Equipment Design: Mechanical Aspects
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Lecture 11
L/D Ratio of Vessel

Welcome to the first lecture of week 3 of course Equipment Design: Mechanical Aspects and in this lecture we will discuss L/D ratio for a pressure vessel. So let us start the discussion.

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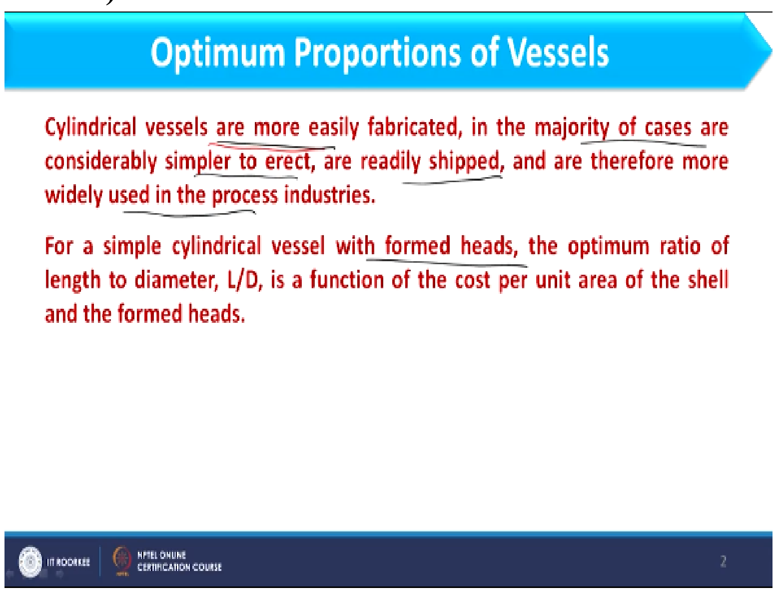

Now, basically when I am saying L/D ratio, it means I am speaking about optimum L/D ratio for the pressure vessel, okay. And till now you must have the idea that pressure vessel which we are going to design it is extensively used in chemical process as reactor, as separator, as heat exchanger, and there are many more equipment comes under the category of pressure vessel. And the vessel which I am going to consider over here for optimum proportion, it is related to cylindrical vessel.

Because proportion will only matter for cylindrical vessel, not for spherical vessel, okay. So, cylindrical vessel are more easily fabricated in majority of cases and these are simpler to erect, erect means assembly, and are easily or readily shipped and therefore more widely used in process industry. So it has advantage in shipping, it has advantage in erecting, and therefore we are considering cylindrical vessel for optimum proportion.

Now, what happens when I am having cylindrical vessel if dimension of cylindrical vessel is significantly high, we can easily transport that in different parts, and different parts of cylindrical vessel can be easily joined, okay. So therefore it becomes; therefore erection as well as shipment for cylindrical vessel becomes easier. Now for a simple cylindrical vessel with formed heads, what is formed heads you must have the idea of that till now. The optimum ratio of length to diameter is the function of cost per unit area of shell and the formed heads.

So, as far as optimum proportion is concerned, we will optimize this ratio of L/D based on economy and economy would be the cost involved for preparation of that vessel and as far as components are concerned we are considering shell as well as formed heads, okay. So in this lecture we will discuss, we will demonstrate how to find out optimum L/D ratio for cylindrical vessel with formed head. In the similar line you can apply that for other geometry also, okay.

So, as far as simple vessel is concerned L/D ratio can be decided based on economy. However, in other or complicated geometry, for example, distillation column, heat exchangers, where inside the column number of trays are involved, number of tubes are involved in heat exchanger, in that case L/D ratio will not be found directly as we are going to demonstrate now. In that case, we have to consider many more parameters. So, that it can have optimum L/D ratio, okay. So, in this lecture optimum L/D ratio we can found based on cost analysis.

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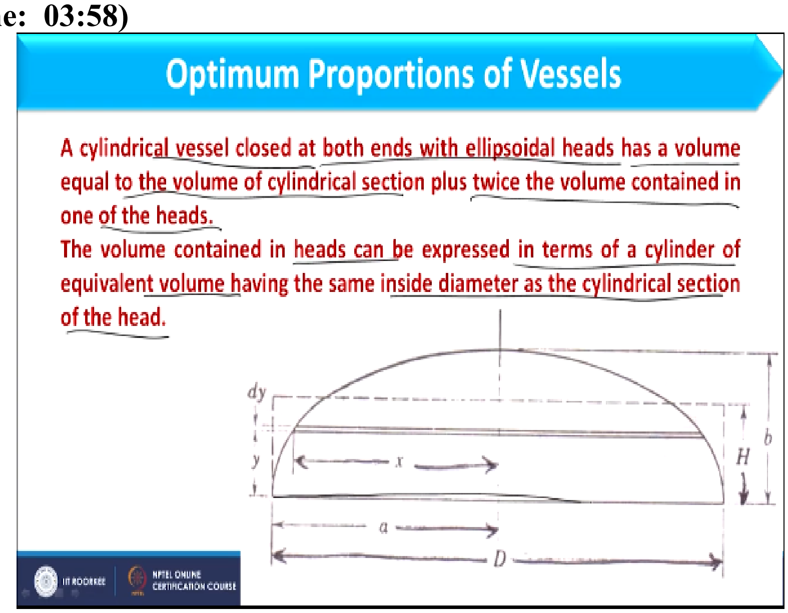

So, for optimum proportion of vessel we have considered cylindrical vessel closed at both ends with ellipsoidal heads, okay. So, formed head we are considering which is ellipsoidal head and has volume equal to the volume of cylindrical section plus twice the volume contains in one of the heads. So, if I am considering total volume of the vessel that will be nothing but volume of shell as well as volume of heads. So, both are ellipsoidal so we can consider volume of one head and consider twice value of that to calculate total volume of the vessel.

And, if you consider volume of the cylinder that we can determine very easily, however, for form section we have to calculate that volume and that volume we can correlate with the cylinder. So, volume contain in heads can be expressed in terms of cylinder of equivalent volume having same inside diameter as the cylindrical section of the head. So, for cylindrical section we have a particular diameter and when we are considering rectangle of same diameter we can represent the volume of head as a rectangle, as you can see from this schematic.

In this schematic, D is the diameter of head and this is basically ellipsoidal head of total height b and radius is a, now total volume which is available in this that can be represented by this rectangle, okay. And this rectangle should have height H. So, volume of this particular rectangle will be equal to volume of the dome section. Now, we need to derive the expression for that.

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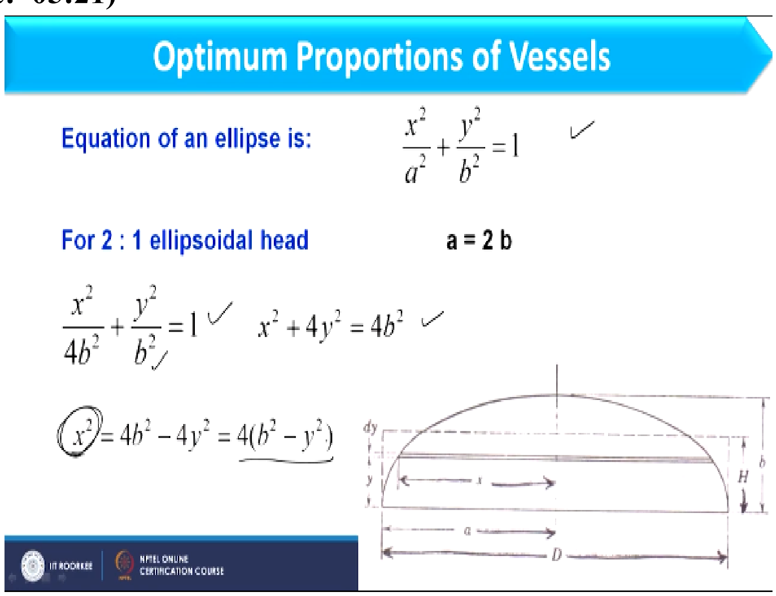

So, in this case as I am considering ellipsoidal head I will use equation of ellipse, which is x 2/a 2 + y 2/b 2 = 1. If I am having 2:1 ellipsoidal head, a should be 2 and b should be 1, so a should be 2b. So, what is a and b, that is already given in this diagram and that we have discussed already.

So, x 2/4b 2, here a we have replaced with 2b and plus y 2/b 2 and that is equal to 1. For resolving this, we have x 2 + 4y 2 + 4b 2. And then we can extract x 2 from this that is 4b 2 – 4y 2, so 4 we can take out and it is b 2 - y 2. So, in this way we can have x 2 in terms of 4b 2 - y 2.

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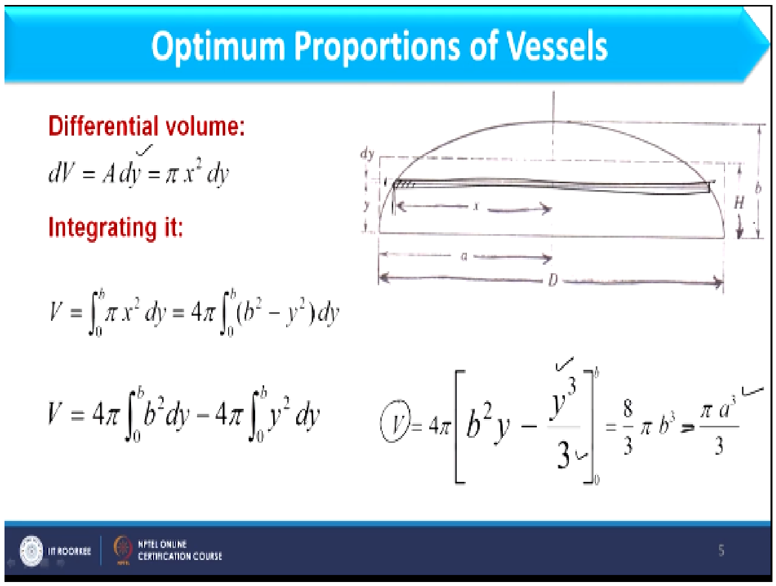

Further, to calculate the volume of this domed head we have to consider the differential part and that differential part will be integrated for whole geometry, okay. So that differential volume would be dV = A dy as it is shown over here. A is basically what, if I am considering A is the cross sectional area of this section, which is x distance from the center, okay. So that Pi x 2 that is the cross sectional area into dy, dy is this height, so that would be the differential volume of this strip.

Integrating this volume from 0 to b, we can have 0 to b Pi x 2 dy, x 2 in terms of y 2 we have already seen in the last slide, so that we can keep over here, so that is 4 Pi 0 to b, b 2 - y 2 * dy. So integrating this and resolving this we have final volume as V = 4 Pi b 2 y - y 3/3, okay. After resolving this, we have 8 by 3 Pi b 3, and b 3 we can write in terms of a and final expression comes as Pi a 3/3. So total volume of domed section will be equal to Pi a 3/3. However, if I want to represent that volume as volume of a cylinder it means Pi a 2 * H that would be total volume of the cylinder, okay.

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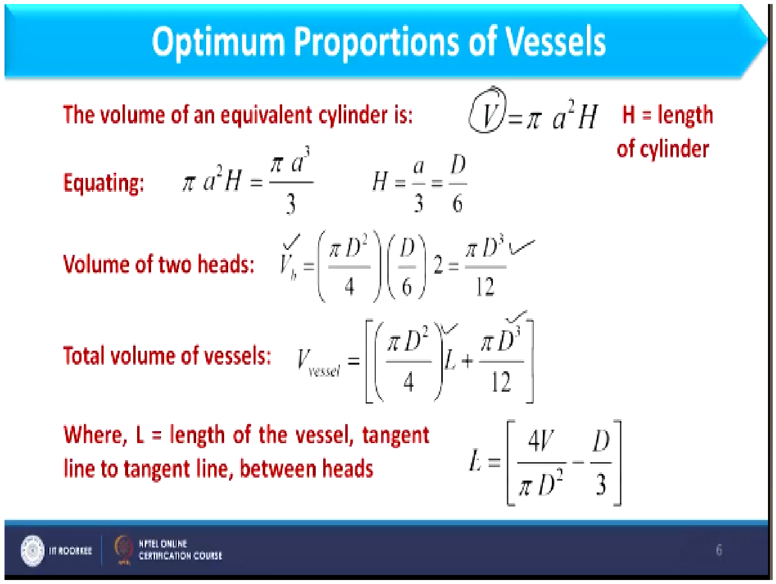

So Pi a 2 * H, where H is the height of the cylinder that we have considered. Now, this volume would be volume of dome section, which we have just derived. So that Pi a 2 H will be equated to Pi a 3/3. And after resolving this, we can find H as a/3 and then a is basically D/2, so in terms of diameter it is D/6. So H we can represent in terms of diameter.

Now volume of 2 heads that is V h that would be equal to Pi D 2/4 * H. So H is D/6 * 2 because 2 heads are there. So after resolving this, we have Pi D 3/12. So this is total volume of the head. Now, total volume of the vessel is the volume of cylinder plus volume of two heads. So here we have volume of the cylinder that is Pi D 2/4 * L + Pi D 3/12, this is for two heads.

So in this way total volume of the vessel we can write. Where L is the length of vessel that is tangent line to tangent line between heads. Now what is tangent to tangent length. If I am considering cylindrical vessel and if domed heads are kept over this. Now what happens when domed head are placed over the cylinder it prepares a tangent, okay. So tangent to tangent length means where the heads are connected. So tangent to tangent length, it means the length of shell only.

So wherever it is written in text that is tangent to tangent length, you should understand that is nothing but the length of shell. So here we can write the expression of L as 4 V/Pi D 2 – D/ 3. Now what is this volume, this volume is the total volume of the vessel, okay. Now, what this volume signifies. For example, if I am having any plant I know the production capacity of that plant, okay. Now, once I know the production capacity, I know the raw material requirement in a given ratio.

So, once I know the raw material requirement I will be aware that how much volume of feed will be used for processing, okay. So that volume we will consider for design purpose and we will consider some extra volume to that so that it will be slightly over designed, so that volume is nothing but whatever V I am considering over here. It means when you are calculating L/D ratio, it means that L/D ratio should be related to the given volume, which you need to handle or for which you need to design the equipment. So this volume is basically used to size the equipment.

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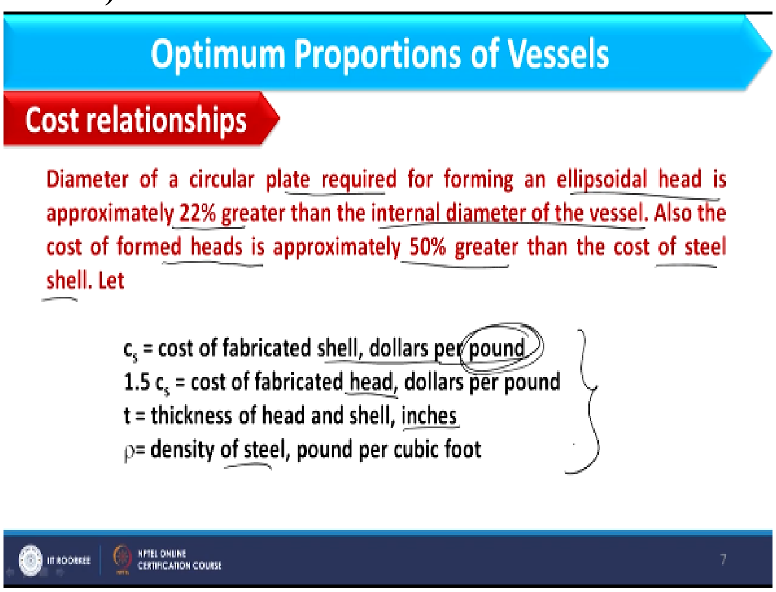

Now we have the cost relationships. Diameter of a circular plate required for forming an ellipsoidal head is approximately 22% greater than the internal diameter of the vessel. What is the meaning of this. If I am having internal diameter of the vessel is D, it means the plate through which ellipsoidal head will be prepared that would be 22% higher than the diameter of the vessel, so it would be 1.22D.

Now what is this 22% greater value. It is approximately 22% higher, so if you remember the lecture of design of heads, there we have discussed the blank diameter. What is that blank diameter. Blank diameter is the diameter of the plate through which formed heads are made. So this 22% extra to the shell diameter we are considering as blank diameter. And also the cost of formed heads is approximately 50% greater than the cost of steel shell. Now, how I will prepare the shell.

Shell is usually prepared when I am having a metal sheet, we roll that metal sheet and then we weld at one edge, okay. So in that way shell will be prepared. But how we will prepare the formed head. Formed heads are prepared for a given plate and then we continuously give a forming section and we continuously give a forming shape to this. So preparation of ellipsoidal head or formed head is very costly in comparison to shell. So that will be taking approximately 50% more than the fabrication cost of shell.

So considering all this factors I am having C s, C s is basically cost of fabricated shell that is dollar per pound, pound means it is defined per unit mass. So C s is the cost of fabricated shell, so 1.5 * C s is the cost of fabricated head; that is dollar per pound, okay. And then we have considered t as the thickness of head and shell and that is given in inches. So in that case I am considering thickness of shell and head as equal and which is equal to t for derivation purpose, however, you can consider different thickness also. And Rho is the density of the steel, which is pound per cubic foot. So these are few parameters which I am going to consider for cost calculation.

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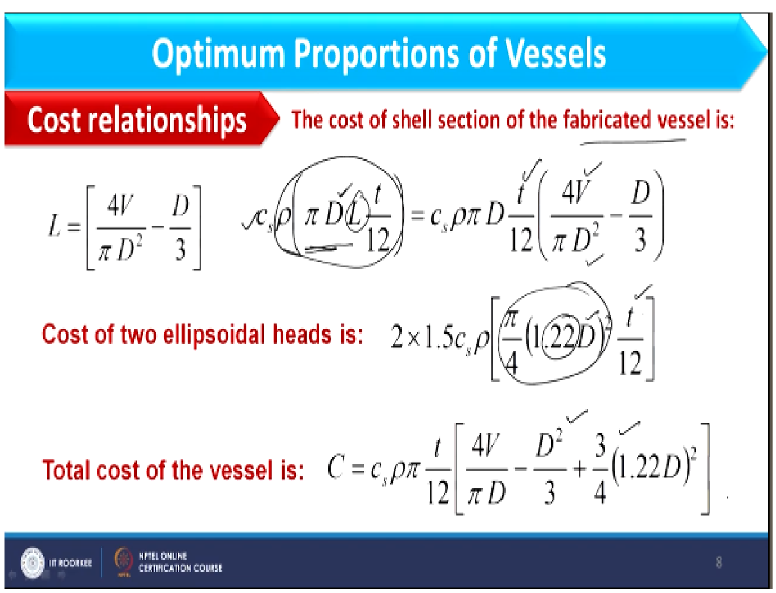

So cost of shell section of the fabricated vessel is C s into this. Now what is this. Because C s is basically dollar per unit pound, so we need to find total weight of the material which is involved to prepare shell and that can be given as Pi DL t/12. So basically Pi D is the periphery of that metal sheet into t/12, so that would be the area and that will be multiplied with L to get the total volume of metal sheet.

Now, why I am taking this t/12 because diameter and length usually I am having in foot and t is in inches, so t/12 I consider to make the uniform unities and then it will be multiplied with the Rho that is density, so it will give total mass of the material used to prepare shell into C s, okay. Further resolving this because we have the expression of L in terms of volume, I am considering in pressure vessel, so replacing the expression of L here, we have C s Rho Pi D t/2, 4V/Pi D 2– D/3. So this is the cost of shell section where L is this, which we have replaced over here.

Now cost of two ellipsoidal heads that is 2 because we have two heads into 1.5 C s because fabrication cost of ellipsoidal head is 50% more than that of shell into Rho that is density of material Pi/4 1.22D whole square. It means this is the cross sectional area of diameter of metal sheet by which head will be prepared, so that is usually 22% higher than the diameter of shell into t/12 because t s the thickness of that head.

And here because we have considered same thickness for shell as well as head, so both t are equal, this t as well as this t, otherwise you can consider different t also. So resolving these expressions, total cost of vessel we can find as C, which is equal to C s Rho Pi t/12, 4V/Pi D-D 2/3 + 3/4 1.22 D whole square. So this is the expression of total cost of the vessel.

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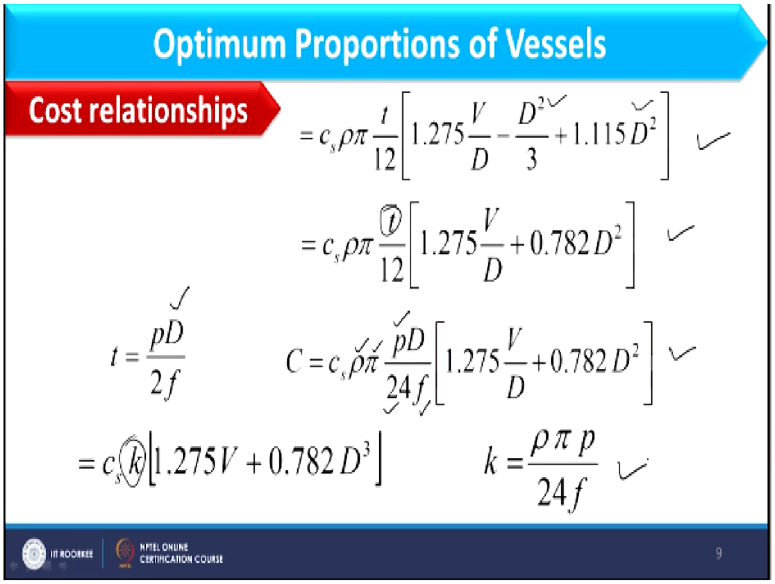

So further resolving this, we have total cost expression like this and then we can again re-write this by because here I am having D 2/3 and here I am having D 2, so further resolving this will give expression like this, okay. And now t, what is this t, tis basically the thickness of shell as well as head, so I can use here the expression for thickness of shell and this PD/2f if you remember this is related to thin walled vessel, okay.

You can use thick walled vessel expression also, so here t we can replace by this expression, so this total cost of the vessel would be C s Rho Pi pD/24f bracket expression will remain same, okay. Further resolving this, we can have this expression where we have considered K as all constants. Now what would be the constants, if you see, here I am having density constant, Pi constant and pressure constant, allowable stress constant and this is constant, so collectively I am calling that as K factor and D we can kept inside, so I am having expression 1.275 V+ 0.782 D 3, okay, where K is this.

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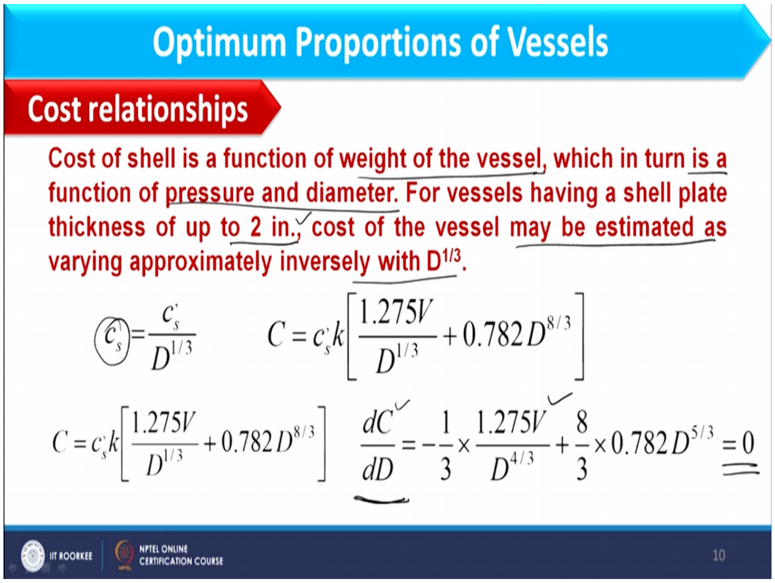

Now, further we will consider cost of shell as a function of operating condition, okay. In that case, cost of shell is a function of weight of the vessel we have already considered, which in turn is a function of pressure and diameter, okay. Now why it is function of a pressure because based on pressure we can find what should be the thickness of the vessel.

So pressure will play its role to compute the thickness because based on thickness as well as diameter I will come to know that how much material will be required. So for vessels having a shell plate thickness up to 2 inches, here I am having the guideline that up to 2 inches cost of vessel may be estimated as it varies inversely to D 1/3, okay. So this is the cost function where C s = C-/D 1/3, so C s we have considered cost of shell, so cost of shell varies with respect to diameter, however, thickness we have considered up to 2 inches, okay.

So this C s value we will put in the previous expression so here we have C s – K 1.275V/D 1/3 +0.782 D 8/3. So this is the same expression and then we can differentiate this function as a function of diameter. Now why I am differentiating this because I have to find out optimum cost and according to the optimum cost, I will find optimum L/D ratio. So to optimize the cost we will differentiate that and that comes as dC/dD = -1/3 * 1.275V/D 4/3 + 8/3 * 0.782 D power 5/3. And for optimization purpose, we will consider DC/D = 0, okay.

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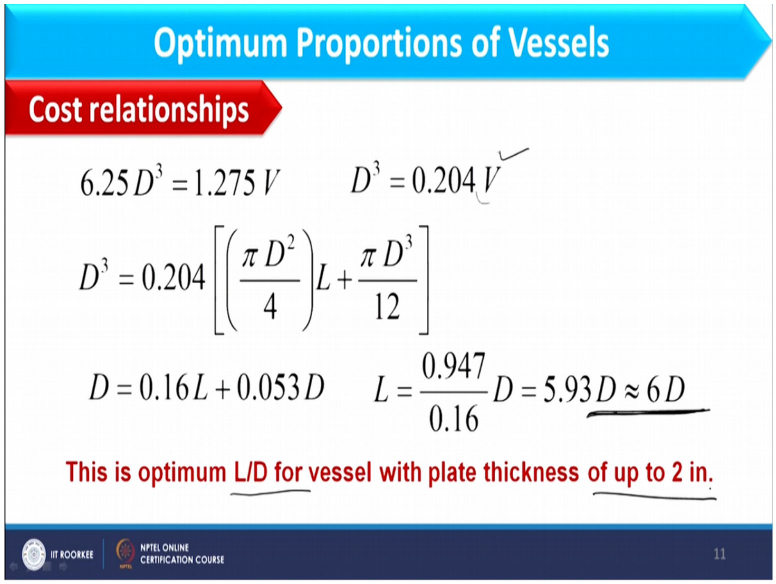

So putting this function to 0 we can have 6.25 D 3 = 1.275 V. So from this expression we can extract D 3 which is equal to 0.204 V and this V expression I already know as the volume of cylinder plus twice the volume of head as we have seen earlier. So that expression of V we can put over here. So D 3 will become 0.204 (Pi D 2/4L + Pi D 3/12), okay.

Further resolving this, I have D = 0.16L + 0.053D and here we can extract L as a function of D and then we can see that L will be approximated to 6D. So L/D ratio in that case is 6. So this is the optimum L/D ratio for the vessel with plate thickness up to 2 inches. So L/D up to 6 we can see when thickness of vessel will be up to 2 inches, okay.

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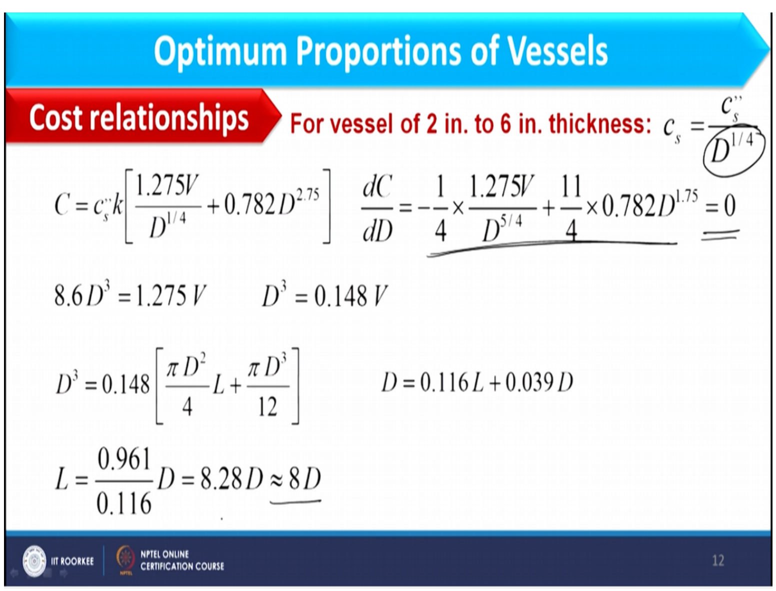

Now we can derive expression for optimum ratio for L/D for thickness when it is varying from 2 inches to 6 inches, and in that case C s will be inversely proportional to D power 1/4, okay. So in that case C would be equal to C s double dash K and then we have the whole expression. Further differentiating it we can obtain this expression and then this expression we have to equate to 0 to get the optimum value. So in that case we have found that 8/6 D 3 will be equal to 1.275V and then D 3 we can find in terms of L as 0.148 (Pi D 2/4L + Pi D 3/12).

Further resolving this, we can have L/D ratio as 8. So when I am having vessel thickness from 2 inches to 6 inches, we can have L/D ratio as 8. So in that way we obtain optimum L/D ratio for cylindrical vessel when ellipsoidal heads are considered. And in the same line we can also derive expression for different geometry, for example if I am having cylindrical shell with conical head etc., so considering all these geometry we can find the optimum ratio of given geometry.

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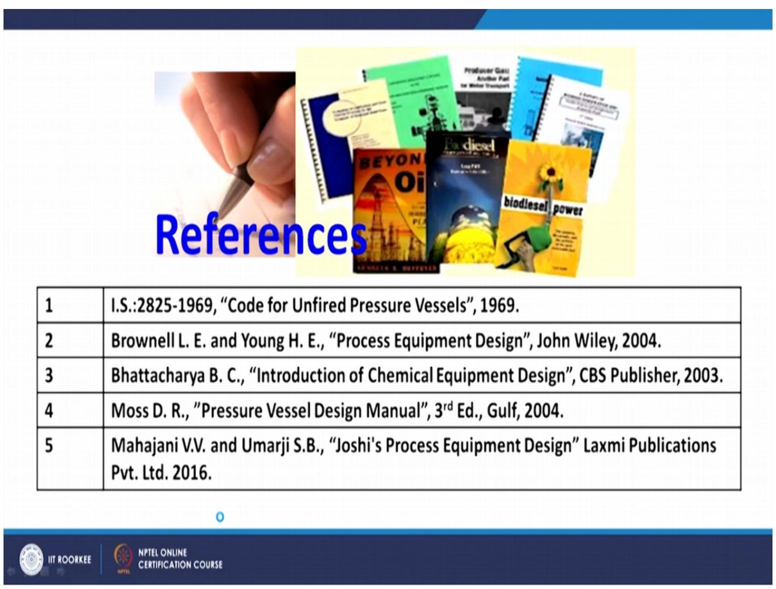

And here I am having some of the references which you can go through.

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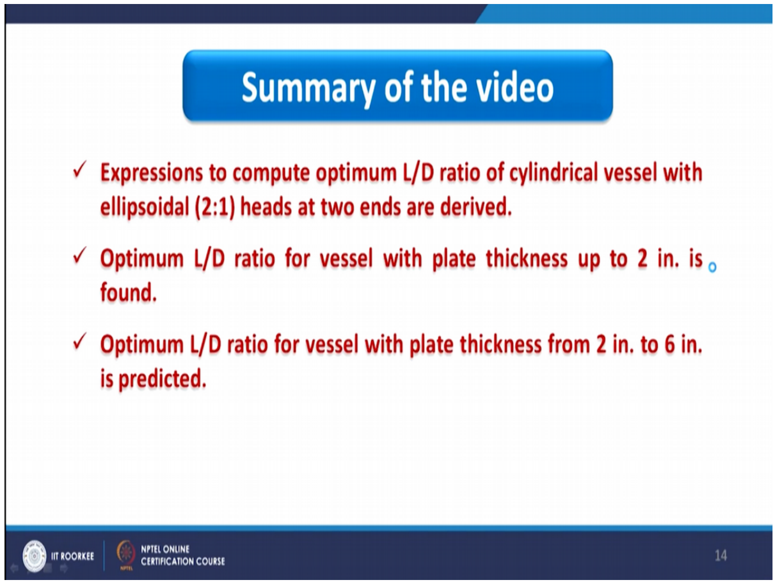

And here I am having summary of this video and it goes as expressions to compute optimum L/D ratio of cylindrical vessel with ellipsoidal heads where major and minor excess ratio is 2:1 at two ends are derived. Then optimum L/D ratio for vessel with plate thickness up to 2 inches is found and finally optimum L/D ratio for vessel with plate thickness from 2 inches to 6 inches is predicted. That is all for now, thank you.