Basic Environment Engineering and Pollution Abatement
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Lecture 23
Air Pollution Control 3

Hello everyone. Now we will start discussion on the topic Air Pollution Control part 3, in part 1 and part 2 of air pollution control, we have discussed on gravity settlers and then packed bag and electrostatic precipitator and now we will discuss on cyclone separator.

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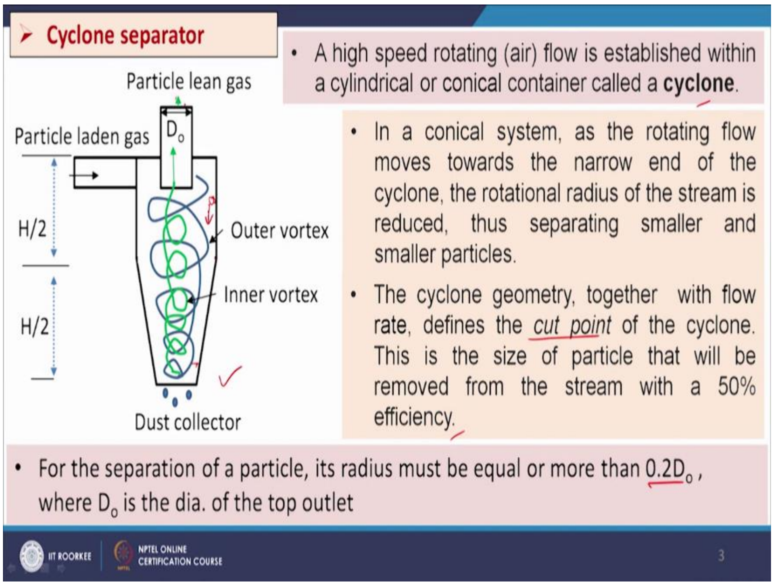

So, as you have seen in case of gravity settlers, the gravitational force plays a significant role to remove the particles from the settler. Here in case of cyclone we will see that a centrifugal force will be important to separate the particles in the system. And a device, as shown here, say here the particle leaden gas is getting entry into the system that is called the cyclone separator.

So, here it is getting some circular movement and the gas will be moving inside the cyclone, which has two part that is upper part and lower part, the upper part is the cylindrical and lower is a conical part. When the gas is entering into it, it will be getting a circular movement as shown here and the bigger particles will be separated easily actually the particles will move from the centre to the outer side and when the particle will reach to the outer side that it will arrest here it radial velocity will be 0 and particlely fall so this is the mechanism for the separation.

Now, one thing very important here to see that the diameter of the lower part is gradually decreasing. So, cylindrical part is converted to conical one, so the finer particles will be moving downwards and may be separated. So, bigger particles will be separated here, smaller particles will be separated here and it will be possible that very fine particles those are may not be able to be separated they are particle size is so small, then they are not getting sufficient centrifugal force and radial velocity is not sufficient, so, that they will not be able to move towards the outer side of this vessel and as we have put one fan at the top.

So, there will be very less or, small vacuum. So, these gas will be going out. So, very fine particles may go out to this gas tube. So, this is the working of a cyclone separator and you see the high speed rotating air flow is established within a cylindrical or conical container called a cyclone. So, here we can have two considerations, one is the size of the particle which can be separated that is minimum particle size which can be separated in the system otherwise, if it is lesser than that value that will not be separated.

Another is there are a number of particles and a range of particles are there, so there will be one particle size that 50 % of the material will be separated which are above that particle size. So, that is the, your cut point or cut diameter as per the definition for cyclone separator. And for the separation of a particle its radius, its radius must be equal or more than 0.2 D_O. So, this is the D_O through which the gas is going out from the separator.

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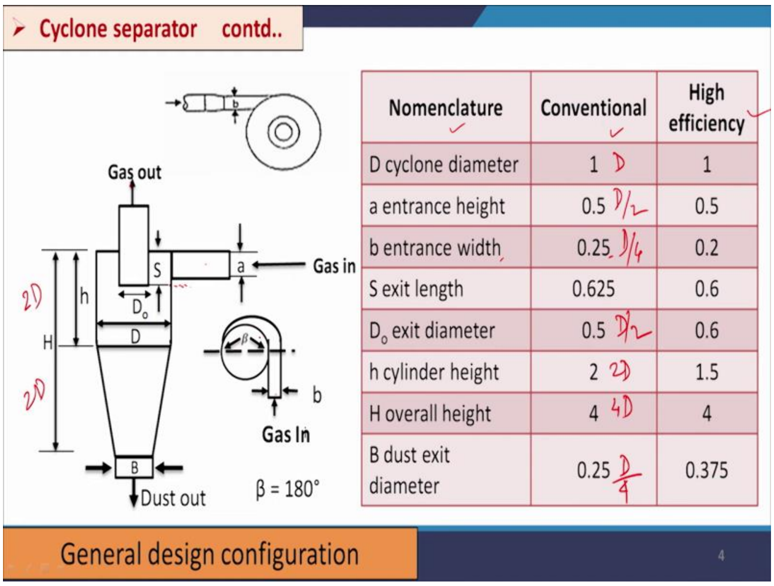

Nomenclature	Conventional	High efficiency
D cyclone diameter	1	1
a entrance height	0.5	0.5
b entrance width	0.25	0.2
S exit length	0.625	0.6
D_o exit diameter	0.5	0.6
h cylinder height	2	1.5
H overall height	4	4
B dust exit diameter	0.25	0.375

Now, we will see different dimension or typical dimension of a separator and in case of Cyclone separator the different dimensions like say height, total height and cylindrical height and your conical height and then the inlet duct, length and width. So, all those things will depend upon the diameter of the cylindrical part. So, what is the diameter of the cylindrical part on the bases that the other lengths will be decided.

So, this table shows us different nomenclature and for conventional separator and high efficiency cyclone separator, what are the relationship, basically D the cyclone diameter that is the basic parameter. So, if D is 1 unit for conventional and for high efficiency in both the cases then entrance height, so, this is the entrance height through which the gas is entering into the separator, that duct will be having some height, so, that height is a and b is entrance width.

So, it is not shown here, here we can see the b. So, when it is going so, 1 will be your say this is the duct. So, this is our b and this height is your a. So, the height and b we are having, so, entrance width that will be 0.25 if 1 unit is our D then b will be 0.25 and a will be 0.5 that means, if it is D.

So, then this will be D by 2, then that will be D by 4 and S is the exit length, that means through this duct the gas will be going out we have a fan at the top. So, it will blow up at the top. So, this will be creating very low vacuum inside this, so this dimension D_O that is exit diameter that will be your 0.5 that is again D by 2 and exit length S this will be 0.625.

So, this a we have 0.5. So, this difference will be there that is equal to D by 8, and total S is equal to 0.625. Now, h is the cylindrical height so, cylindrical portion height is h and that is equal to 2 that means 2D and H is the overall height that is equal to 4D. So, we can say this part is also 2D and this is 2D. So, total is 4D so this one. Now B so dust exit diameter. So, this may be arbitrary or it may be like this 0.2 that means D by 4 and in other case high efficiency 0.375.

So, this is for conventional and these are for high efficiency cyclone separator So, this is a very typical dimension of a cyclone separator and here this β is 180° angle that means, gas will enter here and it will rotate 180° and again it will get a circle, it will get a circular movement and will move just shown in the previous diagram. So, this way the cyclone works.

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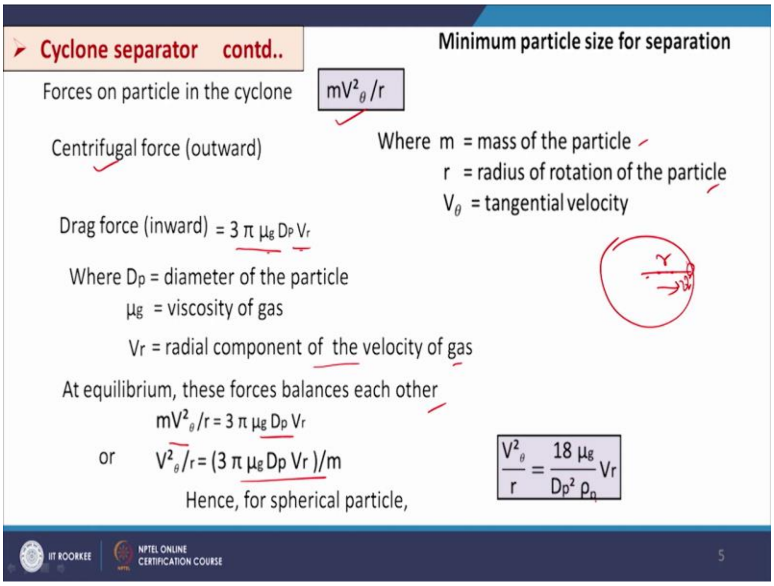

And now we will see the minimum particle size for separation. Now, if we think about different forces working on the particle in the separator, then we will see that one centrifugal force is working on it on the particle, the outer direction so that it centrifugal force and certainly one drag force will be in the opposite direction that is invert direction.

So, then centrifugal force we know that mV_Θ/r

V_Θ is tangential velocity, m is the mass of the particle and r is the radius of rotation of the particle. So, if any particle is there any position from centre to this position same, so, this equal to r so, this will be having this relationship that we know it very well and drag force will be working on it this particle maybe at the outer it may be any place in between.

So, that will be the r value will change and then drag force will be $3\pi\mu g D_P Vr$.

When Vr is a radial component of the velocity of the gas. So, this direction, what is the velocity that is called Vr and then µg viscosity of gas and Dp diameter of the particle. Now, at equilibrium these two forces balance each other.

So, in this case gravitational force will also be there, but that will be not that much it may be ignored, so at equilibrium these two forces balance each other.

$mV_\Theta/r = 3\pi\mu g D_P Vr$.

Vr is the radial component of the velocity of gas. So we can get

$V_\Theta/r = (3\pi\mu g D_P Vr)/m$

If we consider this the particle as a spherical particle. So, m we can get by knowing the particle diameter, that is volume of the particle diameter into density. So, by replacing that volume $4/3\pi r^3$. Hence for spherical particle this will be

$$\frac{V_\theta^2}{r} = \frac{18\ \mu g}{Dp^2\ \rho_p} Vr$$

So, this is the relationship already we have discussed this relationship in case of settling chamber also.

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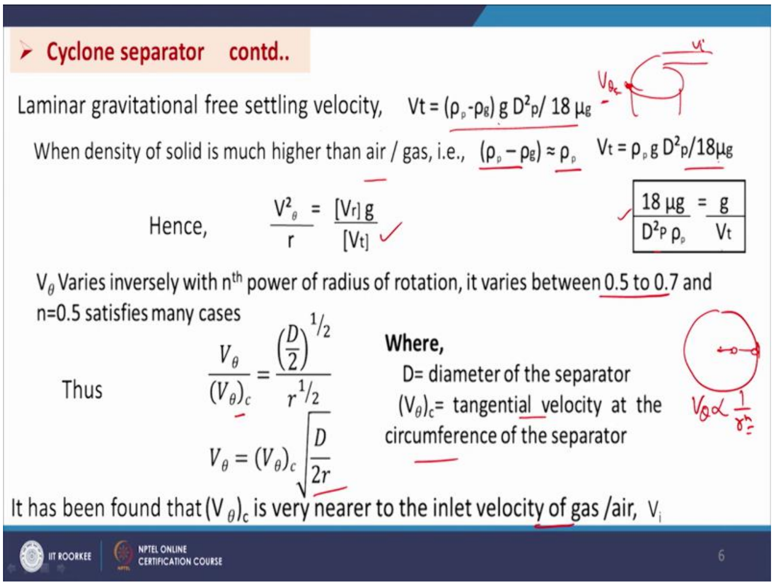

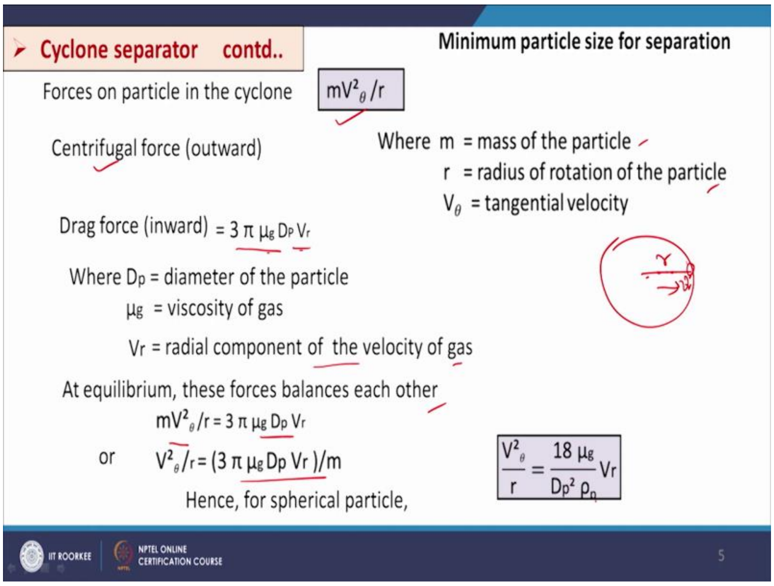

Now, laminar gravitational free settling velocity, as you know particle is inside the cyclone centrifugal force is working on it. So, drag force is also working on it, but still there will be some forces gravitational forces that will be working, but we ignored it, but in that case, if we want to get the Vt value, so, that

$$V_t = (\rho_p - \rho_g) g\, D_p^2 / 18\mu_g$$

If it is a laminar flow already we have discussed in our previous classes.

So, that is this is our Vt and when the density of solid is much higher than air if the particle density is very high and air density as we know it is less. So, in that case this difference can be approximated at

$$V_t = \rho_p g\, D_p^2 / 18\mu_g$$

So, by rearranging

$$\frac{g}{V_t} = 18\mu_g / \rho_p D_p^2$$

This relationship we are getting by the definition of Vt and another relationship we have got

$$\frac{V_\theta^2}{r} = \frac{18\mu_g}{\rho_p D_p^2}$$

So, this right-hand side term is same for these two equations this equation and this equation the right-hand side term here and left-hand side term here are same.

$$\frac{V_\theta^2}{r} = \frac{[V_r]\, g}{[V_t]}$$

Now, three velocities are correlated by this expression, V_Θ is the tangential velocity, Vr is the radial velocity and Vt is the terminal settling velocity.

Now, this V_Θ, Θ value will change. The V_Θ theta means tangential velocity particle may be available at any position that is r value may vary, so V_Θ, it has been shown that it is a function of radius so of rotation, radius of rotation. So, that is inversely proportional, inversely with n^{th} power of radius that means, V_Θ is proportional to $1/r^n$.

So, that will be some constant by r^n, this n value is normally 0.5 to 0.7. And if we take the approximation of 0.5.

$$\frac{V_\theta}{(V_\theta)c} = \frac{\left(\frac{D}{2}\right)^{1/2}}{r^{1/2}}$$

what is the D is the diameter of separator that means, the particle is the outer surface we are considering that is maximum, $(V_\Theta)c$ that is the tangential velocity at the circumference of the separator.

So, tangential velocity to the circumference of the separator, that is at distance small ‘r’ is equal to D/2. So,

$$\frac{V_\theta}{(V_\theta)c} = \sqrt{\frac{D}{2r}}$$

$$V_\theta = (V_\theta)c\sqrt{\frac{D}{2r}}$$

So, this expression we are getting. So, it had been found that this $V_\Theta c$ which is the tangential velocity at the circumference of the cyclone separator that is almost equal to the velocity of gas. That is say if this is the cyclone separator here it is getting entry and then it is moving like this.

So, in that case what velocity we are having that is equal to Vi. So, Vi will be having the same if the particle is here. So, the V_Θ value will be similar to that, that will be the $V_\Theta c$ at this will be equal to Vi. But this position is inside any place then it is not. So, only at the circumference that V_Θ value that is $V_\Theta c$ is equal to Vi this is one approximation as it had been found that these values are very near to each other.

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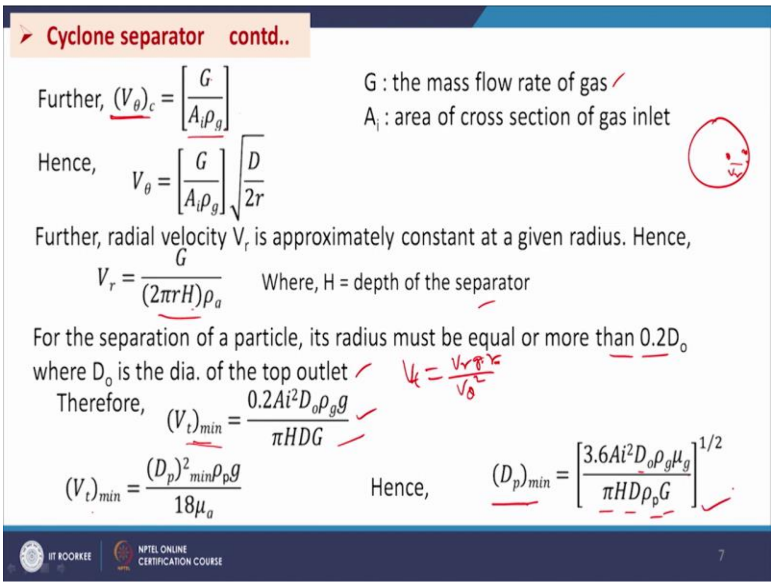

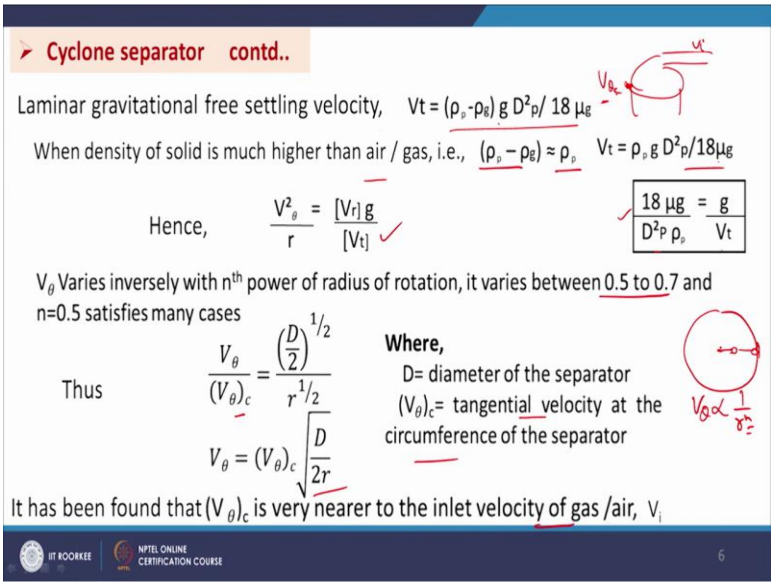

Now, in this case, then $V_\Theta c$ can be calculated from the mass balance, so if G is the mass flow rate of gas then

$$(V_\theta)c = \left[\frac{G}{A_i\ \rho_g}\right]$$

That is Ai is the cross-sectional area of the gas inlet. So, inlet duct cross sectional area * tangential velocity that is the volume*ρ_g that is the mass so, that these expressions we are getting.

So, V theta if I want to get tangential velocity at any location, so, it may be at the circumference or it may be at the inside any position. So, that co-relationship we have got

$$V_\theta = (V_\theta)c\sqrt{\frac{D}{2r}}$$

So, in that case we will replace this $V_\Theta c$ in terms of G/Ai ρ_g. Because V_Θ it is very difficult to measure actually, but these are we can measure easily these are measurable parameter. So, now, we are able to convert this term into a measurable quantity, so,

$$(V_\theta) = \left[\frac{G}{A_i\ \rho_g}\right]\sqrt{\frac{D}{2r}}$$

Now, the radial velocity Vr say here I want to get the radial velocity that is the Vr. So, how can we calculate the Vr. So, again by mass balance we can calculate, why, because if this is the r distance, so, 2πr this will be the perimeter*length, the height of the cyclone.

So, this will be the surface area*Vr, this direction that will be the volume ρ_a that is the mass so, that is equal to G. so what is the mass of air getting entry into the system, we are assuming that the same mass is crossing through the periphery of the surface at distance r.

$$Vr = \frac{G}{(2\pi rH)\rho a}$$

So, this mass flow rate actually has come through the entry point, entry duct to the separator. So, then we are getting Vr equal to this one, where H is the depth of the separator. Now, for the separation of a particle its radius must be equal to or more than 0.2 of Do that is through which the gas is going out.

So, that diameter into 0.2 that should be the critical value for the separation of the particle. And Do is the dia of the top outlet therefore, what will be the Vt minimum, so, Vt minimum will be this much and how why Vt minimum will be this much because we have relationship

$$V_t = \frac{V_r g}{V_\theta 2} r$$

So, now, we will replace this Vr value here, that is $G/2\pi rH\rho_a$ and we will replace V_Θ value

$$(V_\theta) = \left[\frac{G}{A_i\ \rho_g}\right]\sqrt{\frac{D}{2r}}$$

if we replace these values Vr and V_Θ now, in terms of these, then we can get Vt and here this r is also there and that r is nothing but 0.2 Do if we replace this value here also, then by rearranging we will get

$$(V_t)\min = \frac{0.2A_i^2 D_o \rho_g g}{\pi HDG}$$

So, if this is our Vt minimum that minimum terminal settling velocity, then in this expression, we can find that

$$(V_t)\min = \frac{(D_p)_{min}^2 \rho_g g}{18\mu_g}$$

If the flow is in laminar zone, then this expression. So, here, since the Vt is minimum, so, Dp will also be minimum. Now, these Vt expressions will replace by these expressions which we are getting here and rearranging will get the Dp expression Dp, that is minimum.

$$(D_p)\min = \left[\frac{3.6A_i^2 D_o \rho_g \mu_g}{\pi H D \rho_p G}\right]^{1/2}$$

So, this is the expression for the minimum particle size, which can we separated in the cyclone.

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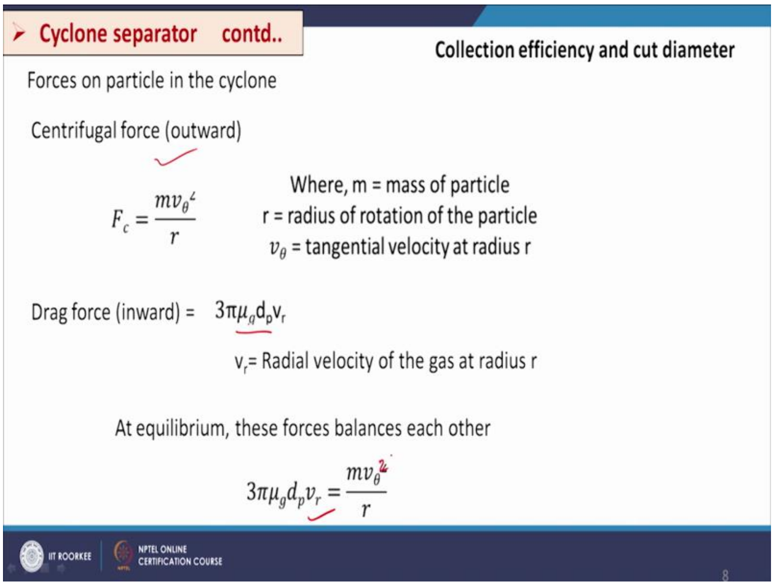

Now, we will discuss the cut diameter and the collection efficiency of Cyclone separator, so here again the same thing the two forces will be there, one is your centrifugal force outward that is

$F_c = mV^2_\Theta/r$.

And drag force (inward) = $3\pi\mu g d_P V_r$

So, these two will be same. So, this is square.

$3\pi\mu g d_P V_r = mv^2_\Theta/r$.

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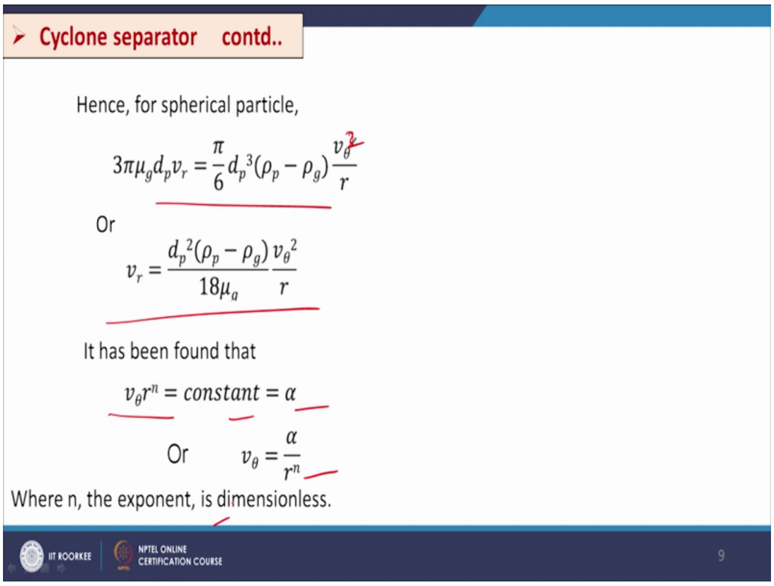

So, again if it is for spherical particle, we can get these two relationship

$$3\pi\mu_g d_p v_r = \frac{\pi}{6} d_p{}^3(\rho_p - \rho_g)\frac{V_\theta^2}{r}$$

$$vr = \frac{d_p^2(\rho_p - \rho_g)}{18\mu_g}\frac{v_\theta^2}{r}$$

It has been found that

$V_\Theta r^n .= constant = \alpha$

$V_\Theta = \alpha/r^n$

So, n is the exponent is dimensionless and its value is normally 0.5.

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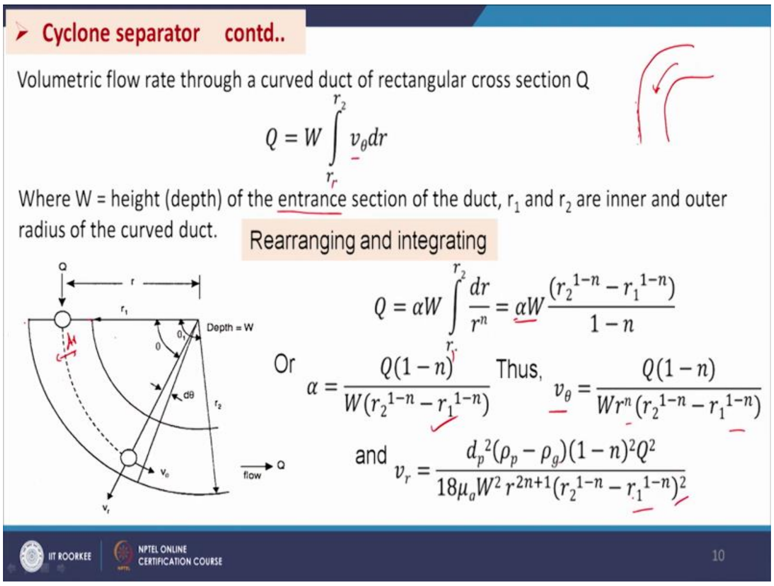

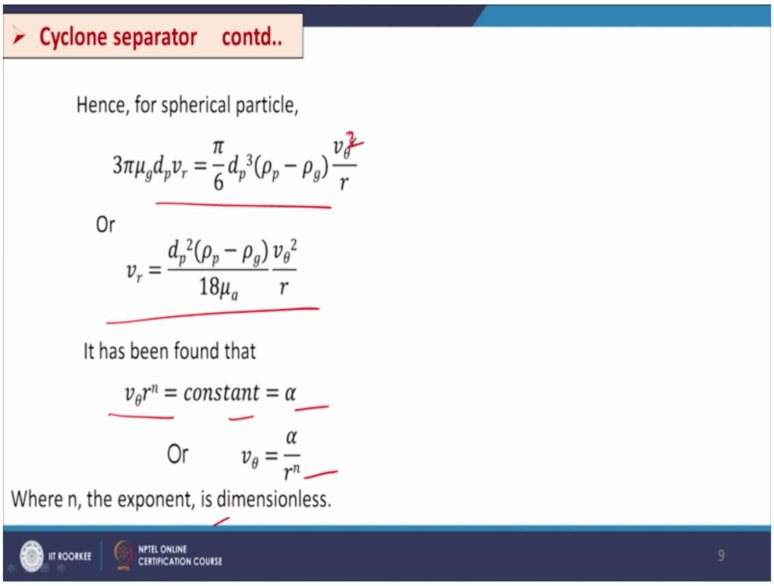

Now, we are going to calculate volumetric flow rate through a curved duct of rectangular cross section. So, if we have a duct like this. So, then we see the volumetric flow rate. And we will be trying to understand how the particles will be collected at the outer surface of the cyclone. Now, the volumetric flow rate here that is equal to we can get this one, we are considering very small like here.

So, this is the particle Q is there. So, from the centre the distance is r for the arc which is produced r and r_1 is the entry r_1 and r_2 is the outer. So, inner and outer diameter. So, this is our duct the particle is coming here with the Q, your volumetric flow rate. So, that volumetric flow rate is how we can calculate. If we have the cross-sectional area and if we have tangential velocity then we can do so, we have W, W is height or depth of the entrance section.

So, depth of the entrance section we have W and we are considering a very small radius small part that is say dr. So,

$$Q = W \int_{r1}^{r2} v_\theta dr$$

So, this is the volumetric flow rate through this small duct with diameter dr.

Now, if we integrate with r_1 to r_2 from this to this, so, then that will give us total volumetric flow rate through this duct. So, these expressions if we can rearrange from r_1 to r_2. So, then if we replace V_Θ, V_Θ is nothing but we have seen α/r^n. So

$$Q = \alpha W \int_{r1}^{r2} \frac{dr}{r^n} = \alpha W \frac{(r_2{}^{1-n} - r_1{}^{1-n})}{1-n}$$

Or by rearranging alpha value we can write

$$\alpha = \frac{Q(1-n)}{W(r_2{}^{1-n} - r_1{}^{1-n})}$$

So, what will be the V_Θ now,

$$v_\theta = \frac{Q(1-n)}{Wr^n\,(r_2{}^{1-n} - r_1{}^{1-n})}$$

And vr will be what? vr is equal to nothing but we have some relationship that is

$$v_r = \frac{d_p^2(\rho_p - \rho_g)\,(1-n)^2 Q^2}{18\mu_g W^2\, r^{2n+1}(r_2{}^{1-n} - r_1{}^{1-n})2}$$

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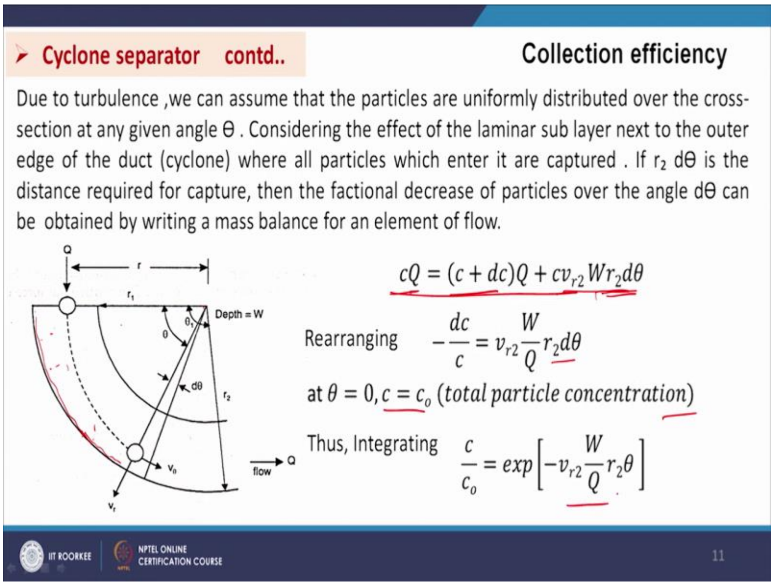

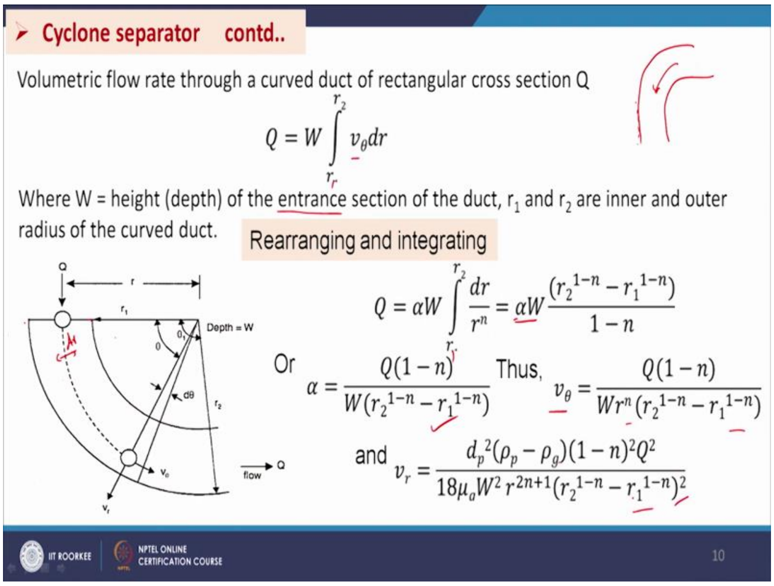

Now, we will see the collection efficiency. So, due to turbulence, we can assume that the particles are uniformly distributed over the cross sections at any given angle Ɵ. So, within this angle we are assuming the particles are almost uniformly distributed here and considering the effect of laminar sub layer next to the outer layer. And see this is our outer layer so, here will be laminar sub layer will be very attached to it, next to the outer edge of the duct cyclone where all particles which enter it are captured.

So, as I mentioned that particles has to come in contact with the surface of this. So, this is a very thin layer we are assuming. So, here if some particle comes that will be trapped that will be settled. Now, if $r_2 d\theta$ is the distance required say this place to this place r_2 and this is our

$d\Theta$ say. So, this volume will be $r_2 d\Theta$ this distance we are assuming is required to trap the particles.

So, within this surface area at the periphery of this separator some particles will be separated, then the fractional decrease of particles over the angle $d\Theta$ can be obtained by writing a mass balance for an element of flow. So, that how what are that mass balance if Q is the volumetric flow rate, so, volumetric flow rate*c that is the mass flow rate and that is getting entry to the duct and which is going out from this that is equal to (c + dc)Q, same volumetric flow rate but density is changing.

So, (c + dc)Q and some particles which are separated so, which particles will be separated that will come at the outlet sub layer. So, that will come at the laminar sub layer which is generated inside the cyclone at its periphery. So, this place, now, then what is that collection or what is that separation, so, cross sectional area*vr*concentration.

So, what is the cross-sectional area we know that W is the height and r_2 d theta is our, this one side so, these $r_2 d\Theta$*W is our cross-sectional area. So, towards this direction if the flow is towards the outer side then this is our cross-sectional area and we will be multiplying with this radial velocity v_{r2} here vr_2 why because, when r value is r_2 then only separation will take place.

So, then vr_2*c, so this is the mass flow rate for the separation of the particles or the particles which are coming at the outer surface and getting settled. So, this is overall mass balance, so by rearranging we can get

$$-\frac{dc}{c} = v_{r2}\frac{W}{Q}r_2 d\theta$$

when theta = 0, c = c_o, that is total particles concentration at that time just what particles are getting entry into the separator. No, decrease. So, that c_o thus integrating we can get

$$\frac{c}{c_o} = \exp\left[-v_{r2}\frac{W}{Q}r_2\theta\right]$$

So, these expressions we are getting.

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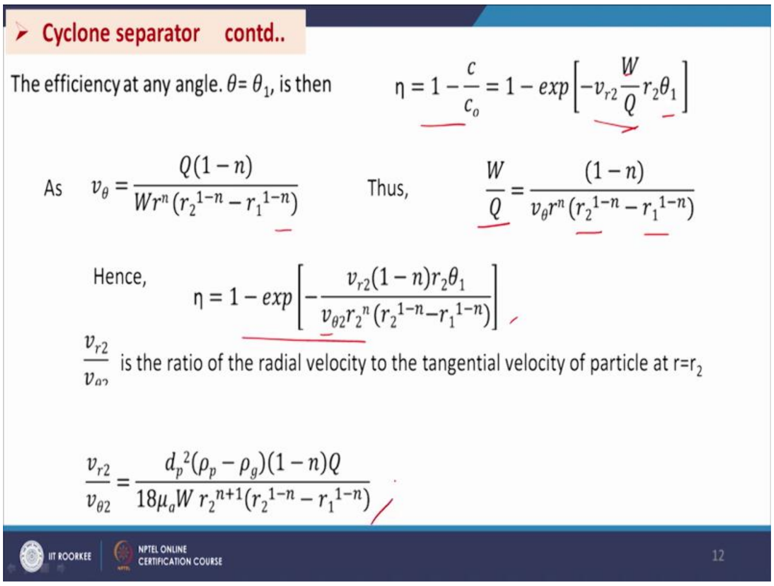

And we know that efficiency is

$$\eta = 1 - \frac{c}{c_o} = 1 - \exp\left[-v_{r2}\frac{W}{Q}r_2\theta_1\right]$$

$$v_\theta = \frac{Q(1-n)}{Wr^n\,(r_2{}^{1-n} - r_1{}^{1-n})}$$

$$\frac{W}{Q} = \frac{(1-n)}{v_\theta r^n\,(r_2{}^{1-n} - r_1{}^{1-n})}$$

Now, we will replace this W/Q value here and if we replace then we will get the efficiency in terms of this expression

$$\eta = 1 - \exp\left[-\frac{v_{r2}(1-n)r_2\theta_1}{v_\theta r_2^n\,(r_2{}^{1-n} - r_1{}^{1-n})}\right]$$

Now, what is this? v_{r_2} by v theta 2 is nothing but when the r is equal to r_2 at that time the radial velocity and the tangential velocity ratio. So, at r equal to r_2, so, now this is equal to we are getting we know by the definition of these forces

$$\frac{v_{r2}}{v_{\theta 2}} = \frac{d_p^2(\rho_p - \rho_g)\,(1-n)Q}{18\mu_g W\,r^{n+1}(r_2{}^{1-n} - r_1{}^{1-n})}$$

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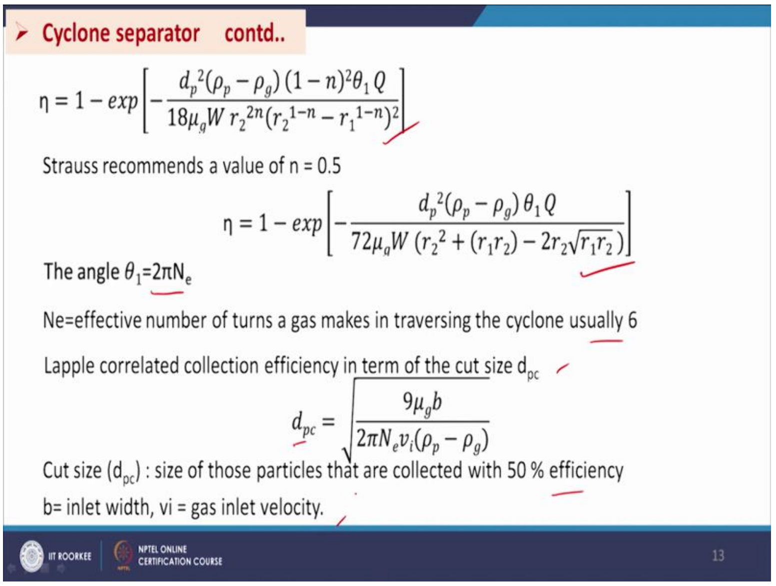

$$\eta = 1 - \exp\left[-\frac{d_p^2(\rho_p - \rho_g)(1-n)^2 Q\theta_1}{18\mu_g W r_2^{2n}\,({r_2}^{1-n} - {r_1}^{1-n})}\right]$$

Now, if we put the value of n is equal to 0.5

$$\eta = 1 - \exp\left[-\frac{d_p^2(\rho_p - \rho_g)Q\theta_1}{72\mu_g W\,(r_2^2 + (r_1 r_2) - 2r_2\sqrt{r_1 r_2})}\right]$$

Now, how we can get the cut diameter for that some empirical relationships are available and if $\Theta = 2\pi N_e$, that is if the revolution is your Ne, Ne is the effective number of tons of gas makes in traversing the cyclone usually it is 6 tones it takes.

So, then Θ_1 will be 2π*6 so, 12π. So, collection efficiency in terms of the cut diameter and that cut diameter is mentioned as

$$d_{pc} = \sqrt{\frac{9\mu_g b}{2\pi Nevi(\rho_p - \rho_g)}}$$

Where the d_{PC} is the cut side or cut diameter and size of those particles that are collected with 50 % efficiency, b equal to inlet width and vi is the inlet gas velocity.

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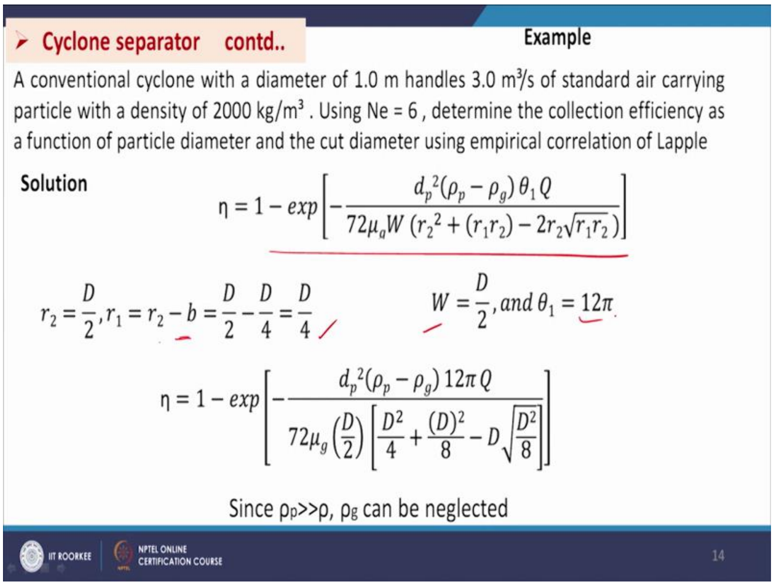

Now, we will see one numerical example, a conventional cyclone with a diameter of 1-m handles 3 m^3/s of standard air carrying particle with a density of 2000 kg/ m^3. Using Ne = 6 determine the collection efficiency as a function of particle diameter and the cut diameter using empirical correlation of Lapple.

So, we have this formula

$$\eta = 1 - \exp\left[-\frac{d_p^2(\rho_p - \rho_g)Q\theta_1}{72\mu_g W\,(r_2^2 + (r_1 r_2) - 2r_2\sqrt{r_1 r_2})}\right]$$

And now, from the discussion which we have made on typical dimensions of different parts of the cyclone separator, we can say

$r_2 = D/2$, where D is the diameter

$r_1 = r_2 - b$, where b is equal to D/4

so, this $r_1 = D/2 - D/4 = D/4$ and W is equal to height that is already D/2 and $\Theta_1 = 6*2\pi = 12\pi$.

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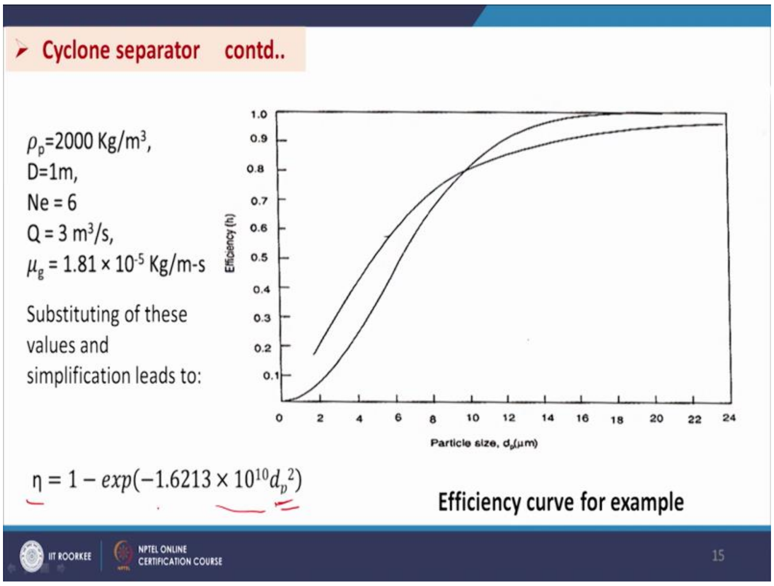

So, we will be putting these values here in this expression.

$$\eta = 1 - \exp\left[-\frac{d_p^2(\rho_p - \rho_g)Q12\pi}{72\mu_g \frac{D}{2}\left(\frac{D^2}{4} + \left(\frac{D^2}{8}\right) - D\sqrt{\frac{D^2}{8}}\right)}\right]$$

$\eta = 1 - \exp(-1.6213*10^{10}d_p^2)$

So, now the first part we are asked to express the efficiency in terms of the d_p^2. So, now this part is solved and if we put a different value of dp, the efficiency value will change as per this graph and as per this expression.

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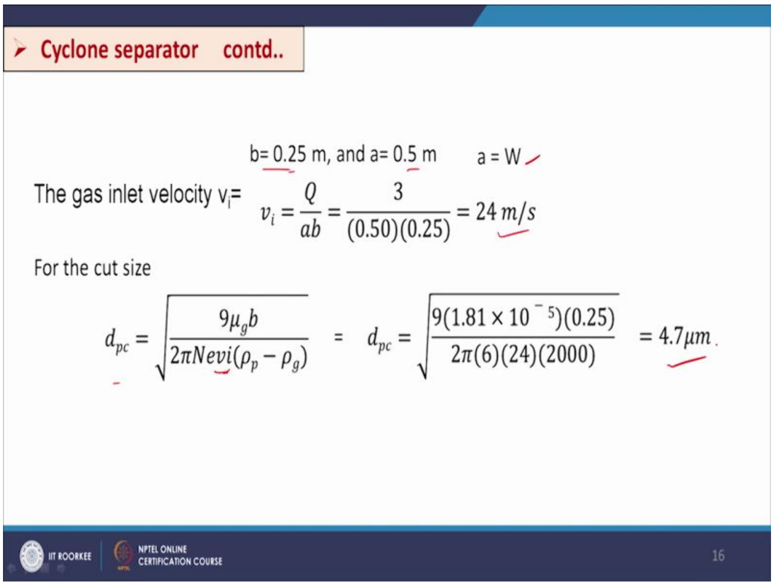

Now, the second part we have to calculate the dpc,cut diameter

$$d_{pc} = \sqrt{\frac{9\mu_g b}{2\pi N e v i(\rho_p - \rho_g)}}$$

So, here we have to calculate the Vi, other values are given,

b = 0.25 m, because D = 1 meter. So, b/4 = 0.25 m and ‘a’ = 1/2 = 0.5 m and ‘a’ = W,

Vi = Q/ab. = 3/(0.5)(0.25) = 24 m/s.

So, all other values are given Vi is calculated we will put it here

$$d_{pc} = \sqrt{\frac{9(1.81 \times 10^{-5})(0.25)}{2\pi(6)(24)(2000)}}$$

d_{pc} = 4.7 µm.

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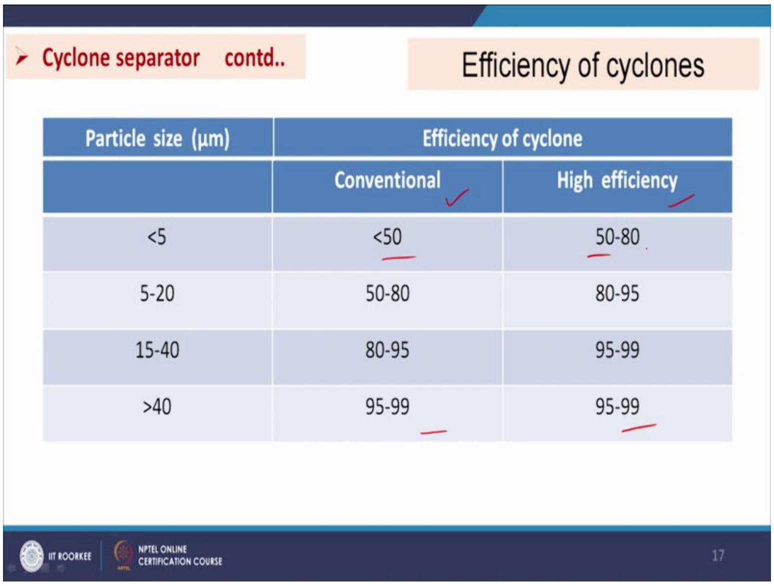

Particle size (µm)	Efficiency of cyclone	
	Conventional	High efficiency
<5	<50	50-80
5-20	50-80	80-95
15-40	80-95	95-99
>40	95-99	95-99

Now, we will see the efficiency of cyclones. So, from this the conventional and high efficiency cyclones if we compare the efficiency for different particle size, then we see that when the particle size is relatively bigger than the efficiency is also very higher, but when the particle size is lesser the efficiency is also less.

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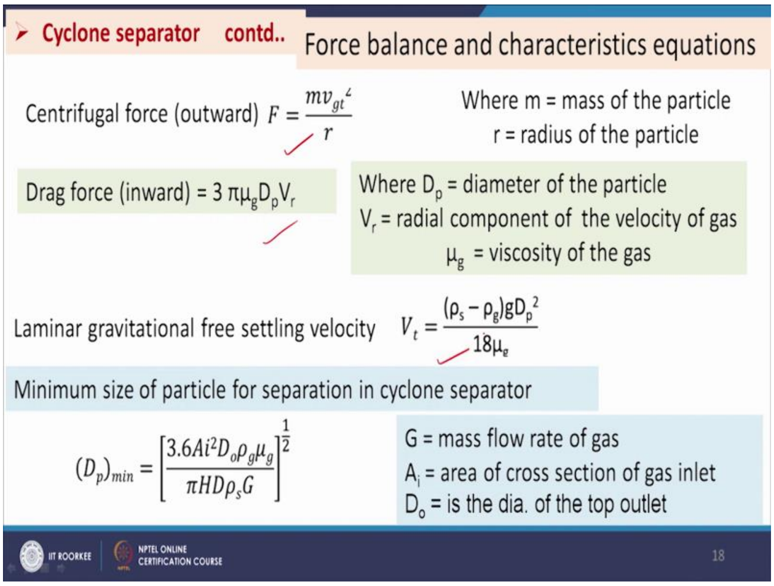

And these are the generalized expressions already we have discussed. So, centrifugal force this one and drag force and then laminar gravitational settling velocity this one and this is Dp minimum.

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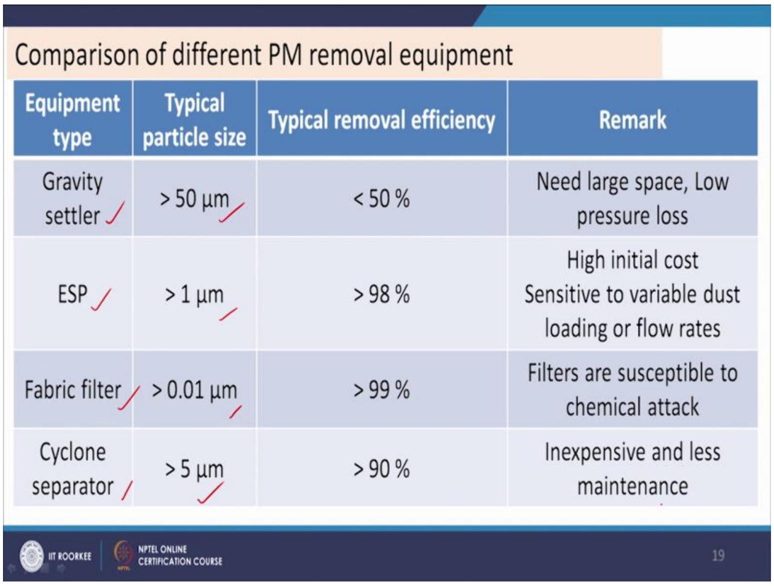
Comparison of different PM removal equipment

Equipment type	Typical particle size	Typical removal efficiency	Remark
Gravity settler	> 50 µm	< 50 %	Need large space, Low pressure loss
ESP	> 1 µm	> 98 %	High initial cost Sensitive to variable dust loading or flow rates
Fabric filter	> 0.01 µm	> 99 %	Filters are susceptible to chemical attack
Cyclone separator	> 5 µm	> 90 %	Inexpensive and less maintenance

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And now we will compare different particulate removal equipment like say gravity settler, electrostatic precipitator, fabric filter and cyclone separator we have discussed all four. And here we can see the comparison likes a typical particle size which can be separated for gravity settler greater than 50 µm for ESP greater than 1 µm for fabric filters greater than 0.01 µm and for cyclone separator greater than 5 µm and typical removal efficiency are also given.

So, there all of the processes have their some characteristics as remark it is given and that is for gravity settler they need large space low pressure loss and for ESP high initial cost sensitive to variable dust loading of flow rates, filters are susceptible to chemical attack for fabric filter and for cyclone separator inexpensive and less maintenance is required. So, up to this in this class. Thank you very much for your patience.