Equipment Design: Mechanical Aspects
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Lecture 03
Stress-Strain Relationship

Welcome to the third lecture of week 1 which is on stress-strain relationship. Discussion on stress-strain relationship we have started from lecture 2 where we have defined stress and strain separately, and discussed types of each, and then we have derived expression for biaxial as well as triaxial system. So, in this lecture, we will solve example related to stress and strain relationship, and then we will discuss a few more stresses. So let us start this.

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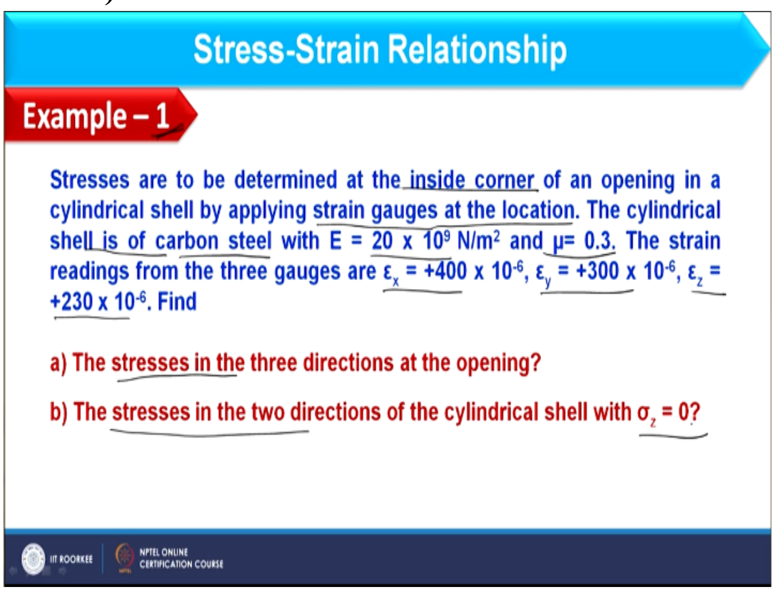

And here I am having example 1, and it states that the stresses are to be determined at the inside corner of an opening in a cylindrical shell by applying a strain gauge at the location. So in this problem, a cylindrical shell is considered, where an opening is made and due to this opening, whatever strains are generated that are measured through strain gauge, which is a instrument to find out the strain at a particular location.

A cylindrical shell is made of carbon steel with E value 20 x 10 power 9 N/m square and Mu is 0.3. The strain reading from the three gauges are epsilon X, epsilon Y and epsilon Z as mentioned here. What we need to compute is, the stresses in the 3 directions at the opening and second part focuses on stresses in two directions of the cylindrical shell where sigma Z will be equal to 0. So, these 2 parts we will solve for example 1. So let us start with part A. So in this part, we have to calculate stress in three directions at the opening.

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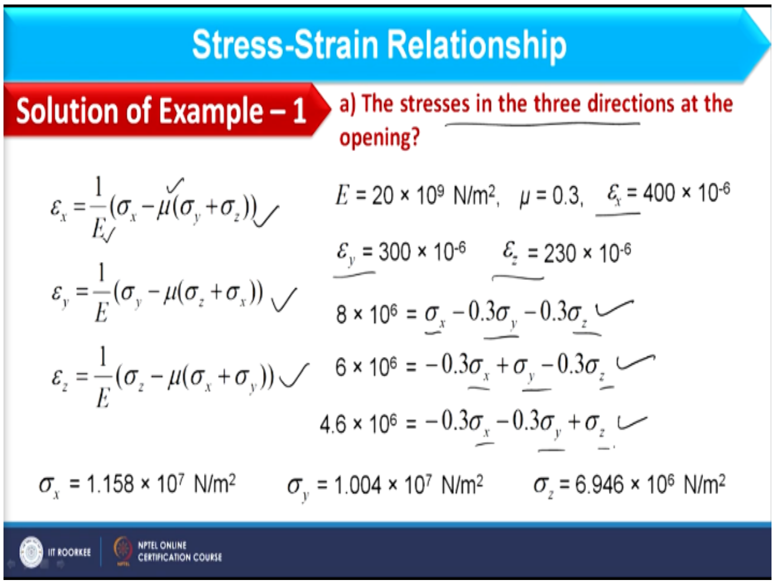

And if you see here, we have epsilon X, epsilon Y, and epsilon Z, so opening whatever we have discussed that is basically of rectangular type and we need to find out stresses considering values of strain. And relation between strain and stress are already derived in lecture 2 for triaxial system, and what are the known values, E and Mu are known values along with strains in all 3 directions.

So, that example is not very much complicated, that is very simple. We have epsilon X, epsilon Y, and epsilon X expressions where I know E and I know Mu. So, putting the value of epsilon, E and Mu, we can have the equation in terms of sigma X, sigma Y and sigma Z, and these are 3 equations where sigma X, Y and Z all three are unknown. So we have three equations and three unknown, so that can be solved simultaneously.

Once we solve that simultaneously, we can obtain the value of stresses in three directions. And sigma X here is 1.158 * 10 power 7 N/m square, sigma Y and sigma Z contain these values respectively. So, in this way, we solve the stresses generated in three directions when I know strain value, and if I know the stress values, I can calculate strain value in all three directions okay. So that is part 1.

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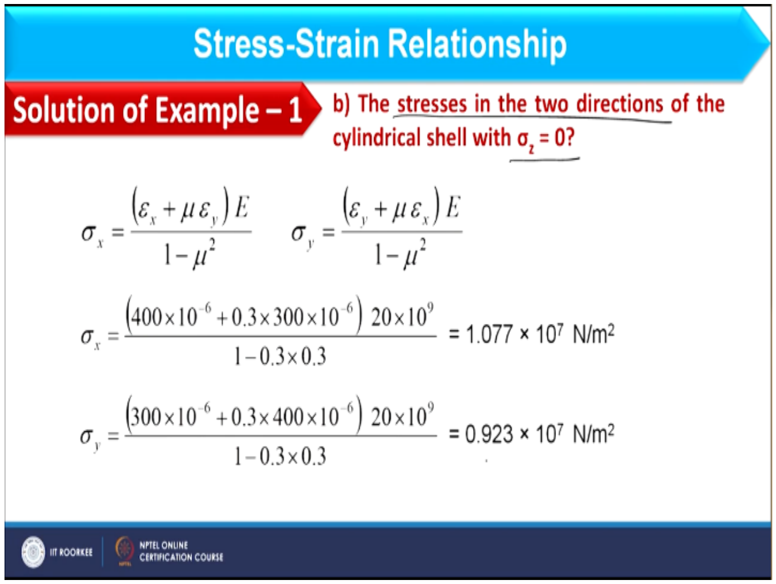

In second part, we need to find out stresses in two directions of cylindrical shell where sigma Z is equal to 0. So here I am having the relation of sigma X as well as sigma Y as a function of epsilon X and epsilon Y. This is biaxial system. These equations we have already derived and while putting the value of strain E and Mu, we can have sigma X as well as sigma Y, okay. So, sigma X, it is simply the expression based, not complicated solution is required for that.

So, sigma X comes out as 1.077 * 10 power 7 N/m square and sigma Y, we can obtain as 0.923 * 10 power 7. So, these values are directly formula based, where stress if we know, we can calculate strain and vice versa also. So, this is the simple example to compute stress whatever we have discussed for biaxial as well as triaxial system. Next, we have the materials of construction that is the mechanical properties.

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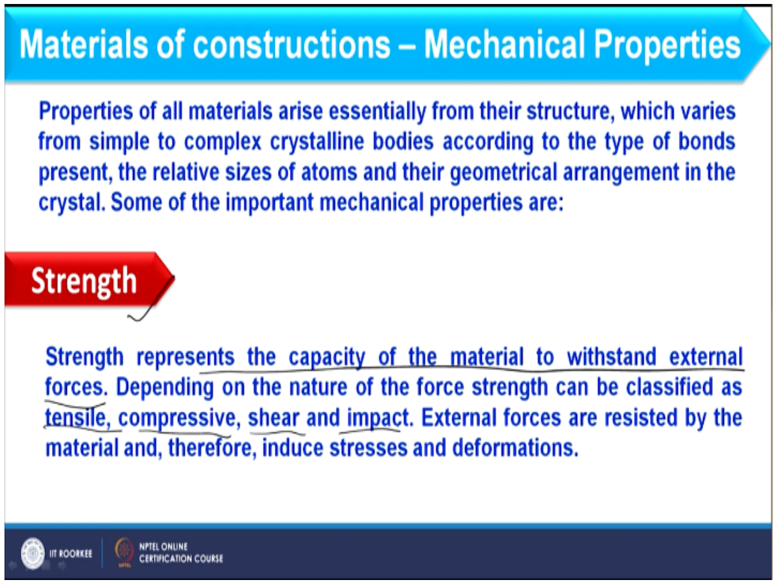

Now what happens, you have seen that stress and strain relationship and that vary for each material. You cannot have two materials have same trend, no it is different. Now why it is different, because each material will have different crystalline structure, either it is very simple or it is complicated, or there may be different size of molecules available in that and geometry of molecules are different. So the crystalline stricture of material vary okay, material to material and so based on that we can define some properties and we call these properties as mechanical properties.

And in this slide, we are focusing on some of the important mechanical properties. The first property is the strength. So the strength represents the capacity of the material to withstand external forces okay. So that when external forces are applied to an object, it will try to deform it okay. So external forces should be resisted by the object. So strength is defined in that way, so that up to what extent it can resist the external forces or deformation okay.

So depending upon the nature of force, strength can be classified as tensile, compressive, shear and impact. So basically strength speaks about that how much a particular material resists the force which are acting on this. So strength can be defined that for how long the material can withstand the external forces or can resist the external forces so that it should not be deformed okay. So this is one property.

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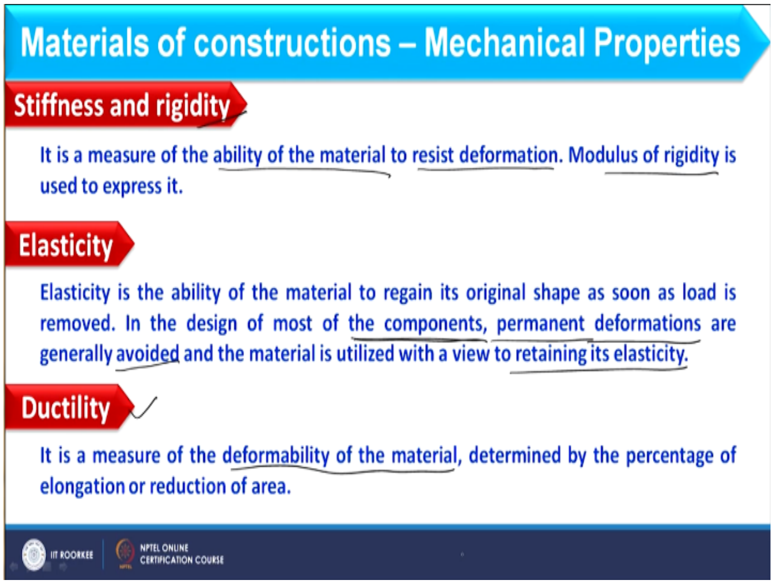

Second property is stiffening and rigidity. It is a measure of the ability of the material to resist deformation and modulus of rigidity is used to express this property. And next, I am having elasticity. Elasticity you understand that when we apply stress to a particular object, it deforms. When we remove that load or stress, that material will regain its original shape, so that we can call as elasticity of the material.

So, in the design of most of the component, permanent deformations are generally avoided and material is utilized with the view to retaining its elasticity. So, while designing we usually avoid permanent deformation because until unless it will be elastic region, if any deformability will appear, it will regain its original shape okay, and that is not possible in permanent failure. So elasticity is important property as it determines that up to what extent we can consider the parameters while designing.

Another property is the ductility, which is the measure of deformability of the material determined by percentage of elongation or reduction of area. What is the meaning of deformability of the material. For example, if I am having this wire, and if I pull this wire from this side, up to how long it will retain its shape though deformation will take place, but it will remain as a wire okay. Up to how long it can maintain its shape without breaking okay. So, as much as length can be increased before breaking of this wire, so that increment in length basically speaks about the ductility of the material.

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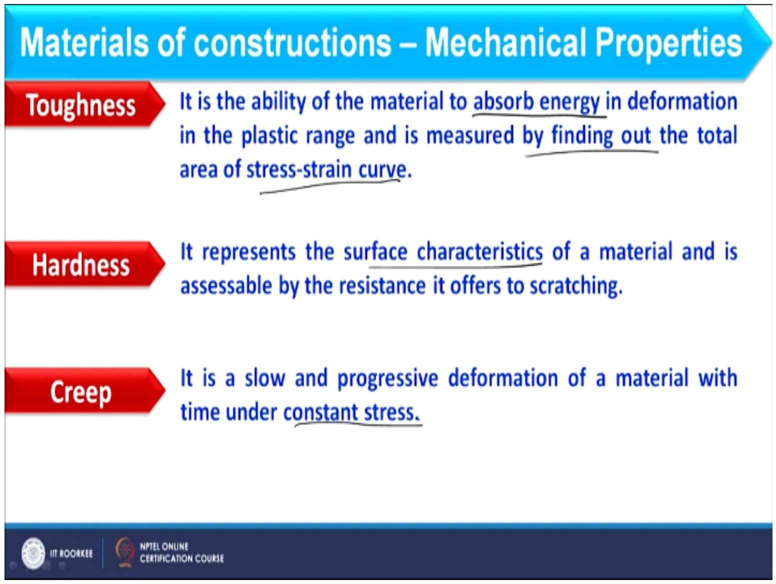

Next property is the toughness. It is the ability of the material to absorb energy in deformation in the plastic range, and is measured by finding out the total area of stress-strain curve. So, this is basically the toughness where how much energy is absorbed in deformation that quantifies okay. So next property I am having is the hardness. Hardness basically speaks about the surface characteristic of the material. It means when the material will be very hard, it will resist the scratching. Scratches are not formed very easily on the very hard material surface, okay. So it is basically the surface characteristic.

And then we have the creep. It is a slow and progressive deformation of a material with time under constant stress. So you see, in this way we have defined different mechanical properties of the material. And now we will focus on those properties which we usually use in designing, okay, and that we can derive through stress and strain relationship. So first property in that case is the proportional limit.

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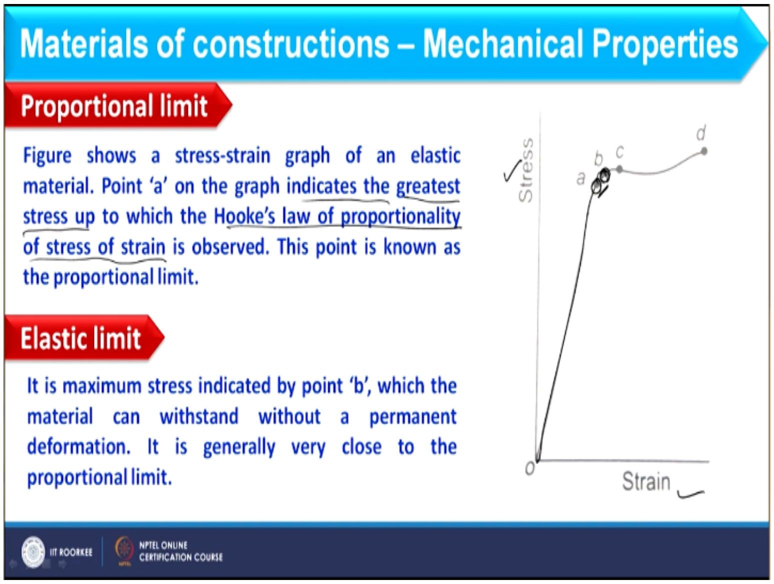

Now, if you consider this particular diagram, here we have stress on Y axis and strain on X axis and here we have trend of a particular material. So point A on this diagram indicates the greatest stress up to which Hooke's Law of proportionality of strain is observed. It means this is almost linear part of a material okay, so where this linearity disappears, we call that point as proportional limit because up to proportional limit, it follows the Hooke's Law okay. So, that point A is basically the extent of linearity you can understand okay. So, this is one property.

Another property is the elastic limit. Elastic limit, you see, so this is basically the maximum stress beyond which permanent deformation will take place, as it is shown through point B. So without having permanent deformation, how much stress can be sustained by the material that is represented by elastic limit and that is very close to the proportional limit. Next, we have the yield stress.

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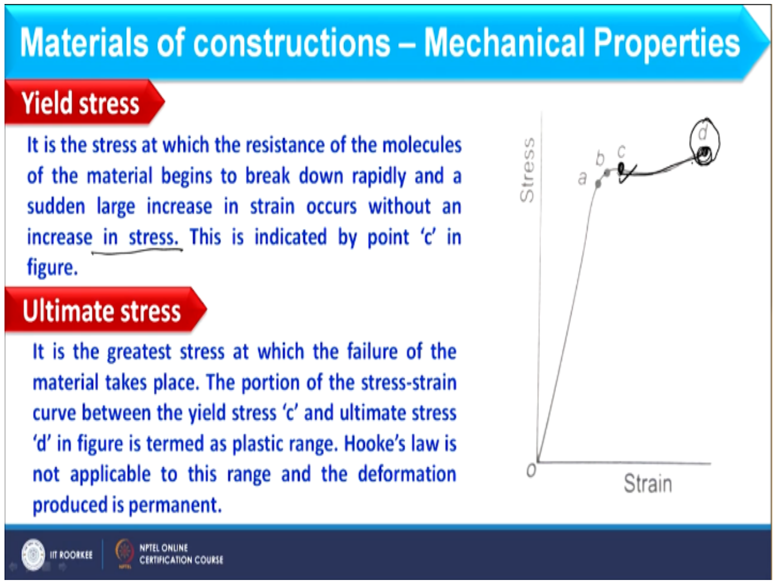

It is the stress at which the resistance of the molecules of the material begins to break down rapidly and sudden large increase in strain occurs without increasing stress okay. So that is denoted by point C in this image. So, what happens, if you see stress at point B as well as point C, stress is almost uniform from B to C, okay. However, strain has significant change, so the point where sudden change in strain occur without having much change in stress that point we can call as the yield stress okay.

After we have ultimate stress. Ultimate stress is basically, it is the greatest stress at which failure of the material takes place. So if you consider this point D that we denote as ultimate stress and from C to D we consider the region as plastic region okay. So, here from C to D we have permanent deformation and D point is basically the maximums stress at which failure of the material takes place permanently, and this is the region from C to D where Hooke's Law will not be applicable. So, these are some of the properties which we will consider in designing. At what level we will consider, that we will discuss when time comes.

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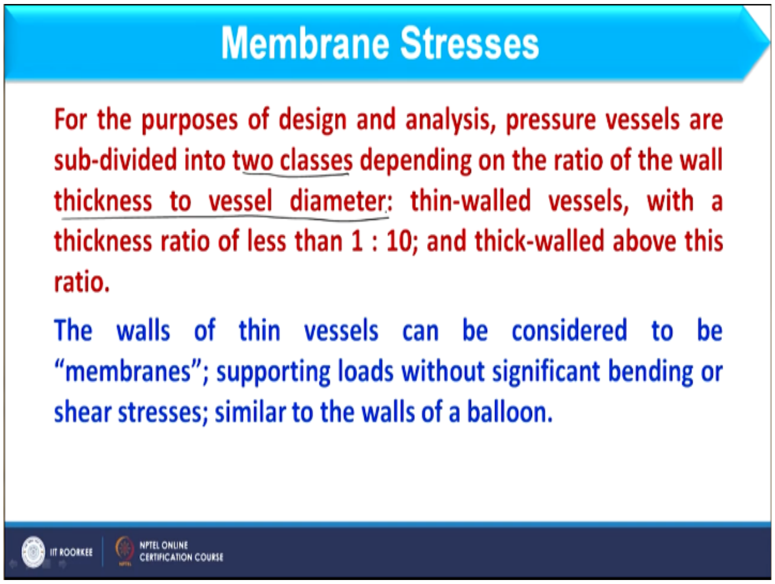

Now, next topic in this series is the membrane stress. Here, we will define what is membrane stress and how the expression related to this stress can be derived for a given system. So for the purpose of design and analysis, pressure vessels are to be subdivided into two classes depending upon the ratio of thickness to vessel diameter. So, ratio of wall thickness to diameter speaks about the class of pressure vessel, okay. So, when this wall thickness divided by diameter, if it is less than 0.1, it comes under thin walled vessel; otherwise, it will be called as thick walled vessel okay.

So, when the ratio of thickness of the wall to diameter is 0.1, whatever stresses are generated in the metal sheet or in the component which we are going to design, that stress is basically called as membrane stress because that is very thin layer and it will be treated as the wall of the balloon, okay. And here, we will derive the expression of membrane stresses on cylindrical shell.

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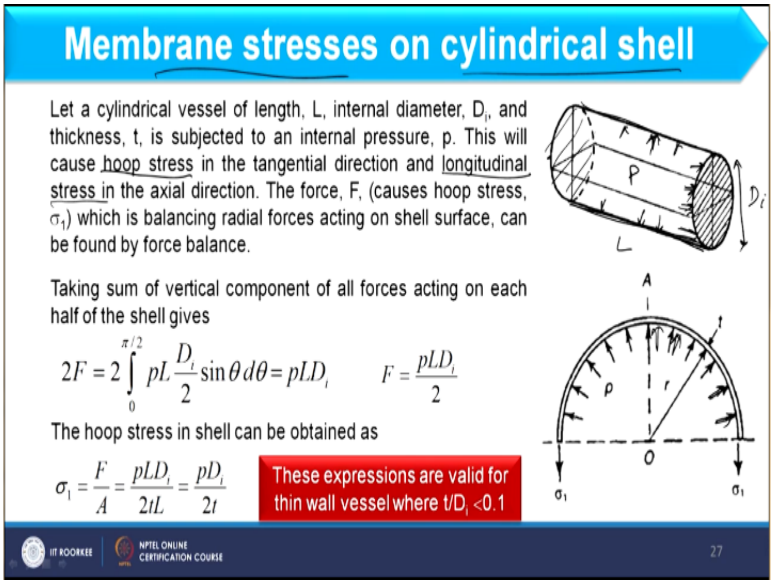

So here I am considering cylindrical shell where the length of the cylindrical shell is L and inside diameter is given as D I okay, and T is the thickness of this, you see, this is basically T, thickness of metal sheet by which this cylindrical shell is prepared, okay, and this system is operated with internal pressure. So, when the system is operated with internal pressure, it means internal pressure will be higher than the outside pressure. Outside pressure may be atmospheric, inside it will be more than atmospheric.

So, once it will be more than atmospheric, it will try to expand it from internal okay, and then we can have the failure if we will not go for proper designing of that vessel okay. So, when internal pressure will act, it has certain stresses or some stresses which are formed and these stresses are usually available along the length and around the circumference okay. Because of internal pressure operation, we have two stresses; one is called hoop stress and second is longitudinal stress in axial direction, okay.

So, to derive the expression how we need to calculate thickness of that, we will make the forced balances around the object, but before forced balances, you should understand what are the stresses. As I have told that there are two stresses, hoop stress as well as longitudinal stress. So, longitudinal stress, how that longitudinal stress will be formed, because if you see this image, L is the total length. From this side, it is entirely covered and from this side also vessel is closed.

So, whatever pressure are acting over here, that pressure would try to expand these walls or we can say try to push this head or this closer. So, once it will push this closer, what happens, it creates stress in longitudinal direction okay because it will try to open those two ends. So, due to this, whatever stress are formed that we can call as longitudinal stress, okay. And what is hoop stress, hoop stress is when internal pressure is acting inside, it will try to expand the diameter.

It will try to increase the diameter because continuously it pushes, so that you can understand through this diagram that all pressure will try to expand this periphery from inside okay. So, what happens, when it will expand, it will create stress at this direction okay, and when pressure is applicable to this, it will give stress in this direction. It means this is basically the circumferential stress. If circular shape I am considering, the pressure will act upwards as well as downwards, so it will try to expand this.

Whenever failure will occur, it be failed from this side. So, all these stresses will create circumferential stresses and we also call this as tangential stress as well as hoop stress. I hope you have understood what is hoop stress, that is basically the circumferential stress, okay So, for design purpose, I need to make the forced balances in this object. So let us start with circumferential stress or hoop stress or tangential stress.

Now, what happens over here, whatever hoop stress I am telling, that is basically sigma 1 here as well as sigma 1 here in this diagram, okay. So, this sigma 1 will be generated due to force F which is applicable to this direction only. So, that force is basically balanced by all vertical component up to this section okay, vertical component of all forces which are acting in this region will be balanced by F.

In the similar line, whatever F is there, that will balance all components of these forces in upward direction okay. So, to make the balance, if you see, here I am taking 2 F, why I am taking 2 F, because I am making the balance in half of this image, okay. So, F is there and F is there, so that would be 2 F. How it is balanced, like 2 F = 2, integrating from 0 to pi/2.

And if you see here, if I am considering this particular section where this angle I have taken as theta okay. So, acting area of pressure in this region would be this distance into length, in this way, okay. So, how I can define this length, that would be D I * 2 sin theta. So, how we can define this distance, that is R = sin theta, R is basically D I/2 * sin theta, and when I am integrating this expression from 0 to pi x2, it will give all forces in this particular section. So, here you see that 2 P, P is the internal pressure, L into this, this would be the acting area for small angle theta and that will be integrated using D theta.

When I resolve this expression, I can have P, L and D i. So, from this I can extract F as PLI x 2. Now the hoop stress in shell can be obtained as sigma 1 = F x A. Sigma 1, you see here, and that is basically F/A. A is what, A is this acting area. I hope you are getting it. So, A is basically this acting area and that is T is the thickness of that sheet along the length. That would be the acting area. So, here we have sigma 1 as PLDI/2TL and that will be PDI/2T, so sigma 1 will be equal to PDI/2T, this is the final expression of hoop stress, okay. And these expressions are valid for thin walled vessel where T/Di is less than 0.1.

So, here this expression, we have derived considering hoop stress in thin vessel, and usually if I am considering thick vessel, there we have radial stress also. Here, we have to only consider circumferential as well as longitudinal and we have not considered radial because in thin cylinder, along the radius, stress will not vary significantly. However, if I am having significant thick cylinder, inner surface to outer surface continuous change in stress is observed and that we call as the radial stress and that we will discuss when we derive the expression for thick cylinder.

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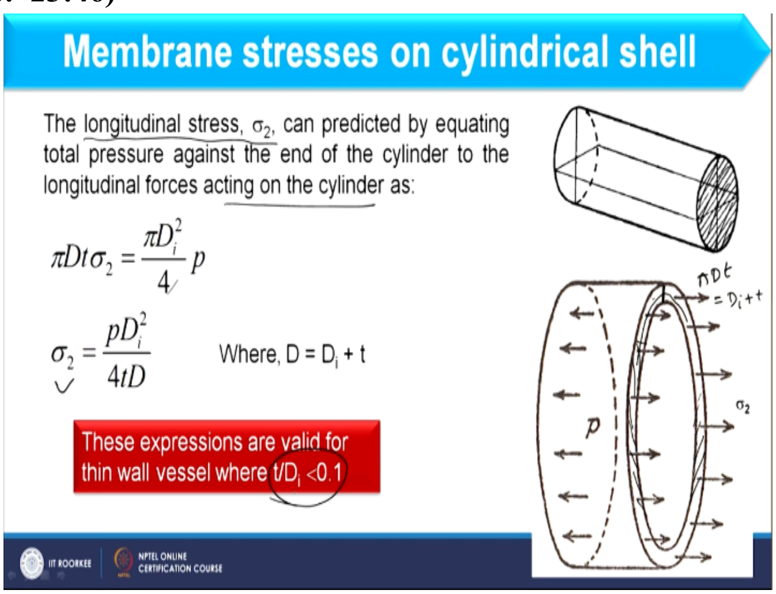

Now, another expression I am considering longitudinal stress that is sigma 2. Here sigma 2 basically represents longitudinal stress and that would be balanced by the forces which are acting on the cylinder in longitudinal direction, okay. So, if you consider this particular image, here we have sigma 2. Sigma 2 will be applicable in this region, okay, in this whole region sigma 2 will be active. So, that sigma would be what, that pressure into acting area. Acting area will be this area.

So, how I can define that pi D * T. what is D, D is basically the average diameter here and that we can write as Di + T or if outer diameter is known, that should be D O - t. So pi D is basically the periphery into T that would be the acting area. So, pi DT sigma 2 will be equal to pi DI square/4 * P, P is the internal pressure, which is acting on whole cross-sectional area of the vessel because it is acting in whole this area, so that would pi Di square/4.

After resolving this, we can have expression of sigma 2 as PD1 square/4 TD where D is DI + T as I have already explained and this is again valid for thin cylinder where T/D O is less than 0.1. So, in this way, we have derived the expression for hoop stress as well as longitudinal stress, and based on this, let us solve one example, so that it will be more clear.

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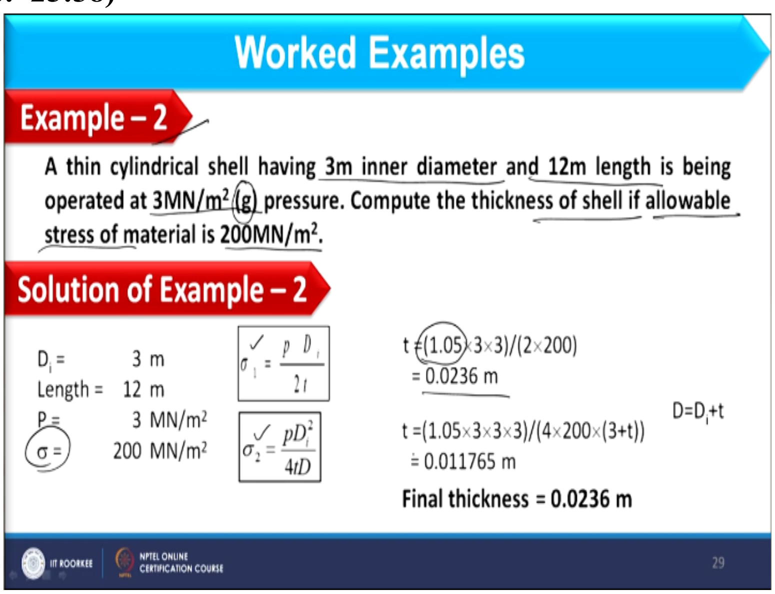

So here I am having example 2, where a thin cylindrical shell having 3 metre inner diameter and 12 metre length is being operated at 3 mega N/m square G. what is this G, G is nothing but the gauge. So, pressure is given in gauge. Why I am considering pressure in gauge that we will discuss later on. What we need to compute is, the thickness of shell if allowable stress of the material is 200 mega N/m square. So, what is allowable stress, that again we will have a separate lecture to define this and there we will discuss this when time comes. Please consider this allowable stress right now as it is okay.

So, let us start the solution. Here I am having DI and length and internal pressure is given 3 mega N/m square and allowable stress we have equated to sigma value, either sigma 1 or sigma 2, whatever expression I am using, in that expression sigma will be replaced by allowable stress. How it is replaced by allowable stress that also will be discussed in upcoming lectures. So, sigma 1 and sigma 2, these expressions we have already defined. Considering sigma 1, we can calculate T as 1.05 * 3 * 3/2 * sigma 1 will come over here once I take T from this expression.

Now, what you need to focus on, what is this 1.05, see when I am operating a vessel at a particular pressure usually I design value higher than this or we can call that as overdesign because failure should not occur; therefore, whatever pressure we are considering, that should be slightly higher than the pressure at which I need to operate and usually that value will take as 5% extra, so 1.05 * 3 resembling to design pressure. So, after solving this, we can have thickness 26.3 mm.

Considering longitudinal stress, we can calculate thickness as T = 1.05, why it is, that we have already discussed, and then 3 * 3 * 3/4 200 because sigma 2 will come at the bottom x D + T. So, here you see, T is coming here also as well as here also, so that you need to solve it properly and the final answer would be 11.77 mm. So, among this, as this would be of higher thickness, we can consider this 23.6 mm as final thickness. So, in this way, we can solve the problem or we can design the system containing membrane stresses, which are applicable for T/D O = less than 0.1. For higher thickness, we will derive the expression separately and that will come under thick walled vessel.

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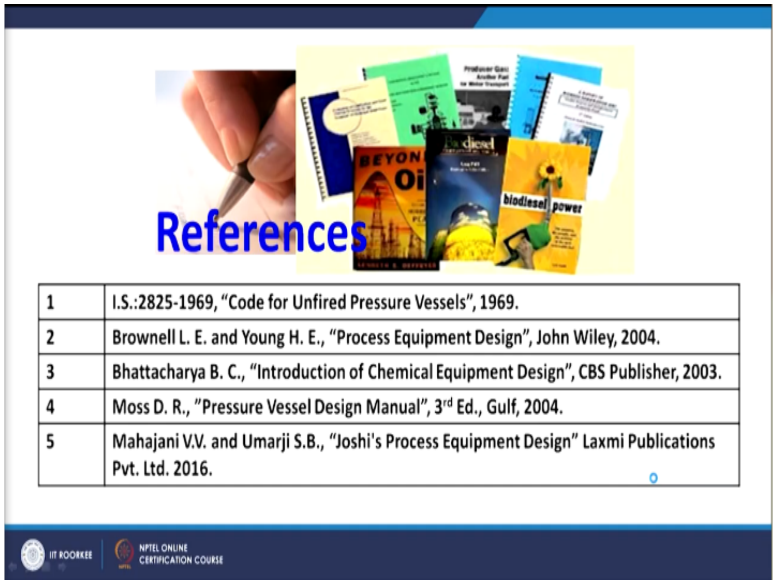

Here, I am having reference books which you can follow to study the topic in detail.

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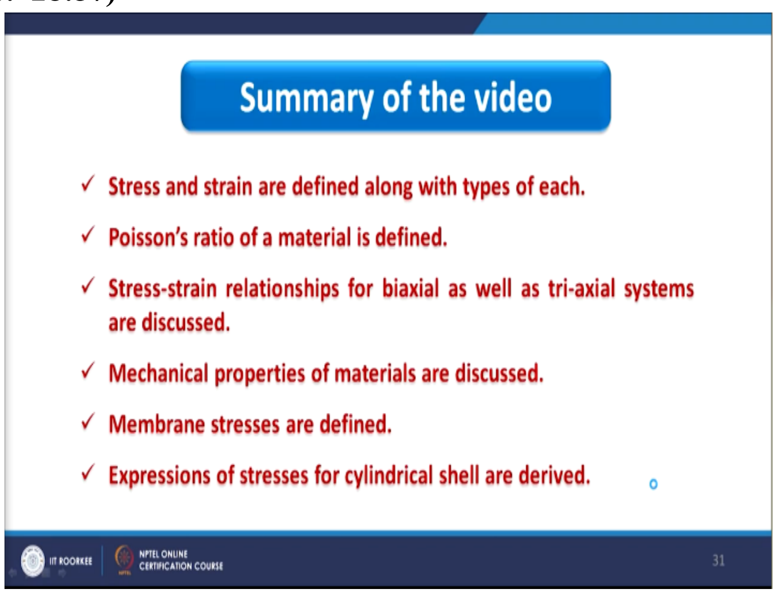

Now, we have the summary of the video and this summary is combinedly written for lecture 2 as well as lecture 3, and here stress and strain are defined along with types of each, Poisson's ratio of a material is defined, stress and strain relationship for biaxial as well as triaxial system are discussed, and mechanical properties of material are discussed. Membrane stresses are defined and expression of membrane stresses for cylindrical shell are derived.

So, I hope you have idea of how to compute the thickness for membrane stresses, how to derive the expression for stress and strain relationship for a given system. That is all for now, thank you.