  minus  $\psi^{2N}$ .

It would help if you also created a table with  $P_{wf}$  and  $Q$  values for this equation. To obtain these values, refer to the Fetkovich equation:  $Q$  equals  $C$ , where the  $C$  value is  $0.0001 \text{ times } 0.001$ , multiplied by  $(P \text{ bar squared minus } P_{wf} \text{ squared})$ . You will get corresponding  $Q$  values when you input  $P_{wf}$  values such as 0, 3000, and others. The values may look like this: 0, 900, 2000 (with corresponding  $Q$  values of 500, 3000, 0). Continue inputting more values, and you will eventually generate a curve. One will represent Vogel's equation, and the other will represent Fetkovich's equation. This concludes today's lecture. Thank you very much.