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ENGINEERING

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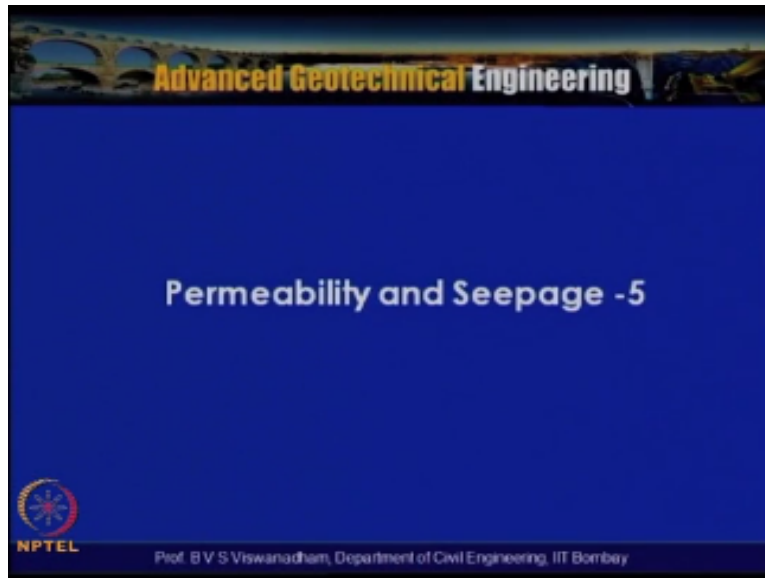
Lecture No. 16

Module-2
Permeability and Seepage – 5

Welcome to a course on advanced to the geotechnical engineering module 2 on permeability and seepage and lecture five, so in previous lectures we have discussed about permeability and factors affecting the permeability and how to determine the permeability in the laboratory and also cases for construction of the flow nets for isotropic soils. But in nature we do have quite often the soils which are actually endoscopic in nature, endoscopic in nature means the soils are having different permeability in horizontal direction as well as vertical direction.

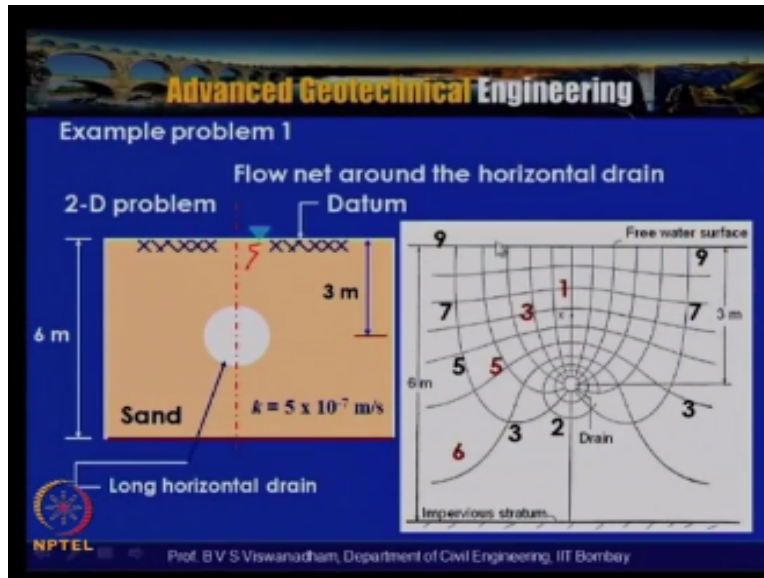
This is possible for stratified soil deposits and when we construct some embankments or dams we have different materials that means that two soils are three soils having different probabilities then what will happen how the flow lines and equipotential lines can be constructed or drawn will be seeing in this lecture.

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So this is a module 2, lecture number 5 in under in permeability and seepage, before discussing anisotropic and non-homogeneous soil conditions let us solve this example problem, this is an example problem where an example of a two dimensional problem where we have got a pipe which facilitates the draining of the water from the soil which is actually shown here.

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So this particular case there is ground surface and two-dimensional problem, because it is basically a plane strain structure where the length is actually perpendicular to the length of this figure it is continuous in nature so we need to construct the flow net around the horizontal drain and at a certain point here above the you know the centerline of the pipe we need to determine what is the you know pore water pressure and also determine the velocity.

Once you know the void ratio you can also determine what is the seepage velocity, so the flow net is constructed it will be like this and this being symmetry you can actually have two identical flow nets and this is also an example of radial flow nets wherein you have the equipotential lines running like this and you have flow channels running towards the periphery in the surrounding the pipe. So the soil which is surrounding the drain is given as $k=5 \times 10^{-7} \text{ m/s}$ and this is a long horizontal drain which actually has got the capacity to drain the water from the surrounding soil.

The strata which is permeable is about 6 meter from the ground surface and the datum is selected here but even the datum can be selected even also here but for convenience the datum is actually selected here, so and this is the impermeable stratum so that is represented here so when we draw the flow channels so you can see this is the first flow channel, second, third, fourth, fifth and then sixth so six flow channels which are actually there and when you look into the head loss which is actually there from the center of the pipeline is about 3 meters.

The drain that is the front the central line to the groundwater table surface it is 3 meters, so the head loss which occurs from this potential line to this potential line is about 3 meters that means

that this is the potential line 9, 8, 7, 6, 5, 4 and then 3, 2, 1 so with this now we will be able to you know calculate discharge and will be able to calculate what is you know seepage velocity and if we know or the elevations we will be able to calculate where is the what is the pore water pressure at different points.

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Flow net around the horizontal drain Example problem

a) Find the discharge through the drain in m^3/day per metre length of drain

- > Consider datum at free water surface
- > This makes total head at top of the ground is 3 m

H_1 (from the surface to the drain) = 3 m

By considering only half of the flow region $N_f = 6; N_d = 9$

$$q = k(h) \frac{N_f}{N_d} = 5 \times 10^{-7} \times (3) \times \left(\frac{6}{9}\right) \times 2 \times (24 \times 3600)$$

0.1426 m^3/day **Per m run**

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So the what we are doing is that we are doing an example problem the flow rate around the horizontal drain the question is that the find the discharge through the drain in m^3/day per meter length of the drain because it is a plane strain condition so per meter length and two dimension analysis is very relevant. And we considered the datum at the free surface so this makes the total head at top of the ground is 3 meters.

Now the head loss hL from the surface to the drain centerline is about 3 meters now by considering the one half of the flow region the number of flow channels we counted as 6 and number of potential drops we counted as 9 by using $q=k(h)N_f/N_d$ and considering the symmetry multiplying by 2 and converting into meter cube per day we will get the discharge as $0.1426 \text{ m}^3/$ so this is the discharge which actually the pipe is capable to drain the water from the surrounding soil.

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Flow net around the horizontal drain Example problem

b) Find the PWP at X, 1.5 m into the soil, directly above the drain

Potential drop between two EQP lines = 3/9

Total head at X = $(5.6/9) \times 3 = 1.867$ m

Elevation head at X = -1.5 m

Pressure head at X = $1.867 + 1.5 = 3.67$ m

PWP at X = $3.67 \times 10 = 36.7$ kN/m²

c) Estimate the velocity of flow at X.

Let the vertical length of the curvilinear square at X is measured as 0.4 m

$$z = \frac{0.333}{0.4} = 0.83$$

$$v = kv = 5 \times 10^{-7} \times 0.83$$

$$= 4.15 \times 10^{-7} \text{ m/s}$$

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Now the question is that how to determine the pore water pressure at X that is 1.5 meter into the soil directly above the drain, so we considered if you recollect the diagram which we have shown the datum is at the ground surface so the point X slice 1.5 meter below the datum so elevation head at X is -1.5 meters, so total head at X if you wanted to write it is 5.6/9 why because, we have said 9, 8 that is 9, 8, 7, 6 and so here we actually have we are counting between approximating between 6th equipotential line and 5th equipotential line so this is somewhat closer to the 6 equipotential line so the, this is actually approximated as 5.6/9, so this is that point which is above the drain.

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Flow net around the horizontal drain Example problem

b) Find the PWP at X, 1.5 m into the soil, directly above the drain

Potential drop between two EQP lines = $3/9$

Total head at X = $(5.6/9) \times 3 = 1.867$ m

Elevation head at X = -1.5 m

Pressure head at X = $1.867 + 1.5 = 3.67$ m

PWP at X = $3.67 \times 10 = 36.7$ kN/m²


c) Estimate the velocity of flow at X.

Let the vertical length of the curvilinear square at X is measured as 0.4 m

$$i = \frac{0.333}{0.4} = 0.83$$

$$v = ki = 5 \times 10^{-7} \times 0.83$$

$$= 4.75 \times 10^{-7} \text{ m/s}$$


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So with this we can actually now when we once you know the total head at X and elevation head at X then total head is equal to pressure head plus elevation head so we need to calculate what is pressure at X so it is nothing but 1.867 meters that is nothing but $5.67/9 \times 3$, 3 is the total head loss so with that we will get 1.867- F- 1.5 you will get with that 3.67 meters now taking γW of the water permeating to the drain is say having 10 kilo Newton per meter cube the pore water pressure at X is given as 3.67×10 that is 36.7 kilo Pascal's or kilo Newton per meter square.

Now next question is that estimation of the velocity of the flow at X once we know the velocity once we know the wild ratio we can also determine what is the seepage velocity of that point so we said that between each potential line the potential drop is nothing bad it is nothing but $3/9$ which is nothing but 0.33 meters so between each that is that means that from 9th equi-potential line to 8th equi-potential line the potential drop is $3/9$ that is 0.33 meters.

And if the length of that a small incremental flow line is known from the safe loan of flow net which is drawn and drawn onto the scale then we can calculate what is the hydraulic gradient that is nothing but $.333/0.4$ which gives .83 by using Darcy's law V is equal to KI , K is given to us and I is actually estimated as 0.83 will be able to calculate what is the discharge velocity or superficial velocity at that particular point.

So that is actually obtained as 4.75×10^{-7} meter per second and if you might if you use $V_s = V/n$ then by using the porosity by knowing the word ratio calculating the porosity we can also calculate the seepage velocity at that point.

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Flow nets in Anisotropic Material

→ On the other hand, if flow occurs at **right angles** to the bedding planes every water particle passes in succession through every one of the layers. Since the flow is continuous the discharge velocity v must be the same in every layer, whereas the coefficient of permeability of the layers is different. Hence the hydraulic gradient must also be different in the different layers.

$$k_{\text{vertical}} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \dots + \frac{H_n}{k_n}}$$

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So coming to the flow Nets in anisotropic soils particularly in sedimentary soil deposits there is always a possibility that the condition of anisotropy occurs that means that the, the soil are pimple soil layers soil layer or soil layers are no longer isotropic in nature they have different trim abilities say for example in this case when we consider the two dimensional cases that is X and say Z Direction the permeability in both exchanger direction is different.

So if a flow occurs parallel to the bedding planes the flow lines are also parallel to these planes and the hydraulic gradient has at every point of every bed the same value of I which is independent of the coefficient of permeability of the beds so the K horizontal combined or equivalent we can actually get by $1/H * K_1 H_1 + K_2 H_2$ so on to K and H 1 very of layers can be accounted suppose we are having say two layers k equivalent horizontal is equal to $1/ H * K_1 H_1 + K_2 H_2$.

Similarly when the flow is occurring right angles to the bedding planes every water particle passes in succession through every one of layers since the flow is continuous the discharge velocity V must be the same in every layer whereas the coefficient of permeability of the layers is different hence the hydraulic gradient must also be different so when the question the permeability is different the hydraulic gradient is different.

So $I = I_1 + I_2 + I_3$ by using again $V = K$ and then simplification we get K vertical equivalent of n number of layers say for example having thickness H_1, H_2 so on to H_n we get $H/H_1 / K_1 + h_2$ by K_2 so on H_n/K_n .

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Flow nets in Anisotropic Material

□ In developing procedure for plotting flow nets, we assumed that the permeable layer is isotropic, i.e.
 $k_{\text{horizontal}} = k_{\text{vertical}} = k$

Consider the case of constructing flow nets for seepage through soils that show anisotropy with respect to permeability
 $(k_{\text{horizontal}} \neq k_{\text{vertical}})$

For 2-D flow problems \rightarrow $k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$

Where
 $k_{\text{horizontal}} = k_x$
 $k_{\text{vertical}} = k_z$
 $k_x > k_z$

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So let us say let us see how we can actually construct flow nets in anisotropic material particularly having a permeability K a horizontal and a vertical different in both horizontal direction and vertical direction previously what we have assumed is that in developing the procedure for plotting flow rates we assumed that the pyramid layer is isotropic that is K horizontal is equal to K vertical is equal to K considering the case of constructing the flow Nets of for seepage to soils that show opacity anisotropy with respect to permeability K horizontal is not equal in to K permeability.

In fact we also should know that K horizontal is more than K vertical the reasons are you know the ease with which water flow and water can flow through the in the horizontal direction is superior than vertical direction the reason is that the σ horizontal which is actually available at any given point for four major types of soil deposits is less than σ_v so this makes you know the K horizontal more than k vertical but it is in such type of soils where they have been you know subjected to a very high pre consolidation pressures.

And in case if there is a locking of pressures takes place then in that case the K horizontal will be less than K vertical but this is not quite common so we actually have discussed the Laplace

equation when you have what K_x is not equal to K_z and for two dimensional problems we can write this $K_x \frac{\partial^2 H}{\partial x^2} + K_z \frac{\partial^2 u}{\partial z^2} = 0$ where K_x horizontal is represented here as K_x and K_z vertical as K_z and K_x is greater than K_z .

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Flow nets in Anisotropic Material

- ◆ In all sedimentary deposits, as well as in most earthen dams, there is a pronounced tendency for the permeability to be greater in the horizontal than in the vertical direction. Generally $k_x \approx 5$ to $10 k_z$

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

← Is not a Laplacian equation, since this equation applies for a soil mass in which stratification occurs.

- ◆ Also results in distorted flow net (not necessary that flow lines and Equip lines are orthogonal)

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Now in all the sedimentary deposits as well as in the most of the earthen dams there is pronounced a tendency for the permeability to be greater in horizontal than in the vertical direction this we have a discuss and the permeability is generally of the order of 5 to 10 times vertical permeability that is K_x is 5 to 10 times the vertical permeability so this case $\frac{\partial^2 H}{\partial x^2} + K \frac{\partial^2 u}{\partial z^2} = 0$ is not an Laplace equation. Since this equation applies for a soil mass in which ref stratification occurs so this results in distorted flow net this results in a distorted flow net.

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Flow nets in Anisotropic Material

By rewriting:
$$\frac{k_x}{k_z} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{-- A}$$

A closer analysis of the above equation indicates that a simple Laplacian equation may be endangered if an adequate change of variable is used.

Let us write the same equation using $X = cx$
(where c is a constant)

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\partial^2 h}{\partial (cx)^2} + \frac{\partial^2 h}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{-- B}$$

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So by rewriting that equation what we done what we have done is that by dividing through the equation at throw out we can write in the form of equation A which is $\frac{k_x}{k_z} \frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Z^2} = 0$ a closer analysis of the above equation indicates that a simple Laplace equation may be endangered if an inadequate change of variable is used so let us now consider a capital X is equal to C times small X and the X small X is the horizontal Direction capital X is you are considered here well C is a constant.

So by substituting X is equal to CX that is by writing this we can write we can write this in the plane like the equation can be X is substituted as the square $\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Z^2}$ this is nothing but a equation in X Z plane that capital X and small Z plane so by writing X is equal to capital X is equal to CX we can write this $\frac{\partial^2 H}{\partial (CX)^2} + \frac{\partial^2 H}{\partial Z^2}$ by those are square so by simplifying this we can write $\frac{1}{C^2} \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial Z^2} = 0$ this is termed as B.

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Flow nets in Anisotropic Material


The constant c is identified by comparing equations A and B.

$$c = \sqrt{\frac{k_z}{k_x}}$$

Equation (B) is of the same form as $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$

♦ Which governs the flow in isotropic soils and should represent two sets of orthogonal curves in the Xz plane.

$$X = \sqrt{\frac{k_z}{k_x}} x$$

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Now the constant C is defined by comparing equations A and B.

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Flow nets in Anisotropic Material

By rewriting:
$$\frac{k_x}{k_z} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{-- A}$$

A closer analysis of the above equation indicates that a simple Laplacian equation may be endangered if an adequate change of variable is used.


Let us write the same equation using $X = cx$
(where c is a constant)

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\partial^2 h}{\partial (cx)^2} + \frac{\partial^2 h}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{-- B}$$

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That is equation A is $\frac{k_x}{k_z} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$ equation B finally $\frac{1}{c^2} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$.

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Flow nets in Anisotropic Material

The constant c is identified by comparing eq

$$c = \sqrt{\frac{k_z}{k_x}}$$

Equation (B) is of the same form as $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$

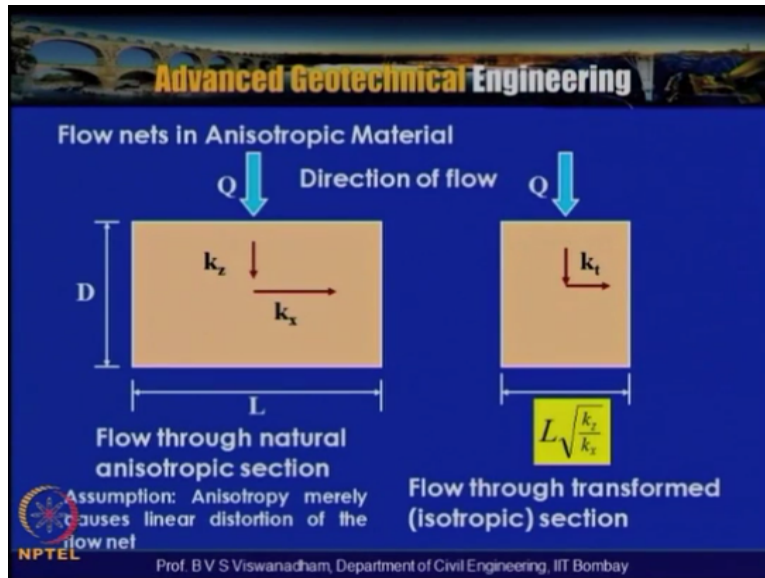
◆ Which governs the flow in isotropic soils and should represent two sets of orthogonal curves in the Xz plane.

$$X = \sqrt{\frac{k_z}{k_x}} x$$

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This gives constant C as root over KZ/ KX so equation B is of the same form as $\partial^2 u / \partial X^2 + \partial^2 H / \partial Z^2 = 0$ which governs that the flow net inflow in isotropic soils and should represent two sets of orthogonal curves in capital X and small Z plane so we can write capital X is equal to root over root over the ratio of the vertical and horizontal permeability is root over KZ/ KX into small X .

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Now by using this concept let us try to write you know the construct you know the anisotropic convert this natural anisotropic section into a transformer section by using the logic what we discuss now as we have said that case is greater than K_z that means that when we transform the L the L is actually multiplied within the transformer section what we do is that we the vertical distance or height same as in the original plane but the horizontal distance are reduced by root over K_z/K_x .

So the transformer flow through natural anisotropic section is given as that is direction of the flow which is shown there and which is nothing but K_z and then K_x and what we are actually doing is that by converting into transform section we are actually making this into the equivalent permeability in both the directions so the anisotropic case is converted into anisotropic section by transforming from an historic section to transformed section. The assumption is that anisotropy merely causes linear distortion of the flow net so that is what under that assumption and this actually has been done.

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Flow nets in Anisotropic Material

Starting with the natural section, the flow quantity per unit length is:


$$Q = (L \times 1) k_z i \quad \text{---- (C)}$$

Within the transformed section, the same quantity of flow is calculated as follows:

$$Q = \left(L \sqrt{\frac{k_z}{k_x}} \times 1 \right) k_t i \quad \text{---- (D)}$$

Comparing (C) and (D), the transformed permeability is then established:

$$k_t = \sqrt{k_x k_z}$$

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Now starting with the natural section that is the flow quantity in the anisotropic section can be given as Q is equal to L into 1 because L is the length and perpendicular to that plane it is one, one unit.

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Flow nets in Anisotropic Material

Q ↓ Direction of flow Q ↓

Flow through natural anisotropic section

Flow through transformed (isotropic) section

Assumption: Anisotropy merely causes linear distortion of the flow net

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So $L \cdot 1$ is the area or which this flow is occurring into this k_z .

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Flow nets in Anisotropic Material

Starting with the natural section, the flow quantity per unit length is:


$$Q = (L \times 1) k_z i \quad \text{---- (C)}$$

Within the transformed section, the same quantity of flow is calculated as follows:

$$Q = \left(L \sqrt{\frac{k_z}{k_x}} \times 1 \right) k_t i \quad \text{---- (D)}$$

Comparing (C) and (D), the transformed permeability is then established:

$$k_t = \sqrt{k_x k_z}$$

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k_z is the permeability in the direction into I so while for the transformed section the same quantity of the flow is calculated as Q is equal to now the L is nothing but L into root over k_z by k_x into 1 into k_t the k_t is nothing but the equivalent permeability in transformed section into I so comparing both C and D we actually get the transformer permeability is established as k_t is equal to root over k_x and k_z .

So the in the in the transformer permeability where we convert anisotropic case into the isotropic case the k_t is equal to root over k_x into k_z that is k permeability in horizontal Direction multiplied by the parameter vertical direction and the square root of that.

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Flow nets in Anisotropic Material

The same expression could have been found using the flow in horizontal direction. Note, then, the hydraulic gradient changes from $i = H/L$ in the natural section to $i = \frac{H}{L} \sqrt{\frac{k_z}{k_x}}$ in the transformed section.

Steps involved for construction of flow net in an anisotropic medium:

1. To plot the section of the hydraulic structure, adopt a vertical scale
2. Determine $\sqrt{\frac{k_z}{k_x}} = \sqrt{\frac{k_{vertical}}{k_{horizontal}}}$

3. Adopt a horizontal scale such that $Scale_{horizontal} = \sqrt{\frac{k_z}{k_x}} (Scale_{vertical})$

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So how to construct you know flow nets for the anisotropic cases the procedure is actually given here the same expression could have been found using flow in the Horizontal direction but note that then the hydraulic gradient changes from $i = H/L$ in the natural section and in the case of transformed section it will be $H/L \times \sqrt{k_z/k_x}$.

So steps involved in the construction of flow net in I said anisotropic medium is that to plot the section of the hydraulic structure adopt a vertical scale so first you adopt a vertical scale and determine $\sqrt{k_z/k_x}$ which is nothing but $\sqrt{k_{vertical}/k_{horizontal}}$ and adopt a horizontal scale so vertical scale is considered as it is and the horizontal scale is modified the scale horizontal is nothing but $\sqrt{k_z/k_x}$ into the whatever the scale which is adopted for the vertical scale.


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Flow nets in Anisotropic Material

4. With the scales adopted in Steps 1 and 3, plot the cross section of the structure.
5. Draw the flow net for the transformed section plotted in step 4 in the same manner as is done for seepage through isotropic soils.
6. Calculate the rate of seepage as:

$$q = \sqrt{k_x k_z} (h) \frac{N_f}{N_d}$$


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So the scale for horizontal is nothing but root over KZ/KX into scale vertical so in the fourth step when the scales adopted in step one and step three plot the cross section to the structure so draw the flow net for the transformed section plotted in stuff for in the same manner as is done for the see phase to the isotropic soils and calculate the rate of the EC page but here in case of in calculating the discharge here we need to use the K_T that is the transformed section permeability which is nothing but root over $KZ KJ^* H^*nm/nd$.

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Flow nets in Anisotropic Material Example problem:

Consider a concrete dam on a layer of a homogeneous anisotropic clay having $k_x = 16 \times 10^{-8}$ m/s and $k_z = 1 \times 10^{-8}$ m/s

Required:
Flow quantity seeping on the downstream side of a dam

$$C = \sqrt{\frac{k_z}{k_x}} = \frac{1}{4}$$

$$k = \sqrt{k_x k_z} = 4 \times 10^{-8} \text{ m/s}$$

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So let us try to look into this example problem wherein we actually have a particular concrete dam with the butress as retaining a head of water of five meters and this is the downstream side and this is the upstream side and this is the vertical scale and this is the horizontal scale the horizontal length here is 12.8 meters and at this point it is 9.6 meters.

And if this point is 10 meters and this is called cutoff wall basically this is provided to direct to the flow and then reduce the uplift pressures and all the considering the concrete dam on a layer of an homogenous anisotropic clay having permeability 16×10^{-8} meter per second in the horizontal direction and k_z is 1×10^{-8} to provide 8 meter per second in the vertical direction.

So the permeability for this clay is, is 16 times more than vertical direction so this could be a normally consolidated clay and the flow quantities seeping on the on the downstream side of a dam is required so the flow takes place in this direction and this is the impervious rock strata so this is the impervious boundary for this and now the first step is to determine the constant C which is nothing but thee for the root over k_z / k_x .

So with that we will get $\frac{1}{4}$ because the k_z / k_x is ratio is $\frac{1}{16}$ root over that we get $\frac{1}{4}$ and the transformed permeability k_t is equal to root over k_z gadget so we get 4×10^{-8} meter per second is the transformer permeability.

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Flow nets in Anisotropic Material Example problem:

Flow net in the natural section

Curvilinear rectangles of a sides ratio $\sqrt{\frac{k_x}{k_z}}$

◆ Equipotential lines and flow lines do not cross at right angles, as the tangent to flow line does not correspond to the normal to the equipotential line as was the case for isotropic soils.

◆ Hence, flow net is harder to sketch because the sides of every quadrangle in the flow net must have a ratio $\sqrt{\frac{k_x}{k_z}}$

⇒ Transformed section

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Now in the when we when we when we do the you know without transforming and the isotropic case when you convert when you put you have convenient angles of a sides ratio of root over a case $\sqrt{\frac{k_x}{k_z}}$ and this is what actually we have told because of the inertia trophy the flow net in the natural sections will be distorted and equipotential lines and flow lines do not cross at right angles and as the tangent to flow line does not correspond to the normal to the equipotential and as was the case for isotropic soils.

So flow rate is not because it because in this case the flow-rate is ordered to sketch because the sides of every quadrangle in the and I must have a ratio of root over $\sqrt{\frac{k_x}{k_z}}$ package it so this type of problem where we cannot actually sketch the flow rate comfortably so for that the transformer section is one option and for that what we said is that represent the vertical scale and adopt the horizontal scale such a way that the scale is $\sqrt{\frac{k_x}{k_z}}$ times scale of the vertical.

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Flow nets in Anisotropic Material Example problem:

$$q = k_t(h) \frac{N_f}{N_d}$$

$$= (4 \times 10^{-8})(5) \frac{5}{10}$$

$$= 10^{-7} \text{ m}^3 / \text{s} / \text{m}$$

Scale_{horizontal} = $\frac{1}{4}$ (Scale_{vertical})

Transformed section →

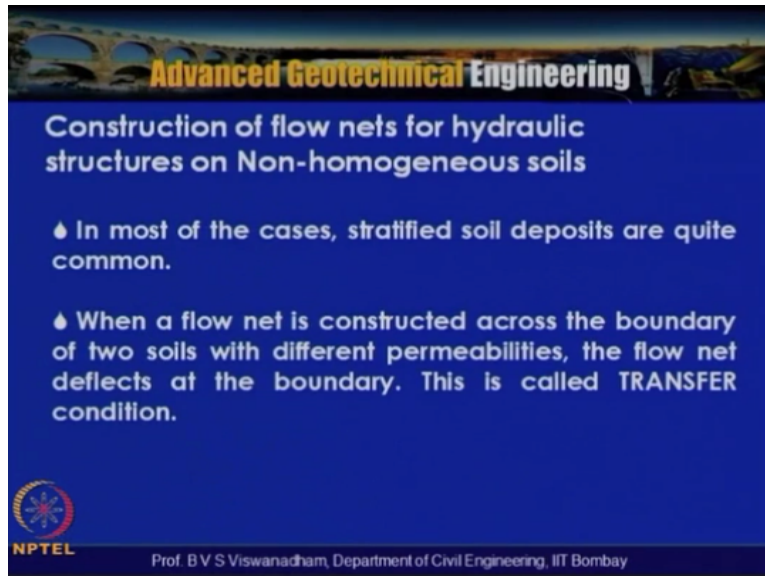
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Now in the next slide when we see that the transformed section flow rate is shown here the 12.8 meters now Stunk by about four because the scale what we adopted is a 3.8 meters and the vertical Heights are remained constant so with this now we constructed the, the, the flow rate or the transformed section the scale for the horizontal is one by four times of the scale for the vertical so with this we constructed and by adopting the similar procedure now considering KT that is the equivalent permeability because of the horizontal and vertical permeability of the credible layer.

We actually have $K_T \cdot H \cdot n$ of by n_d and we have got ten potential drops here the same procedure so here the this is the tenth equi-potential line and this is the 0 the equi-potential line and what we can see is that here the head available is about five meters then as the water progresses it loses the head and by the time it comes here the head loss is about is completely lost so with that q can be obtained so this is the you know how the transformer sections are a handle and so the important is that by adopting a vertical or horizontal.

We actually have to construct from isotropy convert into a transformer case and taking K_T that is equivalent permeability in that is transformed permeability better refer it has is Q is equal to $K_T \cdot H \cdot n_d$ and the problem is actually handled in a similar manner as that of the isotropic sides.


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Construction of flow nets for hydraulic structures on Non-homogeneous soils

- ◆ In most of the cases, stratified soil deposits are quite common.
- ◆ When a flow net is constructed across the boundary of two soils with different permeabilities, the flow net deflects at the boundary. This is called **TRANSFER condition**.

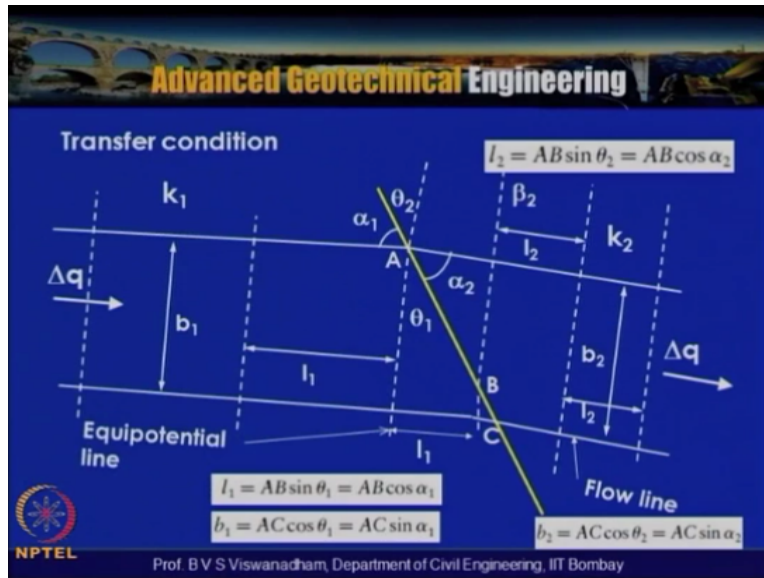
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Now we have said that we have a case we have discussed about the isotropic essence but now we may actually have hydraulic structures resting on non homogeneous soils that means that most of the cases stratified soil deposits are quite common and particularly when we have soils having two different permeability there is a possibility that the deviations of you know flow lines flow Nets occur.

So when a flow net is constructed across the boundary of two soils with the different variability's the flow net deflects at the boundary here is something like refraction type it actually takes place. And the condition where the deflection of you know the flow net happens and this is called the transfer condition so how to handle this transfer condition particularly for flow rates for hydraulic structures on non homogeneous soils.

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So in the transfer condition is considered we have say on this side this is the boundary the yellow line which is actually shown is the boundary and this side is actually having a soil with permeability K_1 and this side we are having a soil with a permeability K_2 so here soil one having permeability K_1 coefficient of permeability K_1 and this side we are actually having soil to having coefficient of permeability K_2 .

So this is the flow channel so this is the flow line what we see these are the flow lines but once it crosses the boundary it undergo a deviation the deviation which is actually shown here now here this distance between two flow lines is represented here as B_1 and distance between these are the equi-potential lines which are actually shown the broken lines here and these are after deviation and here the B_1 is this dimension between two flow lines.

And this is the L_1 is the dimension between two edges and equi-potential on the side of the boundary and this side of the boundary that is soil to side with this distance is L_2 and this distance is B_2 and this distance is L_2 but as the soil they discharge is same where we have got ΔQ enters and then ΔQ comes out we are using the equation of continued equation of continuity so here the let α_1 be the angle which makes with the boundary and α_2 is the angle which actually makes the flow, the flow line which makes with the boundary.

Similarly here this is the equi-potential line so this is this angle is θ_2 in boundary 2 in that is in the soil 2 and this angle is θ_1 in soil 1 so this is this, this angle is θ_1 now this boundary point on the, the point where the deflection of flow line is occurring here is termed as a at this point is

termed as B and this point is termed as C now by using the geometry we can say that that L 1 is equal to AB sin θ1 is equal to AB cos α cos A B cos α1.

Similarly B1 is equal to that is B1 is equal to AC cos θ 1 is equal to AC sin α 1 similarly we can write L2 is equal to AB sin θ 2 is equal to a B cos α2 and B 2 is equal to AC cos θ 2 is equal to AC sin α2 so by using this we can now handle this cases.

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Transfer condition

Let Δh be the loss of hydraulic head between two consecutive equipotential lines. Considering a unit length perpendicular to the section shown, the rate of seepage through the flow channel is:

$$\Delta q = k_1 \frac{\Delta h}{l_1} (b_1 \times 1) = k_2 \frac{\Delta h}{l_2} (b_2 \times 1)$$

or

$$\frac{k_1}{k_2} = \frac{b_2/l_2}{b_1/l_1}$$

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Now the let ΔH be the loss of hydraulic head between two consecutive accommodation lines considering a unit length perpendicular section shown the rate of this seepage the flow channel is given as ΔQ is equal to k_1 is the permeability of the soil 1 ΔH which the head loss and L_1 which is actually there in the soil 1 that the distance between the there is the distance between any two potential line sand B_1 is the distance between two flow lines into 1 is equal to K_2 into ΔH by L_2 that is the hydraulic gradient in soil 2 into B_2 into 1.

This is because of the equation of continuity and by simplifying that we can write K_1 by K_2 is equal to B_2 by L_2/B_1 by L_1 so by using this expression it is possible for us to construct the flow lines or understand about how the flow lines deviate when K_1 is k_1 is greater than k_2 or k_1 is less than k_2 when k_1 is equal to k_2 it is which is nothing but B_2 by n_2 is equal to B_1 by L_1 is equal to 1 so what is that ratio B_1 by L_1 is nothing but the aspect ratio the which is in for isotropic cases the, the mostly the, the you know the aspect ratio is 1.

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Transfer condition

$$\frac{b_1}{l_1} = \frac{\cos \theta_1}{\sin \theta_1} = \frac{\sin \alpha_1}{\cos \alpha_1}$$

$$\frac{b_2}{l_2} = \frac{\cos \theta_2}{\sin \theta_2} = \frac{\sin \alpha_2}{\cos \alpha_2}$$

Flow nets in non-homogeneous subsoils can be constructed using:

$$\frac{k_1}{k_2} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\tan \alpha_2}{\tan \alpha_1}$$

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Now the transfer condition where we have B_1/L_1 is equal to by using the deliberations what we have discussed here by using these conditions and now once we substitute in the expression what we said is that K_1/K_2 is equal to B_2/L_2 by B_1/L_1 by substituting there we will actually get B_1/L_1 is equal to $\cos \theta_1 / \sin \theta_1$ is equal to $\sin \alpha_1 / \cos \alpha_1$ and then B_2/L_2 is equal to $\cos \theta_2 / \sin \theta_2$ is equal to $\sin \alpha_2 / \cos \alpha_2$ do so by K_1/K_2 is equal to B_2/L_2 by B_1/L_1 and substituting this we will get K_1/K_2 is equal to $\tan \theta_1 / \tan \theta_2$ and is equal to $\tan \alpha_2 / \tan \alpha_1$.

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Flow channel at the boundary between two soils with different coefficients of permeability

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So this will tell us you know how the deviations of you know the flow lines happen our equipotential lines suffer when you actually have say to two cases which are disgustedly here one is let us assume that this is the boundary this is the boundary and this is the soil one having permeability K_1 and this is the soil actually having permeability K_2 that is soil 2 which is shown here so when K_1 is K_2 then the B_2 by L_2 is actually greater than 1.

So we have here you get large rectangles and not necessarily that year on the soil one boundary where the square elements are ensured in one of the soils that is soil one it can be vice versa if you take soil 1 as soil 2 and dense this thing so here you have square elements and here you have rectangular elements when K_1 is K_2 you get large rectangles but similarly when you have K_1 less than K_2 and with the ratio is less than 1.

So B_2 by L_2 is less than 1 so you get here the deviation will become narrow here and where you actually get small rectangles but on the soil one side again it maintains the square elements which is equal to B_1 is equal to L_1 the aspect ratio is maintained as 1 in fact we will see this through a numerical simulation also.

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Flow channel at the boundary between two soils with different coefficients of permeability

$$\frac{k_1}{k_2} = \frac{b_2/l_2}{b_1/l_1}$$

- ◆ If $k_1 > k_2$, we may plot square flow elements in layer 1. This means that $l_1 = b_1$. So $k_1/k_2 = b_2/l_2$. Thus the flow elements in layer 2 will be rectangles and their width-to-length ratios will be equal to k_1/k_2 .
- ◆ If $k_1 < k_2$, we may plot square flow elements in layer 1. This means that $l_1 = b_1$. So $k_1/k_2 = b_2/l_2$. Thus the flow elements in layer 2 will be rectangles and their width-to-length ratios will be equal to k_1/k_2 .

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So what the case we discussed is that the flow channel at the boundary between two soils with different coefficients of permeability we can write K_1 by K_2 is equal to B_2 by L_2 by B_1 by L_1 . B_2 by L_2 is in soil two region where B_2 is the distance between two flow lines after

deflection in soil 2 L_2 is the you know distance between two equi-potential lines and similarly B_1 is the B_1 by L_1 is nothing but the aspect ratio in soil 1 if K_1 is K_2 that is if you are having two soils where the permeability of the soil 1 is greater than soil 2 we may plot square flow elements in layer 1 this means that L_1 is equal to B_1 and so K_1 and K_2 is $B_2 L_2$.

So does the flow elements in layer 2 will be rectangles and there with 2 ratios will be equal to K_1 by K_2 that is what it says because in this when we put the B_1 by L_1 is equal to 1 the K_1 by K_2 it is going to be L_2 so whatever the ratio is there the perm value having two different permeabilities we can actually plot this as two ratio which is actually B_2 by L_2 is equal to in the ratio of K_1 by K_2 if say K_1 is less than K_2 we may plot the squares square flow elements in the layer 1 that is similar and this means that again the aspect ratio is 1 in soil 1 and so K_1 by K_2 is going to be L_2 so this the flow elements in layer 2 will be rectangles again and the width to length ratios will be equal to K_1 by K_2 .

So now having discussed and so we actually have two types of conditions we had discussed anisotropic condition and other one is non homogenous soils and whenever we actually construct the embankment dams and this condition is actually referred as unconfined seepage suppose if you are having a hydraulic structure in the form of a sheet pile wall or a coffer dam the flow surrounding that is actually referred as a confined seepage. So in example for of an unconfined seepage is nothing but one boundary of the flow region being periodic surface on which the pressure is atmospheric.

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Seepage through embankment dams

- Example of an unconfined seepage
 - One boundary of the flow region being a phreatic surface on which the pressure is atmospheric.
 - (i.e cross-sections with top flow line at atmospheric)
- The phreatic surface constitutes the top flow line and its position must be estimated before drawing the flow net

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So the cross-sections of the top flow line are at atmospheric so because of this, this condition is referred as unconfined seepage condition so the cheapest embankment dams the example is an example for unconfined seepage condition the phreatic surface constitutes the top flow line and its position must be estimated before drawing the flow rate suppose if you are having an embankment dams.

For example in practical cases these structures are like levees or dams or nowadays the modern structures like ash pond dams or you have some tailing dams these are the examples of these you know embankment dams which are actually constructed for different purposes so in which case when have got water retained on the upstream side of stream surface and in downstream side if it is actually having say water level and if it is actually constructed with earth then it is called as a tendon at estimate a major construction material.

And so the phreatic surface which constitutes the top flow line and it is must be estimated before they trying the flow rate so important you know requirement is that how to estimate the phreatic line when you are actually have ϕ which is retaining a certain head of water so for that in order to calculate the idea is that once we construct.

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Calculation of seepage through an earth dam resting on an impervious base

➤ Several solutions have been proposed for determination of the quantity of seepage through a homogeneous earth dam.

- Dupuit's solution
- Schaffernak's solution
- Casagrande's solution

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And then to estimate the seepage through an earthen dam resting on impervious base suppose impervious base because the otherwise the flow can actually lead into the subsoil so several solutions have been proposed to for determining the quantity of seepage through a homogeneous earthen dam to name a few we actually have we discussed here Dupuit's solution and the second one being Schaffernak's solution and third one is Casa Grande solution all actually have to have given methods which are actually for constructing the, the uppermost flow line that is the phreatic surface which actually can form and this can be also estimated from the numerical simulation which are actually inbuilt and also on the physical simulation.
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Advanced Geotechnical Engineering

Dupuit's solution for flow through an earth dam

Assumption: Hydraulic gradient $i = \text{slope of the free surface}$ and is constant with depth. $i = \frac{dz}{dx}$

$q = k \frac{dz}{dx} [z \times 1]$

$\int_0^d q dx = \int_{H_2}^{H_1} kz dz$

$q = \frac{k}{2d} (H_1^2 - H_2^2)$

Equation of a parabolic free surface

Uppermost flow line (i.e. phreatic surface ab)

- No attention for entrance and exit conditions

- If $H_2 = 0$, phreatic line would intersect the impervious base.

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So in this particular slide in this particular slide we are actually discussing, discussing going to discuss about the Dupuit-Darcy solution for flow through an earthen dam solution for flow through an attendant so here we actually have an embankment which is constructed on an impervious base and the value of the x-axis is shown here and H_1 is the head of water upstream water level and this is the head of water that is tail stream water level so AB this but this part this is an equi-potential line and again this is an equi-potential line with a head that is H_2 .

So the portion a B is the uppermost flow line or the is called as phreatic surface AB so this is the point where is a conflict point where you actually have a flow net portion a flow line portion and then equi-potential line portion which actually commences at this level so this ordinate being said here the axis is shown like this vertical axis Z and X and this is a two dimensional case so the slope which is actually indicated as DZ/DX that is nothing but hydraulic gradient.

So our assumption is that hydraulic gradient I is equal to the slope of the free surface and is constant with the depth now when we consider the parameter section if you are actually having height Z then the permeability is the seepage can be written as Q is nothing but K into DZ by DX into J into 1 now by integrating this X that is from X is equal to 0 here and to the point D that is the point where the upstream water level B meets the or where the water phreatic surface meets this point and this distance is d by integrating up to that and the Z is actually integrated with H_1 and H_2 because this is $H_1 - H_2$ is nothing but the differential water level so 0 to DQ DX is equal to H_2 to H_1 KJ DJ by simplifying that we get Q is equal to $K/2 D \cdot H_1^2 - H_2^2$.

So if you look into this, this is the equation of the parabolic surface which is actually estimated by Dupuit in the solution with assumption that the hydraulic gradient is nothing but the slope of the free surface and is constant with the depth so that is what actually assumption which has been made in the Dupuit solution for estimating the flow-through thinner than that but one thing we must observe here is that no attention for the entrance and exit conditions for the phreatic surface were considered and if h_2 is equal to 0 .

Let us say that this tail water level is at this point that is if the tail water level as is at this point the phreatic line will actually intersect the impervious space the phreatic line is actually intersecting the impervious base.

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Schaffernak's solution for flow through an earth dam

Assumption: Phreatic surface will be like line ab and will intersect at a distance l from the impervious base.

BY considering triangle obd: $i = \frac{dz}{dx} = \tan \beta$

$$q = kz \frac{dz}{dx} = k(l \sin \beta)(\tan \beta)$$

$$\int_{l \sin \beta}^H z dz = \int_{l \cos \beta}^d (l \sin \beta)(\tan \beta) dx$$

$$l = \frac{d}{\cos \beta} - \sqrt{\frac{d^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}}$$

By knowing $l \rightarrow q = k(l \sin \beta)(\tan \beta)$

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The coming to the next solution that is the Schaffernak's solution for flow through a tandem and way the periodic surface is assumed like line a B that is actually which is shown here line a B and will intersect at a distance L along the along the point which is from O to B that is the L here which is actually given from the impervious base and by considering the triangle OBD the slope which is actually given is equal to DZ /DX.

And this angle which is actually subtended by this OB with OD is β so we can write DZ/ DX is equal to $\tan \beta$ so the hydraulic gradient is considered as DZ/ DX is equal to $\tan \beta$ so we can write Q is equal to kg into DZ /DX and now that we can write now by substituting for DZ /DX is $\tan \beta$ and Z we can write in terms of $L \sin \beta$ by taking the triangle OBD that is here OB d and with that by integrating from the distance $L \sin \beta$ to $L \sin \beta$ is this vertical distance.

That is this distance to this that is H and then this, this one is that $L \cos \beta$ $L \cos \beta$ to D that is the horizontal distance from this point to this point where it meets the upstream water surface so by simplifying that we will actually get L that is the L is the length along the downstream slope surface which is nothing but L is equal to D by $\cos \beta$ minus root over d^2 by $\cos^2 \beta - H^2$ square by $\sin^2 \beta$.

Once we knew L by using the expression now here we can calculate the flow which is nothing but Q is equal to $KL \sin \beta \tan \beta$ so this in the Schaffernak's solution pre attic surface was assumed to be like in line AB and will intersect at a distance L from the in previous base so to

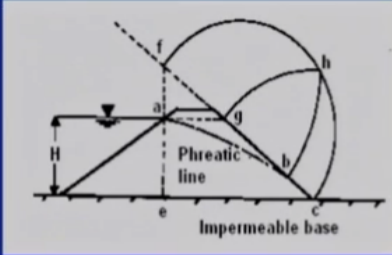
some extent what Dupitt could not consider has been improvised in this particular solution the graphical construction for determining L is also given previously.

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Graphical construction for Schaffernak's solution

1. Extend bc downwards
2. Draw a vertical line ae and will intersect projection of line bc at f .
3. With fc as diameter, draw a semi circle fhc
4. Draw a horizontal line ag
5. With c as the center and cg as radius, draw an arc gh
6. With f as center and fh as radius draw an arc hb .
7. Measure $bc = l$



Phreatic line
Impermeable base

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We have seen the analytical solution and the Schaffernak actually has given the graphical solution also so here in this the graphical solution goes like this the extender BC downwards that means that the along the slope line this line is to be extended and draw a vertical line AE and will intersect the projection line BC at EF.

So this is that point F with FC as the diameter draw a semicircle f XC and the draw a horizontal line a G that is the point where C as the center C as the center CG as the diameter as is sorry C G as the radius draw, draw an arc G H so it cannot it cuts at H so with a pheasant a pheasant centre FS centre a fetch as radius draw an arc HB so B is the point by measuring the distance graphically.

We can determine VC is equal to L so how to look determine L once we get the L then there is a possibility that we will be able to estimate again the flow through an earthen dam by using Schaffernak's solution.

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Casagrande's solution

Casagrande (1937) shown experimentally that parabola ab actually starts from the point a' . $aa' = 0.3\Delta$
(with this modification value of $d = a'c$)

Casagrande suggested: Considering the triangle bcd

$i = \frac{dz}{dx} = \sin \beta$ Instead of $i = \frac{dz}{dx}$

Using $q = KI$

$q = k(\sin \beta)(l \sin \beta) = kl \sin^2 \beta$

By solving for l we get:

$l = s - \sqrt{s^2 - \frac{H^2}{\sin^2 \beta}}$ Whereas, $s = \sqrt{d^2 + H^2}$

Once l is known, rate of seepage q can be calculated.

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s = length of the curve a'bc

Diagram labels: H , Δ , 0.3Δ , a' , a , b , c , d , s , l , β , z , x , Δ , d , s , l , β , Impermeable layer, Phreatic line, $\frac{dz}{dx}$.

Now third solution what we discussed and put forward is the and Casagrande's solution in Casagrande in 1937 as shown experimentally that the parabola AB actually starts from point A' which is that is A' is equal to 0.3 Δ so the parabola was assumed is assumed in the Casagrande solution as that parabola starts instead of at this point it actually so the entrance correction which is actually given herewith the .3 Δ and the slope of this line again DZ /DX and the DS which is shown and this length is L which is considered and this angle is β and this point is the this is the vertical axis and this is the original axis.

And this Δ is nothing but from the upstream point where it meets this distance and this distance this is Δ so it is one third of this distance was considered by Casa Grande and with the modification of this modification value of D is equal to A'C that is the value of D will be equal to A'C that is this particular distance so considering now the triangle BCD we can write I is equal to DZ/DS is equal to sin β considering BCD instead of DZ /DX so here he has been is considered DZ/DS is equal to sin β and using Q is equal to KI where flow is actually happening perpendicular to the plane and where Q is equal to K sin K sin β into that is sin β is nothing but

DZ/DS and, and then a that is area of the cross section which is nothing but the vertical distance is nothing but $L \sin \beta$.

So with that we can write Q is equal to K and square $\sin \beta$ by solving for L we get L is equal to s minus root over $S^2 - H^2$ by $\sin^2 \beta$ where s is equal to root over d^2 plus h^2 where the s is nothing but the length of the curve $A'A'BC$, $A'BC$ length of the curve is obtained nothing but nu square plus root over d^2 plus h^2 once L is known the rate of seepage q can be calculated once they L is estimated the rate of seepage can be estimated.

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Casagrande's solution (Plotting of phreatic line for seepage through earth dams)

For construction of flow nets for seepage through earth dams, the phreatic line needs to be established first:

- Curve $a'efb'c$ is a parabola with its focus at c ; the phreatic line coincides with the parabola (with some deviations at U/s and D/s faces).
- At point a , the phreatic line starts at an angle of 90°

The diagram shows a cross-section of an earth dam with a phreatic line. A parabola is drawn with its focus at point c and its vertex at point a . The phreatic line is shown as a solid line that follows the parabola closely. The diagram also shows the directrix and the coefficient of permeability k .

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Cause Casagrande's solution for the plotting of the periodic line for the seepage to endure For construction of the flow nets for C pays to attendance the periodic line needs to be established first so the curve $A'E B'C$ is a parabola with its focus at C that is the focus at C so when the parabola runs like this, this is the way it actually erupts and the phreatic line coincides with the parabola with some deviation upstream and downstream phases and this is the directory so all point at Point A, at Point A the phreatic line starts at an angle 90° .

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Casagrande's solution Procedure for constructing parabola a'e'fb'c

1) Let $CC' = p$ $AC = \sqrt{x^2 + z^2}$ $AD = 2p + x$

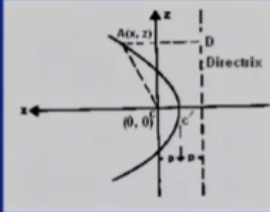
$AC = AD$ (based on the properties of parabola)

Thus, $\sqrt{x^2 + z^2} = 2p + x$ At $x = d; z = H$ $p = \frac{1}{2}(\sqrt{d^2 + H^2} - d)$

By knowing d and H , p can be calculated.

□ With p value, for values of x , various values of z can be calculated. $x = \frac{z^2 - 4p^2}{4p}$

When $\beta < 30^\circ$, l can be calculated using

$$l = \frac{d}{\cos \beta} \sqrt{\frac{d^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}}$$


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So with this Casagrande's solution the procedure for constructing parabola is given as follows so parabola here is nothing but A'E FB'C so here CC' is equal to P that is CC' is equal to P the from the distance which is equidistant from this side is again P so AC which is nothing but root over X square plus Z square is a point with coordinates X and Z AB is nothing but nothing but 2p plus X so AD distance horizontal distance is nothing but 2 P that is from the directrix it is 2 P plus X.

So AC is equal to ad based on the properties of the parabola so we can write root over X square plus root over X square plus y square is equal to 2P, 2P+ X by using the properties of parabola we can write root over X square plus X square is equal to B plus X so at egg at X, X is equal to D that is X is equal to D and H is equal to H we can write P is equal to 1 by 2 root over d square plus h square minus d by knowing D and HP can be calculated so when P with P value the value of the X various values of Z can be calculated with what by knowing the p value of the PE and with various values of Z can be calculated.

So for different values of x for values of x various very subject can be calculated when β less than 30 L can be calculated by using the expression which is L is equal to D by $\cos \beta$ minus root over d^2 by $\cos^2 \beta - \sin^2 \beta$.

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Casagrande's solution

For $\beta \geq 30^\circ$, Casagrande proposed

β	30°	60°	90°	120°	150°	180°
$\frac{\Delta l}{(l + \Delta l)}$	0.36	0.32	0.26	0.18	0.10	0

After locating, $b'b = \Delta l$; $bc = l$; curve fb can be approx. drawn by hand

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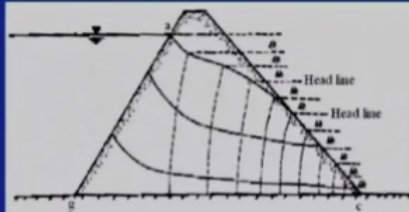
Otherwise the table which is actually given also allows to estimate the length Δ , ΔL and for β these this is for $A \beta$ less than 30° in case when the β is greater than 30° Casa Grande proposed this table when you have got $\beta 30^\circ$ this is actual .36 but when you have $60^\circ 90^\circ \Delta L / 1 + \Delta L + \Delta L$ so after locating $B' B$ is equal to ΔL BC is good here Schaffernak B of B can be approximated so this is the condition what will happen is that how the, the estimate the periodic surface which is given by from the Casagrande's solutions.

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Advanced Geotechnical Engineering

Flow net construction for an earth dam

1. Draw the phreatic line, since this is known
2. Note that ag is an EQP line and that gc is a flow line
3. Pressure head at any point on the phreatic line is zero; hence difference of total head between any two EQP lines should be equal to the difference in elevation between points where these EQP lines intersect the phreatic line.
4. $\Delta h = H/N_d$
5. Draw head lines for the c/s of the dam
6. Draw flow net
7. $q = k(H) \frac{N_f}{N_d}$



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So when we construct a flow net construction through an earthen dam basically what we need to do is that we need to construct the phreatic surface so for that three methods have been discussed one is Dupitt solution other one is Schaffernak but Casagrande's solution is widely used and note that AG is an equi-potential line and GC is a flow line and pressure head at any point on the phreatic line is 0 and the difference of the total head at any two equi-potential lines should be equal to the difference in the elevation.

Because the total head between any two equipotential lines should be equal to the pressure at any point and because it is on the atmospheric the pressure head is zero so just what we need to do is that by knowing the elevations we will be able to calculate what is the total head between any two equipotential lines so ΔH is nothing but you know in the similar way we once we construct the flow net, the upper boundary is the phreatic surface or the flow line and then construct these flow channels.

So here channel 1, channel 2, channel 3 and this is the equipotential line say 1, 2, 3, 4, 5, 6 so these are the potential drops and we can actually draw the headlines which is nothing but ΔH is nothing but H/N_d so by knowing the number of drops we can actually calculate what is ΔH because this case being very easy and we can construct the flow net and then determine Q is equal to which can be seen on page or it can be leakage and suppose discharge can be in the form of for a reservoir where K is equal to q by d and N_d .

So in this particular lecture we actually have discussed a case for an anisotropic case and when you have actually how what then we said that it has to be changed to transform the condition and when you have got say non homogeneous soils and how the flow rates deviate and all we have discussed and, and then when we have say embankment dams or unconfined seepage conditions we have discussed about how a flow net can be constructed.

So in the next lecture we will try to look into some numerical and physical simulations of these examples and then we will try to see how you know the use of filters or the chimney drains helps in you know changing the course of these periodic surfaces which are periodic surface which will be there within the turn dam or when you actually have a concrete dam with, with or without cut-offs how there is a direction of the how the flow rates will change.

And then we will also try to see the examples of how to estimate an upstream uplift pressures and then calculate the factor of safety against uplift pressure incase if there is a case of uplift then how this can be solved by using some case studies we will discuss in the next lecture of this particular permeability and seepage module.

**NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

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