Equipment Design: Mechanical Aspects
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Lecture 02
Stress–Strain Relationship

Welcome to the second lecture of week 1 and this lecture will be on stress-strain relationship. And this particular lecture I have divided in two part where in both part I will discuss and stress and strain relationship. In first part, we will discuss basic of stress and strain and derive the expression of relation between stress and strain. And in next part we will discuss some of the examples with few more stresses okay.

So let us start with definition of stress. So you all have the idea what is stress okay because I need to use this stress in determining or in deriving the expression of stress and strain relation. We will recall the stress-strain first okay.

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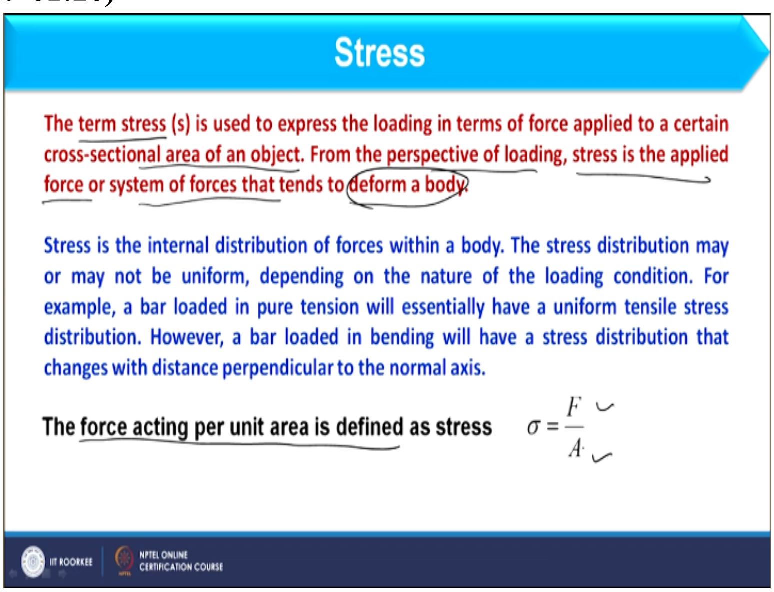

So if you see the term stress is used to express the loading in terms of force applied to a certain cross-sectional area of an object. So what is a stress, that is basically the force when on applying a force on a particular object, but in a particular cross-sectional area of that object not in whole object. Whatever would be the cross-sectional area and whatever force is applicable to that area based on that we can define the stress okay. So from the perspective of loading stress is applied force or system of forces that tends to deform a body okay.

So this stress is basically the force or system of forces applied to a particular object till it deform okay. It gives the deformation and that is occurring due to the stress. So as I have already told that it deforms the body, it deforms the object. So what happens, how it will deform. It will deform because whatever force is applied to a particular cross-sectional area that force will be distributed internally to that object.

And if force is applied to stretch it, for example if this is the bar here I am applying the force, so what happens it will be pulled or stretched in this direction. So whatever force I am applying that is distributed internally in this object and after sometime when force will increase that distance between molecules available in that aparts or that distance increases and therefore the deformity occur. So according to the stress distribution we can say the deformity will be uniform or it will be non-uniform.

For example, if I am having this bar okay and if I apply force to this side to a given cross-sectional area okay, so what happens here whatever force I am applying that would be uniformly distributed in this object. So according to that uniformity whatever change will occur that will be uniform, like if I pull in this way the whole section or whole object will have increment in length not in a particular point okay.

And in the similar line if I tried to bend this or bend a wire so distribution will be non-uniform and stress would be higher at centre and it will keep on decreasing as we move towards the bending side okay. So according to the internal distribution we have uniform change in shape or that shape may be non-uniform okay. And mathematically if you define the stress that is force acting on a particular area, we call this as stress that almost all of us know this so that is mathematically it is represented as sigma = F/A.

Symbol to be used for stress, F is the force applied and A to a given cross-section. So this is the basic expression for stress. There are many types of stresses, but primarily we have three types which we will consider in this particular course.

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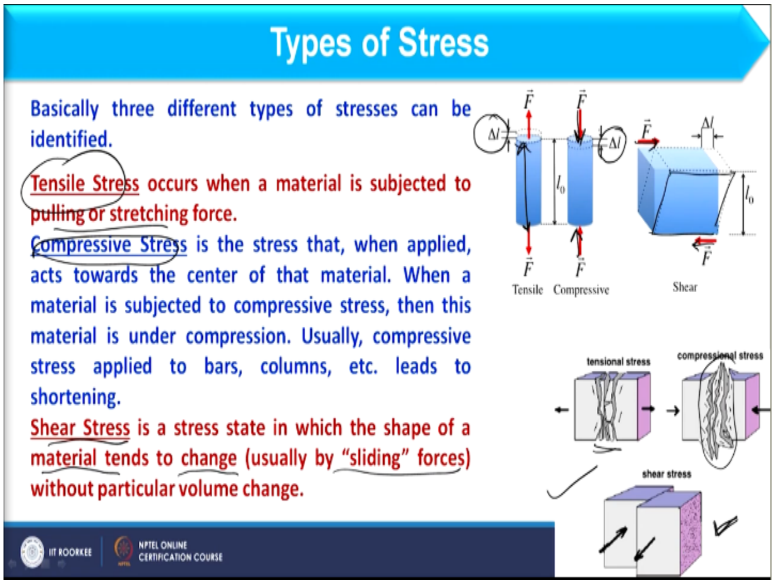

The first one is the tensile stress okay. Tensile stress occurs when the material is subjected to pulling or stretching. So if I need to pull the object, okay, from one side that will give the tensile stress in this object okay. And next stress I am having is the compressive stress. So what happens compressive stress basically works because I am compressing from here as well as well as from here. If I can compress from both side may be one side is fixed from another side I am pushing it.

So what happen that stress collected at the center okay and it reduces the length of that object okay, so whatever dimension either tensile or compressive stress will be applied. In tensile that dimension will increase and in compressive stress that dimension will be decreased, and the third stress I am having is the shear stress. So shear stress is the stress state at which shape of material tends to change okay, like sliding force.

If I am having two objects like this or if I am having one object of this shape and here I have to apply force in this direction and in this object I have to apply force in this direction so it will be slide over each other. So in that case shape of the object will change. In tensile and compressive shape will remain as it is and only that dimension where that force is applicable that will change. If it is a bar it will remain bar at the end; however, if it is Shear stress that shape may be changed okay.

So if you see this image here we have this, this is the original length that is L O, this is the original length when I am pushing that or when I am applying tensile load will have change in length that is basically delta L, so through delta L whole length will be changed. And in the similar line when I am applying compressive load or when I compress this overall length will be reduced by delta L. And in sliding or in shear or in tangential stress we have to apply two opposite forces and then it changes the shape of the object like this okay.

So in shear volume of the object will remain as it is; however, that volume vary when I am applying tensile as well as compressive. So it is another example of tensile like here we have the broken lines at the center, here again I have a broken line, but that lines are joining each other and in shear we have the applied forces. And in shear the sliding occur in the object. So these are different types of stresses.

Usually in our day today life we see example of such stresses, but I can give you very common example of compressive stress which I think all of us use okay, and that can be given through shoes which we are wearing. For example, if I am purchasing a shoe, and at that time when I have purchased that at that time when we wear that shoe we have very cushioning effect, very comfortable effect we observe in our feet.

But after continuous operation or after continuous use, use in what way, if I continuously use that in exercising, walking, running, so what happens whatever material available in the sole that compacts or what we say that the material will be compact. So after certain time when that compactness will keep on increasing that cushioning effect will keep on destroying. So after continuous operation that cushioning effect will not be as we have felt at the time of purchase of that shoes.

So this is a very common example of compressive stress because continuously putting force to that sole makes that sole more compact okay. Next we have to define the strain. It is basically generated due to stress okay.

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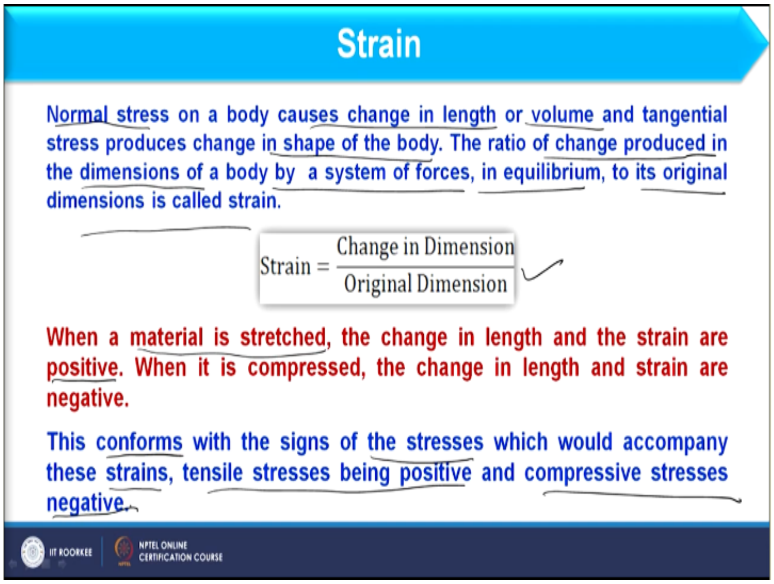

So strain is basically normal stress on a body causes change in length or volume, and tangential stress produces change in shape. So this we have already discussed that when normal stress will be applicable the shape of the object will remain as it is, only that particular dimension will change where that stress is applicable. However, when I am using shear stress, the shape of the object will change. So the ratio of change produced in the dimension of a body by a system of forces in equilibrium to its original dimension is called strain.

So strain is basically change in dimension divided by original dimension, as you can see here, okay. So when the material is stretched, change in length and strain are positive okay. So when we are pulling any object or stretching any object we have change, whatever change I am expecting that change is positive. How that change is computed that final length minus initial lengths so obviously when we have the increment in length that delta L would be positive.

In the similar line when I am having compressive stress, final length would be lesser in comparison to original, so in that case strain would be negative. So this confirms with the signs of stresses which would accompany the strains. Tensile stress being positive and compressive stress being negative. So when I am applying tensile stress then whatever change will occur in strain that would be positive and therefore strain will be called as positive strain or tensile strain. I think you are getting me.

And if I am applying compressive stress, it means that change in length would be negative, so compressive stress will reduce the length and then strain would be negative and then that strain will be called as compressive strain. And here I am having longitudinal strain. Longitudinal strain means what. If I am having original length L, I stretch this and then increment in length would be delta L, so delta L/L would be the longitudinal strain as can be seen from this particular expression.

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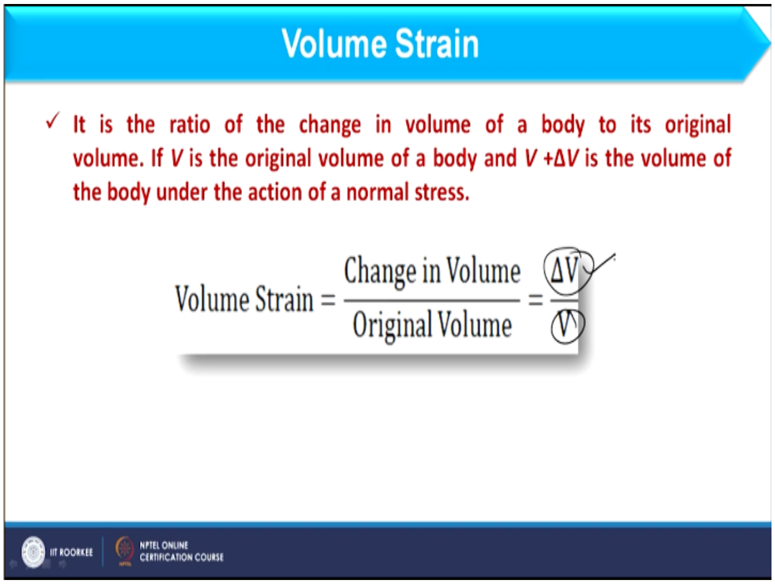

And now we have volume strain. Volume strain means how you can define that change in volume divided by original volume okay. So when we have observed change in volume either I am using tensile stress or compressive stress. So volume strain would be delta V/ V, as delta V is the change in volume which is observed through stress. So till now we have discussed what is stress, what is strain, and what are the types of stress and strain.

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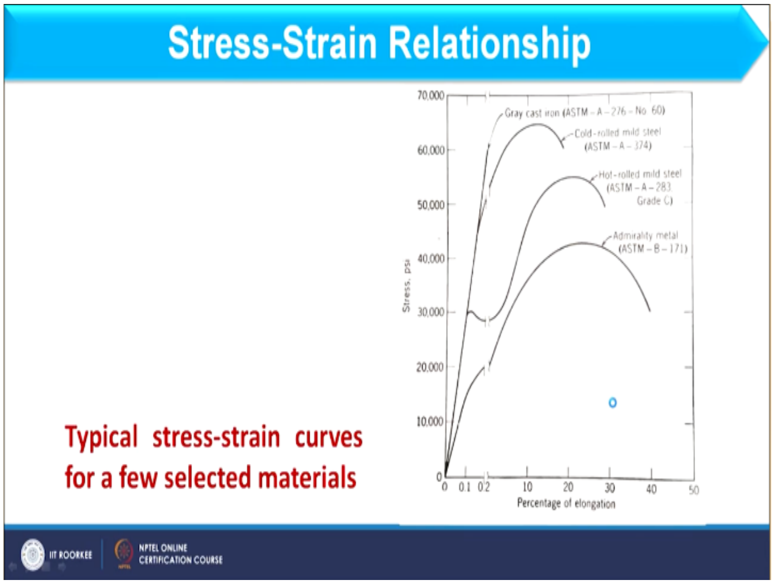

And now we will focus on stress-strain relationship okay, and for this we will discuss induced stresses. Induced stresses means the stresses which are made in the system which are induced, which are formed in the system or object. For that case, elastic deformation is induced by a load such that when the load is removed the part resumes its original shape. So here we have elastic deformability where when we remove the load the object will come to its original shape okay, so that is basically elastic deformation; however, when we have different operations or services in different parts of the equipment.

There are different stresses which are available or which are made in the object, and these stresses are found due to different loading conditions, and these stresses are tensile, compressive, shear, bending, torsion, etc. So all these stresses are induced in the system when we apply the force, but when we remove that force all that stress will go out and object will take its original shape. So materials undergo strain when they are subjected to stress okay, so when we apply the stress strain occurs in the system.

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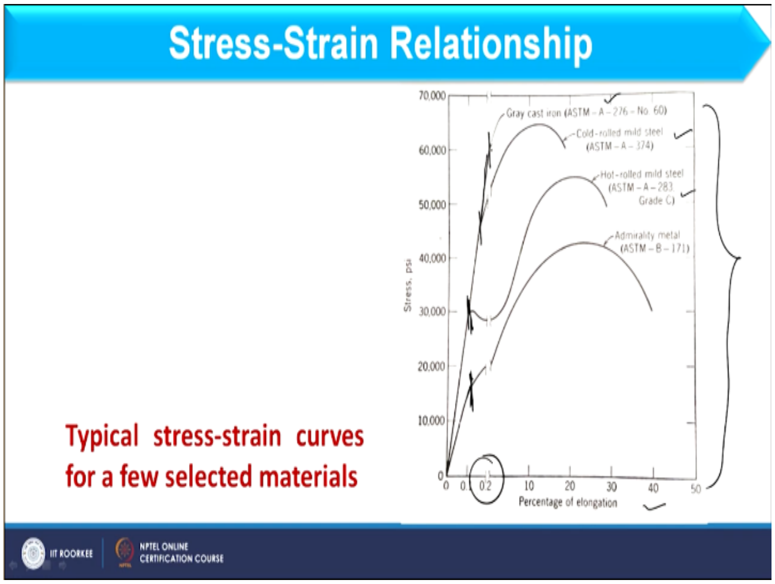

And in this slide if you see here I am having particular graph, where stress and strain relationship are plotted for different material. On Y-axis we have stress and on X-axis we have strain, and that X-axis is broken because we need to show large scale at X-axis okay. Now if you observe all these graphs up to certain value we have straight line okay. If you see this is the straight line, this is a straight line, and after that we have non-linearity in the curve okay.

So this linearity gives the elastic condition for each material and that elastic condition vary for different material. For example, in first this curve only this section is linear, this curve we have this much linearity, and similarly for you know Gray cast iron we have highest linearity okay. So each object will have certain linearity and then the deformation, whatever happen that will be permanent. Wherever it is linear it is basically fall in the region of elastic deformation okay. So we will use this point to derive the mathematical expression of stress and strain relationship.

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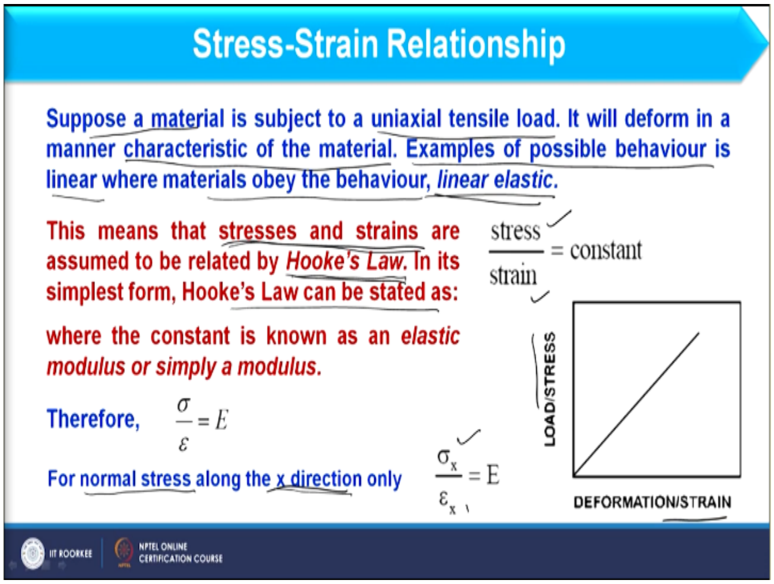

Suppose a material is subjected to a uniaxial tensile load okay, we have applied tensile load in one direction only, it will deform in a manner characteristic to the material. So according to the material uniformity takes place, when I am having uniaxial tensile load to a particular object and that characteristic will depend on type of material okay. So examples of possible behavior is linear when the material obeys the behavior linear elastic.

So in the last slide we have discussed that each material have certain linearity okay, that linearity section may be small or larger that depend on type of material okay, but each material have this linearity and which we call us linear elastic behavior. As shown in this particular figure where we have the strain and stress and linear linearity occur. So this means that stresses and strain are assumed to be related by Hooke's law. In its simplest form, Hooke's law can be stated as where the constant is known as an elastic modulus or simply a modulus.

So till the material follows linearity that stress and strain can be represented by a particular constant okay. So according to Hooke's law, we have stress divided by strain and that should be equal to constant because linearity occur okay, and that constant we basically call as modulus of elasticity or simply the modulus. If in this case that is sigma by epsilon, sigma is the notation for stress, epsilon is the notation for strain and that will be equal to E, where E is the modulus or modulus of elasticity okay.

If force is applicable in x direction, it means if normal stress is applicable in x direction we can have the expression sigma x/epsilon x and that would be equal to E okay.

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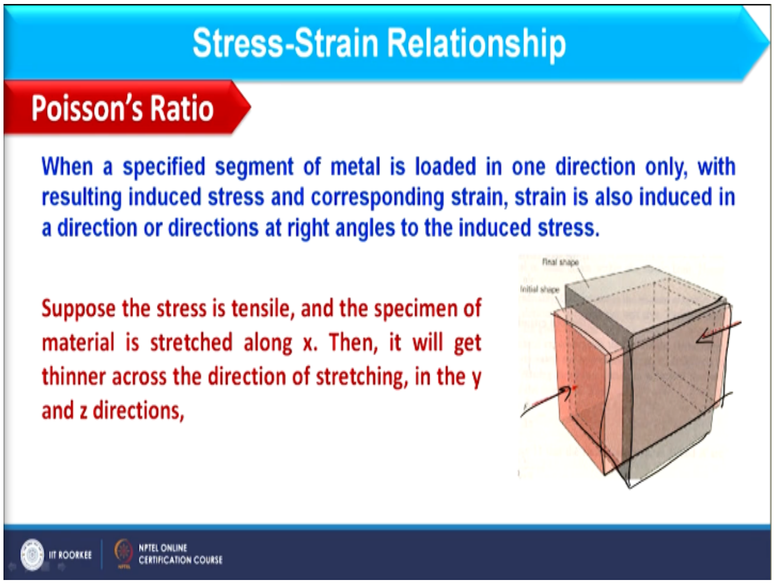

So next parameter which I need to define is the Poisson's ratio okay. So let us focus on this definition and then I will explain what exactly it is. So when I specified segment of metal is loaded in one direction only with resulting induced stresses and corresponding strain, strain is also induced in a direction or directions at the right angle to the induced stress okay. So what is the meaning of this that we can understand through this object.

If you see this is the 3D object, where this is the X-axis, this is Y-axis and this is Z-axis. If I am applying stress in x direction only, so what happens. If I am applying load in X-axis it means if I am pulling it in X-axis it means tensile stress will be applicable in X-axis, what happens length along the X-axis will increase okay. However, we can also observe change in y and z direction. So in x direction we have the strain, along with this we have strain in y as well as z direction. Okay.

So that basically when we stretch from one direction, another two direction will have the reduction in more respect to length okay. And that you could also understand through this particular image. If you see this is that red section, this is basically the original shape okay. And when I apply tensile stress in x direction length in y as well as z direction will also be decreased, however length in x direction will increase.

So though stress is applicable only in one direction, but it affects another two directions also. And here I am having another image where the original shape is this and from both side I am compressing this okay. Stress is applicable in x direction but it compresses, so final length would be decrement in x direction, however, increment we can observe in y as well as in z direction. This is the direction okay. So that is the phenomena when stress is applicable in one direction only.

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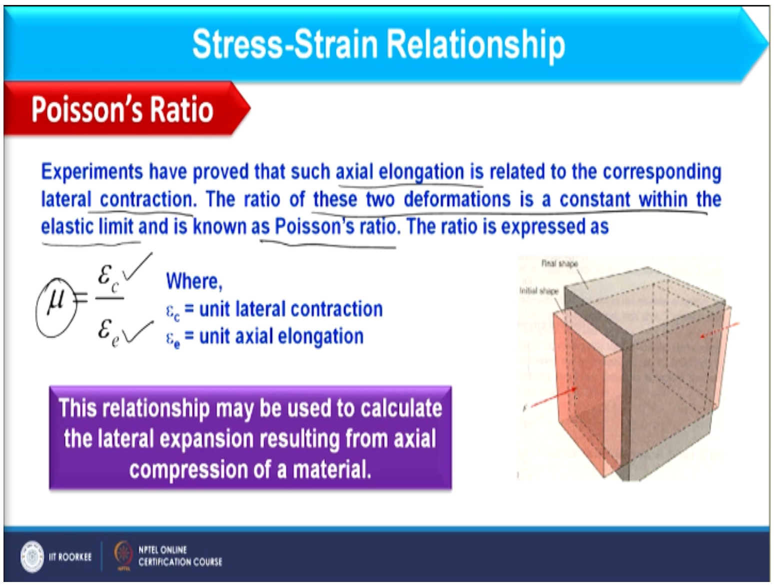

So experiments have proved that such axial elongation is related to corresponding lateral contraction. So when I am having 3D object and force is applicable only in one direction, so there will be elongation in that particular direction, however, contraction will occur in other directions also whether stress is applicable there or not. So ratio of these two deformation is a constant when it is falling within the elastic limit and that constant is known as Poisson's ratio okay.

So mathematically how we can define Poisson's ratio is that lateral contraction by axial elongation okay. Lateral contraction by axial elongation is nothing but the mu. So this mu is basically Poisson's ratio and that depends from material to material. When I am applying compressive stress that mu how we can define is lateral expansion divided by axial compression okay. So in Poisson's ratio we need to consider lateral divided by axial whether it is contraction or elongation.

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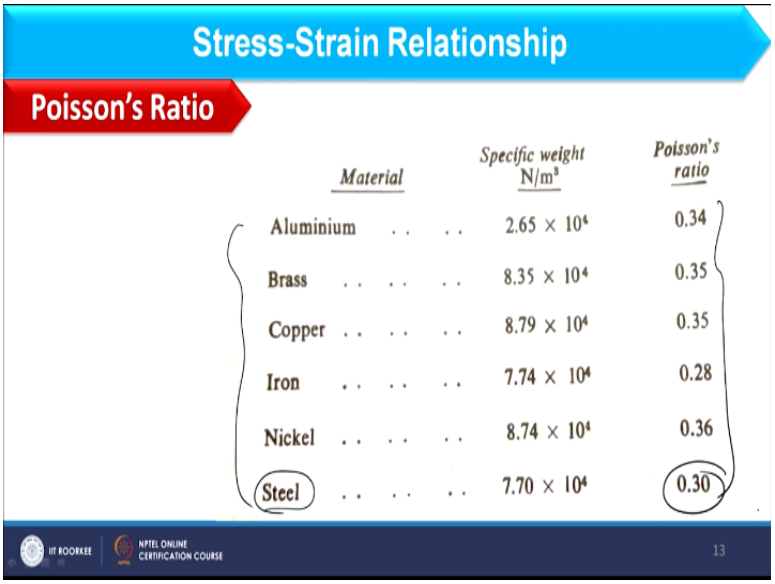

So here we have different Poisson's ratio for different material. So you see for steel which we will consider in manufacturing of pressure vessel, the mu value should be 0.3. So here this table summarizes some of the values of Poisson's ratio okay.

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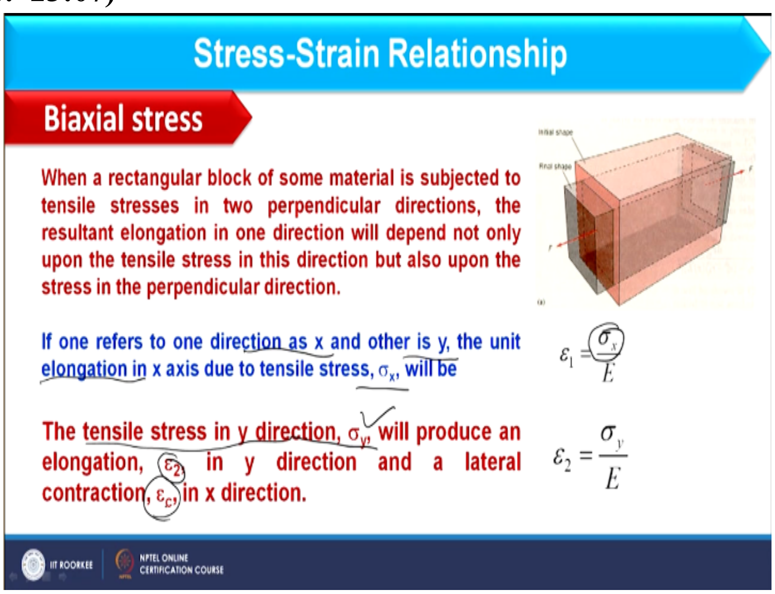

And now we will discuss stress and strain relationship mathematically. So first of all, I am having biaxial system and then we will focus on triaxial system. So for example if I am having this, I am taking example of this box again what happens in biaxial system it means stress is applicable in x direction as well as in y direction. So this is biaxial system we are considering, it means stress is applicable in x direction as well as in y direction also okay. So what happen when I am applying stress in x direction and that will be denoted as sigma x, I am applying stress in y direction and that will be denoted as sigma y. So due to sigma x we can observe elongation in x direction okay, due to sigma y we can observe elongation in y direction okay.

But at the same time, this y will give some contraction in x direction, and similarly when I am applying force in x direction, and if I want to calculate the strain in y direction there will be elongation in y direction due to sigma y and contraction due to sigma x. I hope you are getting this. So if one refers to one direction as x another direction is y, the unit elongation in X-axis due to tensile stress sigma y will be epsilon 1 = sigma x/E okay. So that is due to sigma x only.

And tensile stress is also applicable in y direction that would be sigma y and elongation due to this in y direction we are representing as epsilon 2 okay, that would be the elongation. And due to this sigma y, we have contraction in x direction, that is epsilon 2 okay. So that we can define as elongation in y direction epsilon 2 will be equal to sigma Y/E.

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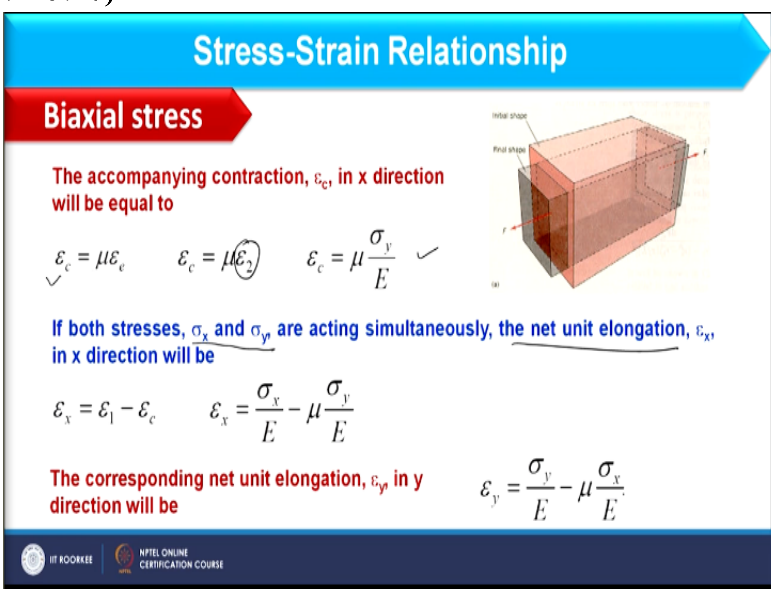

And along with this we have contraction in x direction so that we can define as epsilon C that will be contraction in x direction and that is equal to mu that is Poisson's ratio into epsilon E. Epsilon E is basically elongation in y direction so that we can replace with epsilon 2. So here we have epsilon C equal to mu into epsilon 2. Epsilon 2 will have a particular expression and that expression we can put over here so that is mu sigma Y/E.

So resultant elongation or net unit elongation in x direction when both sigma X and sigma Y are applicable will be epsilon X is equal to epsilon 1 - epsilon C okay, that is axial elongation minus lateral contraction. We have expression of epsilon 1 and epsilon C, so that we can put over here where epsilon X will be equal to sigma X/E - mu sigma Y/E okay. And accordingly we can have resultant elongation in Y-direction as epsilon Y = sigma Y/E - mu sigma X/E okay.

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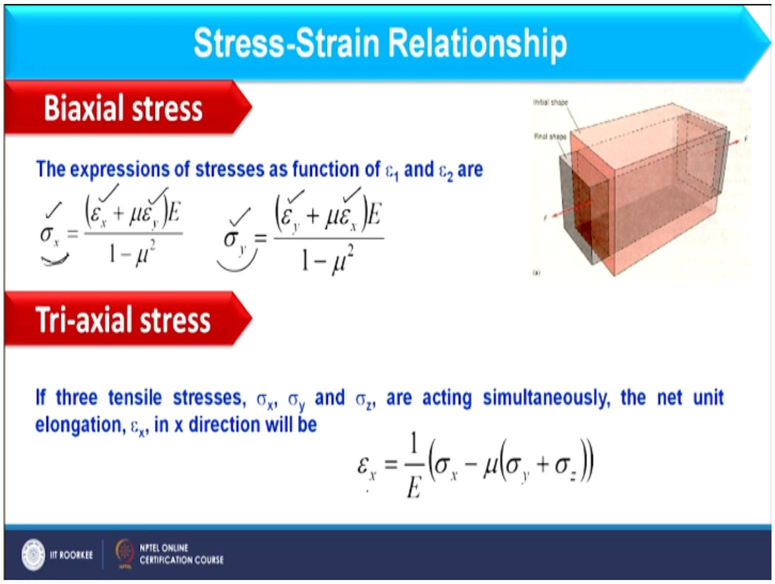

Solving these two equations, we can have expression of sigma X as a function of epsilon X and epsilon Y, and sigma Y as a function of epsilon Y as well as epsilon X. So here we have sigma X and sigma Y as a function of epsilon X as well as epsilon Y. So this is basically biaxial system. In the similar line I can derive expression for triaxial system, where sigma X, sigma Y and sigma Z all three are applicable. So in that case resultant elongation in X-direction would be elongation due to sigma X minus contraction due to sigma Y and sigma Z okay. So mathematically we can represent epsilon X as 1/E (sigma x - mu (sigma y + sigma z).

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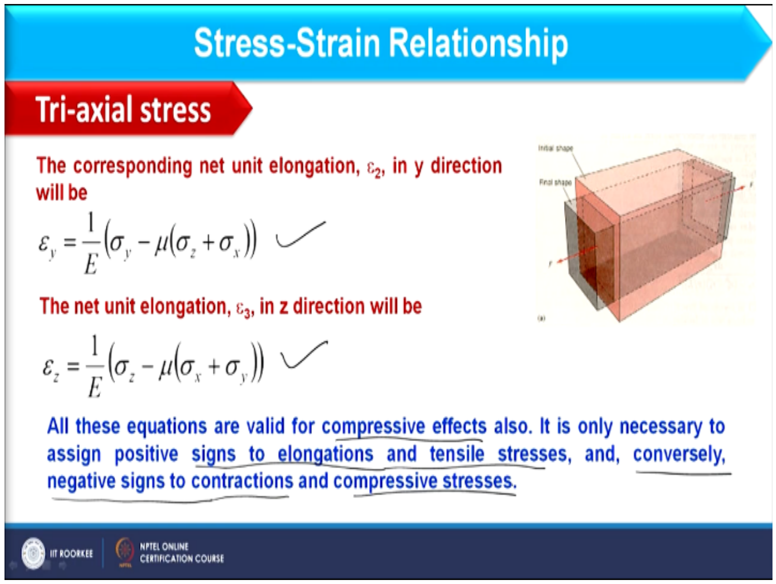

And in the similar line, I can define resultant elongation in Y-direction as well as resultant elongation in Z-direction okay. After solving these three equation we can have sigma X, sigma Y and sigma Z as a function of epsilon X, epsilon Y and epsilon Z. So in this way we derive the expression for triaxial system also and here we have mainly focussed on tensile stress.

However, all these equations are also valid for compressive effects. Compressive effect how we need to consider, that in compressive stress we have negative strain okay, so that compressive stress as well as that strain would be represented by negative sign. So it is only necessary to assign positive signs to elongation and tensile stress and conversely negative signs to contraction as well as compressive stress.

So these points we need to consider when we have compressive stress and when we have combination of tensile as well as compressive stress. So where compressive stress is there take negative sign for stress and strain, and similarly in opposite to that, take positive sign in tensile stress as well as tensile strain okay. And here I am stopping lecture two of week one. We will consider discussion on stress and strain relationship in next lecture. And that is all for now, thank you.