

Basic Environmental Engineering And Pollution Abatement
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Lecture: 40
Tutorial 8

Hello, everyone. Now, we will have a tutorial session and, in this class, will solve some numerical problems based on our discussions in the last four classes.

(Refer Slide Time: 00:48)

Problem 1

Find the flow rate of water in $\text{m}^3/\text{m}^2\text{-day}$ when it passes through a porous polyethylene membrane by a pressure differential at 70°C . The membrane is of 20% porosity, with 0.5 mm diameter pores and a tortuosity of 1.5. Pressures on either side of the membrane are 525 kPa and 100 kPa. Assume membrane is 2 micron thick and flow is fully developed laminar flow. At 70°C the viscosity and density of water is 0.00042 Pa.s and $1000\text{ Kg}/\text{m}^3$

Solution

For laminar flow:

$$v = \frac{Q}{A} = \frac{N}{\rho}$$
$$\frac{N}{\rho} = \frac{\epsilon * D^2 * (P_0 - P_{IM})}{32 * \mu * lm * \tau}$$

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The first problem statement is find the flow rate of water in $\text{m}^3/\text{m}^2\text{-day}$ when it passes through a porous polyethylene membrane by a pressure differential at 70°C . The membrane is of 20 % porosity with 0.5 mm diameter pores and a tortuosity of 1.5. Pressures on either side of the membrane are 525 kPa and 100 kPa. Assume membrane is 2 micron thick and flow is fully developed laminar flow. At 70°C , the viscosity and density of water is 0.00042 Pascal second and $1000\text{ kg}/\text{m}^3$.

So, this is a statement and we have to find out the flow rate in $\text{m}^3/\text{m}^2\text{-day}$ that means, volumetric flow rate per cross sectional area per day that is Q by A that we have to determine.

For laminar flow:

$$v = \frac{Q}{A} = \frac{N}{\rho}$$

$$\frac{N}{\rho} = \frac{\epsilon \cdot D^2 \cdot (P_0 - P_{IM})}{32 \cdot \mu \cdot l_m \cdot \tau}$$

where this E or epsilon does this is your porosity, D is the diameter of the pore P₀ is the pressure of one side and this is P_{IM} other side pressure and 32 and then mu the viscosity of the water, l_m is the thickness of the membrane layer. And τ is the tortuosity. So, that we have discussed in the previous class.

$$\mu = 0.42 \text{ or } 0.00042 \text{ Pa}\cdot\text{s}$$

$$\rho = 1000 \text{ Kg/m}^3$$

$$P_0 = 525 \text{ kPa} = 525000 \text{ Pa}$$

$$P_{IM} = 100 \text{ kPa} = 100000 \text{ Pa}$$

$$\epsilon = 0.2$$

$$D = 0.5 \text{ }\mu\text{m} = 5 \times 10^{-7} \text{ m}$$

$$l_m = 2 \times 10^{-6} \text{ m}$$

$$\tau = 1.5$$

Substituting the values in the above formula: -

$$V = \frac{0.2 \cdot (5 \times 10^{-7})^2 \cdot (525000 - 100000) \cdot 3600 \cdot 24}{32 \cdot 0.42 \cdot 10^{-3} \cdot (2 \times 10^{-6}) \cdot 1.5} \frac{\text{m}^3}{\text{m}^2 \cdot \text{day}}$$

$$= 45536 \frac{\text{m}^3}{\text{m}^2 \cdot \text{day}}$$

(Refer Slide Time: 03:10)



$\mu = 0.42$ or $0.00042 \text{ Pa}\cdot\text{s}$
 $\rho = 1000 \text{ Kg/m}^3$
 $P_0 = 525 \text{ kPa} = 525000 \text{ Pa}$
 $P_{lm} = 100 \text{ kPa} = 100000 \text{ Pa}$
 $\epsilon = 0.2$
 $D = 0.5 \mu\text{m} = 5 \times 10^{-7} \text{ m}$
 $l_m = 2 \times 10^{-6} \text{ m}$
 $\tau = 1.5$

Substituting the values in the above formula:-

$$v = \frac{0.2 \cdot (5 \times 10^{-7})^2 \cdot (525000 - 100000) \cdot 3600 \cdot 24}{32 \cdot 0.42 \cdot 10^{-3} \cdot (2 \times 10^{-6}) \cdot 1.5} \frac{\text{m}^3}{\text{m}^2 \cdot \text{day}}$$

$$= 45536 \frac{\text{m}^3}{\text{m}^2 \cdot \text{day}}$$

$$v = \frac{\epsilon \cdot D^2 \cdot (P_0 - P_{lm})}{32 \cdot \mu \cdot l_m \cdot \tau} \cdot \frac{\delta}{A}$$



3

So, we are asked to calculate this one and now we are able to solve it.

(Refer Slide Time: 05:11)

Problem 2

Each 100 ml solution containing 20 mg/l fluoride is treated with two different adsorbent doses i.e, 6 g/l and 4 g/l separately. The % removals of the fluoride for these two cases are 60 % and 50 % respectively. Calculate the specific uptake for both the cases and explain the trend.

Solution



Case 1

Initial concentration of F = 20 mg/l, final concentration of F = 20*(1-0.6) = 8 mg/l

Adsorbent used = 6*100/1000 = 0.6 g

F removed by the adsorbent = (20-8)*100/1000 = 1.2 mg

Specific uptake = 1.2/0.6 = 2 mg/g



4

Now, we will see problem number 2. Statement is, each 100 ml solution containing 20 mg/l fluoride is treated with two different adsorbent doses, that is 6 g/l and 4 g/l separately. The percentage removals of the fluoride for these two cases are 60 % and 50 % respectively. Calculate the specific uptake for both the cases and explain the trend.

So, we have to calculate the specific uptake of the adsorbent which is used to remove the fluoride from the water. Two different cases are given and adsorbent doses different, initial concentration

is different and removal of the fluoride is also different. So, we will be calculating the specific uptake, then specific uptake, we know that the amount of adsorbate taken up by unit mass of the adsorbent.

Case 1

Initial concentration of F = 20 mg/l, final concentration of F = $20*(1-0.6) = 8$ mg/l

Adsorbent used = $6*100/1000 = 0.6$ g

F removed by the adsorbent = $(20-8)*100/1000 = 1.2$ mg

Specific uptake = $1.2/0.6 = 2$ mg/g

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Case 2

Initial concentration of F = 20 mg/l, final concentration of F = $20*(1-0.5) = 10$ mg/l

Adsorbent used = $4*100/1000 = 0.4$ g

F removed by the adsorbent = $(20-10)*100/1000 = 1$ mg

Specific uptake = $1/0.4 = 2.5$ mg/g

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Case 2

Initial concentration of F = 20 mg/l, final concentration of F = $20*(1-0.5) = 10$ mg/l

Adsorbent used = $4*100/1000 = 0.4$ g

F removed by the adsorbent = $(20-10)*100/1000 = 1$ mg

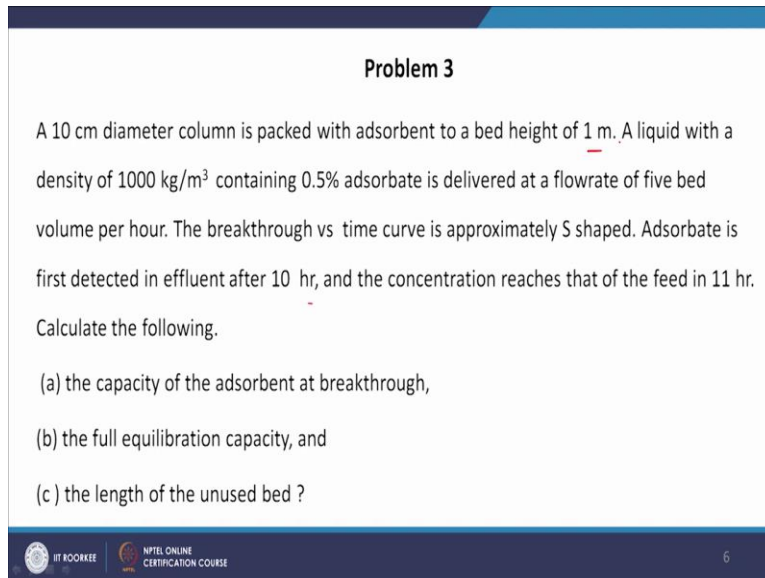
Specific uptake = $1/0.4 = 2.5$ mg/g

So, the second case specific uptake is 2.5 mg per gram, but in the first case the specific uptake is 2 mg per gram.

So, you see, in this case with the increase of initial concentration, we are getting lesser percentage removal. However, we are getting more specific uptake, this is apparently in opposite trend with a decrease in adsorbent dose, the adsorbent is getting more available adsorbate molecules and capturing more adsorbate. So, that is why especially uptake is being increased with the decrease of adsorbent dose.

But percentage removal is not giving the same trend because percentage removal is the factor final and initial concentration. Thus, this is not giving the same trend with respect to specific uptake. So, the trend of specific uptake and percentage removal is not similar, it is opposite. Now, we are coming to problem number 3.

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Problem 3

A 10 cm diameter column is packed with adsorbent to a bed height of 1 m. A liquid with a density of 1000 kg/m^3 containing 0.5% adsorbate is delivered at a flowrate of five bed volume per hour. The breakthrough vs time curve is approximately S shaped. Adsorbate is first detected in effluent after 10 hr, and the concentration reaches that of the feed in 11 hr. Calculate the following.

- (a) the capacity of the adsorbent at breakthrough,
- (b) the full equilibration capacity, and
- (c) the length of the unused bed ?

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Statement is a 10 cm diameter column is packed with adsorbent to a bed height of 1 meter. A liquid with a density of 1000 kg/m^3 containing 0.5 % adsorbate is delivered at a flow rate of 5 bed volume per hour. The breakthrough versus time curve is approximately S shaped. Adsorbate is first detected in effluent after 10 hour, and the concentration reaches that of the feed in 11 hour. Calculate the following.

The capacity of the adsorbent at breakthrough, the full equilibrium capacity, and the length of the unused bed.

So, this is again a problem of the adsorption in packed bed column. So, here height is given, liquid density given, the solid content is given, and its flow rate is also given. So we have to calculate these parameters, so, we will be using the mass balance.

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Solution

a) The bed volume is $\pi r^2 h \rightarrow \frac{\pi d^2}{4} L$

$$\frac{\pi \cdot (0.1)^2}{4} \cdot 1 = 0.0079 \text{ m}^3$$

The flow rate is 5 bed /h

The volumetric flow rate = $5 \cdot 0.0079 = 0.0395 \text{ m}^3/\text{h}$

The adsorbate feed rate is $0.0395 \text{ m}^3/\text{h} \cdot 1000 \text{ kg/m}^3 \cdot 0.005 = 0.1975 \text{ kg/h}$

The adsorbate capacity at breakthrough = $\frac{0.1975 \cdot 10}{0.0079} \text{ kg/m}^3 = 250 \text{ kg/m}^3$

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b) The capacity at full equilibration : the calculation are same but in this case we use the stoichiometric time which is the mid value of breakthrough time and equilibration time i.e. 10.5 hr.

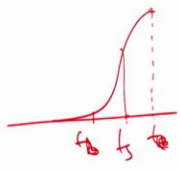
So repeating above we get $250 \times \frac{10.5}{10} = 262.5 \text{ kg/m}^3$

c) Length of unused bed(LUB):

We know that $LUB = \text{Ideal wave front velocity} \cdot (t_s - t_b) = (L_e/t_s) \cdot (t_s - t_b)$

$$\frac{LUB}{L_e} = 1 - \frac{t_b}{t_s} \text{ where } t_b \text{ is the breakthrough time and } t_s \text{ is the stoichiometric time}$$

Thus, $LUB = 1 \cdot (1 - \frac{10}{10.5}) = 0.048 \text{ m}$



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The bed volume is

$$\frac{\pi \cdot (0.1)^2}{4} \cdot 1 = 0.0079 \text{ m}^3$$

The flow rate is 5 bed /h

The volumetric flow rate = $5 * 0.0079 = 0.0395 \text{ m}^3/\text{h}$

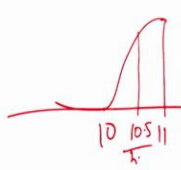
The adsorbate feed rate is $0.0395 \text{ m}^3/\text{h} * 1000 \text{ kg}/\text{m}^3 * 0.005 = 0.1975 \text{ kg}/\text{h}$

The adsorbent capacity at breakthrough = $\frac{0.1975 * 10}{0.0079} \text{ kg}/\text{m}^3 = 250 \text{ kg}/\text{m}^3$

Problem 3

A 10 cm diameter column is packed with adsorbent to a bed height of 1 m. A liquid with a density of $1000 \text{ kg}/\text{m}^3$ containing 0.5% adsorbate is delivered at a flowrate of five bed volume per hour. The breakthrough vs time curve is approximately S shaped. Adsorbate is first detected in effluent after 10 hr, and the concentration reaches that of the feed in 11 hr. Calculate the following.

(a) the capacity of the adsorbent at breakthrough,
(b) the full equilibration capacity, and
(c) the length of the unused bed ?



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b) The capacity at full equilibration: the calculation are same but in this case we use the stoichiometric time which is the mid value of breakthrough time and equilibration time i.e. 10.5 hr.

So repeating above we get

$262.5 \text{ kg}/\text{m}^3$

c) Length of unused bed(LUB):

We know that

$$\text{LUB} = \text{Ideal wave front velocity} * (t_s - t_b) = (L_e/t_s) * (t_s - t_b)$$

$$\frac{\text{LUB}}{L_e} = 1 - \frac{t_b}{t^*} \text{ where } t_b \text{ is the breakthrough time and } t^* \text{ is the stoichiometric time}$$

$$\text{LUB} = 1 * \left(1 - \frac{10}{10.5}\right) = 0.048 \text{ m}$$

(Refer Slide Time: 17:12)

Problem 4

100,000 L/day of liquid containing 0.1% adsorbate is to be treated . During pilot plant testing of adsorbent, performance was investigated at a superficial velocity of 0.003 m/s, with total tolerable pressure drop and adsorption capacity of 0.3 kg adsorbate /kg adsorbent. The feed density is 1,000 kg/m³ ,the adsorbent bulk density is 640 kg/m³ and the desired online time is 48 hr.

Determine column diameter and packed height ?

Then, problem 4. 100,000 L/day of liquid containing 0.1 % adsorbate is to be treated. During pilot plant testing of adsorbent performance was investigated at a superficial velocity of 0.03 m/s with total tolerable pressure drop and absorption capacity of 0.3 kg adsorbate/ kg adsorbent the feed density is 1000 kg/m³. The adsorbent bulk density is 640 kg/m³ and the desired online time is 48 hour. And then determine column diameter and packed height.

So, again adsorbent based problem, we have to determine the column diameter and packed height. So certainly, in that case, we will be using the mass balance.

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Solution

The volumetric flow rate in m³/s is :

$$100000 \text{ L/day} * \frac{1 \text{ day}}{86400 \text{ s}} * \frac{1 \text{ m}^3}{1000 \text{ L}} = 0.00116 \text{ m}^3/\text{s}$$

The required cross-sectional area is:

A= volumetric flow rate/superficial velocity

$$= 0.00116 / 0.003 = 0.386 \text{ m}^2$$

Diameter = $\sqrt{4 * A / \pi} = \sqrt{4 * 0.386 / \pi} = 0.7 \text{ m}$.

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The volumetric flow rate in m³/s is :

$$100000 \text{ L/day} * \frac{1 \text{ day}}{86400 \text{ s}} * \frac{1 \text{ m}^3}{1000 \text{ L}} = 0.00116 \text{ m}^3 / \text{s}$$

The required cross-sectional area is:

A= volumetric flow rate/superficial velocity

$$= 0.00116 / 0.003 = 0.386 \text{ m}^2$$

$$\text{Diameter} = \sqrt{4 * A / \pi} = \sqrt{4 * 0.386 / \pi} = 0.7 \text{ m}$$

So diameter, we are getting. Now, we have to determine the height also.

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The amount of adsorbate delivered to the column per cycle:

$$48 \text{ hr} * \frac{100000 \text{ L}}{24 \text{ hr}} * \frac{1 \text{ m}^3}{1000 \text{ L}} * \frac{1000 \text{ kg}}{\text{m}^3} * 0.001 = 200 \text{ kg} \checkmark$$

The adsorbent needed is :

$$\frac{200 \text{ kg}}{0.3 \frac{\text{kg adsorbent}}{\text{kg adsorbate}}} * \frac{1 \text{ m}^3}{640 \text{ kg}} = 1.04 \text{ m}^3$$
$$H = 1.04 / 0.386 = 2.7 \text{ m. } \checkmark/A$$

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The amount of adsorbate delivered to the column per cycle:

$$48 \text{ hr} * \frac{100000 \text{ L}}{24 \text{ hr}} * \frac{1 \text{ m}^3}{1000 \text{ L}} * \frac{1000 \text{ kg}}{\text{m}^3} * 0.001 = 200 \text{ kg}$$

The adsorbent needed is :

$$\frac{200 \text{ kg}}{0.3 \frac{\text{kg adsorbent}}{\text{kg adsorbate}}} * \frac{1 \text{ m}^3}{640 \text{ kg}} = 1.04 \text{ m}^3$$

$$H = 1.04 / 0.386 = 2.7 \text{ m.}$$

(Refer Slide Time: 21:42)

Problem 5

Consider a fixed bed adsorption before breakthrough in which $L_{ideal}/L_b = 0.5$, take $Q=1$ L/s, $t_{ideal}=100$ s. The mass of the adsorbent used equals to 100 gm and inlet concentration of 20 kg/m³. Adsorption follows the Freundlich Adsorption Isotherm with $q = 5 * C^{1/n}$,
Calculate the mass of adsorbent required for the inlet concentration of 25 kg/m³.



Next, problem number 5. The statement, is consider a fixed bed adsorption before breakthrough in which L_{ideal}/L_b is equal to 0.5. Take Q equal to 1 liter per second, t_{ideal} equal to 100 second, the mass of the adsorbent used equals to 100 gram and inlet concentration of 20 kg/m³. Adsorption follows the Freundlich Adsorption Isotherm with q equal to 5 into C to the power 1 by n . Calculate the mass of adsorbent required for the inlet concentration of 25 kg/m³

So it is again a problem of adsorption-based process. So, we have a fixed bed, L_{ideal}/L_b , total length, it is given, flow rate is given, t_{ideal} is given, and mass of the adsorbent used is also given. And initial concentration is given. And the specific uptake, how to calculate this formula is also given.

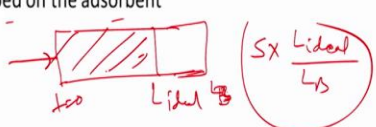
So, by doing the mass balance, we can calculate the value of n , that is the Freundlich, isotherm constant for the first case. And the second case, we will be using that n value to calculate the q . And then we will be doing the mass balance and we will get the desired values, the mass adsorbent required for the inlet concentration of 25 kg/m³. So, this way we will proceed.

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Solution:
Material balance of the adsorbate before breakthrough occurs :
Solute in entering feed = adsorbate adsorbed on the adsorbent

$$Q * c * t_{\text{ideal}} = q * S * \frac{L_{\text{ideal}}}{L_b}$$

$q = 5 * C^{\frac{1}{n}}$

$$Q = 1 \text{ L/s} = 0.001 \frac{\text{m}^3}{\text{s}}$$
$$c = 20 \text{ kg/m}^3$$
$$t_{\text{ideal}} = 100 \text{ s}$$
$$S = 100 \text{ gm} = 0.1 \text{ kg}$$
$$L_{\text{ideal}} / L_b = 0.5$$


Now, say material balance of the adsorbate before breakthrough occurs, that we know, in our previous class we have discussed, that is solute in entering feed equal to adsorbate adsorbed on the adsorbent. So, what was the solute in the incoming feed, we have volumetric flow rate, we have time and we have concentration of it.

Material balance of the adsorbate before breakthrough occurs :

Solute in entering feed = adsorbate adsorbed on the adsorbent

$$Q * c * t_{\text{ideal}} = q * S * \frac{L_{\text{ideal}}}{L_b}$$

$$q = 5 * C^{\frac{1}{n}}$$

$$Q = 1 \text{ L/s} = 0.001 \frac{\text{m}^3}{\text{s}}$$

$$c = 20 \text{ kg/m}^3$$

$$t_{\text{ideal}} = 100 \text{ s}$$

$$S = 100 \text{ gm} = 0.1 \text{ kg}$$

$$L_{\text{ideal}} / L_b = 0.5$$

So, we will be putting all those values in this expression and we will get the value of n.

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$$q = \frac{Q * c * t_{ideal}}{S * L_{ideal} / Lb}$$

$$= \frac{0.001 * 100 * 20}{0.1 * 0.5}$$

$$= 40 \frac{kg}{kg-adsorbent}$$

Now

$$q = 5 * C^{\frac{1}{n}}$$

$$40 = 5 * 20^{\frac{1}{n}}$$

$$8 = 20^{\frac{1}{n}}$$

Taking log



$$q = \frac{Q * c * t_{ideal}}{S * L_{ideal} / Lb}$$

$$= \frac{0.001 * 100 * 20}{0.1 * 0.5}$$

$$= 40 \frac{kg}{kg-adsorbent}$$

Now

$$q = 5 * C^{\frac{1}{n}}$$

$$40 = 5 * 20^{\frac{1}{n}}$$

$$8 = 20^{\frac{1}{n}}$$

Taking log

$$\frac{1}{n} = \frac{\ln 8}{\ln 20}$$

$$n = 1.44$$

So for $c = 25 \text{ kg/m}^3$ and assuming all condition same the weight of adsorbent needed is

$$Q * c * t_{ideal} = q * S * L_{ideal} / LB$$

$$S = \frac{Q * c * t_{ideal}}{q * L_{ideal} / LB}$$

$$q = 5 * 25^{\frac{1}{1.44}}$$

$$q=46.74 \text{ kg/m}^3$$

$$S = \frac{0.001 * 25 * 100}{46.74 * 0.5} \text{ kg} = 106.97 \text{ gm}$$

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$\frac{1}{n} = \frac{\ln 8}{\ln 20}$ ✓
 $n = 1.44$ ✓
 So for $c = 25 \text{ kg/m}^3$ and assuming all condition same the weight of adsorbent needed is
 $Q * c * t_{\text{ideal}} = q * S * \text{Lideal/LB}$
 $S = \frac{Q * c * t_{\text{ideal}}}{q * \text{Lideal/LB}}$
 $q = 5 * 25^{\frac{1}{1.44}}$
 $q = 46.74 \text{ kg/m}^3$ ✓
 $S = \frac{0.001 * 25 * 100}{46.74 * 0.5} \text{ kg} = 106.97 \text{ gm}$ ✓

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So up to this in this class. Thank you very much for your patience.