Biological Process Design for Wastewater Treatment
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Lecture no. 15
Reactor Hydraulics - I

Welcome everyone in this NPTEL online certification course on Biological Process Design for Wastewater Treatment. So, today we are going to start another important section which is called as Reactor Hydraulics, a mass and heat balance. So, first we will be concentrating on the reactor hydraulics.

In any reactor or a system where some convergence are happening the flow of the system how the flow what is the flow pattern and what is the mixing pattern both have very important impact on the efficiency of that particular reactor. Similarly, for wastewater treatment also the reactants may be considered as the water which is coming having some organic compounds. So, these organic compounds or like reactants which are getting converted into CO_2 and H_2O which is the main target in aerobic system. So, that means these wastewater treatment units can be considered as reactors.

So, in these reactors also similar to the chemical engineering reactors, they have very important consideration with respect to what is the flow pattern and what is the mixing pattern. So, both have very important impact on the efficiency of the wastewater treatment during the processing of the wastewater or during the treatment of the wastewater in these units.

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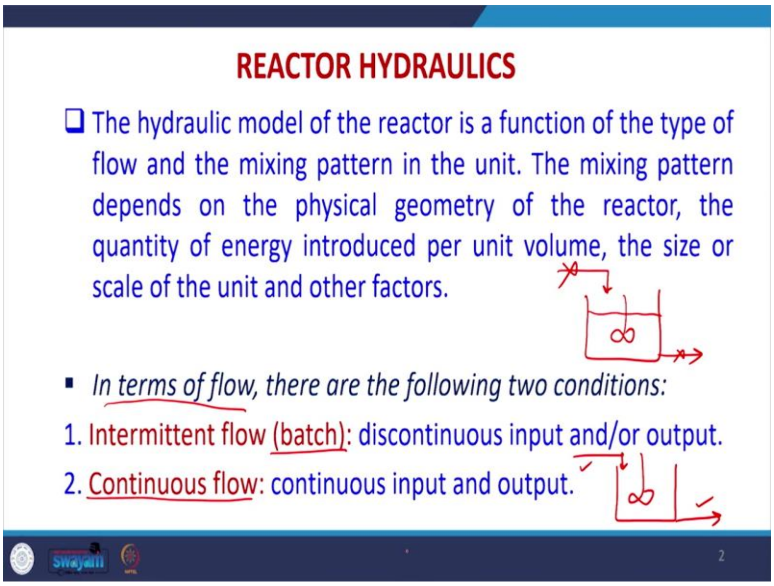

So, reactor hydraulics of any reactor is a function of the type of the flow it is what is the how the flow is happening and the mixing pattern in the unit itself. So, any wastewater treatment plant can be modeled with respect to hydraulic model by determining its flow pattern and the mixing pattern. The mixing pattern depends upon the physical geometry of the reactor, the quantity of energy introduced per unit volume and the size and the scale of the unit and various other factors. So, this is a reactor hydraulics is the thing that we are going to study in today's lecture.

In terms of flow, so, we can characterize the systems into two categories either batch or continuous. So, intermittent flow or batch, so, in this case we have discontinuous input and or output. So, in one of the cases like in simple batch process, what we do is that we have a reactor in which the initially the reactants are the wastewater flows and remains inside the reactor until the treatment happens for certain period of time, then that will be dependent upon the hydraulic time or detention time.

So, after treatment the wastewater will be taken out. This is a process which is called a batch process. In this there is no flow in or flow out during the treatment process. It is possible that flow may be out or it is possible that flow may so, that in those condition it is called as semi batch reactor.

Now when a reactor is there like this reactor in which there is a continuous flow of reactant and the products are also being taken out continuously. So, both are continuously in process. So, that condition is called continuous flow, where we have continuous input and continuous output.

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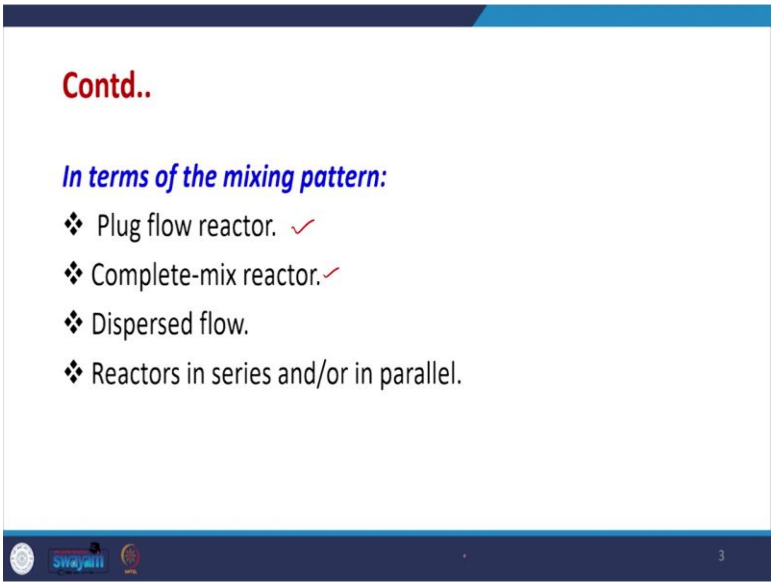

Now, in terms of mixing pattern, we can define various types of reactors and these reactors are called plug flow reactor, completely mixed reactor then we have dispersed flow reactor and then reactor in series or in parallel. So, we will be trying to learn each of them a little bit so, as to we can understand the concept and how they can be useful during the biological treatment of wastewater. So, we are going to learn them each of them in greater detail.

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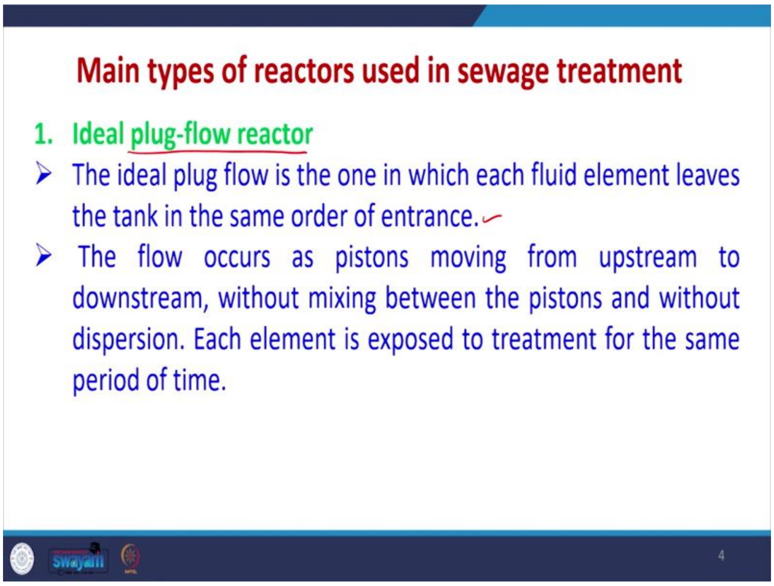

So, main types of reactor used in the sewage treatment or wastewater treatment. Firstly the ideal plug flow reactor. So, plug flow reactor that we are going to study so, ideal is the one which is same as what is the in during the ideal conditions. So, ideal plug flow is the one in which each fluid element leaves the tank in the same order of entrance. That means the fluid element which first enters in that tank leave first, the second one lives in the second order and similarly third, fourth, fifth, sixth. So, this is the meaning of this particular sentence.

The flow occurs as pistons moving from upstream to downstream without mixing between the pistons and without dispersion, each element is exposed to treatment for the same period of time.

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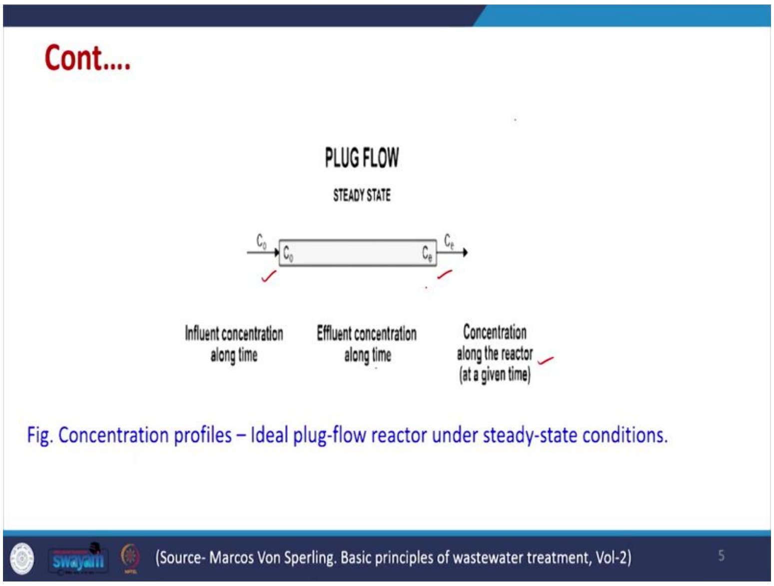

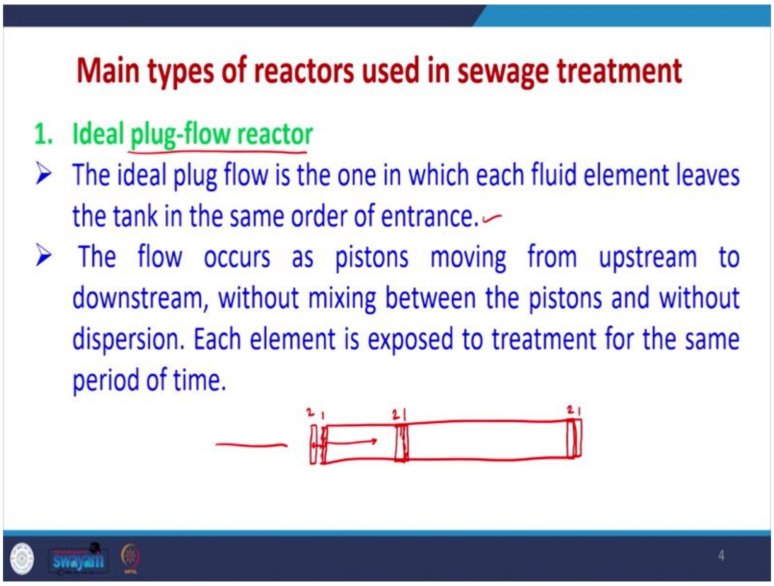

So, in this case this is the plug flow reactor, so, in this case, what we will assume that we can see here the there is a concentration C_0 which is entering and C_e is the effluent concentration. So, this is the influent concentration along time. This is effluent concentration and concentration along the reactor at given time.

So, what we do is that, in this case like for understanding this particular sentence, so, this is the reactor. So, in this case the flow which is coming so, first there will be a first element which is enterings like a piston so, this piston will enter and after some time this piston will move at this point, then there will be another piston which will be entering which is just about to enter at this point and after some time this piston will be here.

So, that means of and there will be no mixing which is happening in between the pistons. The piston's fluids are not mixing among each other and there is no dispersion of the fluid any side. So, this is so, this first element which is there and the second element this is first and second. After a certain time, the first element will come out and the second element is about to come out. So, there is no mixing between the pistons or the fluid pistons we are considering. So, this is called as plug flow reactor.

Now, this is what you described depending upon whether the reactants are reacting or non reacting, the profiles may be different. So, this is what we are going to study.

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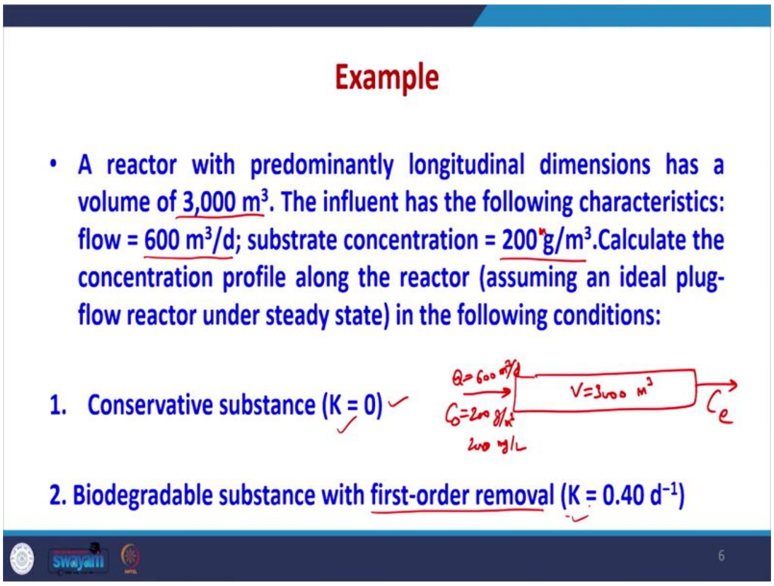

We will take example and understand this. So, a reactor with predominantly longitudinal dimension has a volume of 3000 meter cube so, it is given that a reactor is there which is having a volume of 3000 meter cube. The influent has the following characteristics. The influent which is coming to the reactor is having a following characteristic which is 600 meter cube per day is the flow rate.

Substrate concentration at the inlet is 200 gram per meter cube, calculate the concentration profile it may be considered as milligram also or gram per meter cube is okay. So, calculate the concentration profile along the reactor assuming an ideal plug flow reactor under a steady state in the following condition. In one condition it is assumed that the substance which is coming is conservative that means, it is not degrading at all and in the second condition it is assumed that the substance is biodegradable substance which is getting removed via first order reaction kinetics and the K is 0.04 per unit day. So, these two conditions are there.

So, we have a reactor, a long ideal flow reactor we do not know exactly the length, but we are assuming this is very lengthy. It is having a volume of 3000 meter cube. The flow is coming which is having 600 meter cube per day is the flow and it is having a concentration of 200 gram per meter cube which is like 200 milligram per liter so, this is same. So, this is the concentration and we have to find out what is the exit concentration under two conditions when K is equal to 0 another when K is equal to 0.4 per unit day. So, this is there.

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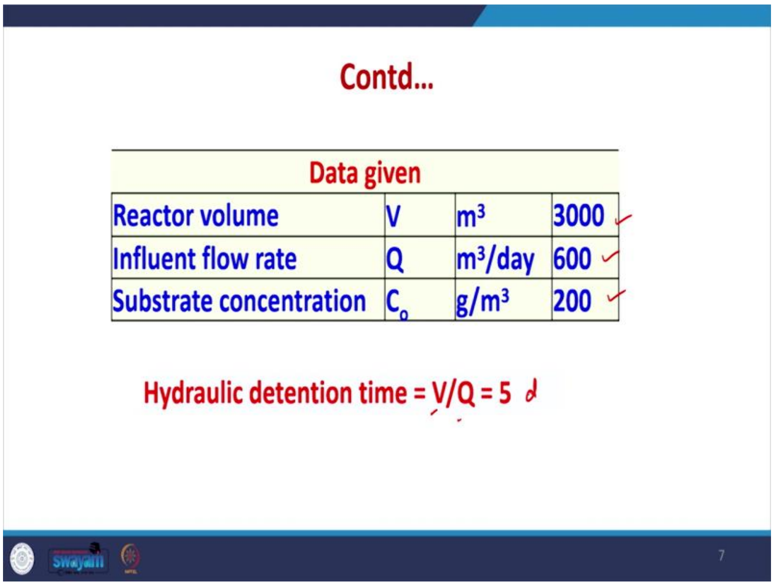

So, we have already given reactor volume, influent flow rate, the substrate concentration, the hydraulic detention time is also known to us because we have volume known to us and Q known to us. So, V by Q is 5 so, we have 5 day is the hydraulic detention time for this particular reactor.

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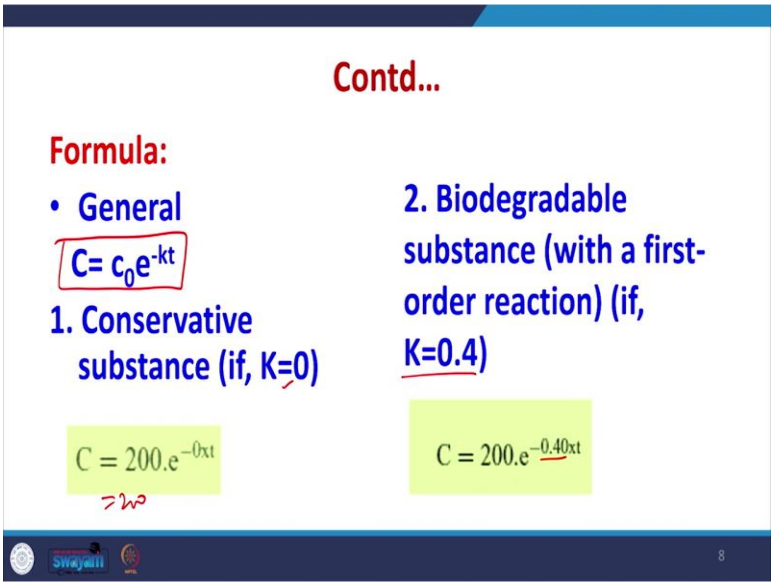

Now, going further if we try to use the equations which have been earlier used, so, for the ideal flow reactor, actually for first order kinetics the expression is C_0 is equal to C is equal to C_0 e^{-kt}. It same as for batch reactor that already we have studied the reactor kinetics. So, this is the now if K is equal to 0 that means the conservative resistance is there and there is no degradation. So, this will be the C final or exit will be same as C_0e^{-0t} that means it will be same as 200. In this case, we have 0.40 is the K value the reaction degradation rate constant. So, we can find out.

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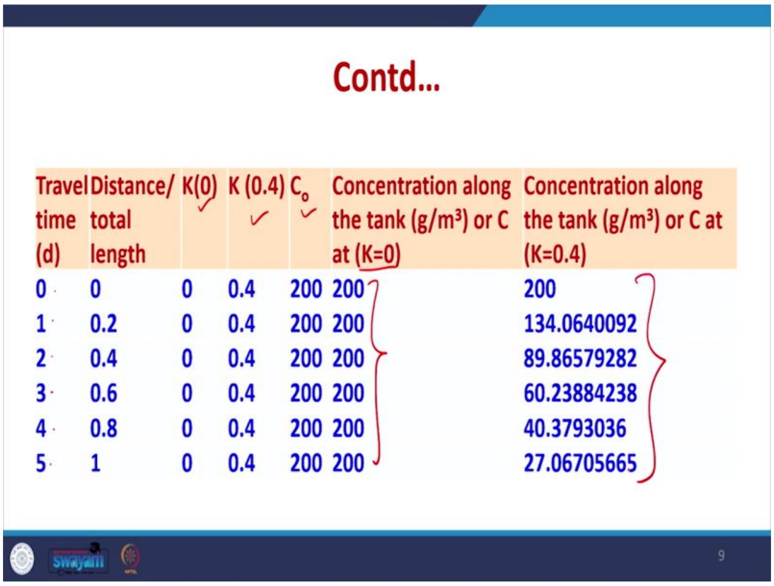

So, with time like time travel time on 1 day, 0 day, 2 day, 3 day, 4 day, 5 day and with this we can find out that distance total upon total length. So, we can take like this and since, for the first case when K is equal to 0 and another K is equal to 0.4 and initial concentration is 200. So, we can find out for conservative conditions when K is equal to 0 the concentration profile remains the same whereas, it decreases with increase in time.

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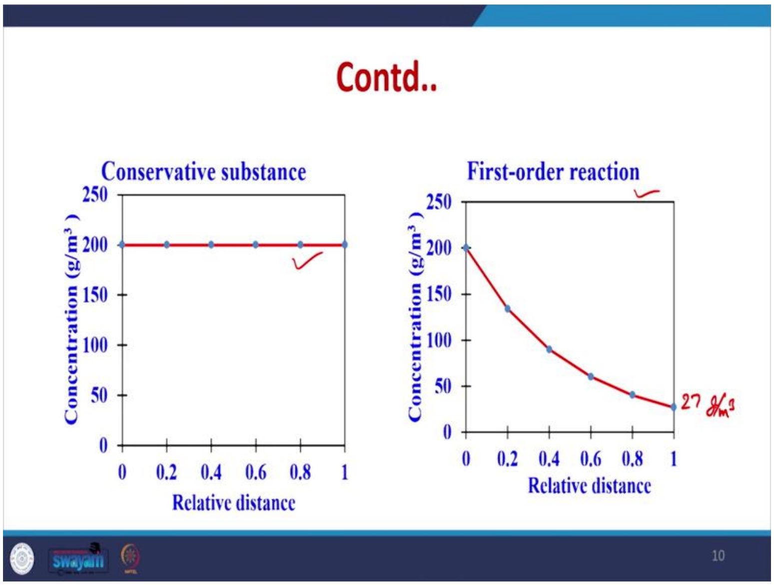

So, we can draw a figure like this the concentration in gram per meter cube remains constant and there is no mixing or nothing is happening with reactor distance whereas, the concentration decreases continuously via the first order reaction kinetics and we can see the at the exit the concentration is around 27 gram per meter cube for 5 day retention time. So, this is what is we are getting the profile is this is the profile and this is for ideal flow reactor.

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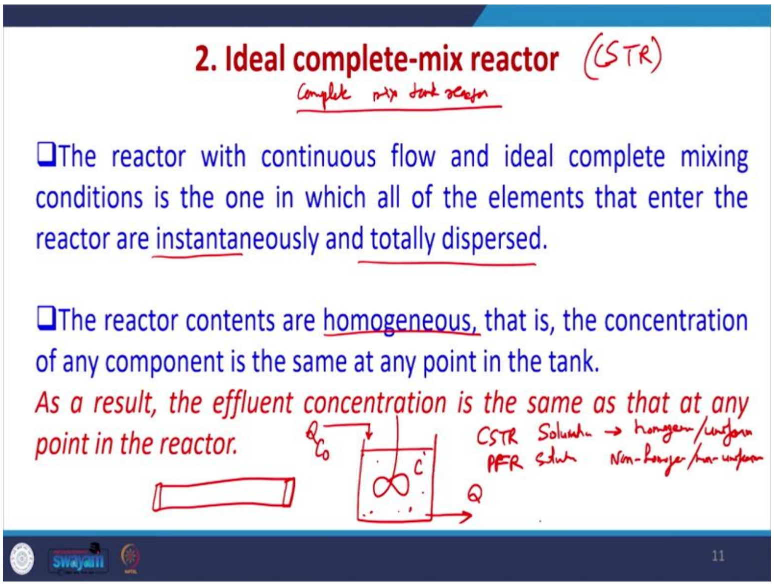

Then there is another type of reactor which is called as ideal complete-mix reactor so, the reactor with this continuous flow or ideal complete-mixing condition is the one in which all the elements that enter the reactor are at the same instant they are totally mixed or dispersed. So, this is a condition which is called the ideal complete-mix reactor. The reactor contents are homogeneous that is a concentration of any component at the same point in the tank is same at any other point. As a result, the effluent concentration is same as that of the point in the reactor. So, this reactor is like this is a mix reactor.

So, we have one thing is that we have very continuous mixing that means, this is all throughout continuous. Now, the influent is coming at some flow rate and effluent is also going outside at some flow rate. It is having a concentration C_0. Now, suppose the concentration is seen inside the reactor. So, inside the reactor there are two conditions which are getting followed. One is that that the this whole solution is homogeneous first thing that it is same or the reactor contents are homogeneous at any time and so, this is at homogeneous or we can call them uniform.

Now, this is true for this is also called CSTR complete mixed tank reactor. So, this is also called a CSTR. Sometimes it is called as complete-mix tank reactor, remember this and for like activated sludge process can be modeled as complete-mixed tank reactor. Now, this is true for CSTR. Now, for ideal for PFR the plug flow reactor that we studied earlier the solution will not be homogeneous, because in this case for CSTR any suppose this is the point whatever is the concentration here same is the concentration at this point, this point, this point at any other point whereas, in the ideal flow reactor, plug flow reactor the concentrations are not uniform and here the concentration is higher whereas, the here the concentration is lower as we have seen earlier.

So, this is that means the for PFR it is not homogeneous non homogeneous is the solution and this is the difference and this is non uniform also or we can see any of these things. So, the solutions in non-homogeneous for the case of plug flow reactor where it is homogeneous for CSTR So, this is the difference.

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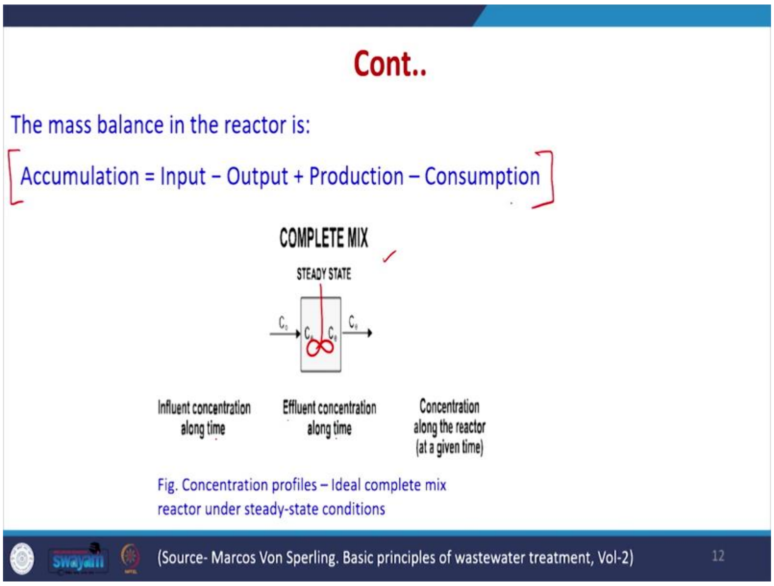

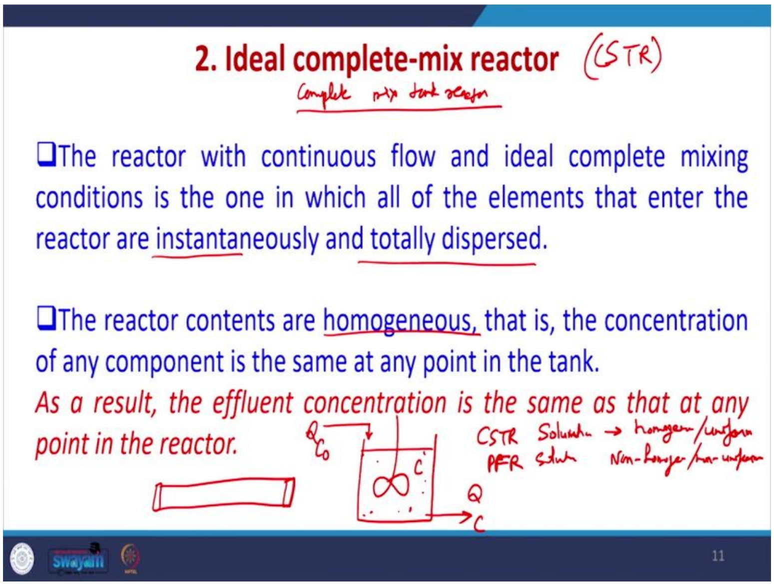

Now, mass balance for the reactor can be written like this. So, this is complete-mix reactor we have C_0 and that means this is another condition that if C is the concentration inside, the C will be the concentration outside and if we are assuming C so, it will be C here also. So, this is the ideal mix reactor complete-mix, influent concentration a long time, effluent concentration, then concentration along the reactor we can find out. So, for this case, the balance in any of the reactor is generally written like this.

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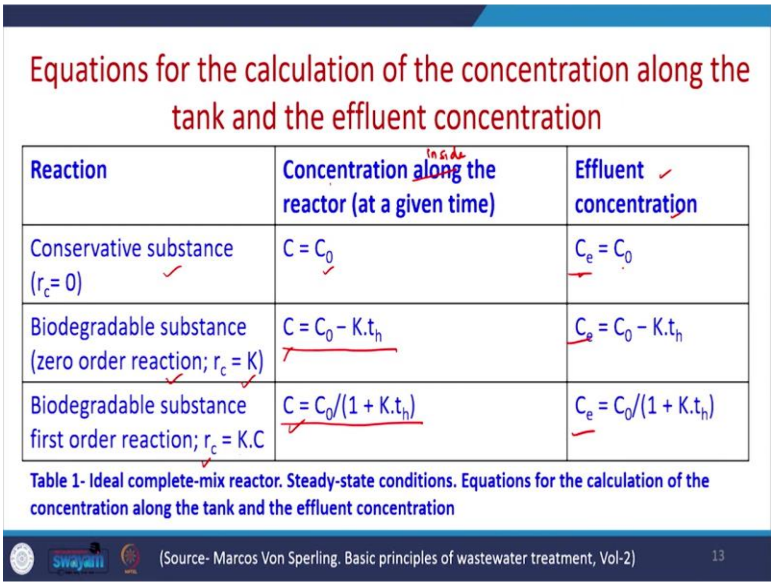

Table 1- Ideal complete-mix reactor. Steady-state conditions. Equations for the calculation of the concentration along the tank and the effluent concentration

Reaction	**Concentration along the reactor (at a given time)**	**Effluent concentration**
Conservative substance (r_c= 0)	$C = C_0$	$C_e = C_0$
Biodegradable substance (zero order reaction; r_c = K)	$C = C_0 - K.t_h$	$C_e = C_0 - K.t_h$
Biodegradable substance first order reaction; r_c = K.C	$C = C_0/(1 + K.t_h)$	$C_e = C_0/(1 + K.t_h)$

So, we can write the balance in terms of actual values using some calculation you can refer to any of the books and we will be finding that the different types of profile we can solve. So, one is for the conservative substances, where the K is equal to 0, then biodegradable substance with reaction order like zero order reaction, then biodegradable substance with first order reaction where the rate of reaction is written as r is equal to KC, so, we can have different conditions.

So, under different conditions, we can have different profile. So, it will be C equal to C_0 or effluent concentration will also be equal to C_0 for the case, where the conservative substances there, so, this is concentration along inside the reactor or we can write that inside the reactor. Along the reactor will be more true for plug flow reactor and then for the case where zero order reaction is the profile will be like C is equal to C0 minus Kth.

For first order it will be C_0, C is equal to C_0 divided by $1+K_{th}$ and the outside concentration will be same as any other inside reactor which is true for CSTR. So, for CSTR the effluent concentration is same as the concentration inside the reactor at any given time. So, these are the equations which can be used for solving problems related to CSTR.

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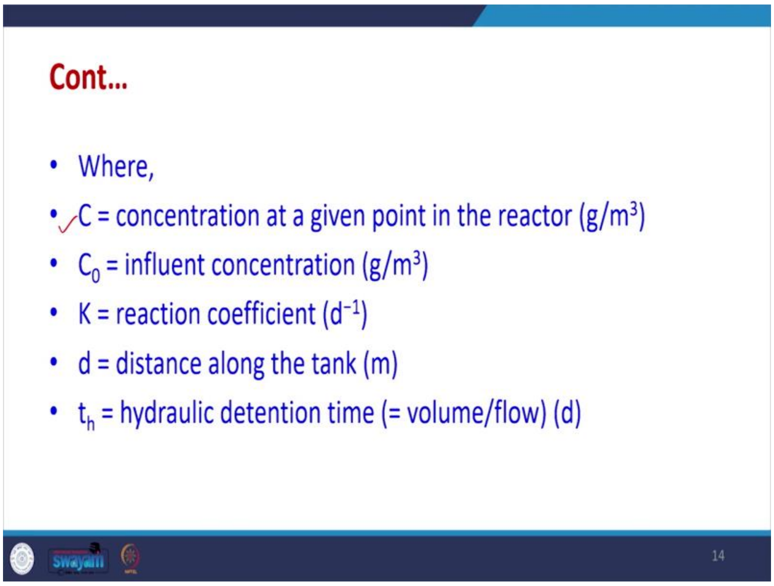

And here in this case in the previous slide, the C is the concentration at any given time in the reactor C_0 is the concentration, influent concentration, which was the initial concentration, K is the reaction coefficient or the reaction constant and d is the distance along the tank which which is only true for the plug flow reactor, th is the hydraulic detention time we can find out.

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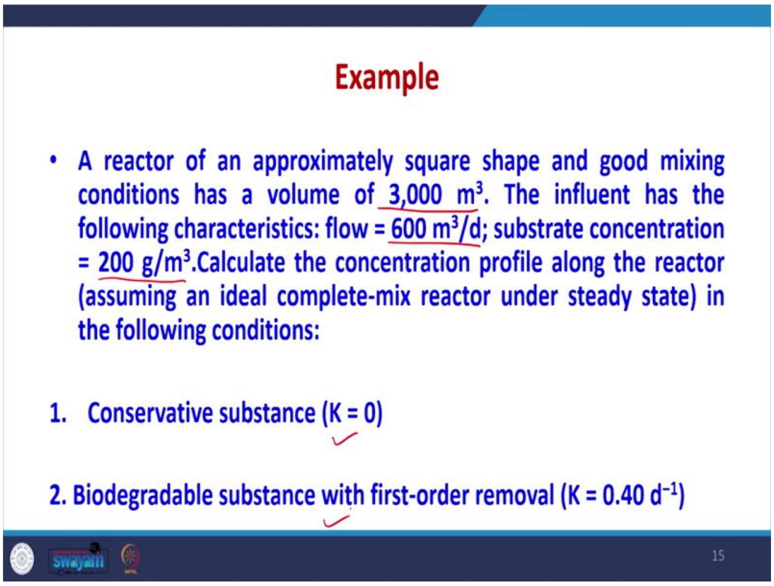

So, this is a question again a reactor has approximately square shape and good mixing condition is having a volume of 3000 meter cube. The influent has following characteristics the flow is 600 meter cube, the substrate concentration is 200 gram. Calculate the concentration profile in the reactor assuming that ideal mix complete reactor complete-mix conditions are there under steady state in the following conditions when the conservative substance is there and then biodegradable substance with first order removal with K is equal to 0.4. So, same as far earlier example, we are taking the actually the same.

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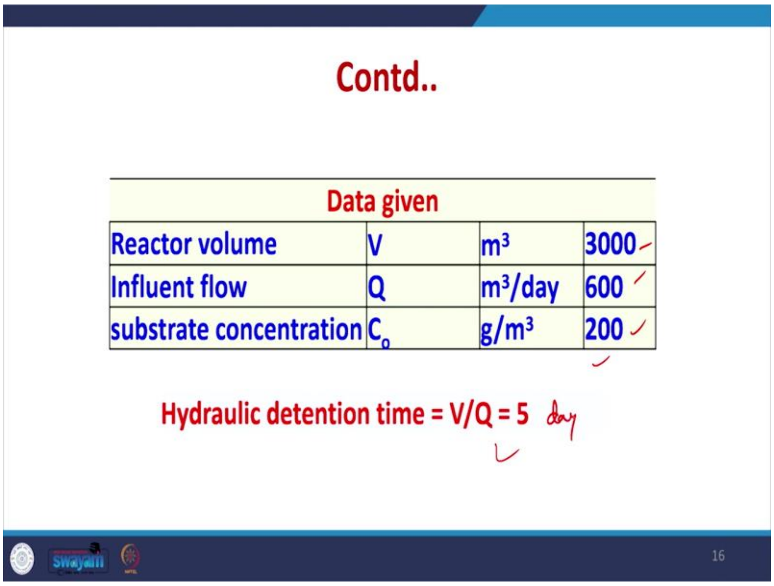

So, again the values are similar, because the volume by Q is again 5, so, we have 5 day, which is the time for the treatment.

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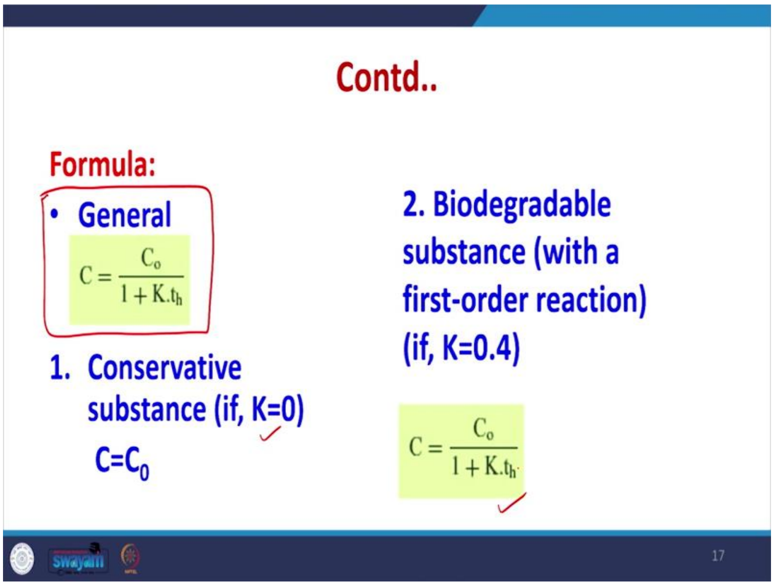

Now, for the first order we found out earlier that the equation can be written like this, the general formula is C is equal to C_0 1+K_{th}. Now for the conservative pollutant K is equal to 0 so, C is equal go to C_0 for biodegradable substance we will be using the same formula.

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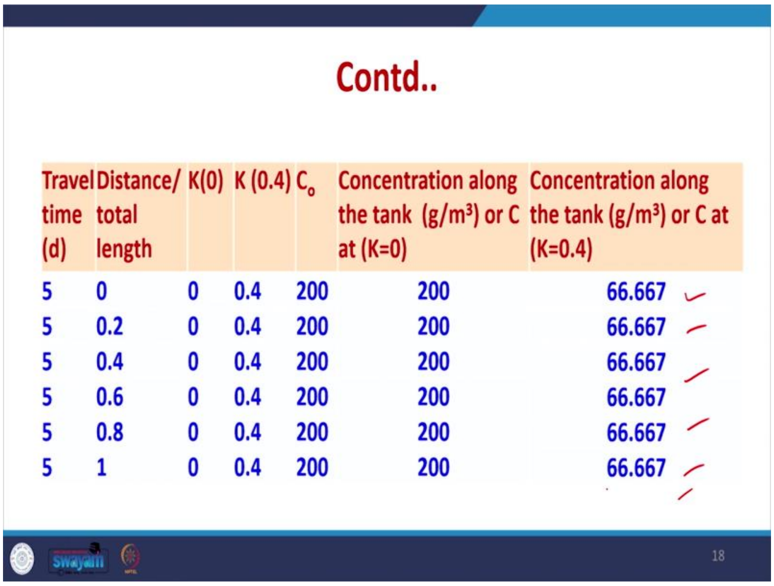

Now, if we solve it again, we will be finding that the concentration remains constant so, and it is 66.667.

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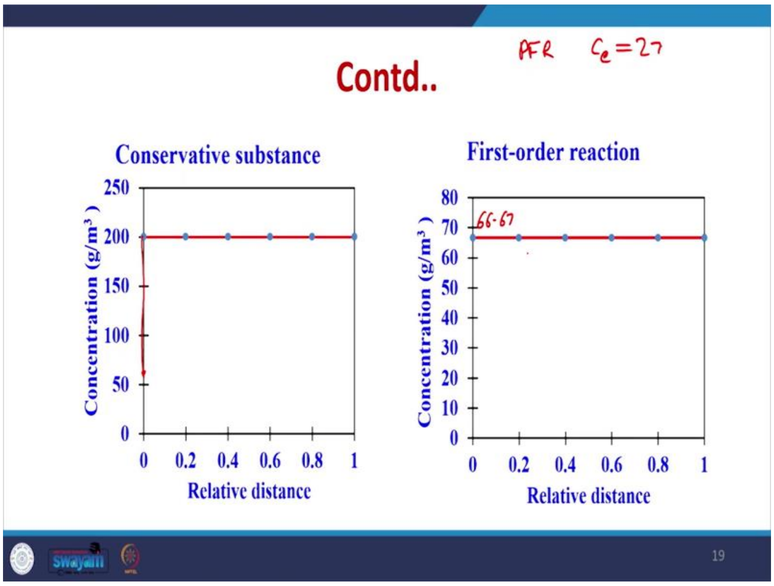

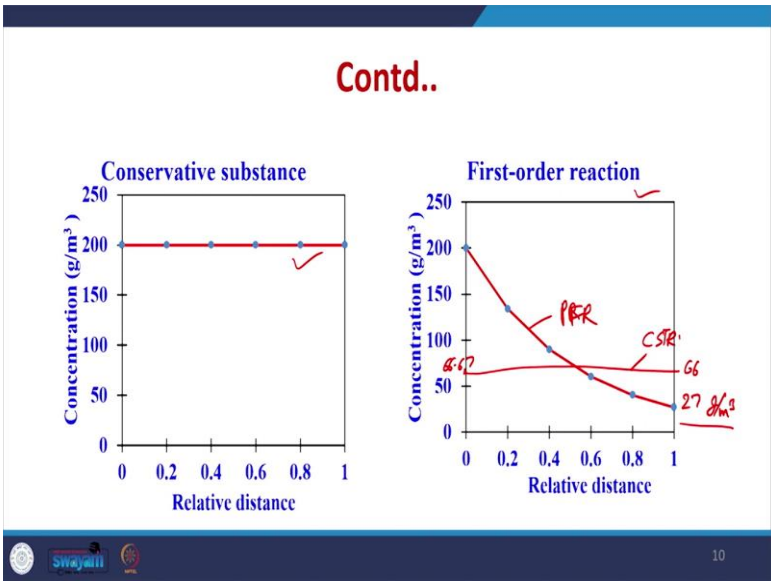

So, the profile is same, only difference is that here it is at 66.67. So, that means, as soon as the reactant enters the reactor, its concentrations become very less that means and if we mix both the slides, so it will be finding that it will be around 66 here. So, that means suddenly there is a drop in the concentration and after that it remains the same and with any relative distance or we cannot presume relative distance in case of CSTR so, with time it will remain as such and it will ultimately the profile is this.

Another thing that we can take the clue is that for the same volume for the same flow for PFR, we found that the exit concentration after the treatment was 27 around 27 gram per liter that we found in the earlier case when we solved a problem so, it was 27 grams per meter cube. The profile was like this whereas for CSTR the profile is like this. So, this is the difference, it is around 66.67 and it is remaining constant, so for first CSTR this is the profile for PFR, whereas, this is the profile for CSTR and for CSTR the concentration exit concentration is much higher that means the plug flow reactors under ideal conditions have better efficiency as compared to the simple CSTR reactor.

So, this is one important key that we can get. So, that means, if we can try to make reactors, which are having flow patterns similar to plug flow reactor, for the same volume, we can have a better efficiency and this is one of the important things that we should have the idea and this is also taken into consideration during the design of reactors also. Certainly there are other points we are worth considering, but this is very important.

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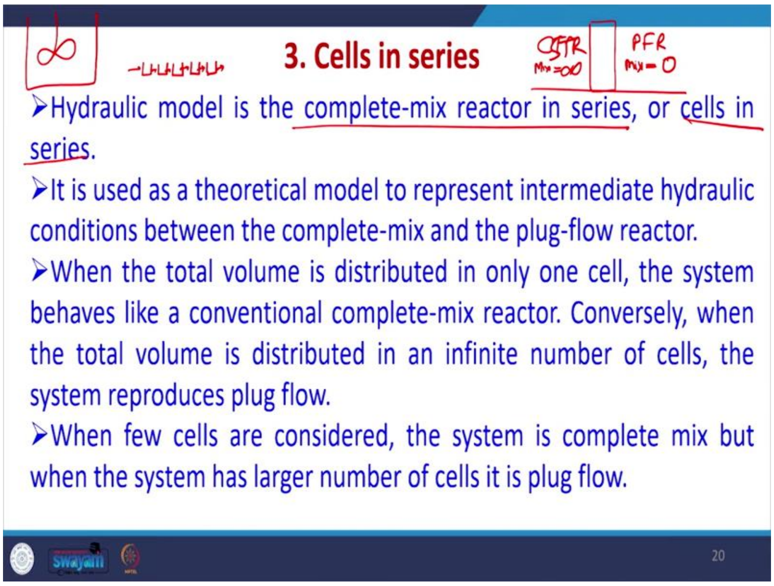

Now, I will try to learn the cells in series. Hydraulic model is like we can have complete-mix reactor in series or cells in series. So, there are many types of situation where we can have cells in series or the reactors in series. So, this is a it is used as theoretical model to represent intermediate hydraulic conditions between complete-mix and plug flow reactor. So, it is possible that our system is neither CSTR and nor PFR. So, it is possible that we can have there is another thing in the consider mixing. So, this here mixing is 0, we are not assuming any mixing whereas, in CSTR we are assuming complete-mixing, so, it is like mixing is up to infinite value.

So, this is these are two extreme cases and it is possible that our reactor may fall in between. So, in this case how to model those reactors, we can do it by modeling them cells in series, when the total volume is distributed in only one cell, the system behaves like a conventional complete-mix reactor. Conversely, when the total volume is distributed infinite number of cells, the system reproduces plug flow. So, and this is more true for wastewater treatment in particular.

So, if we have to see that we want to make a PFR condition because the efficiencies are higher. So, in one condition we may have a big reactor, may be a second condition we may have small CSTR in series. So, if this is done for the same value if the total volume remains same, we will be finding that the CSTRs in series will be having much higher efficiency as compared to a bigger CSTR this is what we are going to know. So, when few cells are considered the system is completely mixed, but when the system has large number of cells, it is plug flow.

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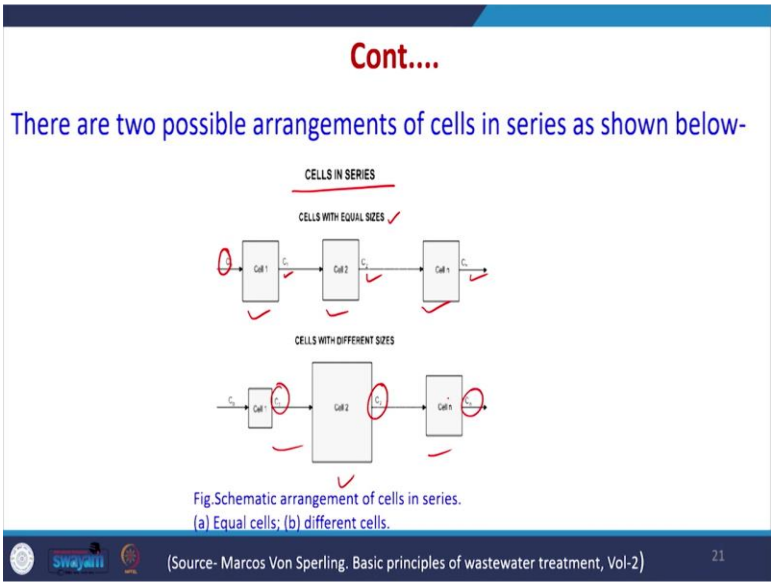

So, there are two possible arrangements of cells in series as shown below, cells with equal sizes, this is the cells in series. So, we have cell 1, cell 2, cell 3, all have equal sizes, C0 is the initial concentration and with time they are decreasing. It is also possible to have different size cells, we can see here again the concentrations are decreasing with the number of cells. So, both the conditions are possible.

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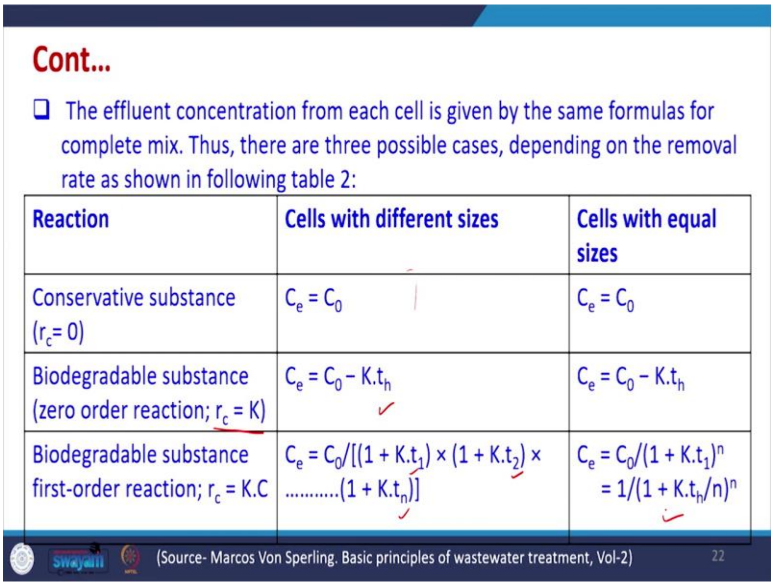

Reaction	Cells with different sizes	Cells with equal sizes
Conservative substance (r_c= 0)	$C_e = C_0$	$C_e = C_0$
Biodegradable substance (zero order reaction; r_c = K)	$C_e = C_0 - K.t_h$	$C_e = C_0 - K.t_h$
Biodegradable substance first-order reaction; r_c = K.C	$C_e = C_0/[(1 + K.t_1) \times (1 + K.t_2) \times$ ………..$(1 + K.t_n)]$	$C_e = C_0/(1 + K.t_1)^n$ $= 1/(1 + K.t_h/n)^n$

Now, the effluent concentration from each cell is given by the same formula for completely mixed reactor. So, we can assume whatever formula we derived for the complete-mix reactor which was like this, so we can use the same formula. And then depending upon the removal rate, we can use the equation. Now we can easily find out for here we can see the exit concentration for zero order reaction C_0 is equal to C_e is equal to C_0-K_{th}. And similarly here, C_e is equal to C_0 1/1+K_{t1}.

So, we can combine different cells together and we can come up with the same formula like this. Here, all the terms are different because the hydraulic detention times are different whereas here all will be combined together. And we can have one simpler formula like this. So, we will try to use this formula for better understanding.

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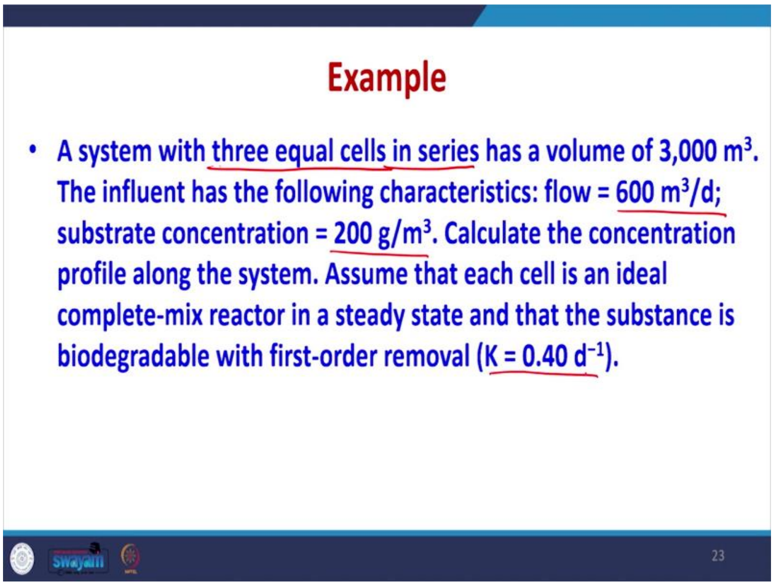

Same question which was earlier, a system with three equals cells in series now, we are assuming remember, when only one cell was there their effluent concentration was 66 when the plug flow reactor was there, the effluent concentration was 27. Now, we are assuming three equal cells in series has a volume of 3000 meter cube, the influent has following characteristics the flow rate is 600 meter cube per day, the substrate concentration is 200, the K value is 0.4 as earlier.

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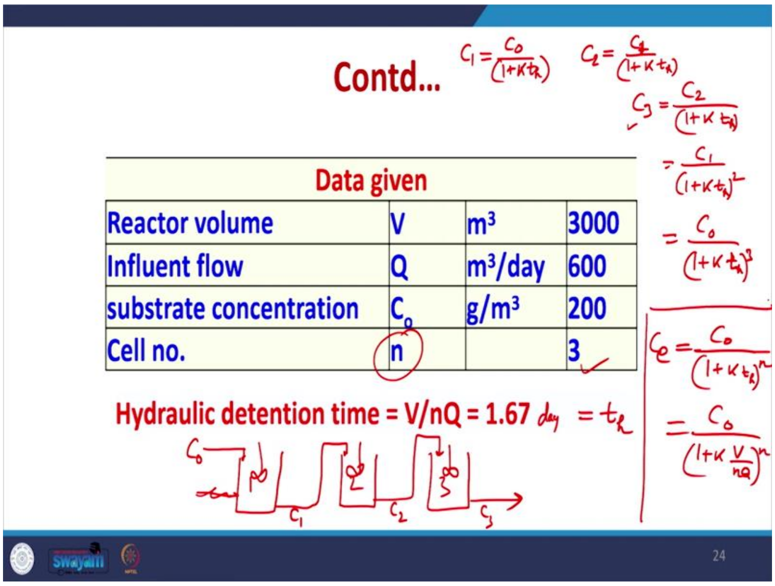

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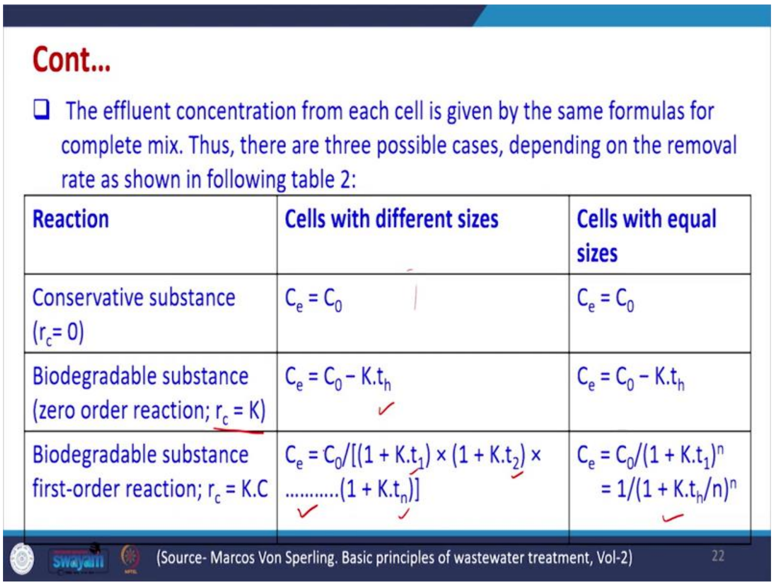
Data given			
Reactor volume	V	m^3	3000
Influent flow	Q	m^3/day	600
substrate concentration	C_0	g/m^3	200
Cell no.	n		3

Hydraulic detention time = V/nQ = 1.67



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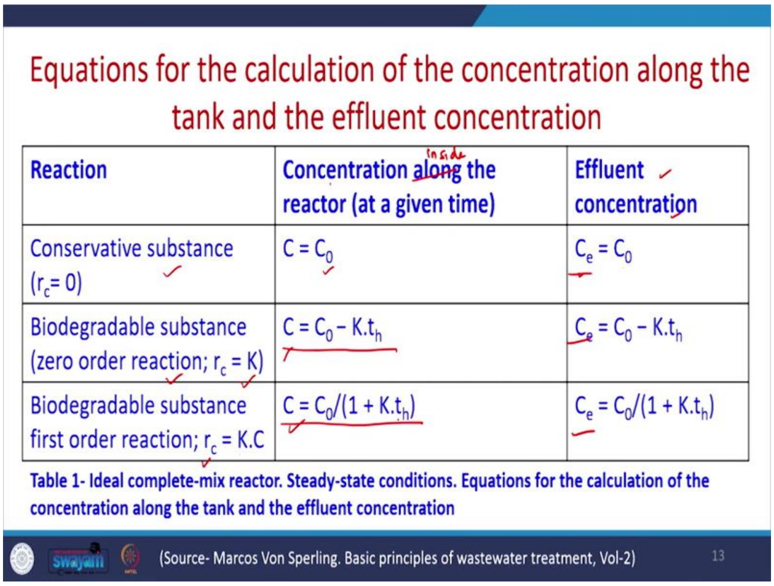
- The effluent concentration from each cell is given by the same formulas for complete mix. Thus, there are three possible cases, depending on the removal rate as shown in following table 2:

Reaction	Cells with different sizes	Cells with equal sizes
Conservative substance (r_c= 0)	$C_e = C_0$	$C_e = C_0$
Biodegradable substance (zero order reaction; r_c = K)	$C_e = C_0 - K.t_h$	$C_e = C_0 - K.t_h$
Biodegradable substance first-order reaction; r_c = K.C	$C_e = C_0/[(1 + K.t_1) \times (1 + K.t_2) \times \ldots\ldots\ldots(1 + K.t_n)]$	$C_e = C_0/(1 + K.t_1)^n = 1/(1 + K.t_h/n)^n$

(Source- Marcos Von Sperling. Basic principles of wastewater treatment, Vol-2)



Equations for the calculation of the concentration along the tank and the effluent concentration

Reaction	Concentration along the reactor (at a given time)	Effluent concentration
Conservative substance (r_c= 0)	$C = C_0$	$C_e = C_0$
Biodegradable substance (zero order reaction; r_c = K)	$C = C_0 - K.t_h$	$C_e = C_0 - K.t_h$
Biodegradable substance first order reaction; r_c = K.C	$C = C_0/(1 + K.t_h)$	$C_e = C_0/(1 + K.t_h)$

Table 1- Ideal complete-mix reactor. Steady-state conditions. Equations for the calculation of the concentration along the tank and the effluent concentration

(Source- Marcos Von Sperling. Basic principles of wastewater treatment, Vol-2)

Now, we have this formula, but we have the number of cells are 3 that means, the hydraulic detention time decreases by 3. So, in place of 5 day it is now 1.67 day in each of the cells. Now, remember the formula that we have the equation is like this we have one cell then we have another cell and then we have third cell. The concentration is like we can draw like this, let C_0 is coming here and after treatment it is going in the second. So, we have 1, 2, 3 all of them are CSTR with good mixing and then the second is going here and the third is coming out. So, we have C_0, C_1, C_2, C_3.

Now, remember if we use this particular formula for first order, so, earlier also we have derived the equation remember this for this we had written C exit equal to C_0 upon 1+ K_{th} so, it was same as this we can write here also. So, for C_1 the formula will be C_1 is equal to C_0 1+K_{th} because all are same, so, I am writing K_{th} otherwise, it could have been Kth1. So, in place of K_{th} we can write th which is this is that th hydraulic detention time.

Now, for the second case second cell it will be C_2 upon C_1 is equal to 1+K_{th} similarly, for C_3 is equal to this will be C_1 sorry this is C_2 upon 1+K_{th}. So, this is the overall formula which is there. Now, these are the three formula that we have to apply. Now, if we want to club them together and we want to find out the final concentration. So, if we start from this so, C_3 is equal to C_2 upon this, but C_2 is equal to C_1 upon 1+K_{th} so, it will become 1+K_{th} square. Now, if you want to replace again C_1 by C_0 upon this so, it will become C_0 upon 1+K_{th} cube.

So, we have the general formula which was given at C_e is equal to C_0 1+K_{th} raise to n. So, this is a formula and we can write for K also like we by given, so, this was a C_0 1 plus K total value upon n q raise to n, so we can use any of the formula here for finding out the concentration and which is given here.

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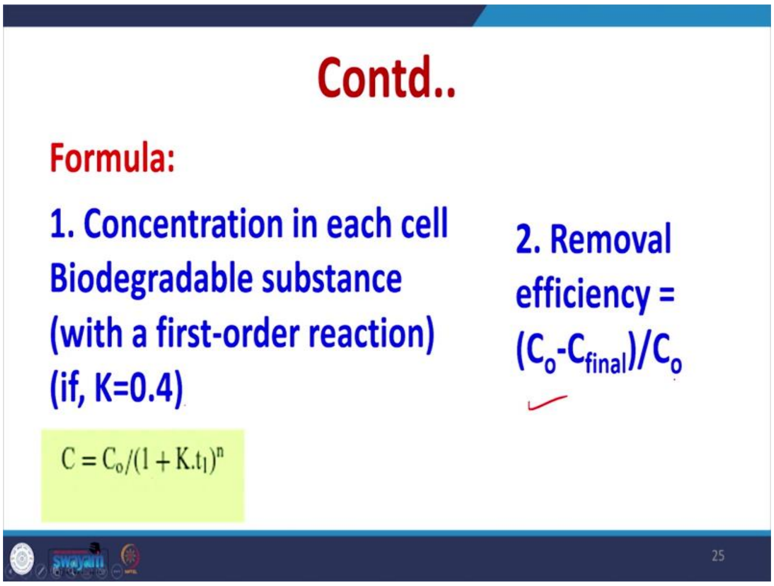

So, concentration is the each cell for biodegradable substance with first order reaction kinetics will be C_0 this and overall efficiency can be $C_0\text{-}C_{final}/C_0$ or $C_0\text{-}C$.

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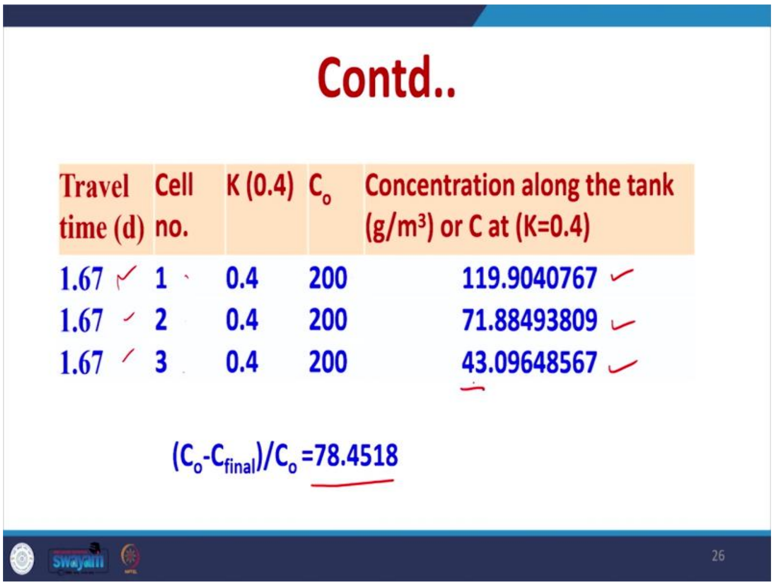

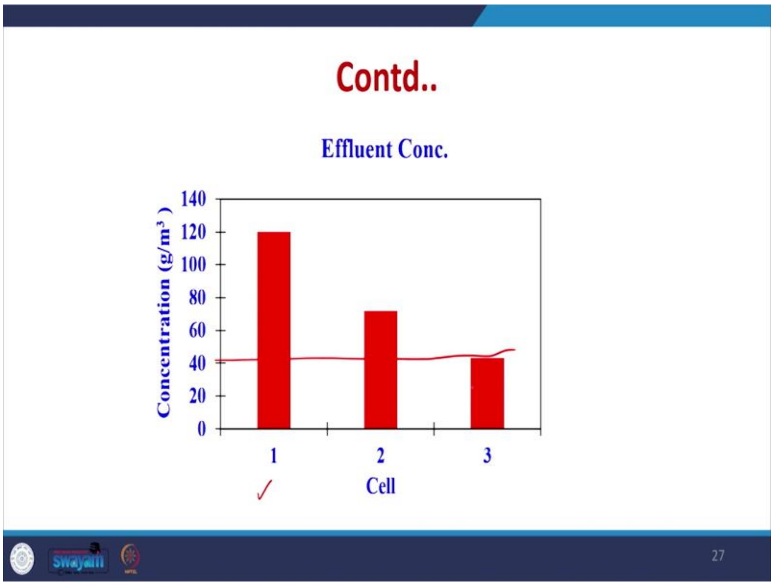

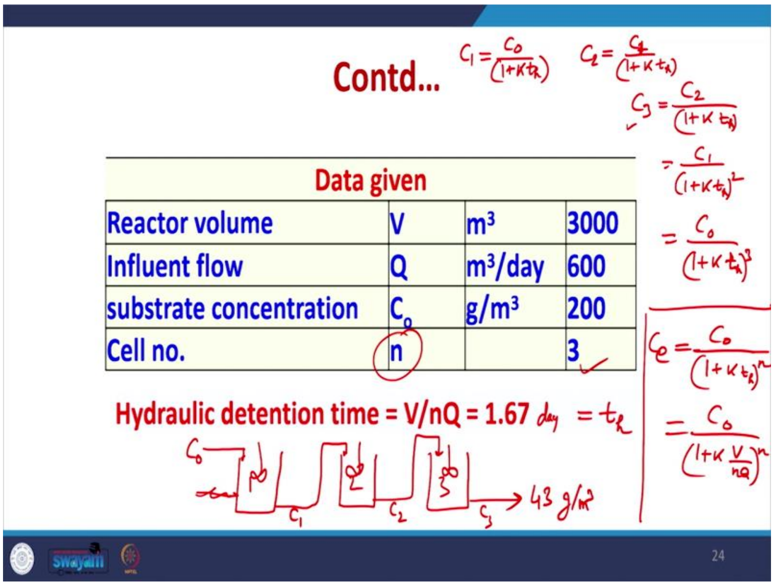

So, we use this formula for this is the d the detention time 1, 2, 3. The K values are there, C_0's who after first cell it will be 119 then 71 then 43. So, overall, this is 78.48 which is the efficiency which is obtained and the exit concentration is 43.

So, this is the difference is that that we are getting the exit concentration as 43 gram per meter cube whereas, it was a 66.67 when only single cell values that means, when we are using three cells, there is a decrease in the effluent concentration that means the overall efficiency is increasing because we have divided a single volume of CSTR into three equal cells. So, this is what we obtained.

So, as we are moving from infinite mixing condition to no-mixing condition that means, we are trying to model our system towards changing our system towards PFR our efficiencies go on increase. So, this is the learning and in each of the reactors the concentration is decreasing we can see here. This is the effluent which is coming at 43. So, we read today we learned a lot of things regarding the idea of plug flow reactor in which is it is assumed that there is no mixing. It is a flow like a piston which is moving from one section to another and there is no mixing in between. Then we have CSTR or complete-mixed tank reactor or complete-mix reactor where infinite mixing takes place.

So, we tried to learn the formula and find out the concentration. Then we model another case where we are assuming that the concentration the CSTR a bigger CSTR is replaced by a number of CSTR in series and we found that when the same volume of reactor is there for a bigger CSTR and three CSTs in series. So, in series reactors have better efficiency and the exit concentration is much lower as compared to single CSTR. So, overall efficiency is increasing.

So, we will continue this particular section with respect to reactor hydraulics in the next slide, and next lecture and we will learn regarding the dispersed flow another thing so, thank you very much.