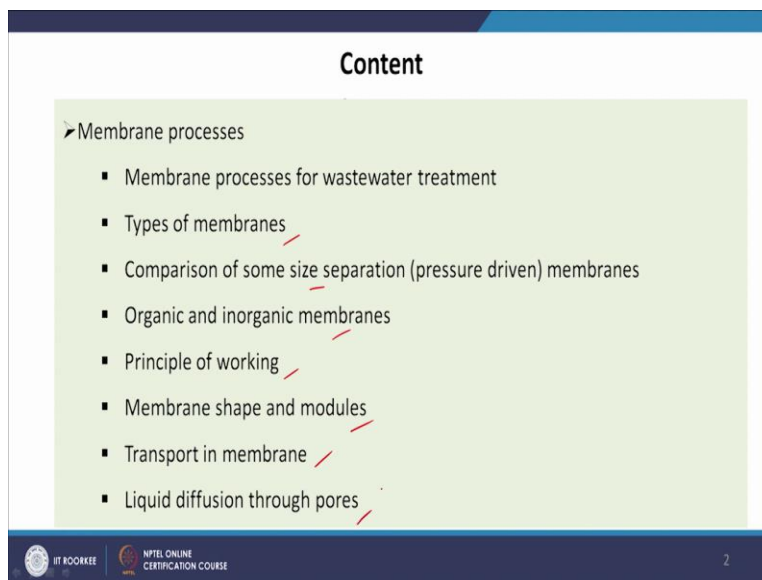


Basic Environmental Engineering And Pollution Abatement
Professor Prasenjit Mondal
Department Of Chemical Engineering
Indian Institute Of Technology, Roorkee
Lecture 38
Tertiary Treatment 2

Hello, everyone. Now, we will discuss on the topic Tertiary Treatment part 2. In this class we will discuss on membrane processes.

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Content

- Membrane processes
 - Membrane processes for wastewater treatment
 - Types of membranes
 - Comparison of some size separation (pressure driven) membranes
 - Organic and inorganic membranes
 - Principle of working
 - Membrane shape and modules
 - Transport in membrane
 - Liquid diffusion through pores

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And content is membrane processes for wastewater treatment, types of membranes, comparison of some size separation membranes, organic and inorganic membranes, principle of working, membrane shape and modules, transport in membrane and liquid diffusion through pores.

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➤ Membrane processes for wastewater treatment

- Membrane processes are physical diffusion processes of particles in water. They function because certain types of membranes allow particles with particular characteristics only to pass through them
- Membranes are thin and porous sheets of material able to separate contaminants from water when a driving force is applied

Advantages:

- High performance ✓
- Compact units: less space needed than conventional treatment schemes ✓
- Simple operation ✓
- Membranes available can be used to separate many kinds of contaminants ✓
- Disinfection can be performed without chemicals ✓

Disadvantages:

- Membrane fouling ✓
- Production of polluted water (from backwashing) ✓
- Membranes have to be replaced on a regular basis

❖ Can be used for production of potable drinking water from ground, surface and seawater sources, as well as for the advanced treatment of wastewater and desalination.

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The membrane processes are physical diffusion processes of particles in water. They function because certain types of membranes allow particles with particular characteristics only to pass through them. And membranes are thin and porous sheets of material able to separate contaminants from water when a driving force is applied.

And this method has many advantages. Like say, performance is very high, it is very compact unit, it requires very less floor area, very less space is needed than conventional treatment schemes, and simple operation, membranes available can be used to separate many kinds of contaminants and disinfection can be performed without chemicals.

But it has some disadvantage also. Like say, membrane fouling. The pollutants are deposited on the membrane surface and also in the pores of the membrane and those are clogged and it loses efficiency. And production of polluted water, water molecule passes through the membrane and one side, we get the permeate, and other side, we get the concentrate. So, this concentrate or retentate, this other side is called retentate so that retentate contains the pollutants and the concentration of the pollutants is very high in that part.




And membranes have to be replaced on a regular basis. So, it is costly method basically. And this process can be used for production of potable drinking water from ground, surface and seawater sources as well as for the advanced treatment of wastewater and desalination. This method can

be used for different types of wastewater treatment basically, from home use to very large scale production.

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Types of membrane

Division factor	Driving force	Type – operation
Size	Pressure	Filtration
		Microfiltration
		Ultrafiltration
		Nanofiltration
Size / Diffusion	Pressure / Concentration	Reverse Osmosis
Ion Charge / Diffusion	Electric field	Electrodialysis Reversal Electrodialysis
Temperature (Hydrophobic membrane)	Steam pressure	Membrane distillation




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Now we will see the types of membrane depending upon different driving forces and division factor, we can have different types of membrane and their operation. Like say when driving force is pressure and division factor is size, we can have filtration, microfiltration, ultrafiltration nanofiltration. So, where the size will be different for different filtration.

And when pressure and concentration are responsible for this we can have the process that is Reverse Osmosis, and size and diffusion both are the division factor. And when we use the electric field as a driving force, we can get electrodialysis and reversal electrodialysis. In that case, ion charge and diffusion, both are considered as division factor.

And when steam pressure is used as the driving force, and then temperature is the division factor for hydrophobic membrane, we can say this is the application for membrane distillation. So, these are the different types of membrane on the basis of application and on the basis of driving force and division factor.

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• Comparison of some size separation (pressure driven) membranes

	Microfiltration ✓	Ultra filtration ✓	Reverse osmosis ✓
Typical membrane materials ✓	Cellulose acetate, poly sulphone, ceramics ✓	Cellulose acetate, polyamides, poly sulphone ✓	Cellulose acetate, aromatic polyamides ✓
Pore size (Angstrom) ✓	100000-200 ✓	200-10 ✓	10-1 ✓
Pore size (microns) ✓	10-0.02 ✓	0.02-0.001 ✓	0.001-0.0001 ✓
Membrane pressure drop (psi) ✓	1-10 ✓	10-100 ✓	100-1000 ✓
permeate ✓	water+ dissolved molecules ✓	Water + small molecule ✓	Water ✓
Retentate (concentrate) ✓	Water + large suspended particles ✓	Water+ large molecule ✓	Water + solutes ✓

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Now, we will see the comparison of some size separation that is pressure driven membranes. So here, the parameters typical membrane material, pore size, membrane pressure drop, permeate and retentate, that is, concentrate. So, pressure driven processes are microfiltration, ultrafiltration and reverse osmosis, three have been chosen. And then, we will see the difference.

Typical membrane material, if you see, the cellulose acetate, poly sulphone, ceramics in case of microfiltration. ultrafiltration, cellulose acetate, polyamides, poly sulphone and reverse osmosis cellulose acetate, aromatic polyamides. That means, the materials are very similar. But pore size is different.

In case of microfiltration, that is 10-to-0.02-micron size, where ultra, this is 0.02 to 0.001 micrometer size, and reverse osmosis 0.001 to 0.0001 micrometer. So, we are seeing even very, very small particles or molecules or the pollutants can also be arrested here or be separated here in case of RO.

So, certainly, it will be requiring different pressure, that is in this case membrane pressure drop psi 1 to 10, here 10 to 100, and 100 to 1000, and permeate, in this microfiltration, maybe water and dissolved molecules and ultrafiltration water + small molecules, and reverse osmosis only water, even small molecules are also not able to pass through because the pore size is very, very small. And retentate, water for large suspended particles for microfiltration water + large



molecule, for ultrafiltration and water + solutes, even the solutes are also not able to pass through the pores in case of reverse osmosis.

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➤ Organic and inorganic membranes

Organic membrane			Inorganic membrane		
Material	Max Temp (°C)	pH range	Material	Max Temp (°C)	pH range
CA	30-40	3-7	Al ₂ O ₃	>900	1-14
PA/PI	60 - 80	2-11	SiC	800	1-14
PES/PS	60-100	1-13	ZrO ₂	400	1-14
PVDF	130-150	1-13	TiO ₂	350	1-14

CA: Cellulose diacetate PA: Polyamide (aromatic and aliphatic) PI: Polyimide
 PES: Polyether sulfone PS: Polysulfone PVDF: Polyvinylidene fluoride



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Now, we will see organic and inorganic membranes, different types of membranes and their applicability. Normally, the organic membranes are applied in lower temperature and inorganic membranes are used at higher temperature. So some examples are given here, that is cellulose diacetate and PA, that is polyamide are aromatic and aliphatic, and PI, polyimide, and PES Polyether sulfone, PS polysulfone, PVDF that is polyvinylidene fluoride.

And here we see the maximum temperature range, this can be used. And for inorganic membranes that is Al₂O₃, silicon carbide, ZrO₂, TiO₂, we see the maximum temperature range is much more than this organic membrane, and this can be used for a wider pH range. So, these are the different available membrane materials.

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➤ **Organic and inorganic membranes**

Polymeric membranes

c1ccc(cc1)S(=O)(=O)c2ccc(cc2)

← PES

*c1ccc(cc1)S(=O)(=O)c2ccc(Oc3ccc(cc3)C(C)C)c2

← PS

C(F)(F)F(C(F)(F)F)C(F)(F)F(C(F)(F)F)C(F)(F)F(C(F)(F)F)C(F)(F)F

← PVDF

Polymer name:	Acronym
Cellulose diacetate and triacetate	CA, CTA
Polyacrylonitrile	PAN
Polyamide (aromatic and aliphatic)	PA
Polysulfone	PS
Polyether sulfone	PES
Polycarbonate	PC
Polyethylene terephthalate (track-etched)	PET
Polyimide	PI
Polyethylene	PE
Polypropylene	PP
Polytetrafluoroethylene	PTFE
Polyvinylidene fluoride	PVDF
Polyvinylchloride	PVC
Polyvinyl alcohol	PVA
Polybenzimidazole	PBI
Polyetherimide	PEI
Polydimethylsiloxane	PDMS

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And here, the structure of some organic membranes are shown here like say PES has this type of structure, PS has this type of structure PVDF has this structure. And these are the different polymer name and they are acronym, which have been applied for different applications not only for water treatment, it has been used for different application.

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➤ **Principle of working and important terms**

- Separation by means of a semi permeable barrier (membrane) through which one or more species move faster than another or other species

General membrane process

Characteristics

- Separating medium is a semi permeable barrier
- A sharp separation is often difficult to achieve
- Low energy requirement

- Retentate: part of the feed that does not pass through the membrane
- Permeate: part of the feed that passes through the membrane
- Sweep: liquid or gas used (optional) to help remove the permeate
- The barrier is most often a thin, nonporous, polymeric film, but may also be porous polymer, ceramic or metal materials or even a liquid or gas
- The barrier must not dissolve, disintegrate or break

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Now, we will see the principle of working and some important terms. As mentioned, in this process, feed mixture is passing through the one side of the membrane and other side, the sweep

is sent as optionally. Otherwise, the water will be passing through it the membrane, and then the pollutants will be arrested here, that will not pass through.

So this part which we are getting, that is where you are getting retentate or reject, and this is concentrate. And here, we are getting the permeate. So, pure water will come here and pollutants will be available in this, concentrated here. So, this is where we are getting retentate and this is where we are getting permeate. So, this is the mechanism.

Then how these, water is passing through the pore size the molecules of the water size is lesser than the pollutants, so pollutants are being arrested, so water is going there. So, different terms we are getting, this retentate part of the feed that does not pass through the membrane. Permeate, part of the feed that passes through the membrane.

Sweep, liquid or gas used to help remove the permeate, and the barrier is most often a thin, nonporous, polymeric film but may also be porous polymer ceramic or metal materials or even a liquid or gas, and the barrier must not dissolve, disintegrate or break. So, these are the essential conditions for the operations of this membrane process.

And some characteristics are that separating medium is semi permeable barriers. So, this medium is semi permeable that means, some material will be allowed to pass through and some will not be allowed to pass through, and the sharp separation is often difficult to achieve and low energy requirements. So, these are the characteristics of this membrane process.

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➤ **Membrane shape and modules**


- Membranes are placed into modules

Important membrane geometry (shapes)

- Flat asymmetric or thin film composite
- Tubular
- Hollow fiber
- Monolithic

- Four major types of membrane modules
 - Plate and frame module (flat-sheet)
 - Spiral-wound module (flat-sheet)
 - (Multi-)tubular membrane (cylindrical)
 - Hollow-fiber membrane (cylindrical)

A membrane module is a complete unit composed of the membranes, a housing, feed inlet, concentrate outlet, and permeate outlet.

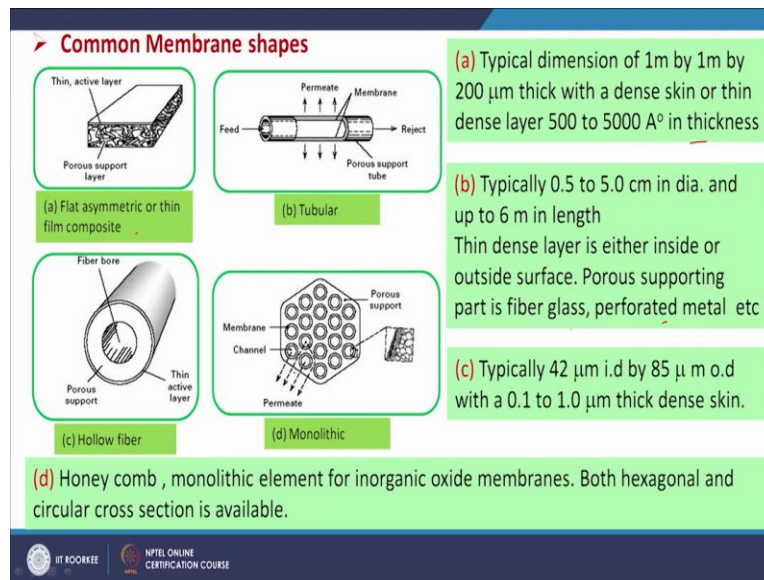


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Now, we will see the membrane shape and modules. So, membrane may have different geometry and different shape like say flat asymmetric or thin film, composite membrane, tubular membrane, hollow fiber membrane, and monolithic structure. And there are four major types of membrane modules that is plate and frame module, spiral wound module, multi tubular membrane module and then hollow fiber membrane module.

Now, what is the module? So, membranes, we see which is used for the separation of the pollutants from it, and this is very thin layer of some organic or inorganic material, and the module is a complete unit composed of the membranes, a housing, feed inlet, concentrate outlet and permeate outlet. So, this is a complete module, and membrane is then important part.

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Now, we will see the common membrane shapes. So, we have seen there are four important membrane shapes, that is, flat asymmetric or thin film composite. Here, we see the thin active layer here, and this is porous support. So, we have porous support an active layer is our main fiber material.

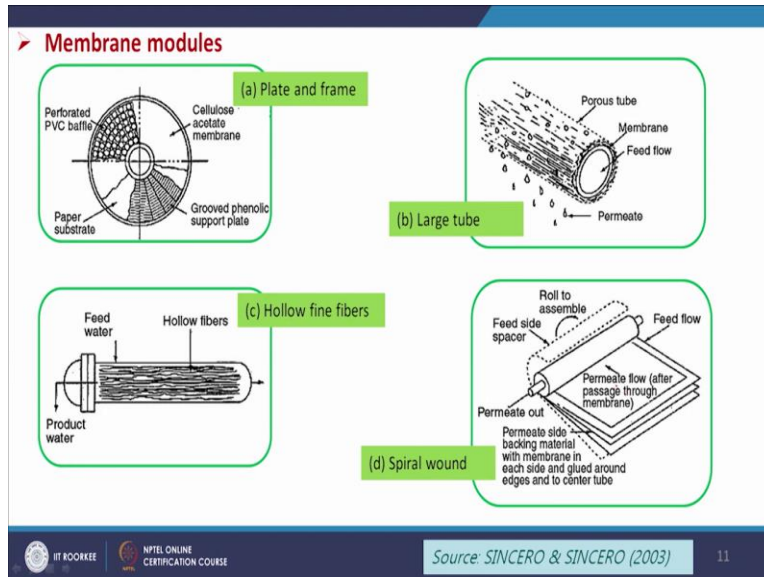
And typical dimension of 1 m by 1 m by 200 μm thick, with a dense skin or with dense layer 500 to 5000 \AA in thickness. So, that, I told you, that it is a very thin layer with this membrane material. And then, we see the tubular. So, we have feed inside, and from the outside we are getting the, permeate and from the inside we are getting the reject or the retentate, so in this case what will be happening.

A typically 0.5 to 5 cm in diameter and up to 6 m in length thin dense layer is either inside or outside surface. So, a porous supporting part is fiberglass, perforated metal etc. So, this is a tubular membrane shape.

And hollow fiber, here we see this is our fiber bore, and thin active layer, and we have a porous support. So, typically 42 μm id by 85 μm od with a 0.1 to 1 μm thick dense skin. So, this is a hollow fiber membrane, the hollow fiber and tubular, these are basically, differ, on the dimension also. And then monolithic type shapes is also available and this is basically for inorganic oxide membranes.

So, this is the Honeycomb we see here. So that monolithic element for inorganic oxide membranes, both hexagonal and circular cross section is available, that maybe, this cross sectional circular or hexagonal as shown here, can be available. So, we have a membrane and these are the channel and porous support. So, we get permeate from this. So this is a membrane shape.

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And then membrane module, we have plate and frame module. There is a plate and frame, just like plate and frame filter press, we have some, one frame and another plate inside we have some membrane. So that is one. And large tube so we have a tube, the porous tube and then either inside or outside, we have a membrane.

Then feed flow, if it goes inside, we will be getting permeate from the outside of it. And then hollow fine fibers so here also the similar way, the dimension is different in tubular and hollow fiber, we get here the feed water is coming from the outside. So, from the inner side will be getting the product.

And spiral wound, this is example of spiral wound. So here, feed flow in that direction, and then from the inner side, will be getting the, penetrate and these are the different permeate side backing material with membrane in each side and glued around edges and to center tube. So, this is a permeate flow from the out to inner, and then we are getting the permeate from the inner side of it.


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➤ **Membrane modules**

- Most lab-scale membrane modules
- Filter presses, MBR (submerged)


Plate and frame module

Ovivo USA, LLC





Source- mi.wea.org

The MBR site



Source- aulabiindicacion.com

Kubota



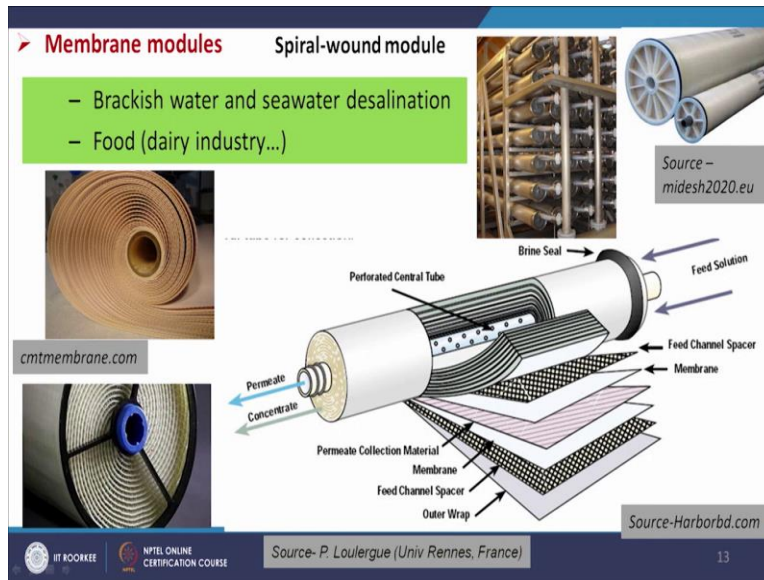
Source- mi.wea.org

BY ROCKEE

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And now we will see some photographs and the concept will be more clear. Now, some example a plate and frame module, most lab-scale membrane modules, so, this is one, so, this is your plate and frame. So, that is Ovivo USA, LLC, their product, and MBR membrane bioreactor also, we have this plate and frame type module, plate and frame module of the membranes, and this is another example. These are also the plate and frame type of module.

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Then spiral wound module, you see here, how it looks. So, this can be used for brackish water and seawater desalination and for treating wastewater of food dairy, industry or distilled industry. So, this type of spiral wound module is used. So, here you see, this is our, permeate and this is our feed solution.




So, that will be our permeate collection material and feed channel spacer, and membrane is here. So, again, we are seeing membrane and feed channel spacer. So, each water, inlet water is coming here. So, it will be from outer to inner side it will move and from the inner, we will get the, permeate, and this type of arrangement is there that is called spiral wound module.

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➤ Membrane modules

Tubular modules

- Mostly ceramic but also polymeric
- Mono- or multi-channel
- Waste and drinking water, Food, biotech...



(a)

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And then tubular modules, so mostly ceramic, but also polymeric. And mono or multi channel, it is possible, and waste and drinking water, food, for the production of biotech application, these modules are used as shown here.


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➤ Membrane modules





Hollow fibers modules

- Drinking water, wastewater (MBR)

View of the lower end of a microfiltration (MF) module for a membrane water treatment system



Cutaway view of a microfiltration (MF) module for a membrane system



Source- spintek.com

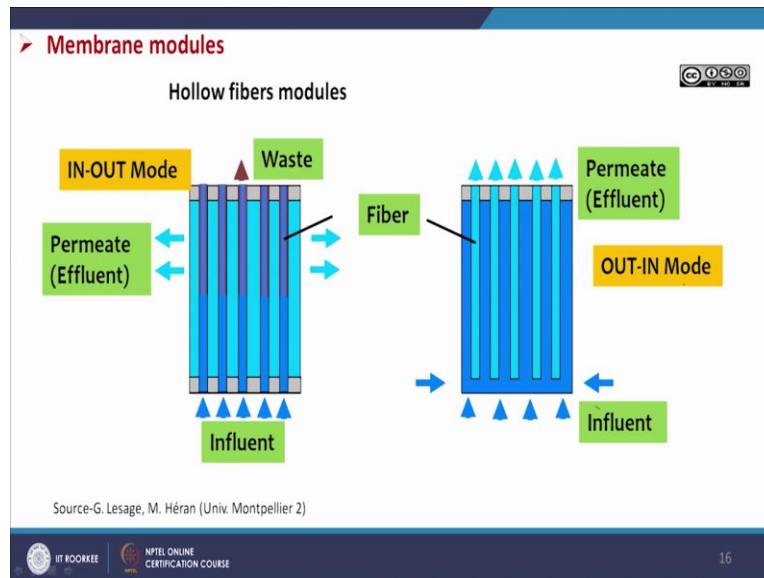
From P. Loulergue (Univ Rennes, France)

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Hollow fiber module, so, that is used for drinking water and wastewater treatment. And you see here, these are the hollow fiber so, number of fibers are taken here. So, water maybe going from

the outside and then from the inner side, we will be getting the, permeate, the treated water. So, this type of modules are available for different applications for drinking water and wastewater also, for the production of drinking water, and treatment of wastewater as well.

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And this figures, shows us how the hollow fiber modules work. So, here, the influent maybe through the inner part of the hollow fiber. So the penetrate will come out from it. So, then this will be our waste or retentate and this is IN-OUT mode. So, influent from the inner side of the hollow fiber and from the outside, we are getting permeate. May be on the reverse case, this may be going from the outside, and from the inner side we can get the permeate. So, this our effluent and this is our influent, influent is coming here and we are getting the permeate here. So, that is OUT-IN mode. So, these are the fiber.

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➤ Membrane modules

Typical characteristic of membrane modules

	Plate and frame ✓	Spiral or wound ✓	tubular ✓	Hollow -fiber ✓
Packing density m^2/m^3 ✓	30- 500 —	200-800 —	30-200 —	500-9000 —
Resistance of fouling	Good ✓	Moderate ✓	very good ✓	Poor ✓
Ease of cleaning	Good ✓	Fair ✓	excellent ✓	Poor ✓
Relative cost	High ✓	Low ✓	high ✓	Low ✓
Main applications	D,RO, UF,MF	D,RO, UF,MF	RO,UF,	D,RO, UF

D: distillation

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Now, typical characteristics of membrane modules like say plate and frame, spiral or wound and then tubular and hollow fiber. So, packing density, m^2/m^3 , so 30 to 500 plate and frame, 200 to 800 per spiral or wound type and tubular type 30 to 200, and 500 to 9000 hollow fiber.

So, hollow fiber, we see we get much packing density with respect to other, and resistance or fouling in plate and frame good resistance and spiral or wound, moderate, tubular, very good resistance and then this is poor resistance. And ease of cleaning, again ease of cleaning is good for plate and frame. After certain interval you can change it.

Spiral or wound, that is fair, and for tubular, excellent, and for hollow fiber, it is again poor. So relative cost, very high cost for plate and frame, spiral and wound, it is low cost, tubular, high and hollow fiber, low cost. And main applications, distillation, reverse osmosis, ultrafiltration, microfiltration for plate and frame, for spiral or wound, distillation, RO, UF, MF, and here for tubular RO, UF and D, RO, UF for hollow fiber membrane module. So, these are the different applications.

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➤ **Membrane modules**

Module geometry	Advantages	Drawbacks
Plate and frame (for both organic and inorganic membranes)	Possibility to analyze permeate at each stage (interesting for scaling)	Low packing density (compactness)
Spiral wound (organic membranes only)	High packing density Low cost	Cleaning-in-place is difficult to handle Pretreatment compulsory
Tubular (for both organic and inorganic membranes)	Limited pretreatment Well adapted to cleaning-in-place	Low packing density High energy consumption
Hollow fiber (mostly organic membranes)	Very high packing density Low energy consumption Low cost	Pretreatment compulsory for in-out mode

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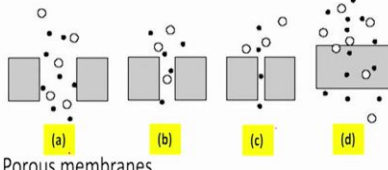
And we can get some advantage and disadvantage, all these types of modules, like say plate and frame, for both organic and inorganic membranes. The advantage possibility to analyze permeate at each stage, but low packaging density, this is the drawback. So, spiral or wound, that is organic membranes only. This is not applicable for inorganic membrane. So, high packing density low cost, and cleaning in place is difficult to handle.

And pretreatment is compulsory. And tubular type, for both organic and inorganic membranes, it can be used. And then limited pretreatment, well adapted to cleaning in place and low packing density, high energy consumption are the drawbacks. And hollow fiber, mostly organic membranes, and then very high packing density, low energy consumption and low cost are advantage, and pretreatment compulsory for IN-OUT mode. So, this is a drawback of it.

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Transport in membrane

Mechanism for the transport of liquid and gas molecules through a porous membrane are :



Mechanism of transport in membranes.
(Flow is downwards).
(a) Bulk flow through pores; ✓
(b) Diffusion through pores;
(c) Restricted diffusion through pores;
(d) Solution-diffusion through dense membranes.

Porous membranes

Bulk flow

Bulk flow of a fluid due to pressure difference through an idealized straight cylindrical pore

If the flow is in laminar regime ($N_{re} < 2100$), which is almost always the case for flow in small diameter pores, the flow velocity v is given by Hagen-Poiseuille pressure drop :

$$v = \frac{D^2}{32\mu L}(P_0 - P_L)$$

Where D is the pore diameter, μ is viscosity of fluid, L is length of pore, P_0 is inlet pressure, P_L is outlet pressure

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Now, we will see the transport in membrane. So what is the mechanism for the transport of the liquid and gas molecules through a porous membrane, that we will discuss here. So, we are showing here different situation when the pollutants and gas molecules or water molecules, both are passing through the pores. So, that is a, there is bulk flow through pores.

So, in that case, we are not getting the efficient separation or the pollutants from the water, but b, is diffusion through pores, through pores it is diffused. So, but c, restricted diffusion. And here some other type of mechanism is possible, that solution diffusion through dense membrane. So, dense membranes, not we are considering the pores, that has solution, that is coming inside it, and it is being diffused.

So, that way the smaller size molecules or which is having more affinity with the membrane material, they can pass through it easily with respect to other molecule. So, that is the mechanism for the purification of the water or the removal of pollutants from the water. And now, we will see the bulk flow.

So, in this case bulk flows. So bulk flow of a fluid due to pressure difference through an idealized straight cylindrical pore. If the fluid in laminar region, which is almost always the case of flow in small diameter pores, the flow velocity v is given by Hagen-Poiseuille pressure drop. That means, it is a flow through pores or porous media.

In that case, the Hagen-Poiseuille equation can be applied their

$$v = \frac{D^3}{32\mu L} (P_0 - P_L)$$

Where P_0 is a inlet pressure, P_L is the outlet pressure, D is the pore diameter, μ is a viscosity of fluid, and L is the length of the pore.

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Transport in membrane

If the membrane contains n such pores per unit cross section of membrane surface area normal to flow, the porosity (void fraction, ϵ) of the membrane is $\epsilon = n\pi D^2/4$

Then the superficial fluid bulk flow flux (mass velocity), N through the membrane is:

$$N = v\rho\epsilon = \frac{\rho \epsilon D^2}{32\mu_M} (P_0 - P_L) = \frac{n\pi\rho D^4}{128\mu_M} (P_0 - P_L)$$

Where l_M is the membrane thickness and ρ and μ are fluid properties

In real porous membrane pores may not be cylindrical and straight and hence pore diameter is replaced by hydraulic diameter

$$d_H = 4 \left(\frac{\text{Volume available for flow}}{\text{Total Pore surface area}} \right)$$

The specific surface area, a_v , which is the surface area per unit volume of membrane material (not including the pores), Thus, $a_v = a/(1-\epsilon)$

$$= \frac{4 \left(\frac{\text{Total Pore volume}}{\text{Membrane volume}} \right)}{\left(\frac{\text{Total Pore surface area}}{\text{Membrane volume}} \right)} = \frac{4\epsilon}{a_v}$$

Further, pore length is longer than the membrane thickness and can be represented by $l_M\tau$, where τ is a tortuosity factor

$$N = \frac{\rho \epsilon^3 (P_0 - P_L)}{2(1-\epsilon)^2 \tau a_v^2 \mu_M}$$

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Now, if the membrane contains n such pores per unit cross sections, if you have a membrane of unit cross section. If we have n such pores or membrane surface area normal to flow, the porosity of the membrane is how much? Πr^2 for a single, so, $\Pi D^2/4$ for a single one. So, $n * \Pi D^2/4$, that is the total surface area covered by this pores. And total surface area of the membrane is 1, we have assumed.

So the void fraction, that will be the volume of the pores divided by volume of the filter. So, if we take the unit length once again, so, $1*1$, the 1 volume and this will be $n* \Pi D^2/4 * 1$, that will be the volume of the pores. So, ϵ is equal to volume of the pores by volume of the membrane, that is equal to 1. So, $n\Pi D^2/4$, that is equal to void fraction.

Then, the superficial fluid bulk flow flux, that is mass velocity, n through the membrane is,

$$N = v\rho\epsilon$$

So, we are considering unit cross section. So, n is the mass flux. So, that is why unit cross section is considered. So, unit cross section $\times v \times E$, that is the volume for the void $\times \rho$.

So, that is the volume of the flux because the water will be passing through the void space only. So, that is equal to, we will replace the value of v now, with a Hagen-Poiseuille expression,

$$N = v\rho E = \frac{\rho E D^2}{32\mu L_M} (P_0 - P_L) = \frac{n\pi\rho D^4}{128\mu L_M} (P_0 - P_L)$$

Now, L_M is the membrane thickness and ρ and μ are the fluid properties. So in real porous membrane, pores may not be cylindrical and straight, and hence pore diameter is replaced by the hydraulic diameter.

So, this is one considerations, we assume when, to represent the real situation. So, d will be replaced by d_H .

$$d_H = 4 \left(\frac{\text{Volume available for flow}}{\text{Total Pore surface area}} \right)$$

And if we divide both numerator and denominator by membrane volume, so,

$$= \frac{4 \left(\frac{\text{Total Pore volume}}{\text{Membrane volume}} \right)}{\left(\frac{\text{Total Pore surface area}}{\text{Membrane volume}} \right)} = \frac{4\epsilon}{a}$$

Where total pore volume/membrane volume is equal to E , and total pore surface area/membrane volume is equal to a .

So, this a can be correlated with the specific surface area like this, $a_v = a/(1-\epsilon)$

The a_v , specific surface area is the surface area per unit volume of membrane material not including the pores. So, when we are considering the pores, then that will be, $a_v = a/(1-\epsilon)$.

So, we will be putting this d in terms of d_H and d_H is in terms of $4E/a$. So, then N value comes as

$$N = \frac{\rho\epsilon^3(P_0 - P_L)}{2(1-\epsilon)^2 \tau a_v^2 \mu L_M}$$

Why this τ ? Because, in this case, the l_M which you have considered, that may not be, the pores may not be a straight line. So, one tortuosity factor is consumed as τ .

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Liquid diffusion through pores

Diffusion through the pores of a membrane from a fluid feed to a sweep fluid when identical total pressure but different component concentration. Bulk flow through the membrane due to pressure difference does not occur. Feed is liquid of solvent and component i is diffused

Transmembrane flux for each solute $N_i = \frac{D_{ei}}{l_M} (c_{i0} - c_{iL})$ Fick's law

D_{ei} is the effective diffusivity, c_i is the concentration of i in the pores at the two faces of membrane

In general $D_{ei} = \frac{\epsilon D_i}{\tau} K_{ri}$

D_i is the ordinary molecular diffusion coefficient (diffusivity) of the solute i in the solution

ϵ is the volume fraction of pores in the membrane

τ is tortuosity,

K_r is restrictive factor that accounts for the effect of pore diameter (d_p) in causing interfering collisions of the diffusing solutes with the pore wall, when the ratio of molecular diameter (d_m) to d_p exceeds about 0.01

When (d_m/d_p) is 0.01, $K_r = 0.96$

When (d_m/d_p) is 0.3, $K_r = 0.24$

When $d_m > d_p$, $K_r = 0$ and solute can not diffuse through the pore. This is size exclusion effect

$S_{ij} = \frac{D_i K_{ri}}{D_j K_{rj}}$ Selectivity for molecules that are not subject to size exclusion

$K_r = \left[1 - \left(\frac{d_m}{d_p} \right)^4 \right], \left(\frac{d_m}{d_p} \right) \leq 1$

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Transport in membrane

If the membrane contains n such pores per unit cross section of membrane surface area normal to flow, the porosity (void fraction, ϵ) of the membrane is $\epsilon = n\pi D^2/4$

Then the superficial fluid bulk flow flux (mass velocity), N through the membrane is:

$$N = v\rho\epsilon = \frac{\rho \epsilon D^2}{32\mu l_M} (P_0 - P_L) = \frac{n\pi\rho D^4}{128\mu l_M} (P_0 - P_L)$$

Where l_M is the membrane thickness and ρ and μ are fluid properties

In real porous membrane pores may not be cylindrical and straight and hence pore diameter is replaced by hydraulic diameter

$d_H = 4 \left(\frac{\text{Volume available for flow}}{\text{Total Pore surface area}} \right)$

The specific surface area, a_v , which is the surface area per unit volume of membrane material (not including the pores), Thus, $a_v = a/(1-\epsilon)$

$$d_H = \frac{4 \left(\frac{\text{Total Pore volume}}{\text{Membrane volume}} \right)}{\left(\frac{\text{Total Pore surface area}}{\text{Membrane volume}} \right)} = \frac{4\epsilon}{a_v}$$

Further, pore length is longer than the membrane thickness and can be represented by $l_M\tau$, where τ is a tortuosity factor

$$N = \frac{\rho \epsilon^3 (P_0 - P_L)}{2(1-\epsilon)^2 \tau a_v^2 \mu l_M}$$

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So, this is the flux through the pores. Now, we will discuss liquid diffusion through pores. So, diffusion through the pores of a membrane from a fluid feed to a sweep fluid when identical total pressure, but different component concentration is there. Here, pressure difference is not there, only concentration difference is there.

So, in that case, we can consider that N_i , transmembrane flux,

$$N_i = \frac{D_{ei}}{l_M} (c_{i0} - c_{iL})$$

Where C_i is the concentration of i in the pores at the two faces, that is inlet and outlet. And D_{ei} is the effective diffusivity. And then D_{ei} can be calculated by this formula

$$D_{ei} = \frac{\epsilon D_i}{\tau} K_{ri}$$

So, D_i is the ordinary molecular diffusion coefficient of the solute i in the solution, and ϵ is the volume fractions of pores in the membrane and τ is the tortuosity. And K_r is restrictive factor that accounts for the effect of pore diameter d_p in causing interfering collisions of the diffusing solutes with the pore wall.

When the ratio of molecular diameter d_m/d_p exceeds about 0.01, then K_r plays a significant role. Otherwise, when you see d_m/d_p , is less than 0.01, in that case K_r is 0.96. So, this values almost 1 this K_r value is almost 1, but when this value is increasing, d_m/d_p is increasing. That means d_m , size is increasing, then 0.3 so K_r 0.24, and when d_m greater than d_p , then K_r equal to 0. That means, there will be no diffusion of the material.

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Example

Find the flow rate of water in m^3/m^2 -day when it passes through a porous polyethylene membrane by a pressure differential at $70^\circ C$. The membrane is of 25% porosity, with 0.3 mm diameter pores and a tortuosity of 1.3. Pressures on either side of the membrane are 500 kPa and 125 kPa. Assume membrane is 1 micron thick and flow is fully developed laminar flow.

For laminar flow:

$$\frac{Q}{A_M} = \frac{N}{\rho} = \frac{\epsilon d^2 (P_0 - P_L)}{32 \mu l_M \tau} \quad (1)$$

At $70^\circ C$, μ of water = 0.42 cP or $(0.42)(0.001) = 0.00042$ Pa-s and $\rho = 1000$ kg/ m^3
 $P_0 = 500$ kPa = 500,000 Pa and $P_L = 125$ kPa = 125,000 Pa
 $\epsilon = 0.25$, $D = 0.3$ $\mu m = 3 \times 10^{-7}$ m, $l_M = 1 \times 10^{-6}$ m, and $\tau = 1.3$

Substitution into Eq. (1) gives:

$$\frac{Q}{A_M} = \frac{N}{\rho} = \frac{(0.25)(3 \times 10^{-7})^2 (500,000 - 125,000)}{32(0.42 \times 10^{-3})(1 \times 10^{-6})(1.3)} (3600)(24) = 41,724 \text{ m}^3/\text{m}^2 - \text{day}$$

So, we will see one example here. Find the flow rate of a water in meter³/m²-day when it passes through a porous polyethylene membrane. By pressure differential at $70^\circ C$. The membrane is of 25 % porosity, with 0.3 mm diameter pores and a totuosity of 1.3. Pressures on either side of the

membrane are 500 kPa and 125 kPa. Assume membrane is 1 micron thick and flow is fully developed laminar flow.

So, we have the formula for laminar flow

$$\frac{Q}{A_M} = \frac{N}{P} = \frac{\rho \epsilon D^2}{32 \mu L_M} (P_0 - P_L)$$

At 70 °C, μ of water = 0.42 cP or $(0.42)(0.001) = 0.00042$ Pa-s and $\rho = 1000$ kg/m³

$P_0 = 500$ kPa = 500,000 Pa and $P_L = 125$ kPa = 125,000 Pa

$\epsilon = 0.25$, $D = 0.3$ μ m = 3×10^{-7} m, $L_M = 1 \times 10^{-6}$ m, and $\tau = 1.3$

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Up to this in this class. Thank you very much for your patience.