Artificial Lift

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Lecture-13 Flow Over a Flat Surface or Flow Through Pipe Part-2

Let's discuss Bernoulli's equation. Bernoulli's equation takes the following form:

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = C$$

This represents the simplest form of Bernoulli's equation, also known as the energy equation. Notice the presence of the v^2 term, which indicates that it deals with energy.

Now, let's relate this equation to pipe flow. Imagine a pipe with conditions labeled as Condition 1 and Condition 2, through which a fluid flows. At Condition 1, we have parameters like p_1, v_1, and z_1. Similarly, in Condition 2, we have p_2, v_2, and z_2. Let's assume this is for single-phase liquid flow, and z is not zero; it may have a non-zero value, such as 1 meter. The elevation, z, is the height from the horizontal plane.

Now, how do we apply Bernoulli's equation in this scenario? Bernoulli's equation is applied as follows:

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho_1 g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho_2 g} + z_2$$

It's important to note that we consistently use the gravitational constant 'g' and do not differentiate it as 'g_1' or 'g_2'. In this equation, the inlet pressure or energy should equal the exit energy. Therefore, at the inlet condition, we have p_1 / (ρ_1g), velocity energy v_1^2 / (2g), and elevation z_1, while at the exit condition, we have p_2 / (ρ_1g), velocity energy v_2^2 / (2g), and elevation z_2. This equation helps us analyze fluid flow and energy changes in pipes.

So, how did Bernoulli's equation come about? You may have already conducted the venturimeter experiment if you're a mechanical engineering student. In this experiment, you have a pipe configuration like this and a U-tube manometer. Fluid flows through the pipe like this, and the U-tube manometer, a simple instrument, contains a single pipe filled with a high-density fluid, such as mercury.

When there's no fluid flow, the mercury levels in the U-tube will be equal and at the same height. However, when fluid flow is initiated, let's say at a flow rate Q, something interesting occurs. The larger-diameter section of the pipe offers lower resistance and, consequently, lower pressure, while the smaller section experiences higher pressure, all with the same flow rate.

Now, if you refer to Bernoulli's equation, you'll notice that as velocity increases, pressure decreases, and as velocity decreases, pressure increases. In this scenario, the high-velocity area will have lower pressure, making the mercury columns read as '2' and '1'. So, in section '2,' the pressure is lower, while in section '1,' the pressure is higher. The height of the columns indicates the pressure difference.

When you observe a certain difference in pressure, you can estimate the pressure drop within the pipe. This equation is known as Bernoulli's equation, which is a universal equation. To demonstrate it, you can conduct fluid flow experiments using manometers. Most mechanical engineering laboratories in colleges include this experiment because it's a straightforward way to determine pressure drops in pipes and the relationship between P_1 and P_2.

Bernoulli's equation, however, comes with certain conditions. It assumes the absence of friction, as we're doing here. The equation holds if we assume there are no pumps or other devices at work and no internal energy transfer, such as heat transfer. Essentially, this is an energy equation, and it remains valid as long as there are no alterations to the energy state. If you introduce or extract energy through a pump or internal means, the equation may not be applicable.

If you need to account for additional factors, use Bernoulli's equation's modified form. So, what does this modified form entail? It includes the following components: P square by 2 g plus V square by 2 g plus z plus the friction factor term and any energy related to pumps or other systems either entering or leaving the system. This modified form ensures the applicability of Bernoulli's equation under these conditions.

However, when using the simplest form, it's important to note that we assume there is no energy exchange with the system, neither incoming nor outgoing. Consider all relevant terms, including friction and pump energy, if you opt for the generalized or modified form. You'll need to calculate friction separately, which can be done using tools like Moody's diagram or Darcy-Weisbach's diagram.

These tools help determine the friction encountered, and you'll also need to account for the energy input from the pump. For instance, a pump will be involved if you're pumping fluid from one floor to another in a building with multiple floors or even to a high-rise structure like the Burj Khalifa, which has 134 floors. Therefore, you need to consider piping friction loss and the energy input from the pump.

So, all of these factors need to be calculated. When dealing with an artificial lifting system in the wellbore, you must also compute the pump's energy output. This includes considering factors such as the net positive suction head and other losses. Many variables come into play. However, the simplest form remains p square by 2 g, v square by rho g, plus z equals a constant.

This formula is universal and won't typically be provided during exams. But occasionally, you might be given the modified form or asked to include certain elements. Now, let's discuss Darcy's formula for pressure drop. Darcy developed a formula to calculate pressure drop, which looks like this: v square by 2 g plus p by rho g plus z equals c. Interestingly, this formula is based on Bernoulli's equation, but Darcy tested it differently.

To measure pressure drop through a pipe, he set up an experiment using a pipe, a reservoir, and an outlet with a storage tank. He also incorporated a piezomanometer, a pressure measurement system. At location 1, he observed very high pressure, while at location 2, he recorded significantly lower pressure. Between these points was a bed of sand, which introduced substantial resistance to the fluid flow.

So, there's a change in pressure from p_1 to p_2. Using that formula, he verified this equation: V square by 2 g plus p by rho g plus z equals 0, which equals a constant. Now, we've seen Bernoulli's formula through Darcy's experiment. Let's try to apply it to our pipe flow.

The pressure drop formula, Δp, equals p_1 minus p_2. How do we calculate p_1 minus p_2? It's like this: If this is point 1, and this is point 2, forming an angle θ, which are the flow rates, q_1 and q_2, with p_1 and p_2, we are assuming single-phase liquid flow with constant mass flow rate. Therefore, the mass flow rate remains constant, denoted as u. Density can also be treated as nearly constant. So, the density is almost constant, making 1 minus rho roughly equal to 0, and the mass flow rate remains constant. Velocity, however, does change, and we consider the diameters, d_1 and d_2, and the speed, u1.

$$\Delta P = P_1 - P_2 = \frac{g}{g_c}\rho\Delta z + \frac{\rho}{2g_c}\Delta u^2 + \frac{2 f_F \rho u^2 L}{g_c D}$$

$$N_{\mathrm{Re}} = \frac{Du\rho}{\mu}$$

$$N_{\mathrm{Re}} = \frac{1.48 q\rho}{d\mu}$$

$$f_F = \frac{16}{N_{\mathrm{Re}}}$$

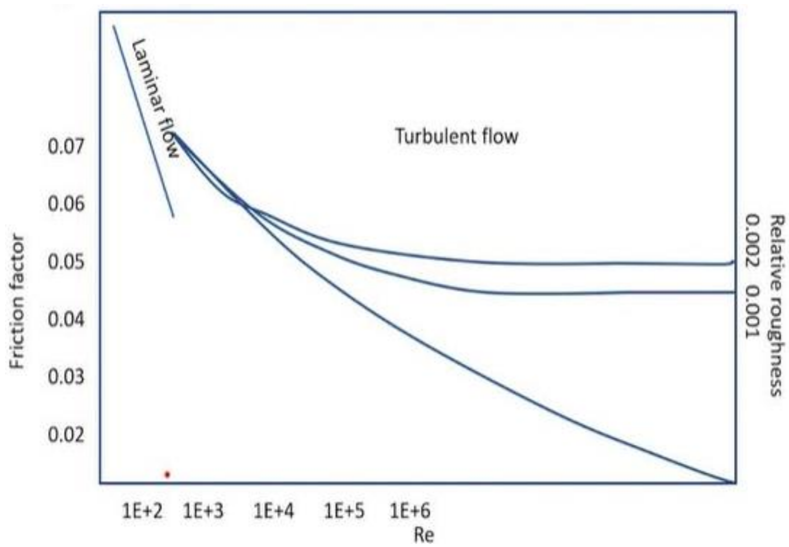
$$\frac{1}{\sqrt{f_F}} = -4 \times \log\left\{\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{\mathrm{Re}}}\log\left[\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{\mathrm{Re}}}\right)^{0.8981}\right]\right\}$$

$$f_F = \frac{f_M}{4}$$

We account for the gravitational constant, g, in FPS units, typically 32 (or, more accurately, 32.17) pounds per second squared. The ratio for gc is normally 1. If there's a location change, then the g_c value may differ, but typically we assume g divided by gc equals 1, which is approximately 32.17. For the exact value, you can refer to your textbook. Rho represents density, and del z is the elevation change. So, rho is the density, g by gc, where gc is again typically 32, and del u represents the change in fluid velocity. Here, we also consider the Fanning friction factor, denoted as f_F.

So, normally, the Fanning friction factor will be obtained from Moody's diagram. I will explain Moody's diagram later. When you use Moody's chart and obtain a specific friction factor value, you have to divide by 4 for the Fanning friction factor. So, 'fm' represents

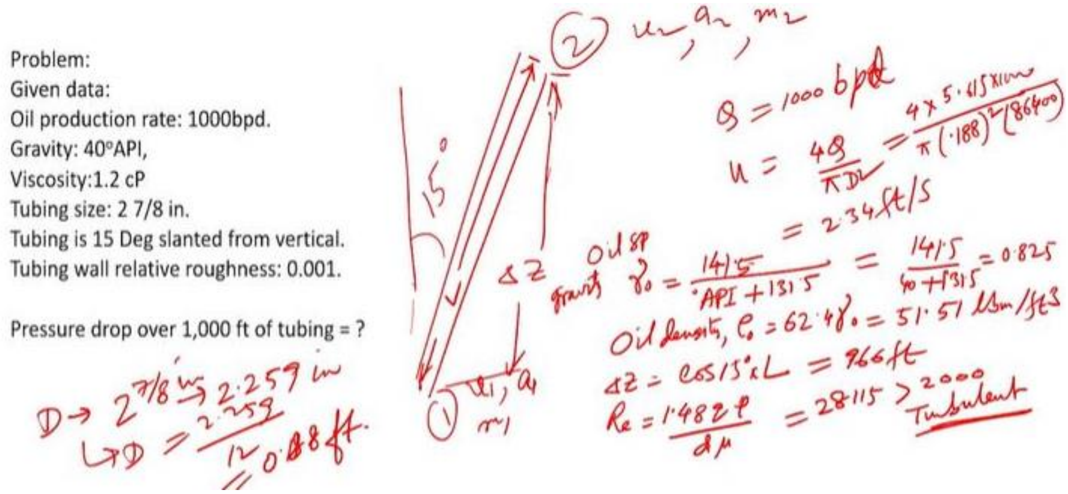
Moody's friction factor, and 'd' represents the diameter. Every time, you must use proper units; if you do not, everything will be incorrect. Here, 'P_1' and 'P_2' are given in psi, so both are in psi. 'g' divided by 'gc' becomes dimensionless, and 'rho' represents density (in pound mass divided by kg per meter). 'ft^3' and 'del z' again have fitting units. 'd' also fits the unit, and 'l' represents the length of the pipe. Moody's graph is shown below.



First, you need to know these values to calculate the pressure drop through the pipe. Friction factor - how to obtain the friction factor? First, you need to calculate the Reynolds number. Why is the Reynolds number required? Because the friction factor depends on the Reynolds number. So, how do you calculate the Reynolds number? For pipe flow, the Reynolds number (NRe) equals 'du' divided by 'mu,' where 'd' represents the diameter, 'u' represents the fluid velocity ('Q' divided by 'A'). Therefore, if 'Q' is the flow rate, you can divide it by the cross-sectional area. 'A1' and 'A2' represent the areas at points 1 and 2. The velocity will be different at locations 1 and 2. You know the pipe diameter, 'rho' will be given, and 'mu' will be given.

If you have laminar flow, you can calculate the friction factor value using a simple manual calculation: '16/NRe.' However, for turbulent flow, we typically use this equation: '1 / (f_f),' a larger equation for turbulent flow. The equation provided above is for laminar flow, referred to as 'failed unit.' So, if you are using the 'failed unit,' you have to incorporate the value '1.48' due to this unit change.

If you are using SI units, it's typically easier, but when data is given in 'failed units,' converting it back to SI units can sometimes be challenging, as you have to remember many digits. Therefore, if you can remember the 'failed unit' conversion, it would be very helpful.



Now, let's try to solve a problem using the previous formula. Given the data: oil production rate of 1000 barrels per day, gravity 40 degrees API, viscosity 1.2 cP, tubing size 2 and 7/8 inches, tubing slanted at a 15-degree angle from the vertical, tubing wall relative roughness 0.001, and pressure drop over 1000 feet of tubing, we need to calculate.

So, the oil production rate is given, and first, you need to draw the tubing. This is the tubing, and I will draw something different: this is the tubing, which is given as 15 degrees, and this is 0.1, this is 0.2, and the tubing length is given as 1000 feet. At one end, assume velocity 'v_1,' 'u_1,' and 'u_2,' area 'A_1,' 'A_2,' mass 'm_1,' and mass 'm_2.'

First, you have to calculate fluid velocity. To calculate fluid velocity, use the formula 'q = 1000 barrels per day (BPD).' Now, 'q' is 1000 barrels per day, and you can calculate it as '4q / (πd^2).' So, 'π / 4 * d^2 / 4q' equals '4 * unit conversion 5.615 * 1000 / (π * (0.188)^2) * 186,400.' This finally gives '2.34 feet per second.' We are assuming the same piping diameter because no information is given about a change from 0.1 to 0.2. 'u' will remain the same from inlet to outlet since the density is constant; we are assuming negligible density change.

Now, we have to calculate the oil-specific gravity. The oil-specific gravity will be calculated as '1.41 * 141.5 / (API + 131.5).' Substituting the API value, it becomes '1.41 * 141.5 / (40 + 131.5),' resulting in '0.825.' So, the oil density (rho oil) can be calculated as '62.4 * gamma O,' finally giving '51.57 lb (pound mass) per cubic foot.'

Regarding the elevation, due to the elevation difference (Δz), it can be calculated as 'cos 15 degrees * L,' where 'L' represents the tubing length. This gives '966 feet.'

You have 'u' and 'd,' and to calculate the Reynolds number, you can use the formula '1.48 * Q * ρ / (d * μ).' This calculation results in '28115,' which is more than 2000, indicating turbulent flow. So, when you get into the turbulent zone, I forgot to mention that when the problem provides tubing size, it usually refers to the outer diameter. You may need to assume the inner diameter or refer to a table or data provided for the inner diameter.

If you do not use the inner diameter and rely on the outer diameter only, your problem and solution will be incorrect. So, for a 2-7/8 inch tubing, the inner diameter can be 2.259 inches. This information can typically be found in API tubing design tables. Normally, data will be provided, but if it's not, you may need to assume a tubing thickness and calculate accordingly. However, it's essential to note that the given size represents the outer diameter, and fluid flows inside the tubing, so you must calculate the inner diameter. You can assume an inner diameter or obtain data from the provided tables. Using the exact inner diameter for your flow calculations will be accurate.

This gives a 'D' value of 2.259 divided by 12, resulting in 0.188 feet. Now, you can utilize Jain's correlation because you have the Reynolds number (Re) value of 28115. In the previous slide, we introduced Jain's correlation. This extensive correlation here, or you can use the chart known as Moody's diagram. Using the chart will provide you with Moody's friction factor (f_m), not the fanning friction factor (f_F). To calculate f_F, you need to divide fm by 4.

In the previous section, we discussed that our Reynolds number is 2 * 28115. In this case, it will be located somewhere, and you need to determine the piping roughness or relative roughness value. What is relative roughness? It is the ratio of the exact roughness value of the pipe to the pipe's diameter. Sometimes, we provide the relative roughness value in the problem. In this case, a pipe roughness value of 0.001 is given. So, with a Reynolds number value in this range, you would consult the chart and find the corresponding relative roughness value.

Relative roughness values are displayed in this chart, and we can obtain the friction factor value. This is a hand-drawn representation, so that the values may vary slightly. We may provide the exact Moody's diagram in exams, allowing you to obtain precise values. However, this illustration closely resembles the actual Moody's diagram.

First, you need to determine the Reynolds number value and then check the Reynolds relative roughness value. The relative roughness value can vary. For instance, it might be 0.02 or 0.003, 0.004, 0.0001, or perhaps 0.001. As you move downward on the chart, the roughness on the surface decreases, resulting in lower friction factor values. Conversely, when you increase the pipe's roughness, the friction factor also increases. If you look at the left side of the chart, you'll notice that the friction values gradually increase from the bottom to the top.

We've seen the formula fF = 16/Re for laminar flow. If your Reynolds number is very low, you can directly use the formula 16/Re or refer to a table or chart like this one. The chart features a straight line, which represents the laminar flow region. However, you enter the turbulent flow region as the Reynolds number increases. In this case, you must consider the Reynolds roughness number, where higher roughness corresponds to a higher friction factor and increased pressure drop.

So, from the actual table, we obtained a roughness value, Moody's friction factor, which is 0.0265. Therefore, when we calculate f_F (Fanning friction factor), we get 0.0066. With these values, we can now calculate the pressure drop (Δp) using the following formula:

$\Delta p = (g / g_c) * \rho * \Delta z + (\rho * g / g_c) * \Delta u^2 + 2 * f_F * \rho * u^2 * (l / g_c) / (d / g_c)$

Since there is no change in the pipe diameter (d) and the mass flow rate remains the same, u remains the same. Therefore, only these terms need to be considered.

With $g / g_c = 1$ and $\rho = 51.57$ (as calculated) * 966 + 2 * 0.0066 * 51.57 * (2.34^2) * 1000 * 32.17 / 0.188

Finally, it comes out to be 350 psi. It is recommended to practice similar problems; some problems are available in the textbook, Guvan Galambar's book on petroleum production engineering, which you should try.