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**ADVANCED GEOTECHNICAL**  
**ENGINEERING**

**Prof. B.V.S. Viswanandhan**

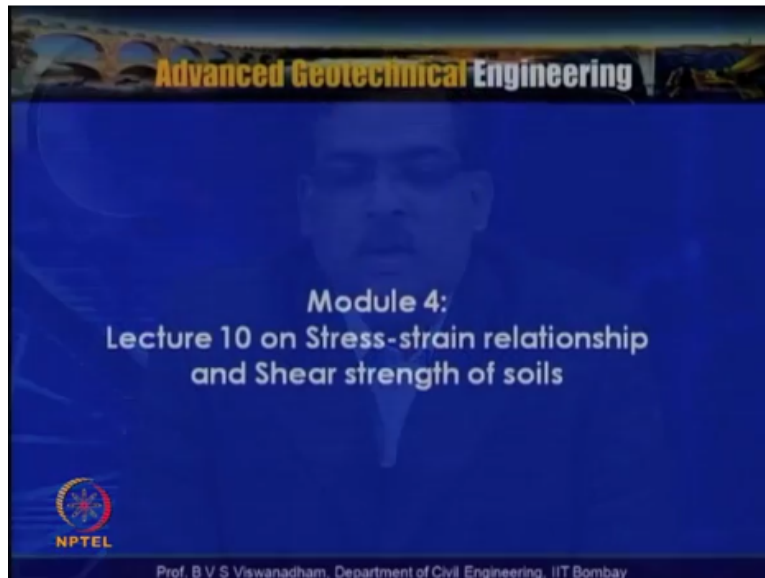
**Department of Civil Engineering**  
**IIT Bombay**

**Lecture No. 39**

**Module-4**  
**Lecture – 10 on Stress-Strain**  
**relationship and Shear**  
**strength of soils**

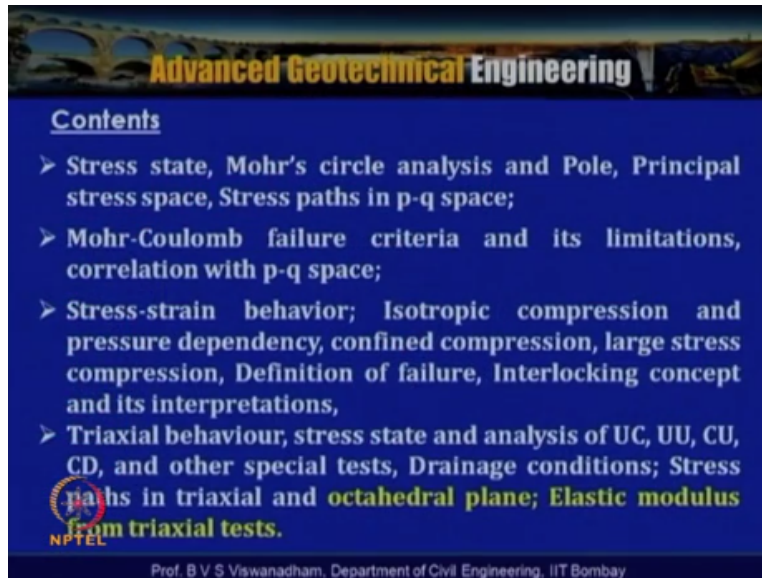
Welcome to lecture series on advanced geotechnical engineering and we are in module 4 stress strain relationship and shear strength of soils.

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And we have discussed in length about the following topics like you know introduce ourselves with the stress state and Mohr circle analysis.

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And then we have defined pole and then we introduce ourselves to principal stress space and the stress paths in p-q space then we have discussed about Mohr Coulomb failure criteria and its limitations and then different stress strain behaviors and then you know under isotropic compression cases and definitions of the failure and interlocking concept and then we introduce ourselves to triaxial behavior and stress state and particularly in difference with you know unconfined compression test, unconsolidated, undrained triaxial test, consolidated undrained triaxial test and consolidated drained triaxial test and other special test.

We are not much covered on the special test and then we also discussed about the drainage conditions. In this particular lecture we will be you know trying to concentrate on octahedral plane and octahedral you know stresses and interpretation of the elastic modulus from the triaxial test. So in the lecture, in this lecture we are going to discuss about the octahedral plane and elastic modulus from triaxial test.

We all know that you know the failure criteria's actually which are used in soil mechanics for actually deduced from advanced materials mechanics of materials.

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**Yield surfaces in three dimensions**

- Comprehensive failure conditions or yield criteria were first developed for metals, rocks, and concrete.
- Application of these yield criteria to soil and determine the yield surfaces in the principal stress space.

**Von Mises (1913) proposed a simple yield function:**

$$F = (\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2 - 2Y^2 = 0 \quad \text{--(1)}$$

where Y is the yield stress obtained in axial tension. However, the octahedral shear stress can be given by the relation:

$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$$

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Especially the comprehensive of failure conditions or yield criteria are first developed for metals, rocks and concrete. So this comprehensive failure conditions or yield criteria basically they are developed for you know metals, rocks and concrete, now let us consider the application of this yield criteria to soil and determine the yield surfaces on the principal stress space.

Von Mises 1913 proposed a simple yield function and which is given as  $F = (\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2 - 2Y^2 = 0$ , so if you if we name this equation as 1 and you know this was actually proposed by Von Mises in 1913 and you know proposed basically a simple yield function and that yield function is F is given by  $(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2 - 2Y^2 = 0$  where Y is nothing but the yield stress obtained in axial tension.

However, the octahedral shear stress can be given by the relationship which is actually given below which is  $\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$  so  $\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$ . Now what we do is that if we square this one then you know we get  $\tau_{\text{oct}}^2 = \frac{1}{3^2} = \frac{1}{9} = \frac{1}{9} [(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2]$ .

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**Yield surfaces in three dimensions**

→ By substituting in (i):  $3^2 \tau_{oct}^2 = 2Y^2$  or  $\tau_{oct} = \frac{\sqrt{2}}{3} Y$

This means that failure will take place when the octahedral shear stress reaches a constant value equal to  $\frac{\sqrt{2}}{3} Y$ .

Let us plot this on octahedral plane (where  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$ )

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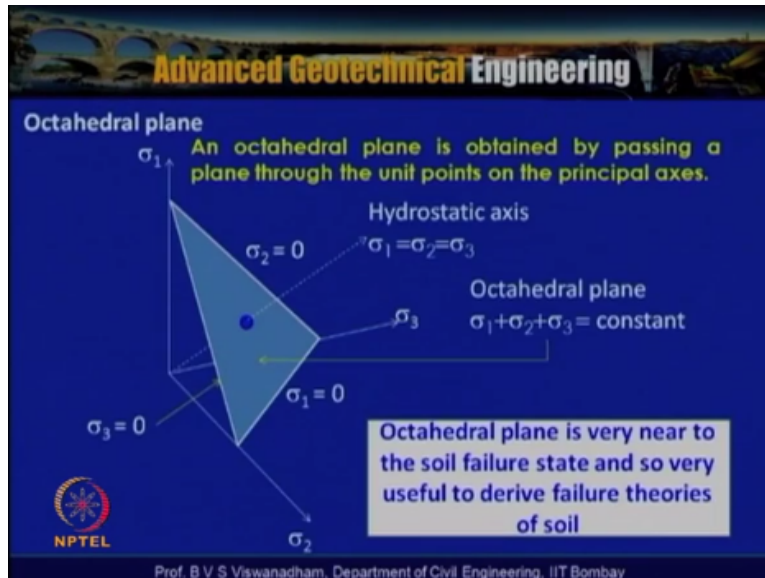
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Now if you substitute this in equation 1 what we get is that  $3^2 \tau_{oct}^2 = 2Y^2$  that means that this term will come outside and this term will become  $\tau_{oct}^2$  okay, then with this what will happen is that  $3^2 \tau_{oct}^2 = 2Y^2$  and  $\tau_{oct}$  can be given by  $\frac{\sqrt{2}}{3}$  into yield stress that is  $\tau_{oct}$  can be given by  $\frac{\sqrt{2}}{3}$  into yield stress. This means that what is the physical significance of this is that the failure will take place then the octahedral shear stress reaches a constant value which is equivalent to  $\frac{\sqrt{2}}{3} Y$ .

So where  $Y$  is the yield stress in tension, so what we have tried to do is that if you, you know when we equate when we substituted you know the  $\tau_{oct}$  in the yield function which was given by Von Mises what we have got is that  $\tau_{oct}$  in terms of  $\frac{\sqrt{2}}{3} Y$  and this indicates that the failure will take place when the octahedral shear stress reaches a constant value which is equivalent to  $\frac{\sqrt{2}}{3} Y$ .

Now what you do is let us plot this, this on the octahedral plane where octahedral plane is the plane on which the  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$ . So let us plot this on the octahedral plane where the  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$ .

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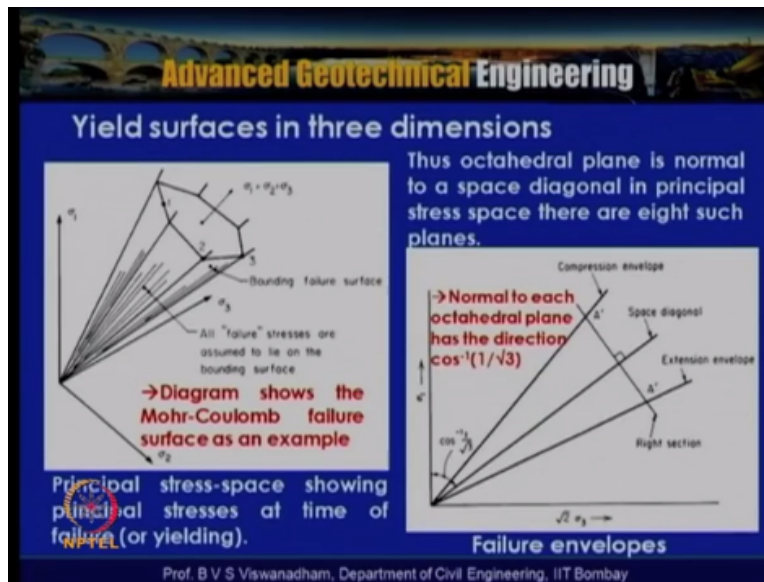
So this is the octahedral plane which is actually shown you know the octahedral plane is obtained by passing a plane through the unit points on the principal axis, so in this particular slide what we see is that  $\sigma_1$  and  $\sigma_2$  and  $\sigma_3$  and this is the hydrostatic axis where  $\sigma_1 = \sigma_2 = \sigma_3$  so this is the hydrostatic axis and the line joining this points which are actually this particular plane is called as octahedral plane.

So the octahedral plane is obtained by passing a plane through the unit points on the principal axis and the principal axis are nothing but  $\sigma_1$   $\sigma_2$   $\sigma_3$  and hydrostatic axis is nothing but  $\sigma_1 = \sigma_2 = \sigma_3$  which actually passes from the origin to the from the center point which actually adopted out which is shown in the octahedral plane here.

So octahedral plane is very near to the soil failure state so very useful basically to derive failure theories of soils, so why octahedral plane has been adopted is that octahedral plane was found to be very you know very near to the soil failure states and so you know where this is adopted to derive this failure theories of soil. So we have discussed that you know these theories actually what deduced for metals initially and then you know these are actually extended for the soils.

So the octahedral plane is very near to the soil failure state and so you know these very useful to derive the failure theories of the soil, so octahedral plane is has been adopted because it represents the you know close to the failure state in the soil and so this is actually used for failure theories derive the failure theories in the soil.

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Now here the yield surface in three dimension is shown here so here with  $\sigma_1$   $\sigma_2$   $\sigma_3$  and for example here the diagram shows the Mohr Coulomb failure surface as an you know hexagonal shape here and this is the bounded failure surface and here all failure stresses are assumed to be in the bounding surface so all failure surface, failure stresses are assumed to be on the, they are assumed to be on this boundary surface.

So this is the principal stress shaped showing principal stresses at time of failure or yielding so this is the principal stress space showing the principal stresses at the time of failure so like this you know octahedral plane is normal to a space diagonal in principal stress space and there are 8 such planes, so this octahedral plane is normal to the space diagonal so you can see that when we have this  $\sigma_1$  verses  $\sqrt{2}\sigma_3$  this is nothing but deduced from the remidlic plots.

And this is compression analog failure and analog in compression and this is the failure analog in the extension slope and this is the space diagonal through which  $\sigma_1 = \sigma_2 = \sigma_3$  and this plane which is right section perpendicular to this on this level the octahedral plane is represented and the normal to each of the octahedral plane has the direction of  $\cos^{-1}\sqrt{1/\sqrt{3}}$ , so the normal to the that is the space diagonal id inclined to the each of the you know each octahedral plane has the direction which is actually equivalent to  $\cos^{-1}1/\sqrt{3}$ .

So what we have done is that in this we have actually represented the principal stress space showing the principal stresses at the time of failure or yielding so on this you know boundary



surfaces you know all failure stresses are assumed to be lie on this surface and this is the Mohr Coulomb failure surface was shown as actually as an example.

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### Yield surfaces in three dimension

Thus any state of stress consisting of three principal stresses may be resolved into two component states of stress,

- a component consisting of equal tensile (or compressive) stresses acting in all directions, and
- a component state of stress consisting of eight octahedral shearing stresses.

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And further the any state of stress consisting of three principal stresses like  $\sigma_1$   $\sigma_2$   $\sigma_3$  may be resulted into two component states of stress and these are called octahedral stresses one is octahedral normal stress octahedral shear stress. A component consisting of equal tensile stresses you know acting on in all directions and a component state of stress consisting of 8 octahedral shear stresses.

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### Octahedral plane

Normal and shearing stresses on the octahedral plane are called 'octahedral stresses'

$$\sigma_{oct} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} = J_1 / 3$$

$$\tau_{oct} = \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}}{3}$$

$$\sigma'_{oct} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} - u$$

$$\tau'_{oct} = \tau_{oct}$$

**Total and effective octahedral shear stress will be equal**

**First invariant**  
 $J_1 = \sigma_1 + \sigma_2 + \sigma_3$

Diagram labels:  $\sigma_1$ ,  $\sigma_2 = 0$ ,  $\sigma_3$ ,  $\sigma_1 = 0$ ,  $\sigma_3 = 0$ ,  $\sigma_2$ ,  $\tau_{oct}$ ,  $\sigma_{oct}$

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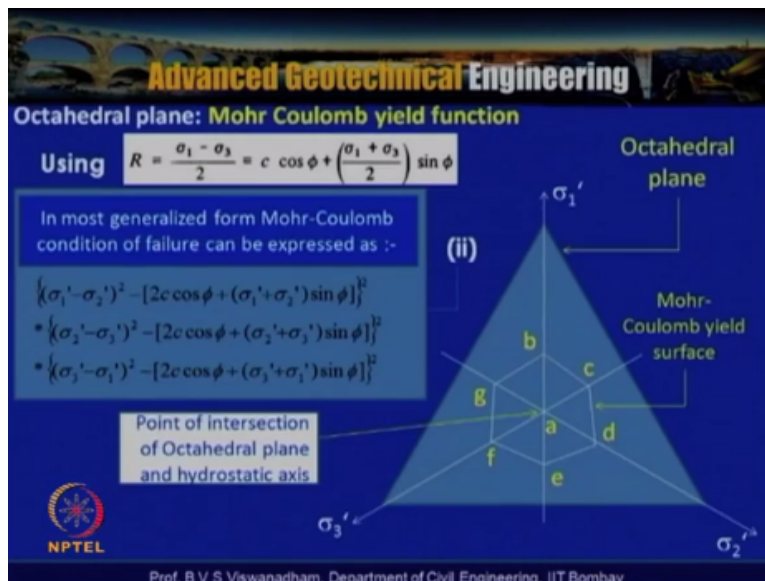


So we have you know this octahedral you know  $\sigma$  octahedral this is the normal stress and this is the shear stress,  $\sigma_{oct}$  and  $\tau_{oct}$ . So any state of stress can be represented by two components states of stress one is you know the component consisting of equal tensile stresses, tensile or compressive stresses acting in all directions and the another one is that component of shear stress consisting of 8 octahedral shearing stresses.

So the normal and shearing stresses on the octahedral plane are called as you know octahedral stresses the normal and shearing stresses on octahedral plane they are actually referred as octahedral stresses and the first invariant is indicated as  $J_1 = \sigma_1 + \sigma_2 + \sigma_3$  and  $\sigma_{oct} = \sigma_1 + \sigma_2 + \sigma_3 / 3 = J_1 / 3$  and  $\tau_{oct} = 1/3 \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ , so  $\sigma'_{oct} = \sigma_1 + \sigma_2 + \sigma_3 / 3 - u$  and  $\tau'_{oct} = \tau_{oct}$ .

So total and effective octahedral shear stress will be equal like we have got  $q' = q$  similarly here  $\tau'_{oct} = \tau_{oct}$  and  $\sigma'_{oct} = \sigma_1 + \sigma_2 + \sigma_3 / 3 - 2u$  that is  $\sigma'_{oct} = \sigma_1 + \sigma_2 + \sigma_3 / 3 - 2u$ .

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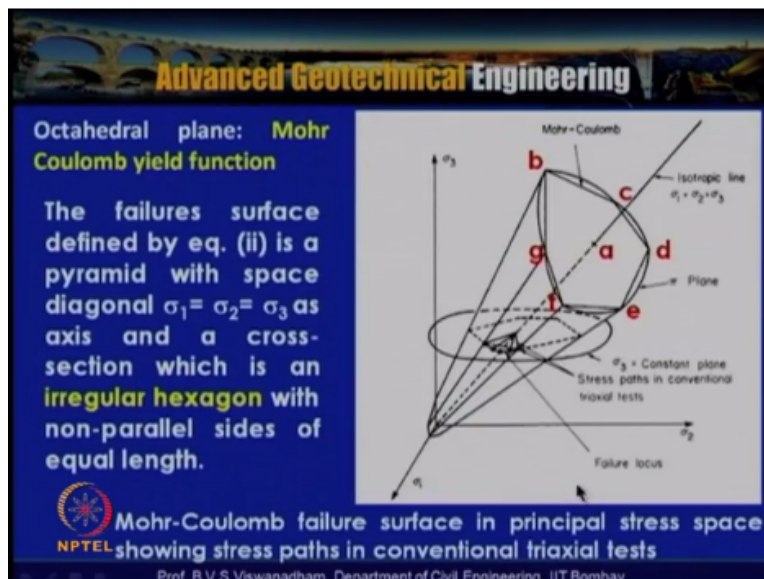
Now using from the Mohr circle if you look into the you know the failure interpretations we can write  $r = \sigma_1 - \sigma_3 / 2 = c + \cos \phi + \sigma_1 + \sigma_3 / 2 \sin \phi$ , so by using this  $\sigma_1 - \sigma_3 = 2c \cos \phi + \sigma_1 + \sigma_3 \sin \phi$ . Now this can be expressed in terms of more generalized condition more Coulomb condition of failure in more generalized form can be

expressed as  $(\sigma_1' - \sigma_2')^2 - 2c \cos\phi + (\sigma_1' + \sigma_2' \sin\phi)^2 (\sigma_2 - \sigma_3)^2 - 2c \cos\phi + (\sigma_2' + \sigma_3' \sin\phi)^2 (\sigma_3 - \sigma_1)^2 - 2c \cos\phi + (\sigma_3' + \sigma_1' \sin\phi)^2$ .

Now you know this particular you know is represented as you know Mohr Coulomb yield surface the point of intersection of the octahedral plane and the hydrostatic axis is actually indicated by a. And bcdefg is the Mohr Coulomb yield surface this is the Mohr Coulomb yield surface and this is the octahedral plane, so what we are seeing is you know when you see this in plan in the, when we have the you know octahedral plane and this is the Mohr Coulomb failure surface.

So this when it is you know super imposed here Mohr Coulomb failure surface is super imposed here we see like this, where bcdefg are the you know vertexes at the points which are shown and a is the point if intersection of the octahedral in hydrostatic axis. So this is actually called as Mohr Coulomb yield surface and this is octahedral plane.

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Now the failure surface is defined by equation 2 that is basically this one is a parameter, basically is a pyramid with space diagonal which is  $\sigma_1 = \sigma_2 = \sigma_3$  as axis that is the isotropic line or hydrostatic axis which is actually called as  $\sigma_1 = \sigma_2 = \sigma_3$  as axis and a cross section which is an irregular hexagon with non parallel sides of equal length.

So this is actually the cross section is basically the equation which is the more general form of in Mohr Coulomb condition which this equation 2 which is you know represents a pyramid with the space diagonal  $\sigma_1 = \sigma_2 = \sigma_3$  as axis and a cross section which is an irregular hexagon with non parallel sides having equal length. So this is the you know this is the  $\pi$  plane what is called this is  $\pi$  plane and this is the  $\sigma_3 = \text{constant}$  plane and the stress point, stress paths in the conventional triaxial test are represented here.

Stress path in conventional triaxial test are represented here, and this is the failure lockers and this is the  $\pi$  plane and this is the Mohr Coulomb failure surface which is actually shown here. So this is basically the more general form of Mohr Coulomb condition, Mohr Coulomb failure condition represents a pyramid with space diagonal  $\sigma_1 = \sigma_2 = \sigma_3$  as axis and a cross section which is an irregular hexagon with non parallel sides of equal length.

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**Octahedral plane: Mohr Coulomb yield function**

- > The projection of this irregular hexagon on the plane  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$  (i.e. plane at right angles to the space diagonal or an octahedral plane)
- > When the yield surface defined by Eq. (ii) is plotted on the octahedral plane, it will appear as an irregular hexagon in section with nonparallel sides of equal length.
- > Point **a** is the point of intersection of the hydrostatic axis with the octahedral plane.
- > Thus the yield surface will be a hexagonal cylinder coaxial with the isotropic stress line.

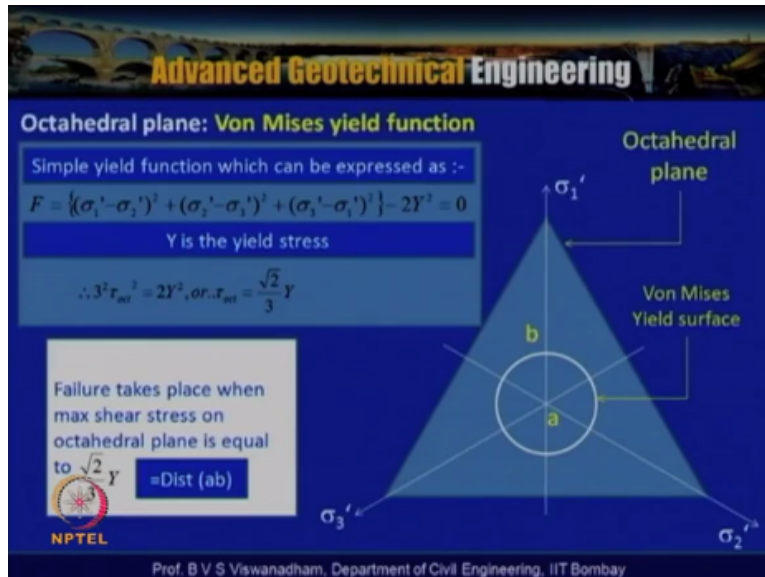
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The projection of this irregular hexagon on the plane  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$  that is the plane right angles to the space diagonal or an octahedral plane. The projection of this irregular hexagon on the plane  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$  that is on the plane at right angle to this space diagonal or an octahedral plane. When yield surface defined by equation 2 is plotted on the octahedral plane it will appear as an irregular hexagon in section with non parallel sides of equal length that we have discussed.

But point a is the, is the point of intersection of the hydrostatic axis with the octahedral plane where point a is the point of intersection of the hydrostatic axis with the octahedral plane. Thus the yield surface will be hexagonal cylinder coaxial with the isotropic stress line, so because of this the yield surface will be hexagonal cylinder coaxial with the, so it is like an hexagonal cylinder coaxial with the isotropic stress line.

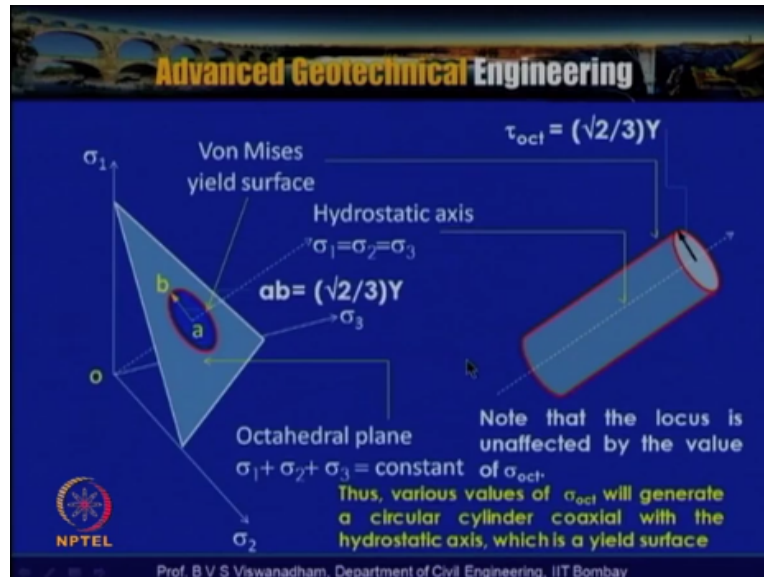
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So the isotropic stress line passes through the center of the hexagon where you know that is it is coaxial with the isotropic stress. See octahedral plane which is the one which is actually we Von Mises according to Von Mises yield function if you look into it and this yield function when we define this one when this simple yield function which can be expressed as  $F = (\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2 - 2Y^2 = 0$  as we have done here so this circle which is actually indicates the Von Mises yield surface and the radius is nothing but the  $\tau_{oct}$  that is nothing but  $\frac{\sqrt{2}}{3} Y$  and Y is the yield stress.

So the Von Mises you know yield stresses is something like a cylinder circular cylinder having you know diameter which is equivalent to  $\frac{\sqrt{2}}{3} \sigma Y$  or yield stress. So failure takes place when maximum shear stress and octahedral plane is equal to when maximum stress on the maximum shear stress on octahedral plane is equal to you know this  $\frac{\sqrt{2}}{3} Y$ , so the distance ab that is nothing but the distance ab that radiant distance ab.

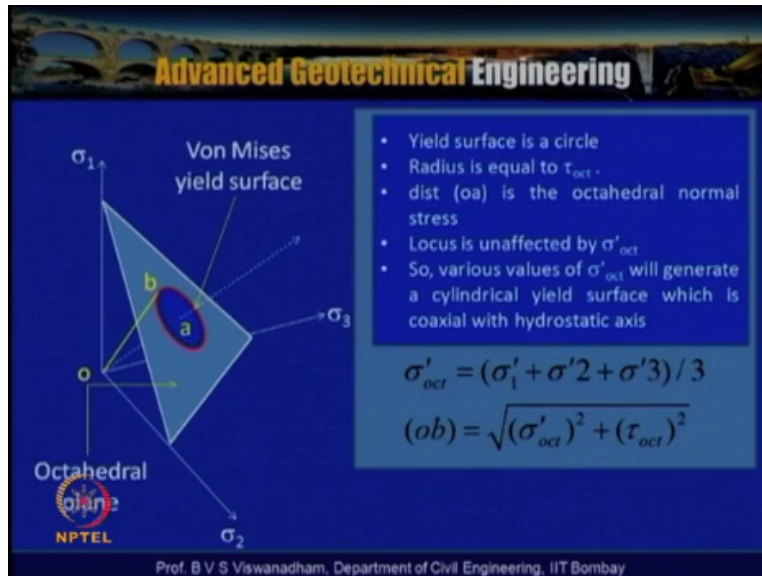
So according to now Von Mises failure surface is actually represented on the octahedral plane and from the earlier discussion whatever we have we actually have determined that  $\tau_{oct} = \sqrt{2/3}y$  this is nothing but this radius of this Von Mises yield surface.  
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And this is represented further in depth here as this is the octahedral plane and this is the Von Mises yield surface and  $ab$  is nothing but  $\sqrt{2/3}Y$  and this is the hydrostatic axis  $\sigma_1 = \sigma_2 = \sigma_3$  and so this Von Mises failure surface is you know you can see like a circular cylinder having with coaxial with you know hydrostatic axis this cylinder is actually the diameter is the radius is equivalent to  $ab$  here, where  $ab = \tau_{oct} = \sqrt{2/3}Y$ .

And note that the locus is unaffected by the value of the  $\sigma_{oct}$ , so that means that various values of  $\sigma_{oct}$  will generate circular cylinders coaxial with the hydrostatic axis which is a yield surface. So we can see that the locus is not getting effected by the values of  $\sigma_{oct}$  that means that the values of  $\sigma_{oct}$  will generate a circular cylinder coaxial with the hydrostatic axis which is a yield surface. So various values of  $\sigma_{oct}$  will generate a circular you know cylinder coaxial with the hydrostatic axis.

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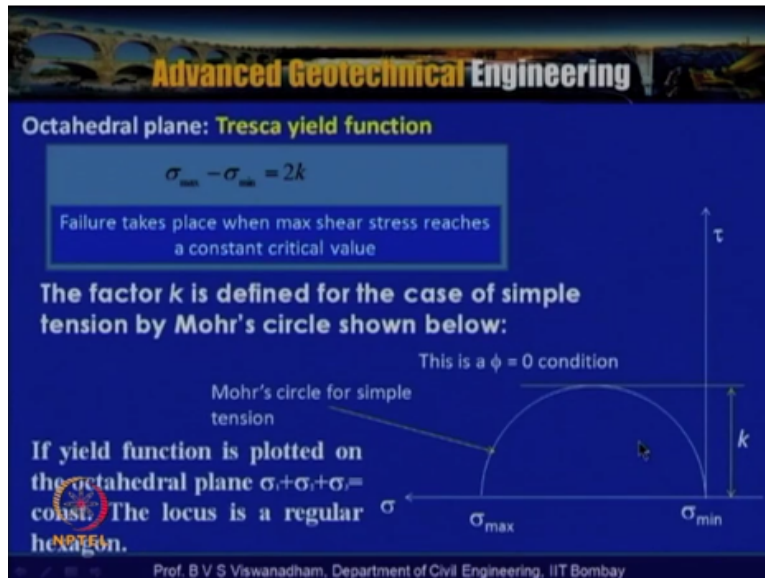


The discussion which we continue further yield surface is a circle Von Mises yield surface is a circle and radius is equivalent to  $\tau_{oct}$  which is nothing but  $\sqrt{2/3}\sigma_Y$  and distance OA is the octahedral normal stress OA is the octahedral normal stress and locus is actually not affected by the values of  $\sigma'_{oct}$  then you have you know the different values we actually have different cylinders coaxial with the you know the hydrostatic axis.

So where as values of  $\sigma'_{oct}$  will generate a circular yield surface which is coaxial with hydrostatic axis and  $\sigma'_{oct} = (\sigma'_1 + \sigma'_2 + \sigma'_3) / 3$  and  $OB = \sqrt{(\sigma'_{oct})^2 + (\tau_{oct})^2}$ . So the OB is nothing but which is distance which is shown here the division which is shown here which is nothing but  $\sigma'_{oct}^2$  that is  $OA^2 + AB^2$ ,  $AB^2$  is nothing but this radius which is nothing but  $\sqrt{2/3}Y$ , so this is OB is given by this  $\sqrt{\sigma'_{oct}^2 + \tau_{oct}^2}$ .

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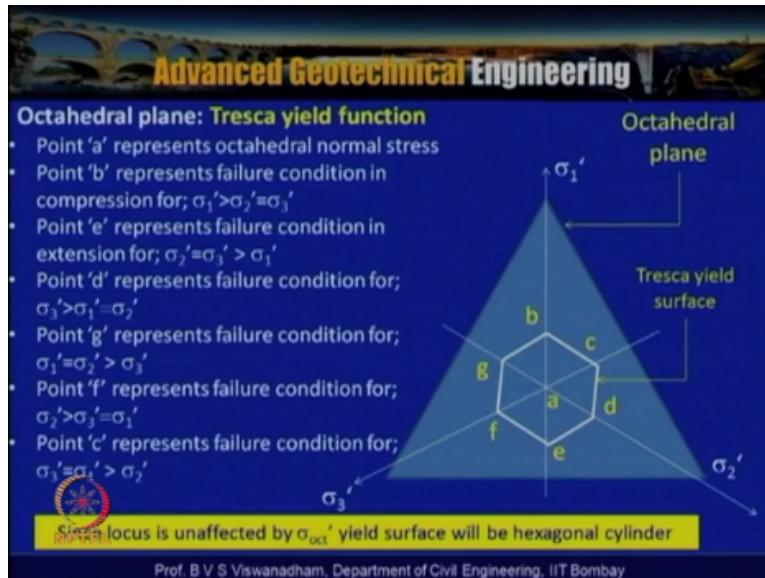
Now the octahedral plane is also given on stress presented by Tresca on the Tresca yield function so which is nothing but the Tresca criterion or Tresca function is defined as  $\sigma_{\max} - \sigma_{\min} = 2k$ , where factor  $k$  is defined by the case of a simple tension by Mohr circle actually shown in the slide. So this indicates that failure takes place when the difference that is max shear stress reaches a constant critical value.

When  $\sigma_{\max}$  and  $\sigma_{\min}$  you know that is the max shear stress reaches say a constant value which is nothing but  $\frac{\sigma_1 - \sigma_{\max} - \sigma_{\min}}{2}$  then Richard says constant critical value. So this constant critical value that factor  $k$  is defined by the for the case of a simple tension which is shown here for a simple tension for unconfined tension which is actually shown here, where  $\sigma_{\min}$  that is  $k$  where  $k =$  this factor.

So if yield function is plotted on the octahedral plane  $\sigma_1 + \sigma_2 + \sigma_3 = \text{constant}$  the locus will be a regular hexagon, this equation actually represents the regular hexagon equation.

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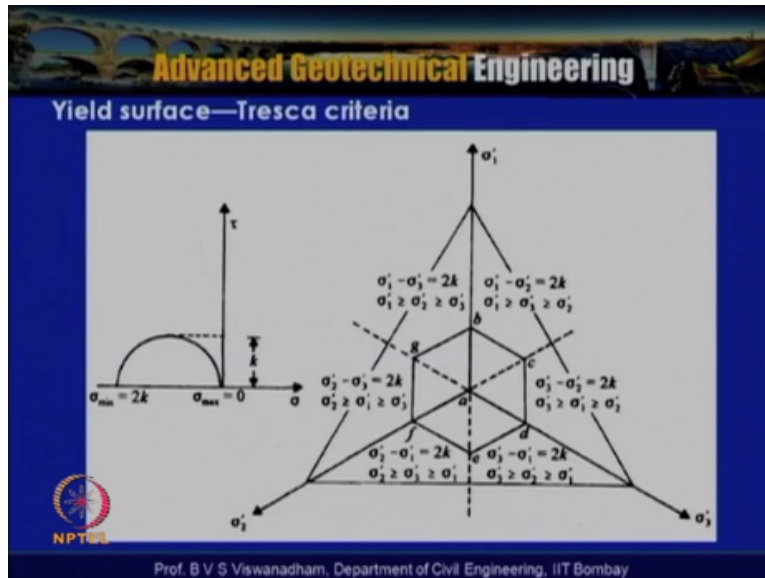


So the tresca yield surface on the octahedral plane is actually represented where bcdefg these shown here which is where similar to Mohr Coulomb failure surface, but a is the you know the point through which the hydrostatic axis or you know where  $\sigma_1 = \sigma_2 = \sigma_3$  is ensured and where bcdef and g so a represents the octahedral normal stress the point passing through the octahedral normal stress through a the octahedral normal stress passes and b represents the failure condition in compression.

b represents the failure condition in compression where  $\sigma_1$  value is greater than  $\sigma_2$  that is  $\sigma_1$  that is axial stress is more than  $\sigma_2$ ,  $\sigma_2 = \sigma_3$ . So this point b represents the failure condition in the compression and similarly point e represents failure conditions in extension where  $\sigma_2 = \sigma_3$  and  $\sigma$  greater than  $\sigma_1$ , and point d represents failure condition for  $\sigma_3$  greater than  $\sigma_1 = \sigma_2$  and point g represents failure condition where  $\sigma_1 = \sigma_2$  and  $\sigma_2 > \sigma_3$ .

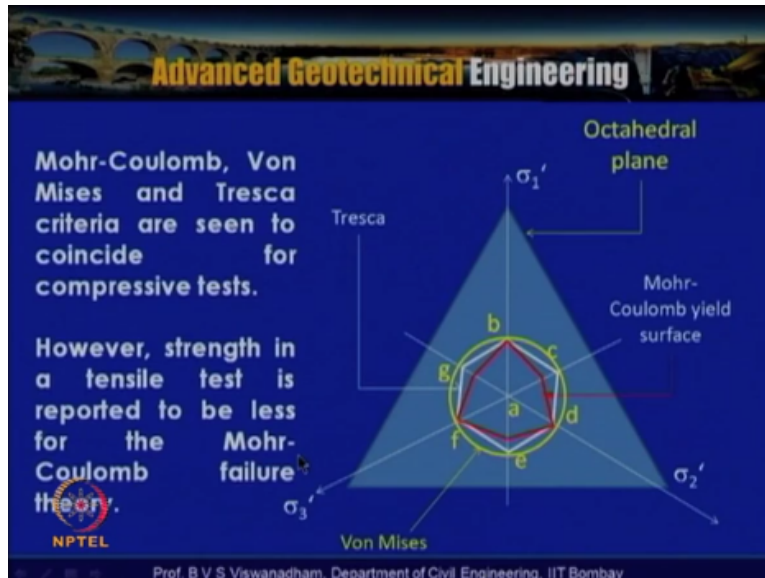
And point f represents failure condition for  $\sigma_2 > \sigma_3 = \sigma_1$  and point c represents failure condition for  $\sigma_3 = \sigma_1$  and where  $\sigma_1$  is actually greater than  $\sigma_2$ . So we have on this tresca yield criterion which is also represented as a hexagon on the octahedral plane and since locus is unaffected by the  $\sigma'_{oct}$  yield surface there will be hexagonal cylinder and here also we have for different values of  $\sigma_{oct}$  the different you know coaxial to the hydrostatic axis we have got number of hexagonal cylinders are possible.

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So the yield surface and Tresca criteria is actually shown here in this zone where  $\sigma_1 - \sigma_2 = 2k$  where  $\sigma_1' > \sigma_3' \geq \sigma_2'$  in this zone  $\sigma_3 - \sigma_2 = 2k$  where  $\sigma_3' \geq \sigma_1'$  greater than equal to  $\sigma_2'$  in this zone  $\sigma_3' - \sigma_1' = 2k$  and  $\sigma_3'$  is  $\geq$  or  $\sigma_2' \geq \sigma_1'$  and similarly here in this zone that is the zone between in this point and then this  $\sigma_2'$  axis  $\sigma_2' = \sigma_2' - \sigma_1' = 2k$  where  $\sigma_2' \geq \sigma_3' \geq \sigma_1'$ . And similarly what we have is that here and here where  $\sigma_1' - \sigma_3' = 2k$ , where  $\sigma_1' \geq \sigma_2' \geq \sigma_3'$ .

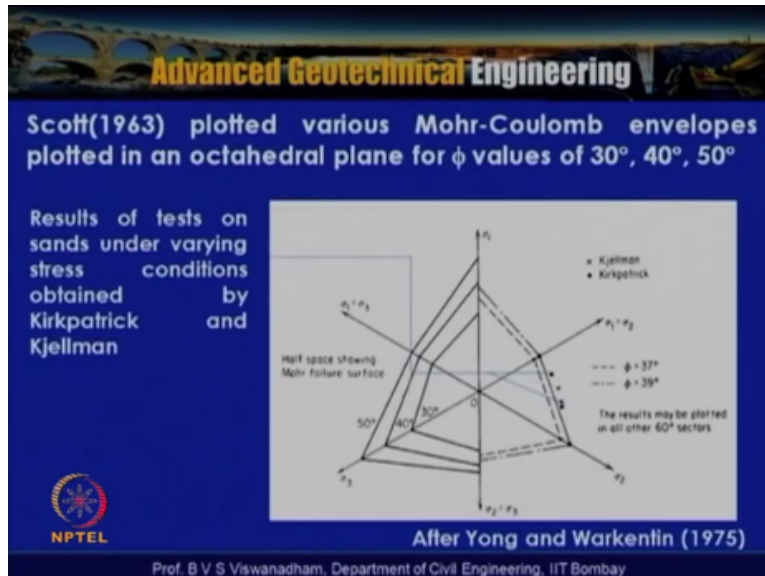
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The Mohr Coulomb Von Mises and Tresca criteria are actually seen to coincide for the compressive test, so the different failure surface are actually shown here and basically here we have the octahedral plane and the tresca surface which is actually shown here and Mohr Coulomb failure surface is actually shown here and the round surface that is actually this is the tresca and this is the Von Mises failure surface which is actually cylinder circular cylinder this is the circular cylinder.

And so if you look into this Mohr Coulomb Von Mises, Mohr Coulomb failure surface yield surface and Von Mises yield surface and tresca yield surface from the tresca criteria they seen to coincide for the compressive test. However, the strength in the tensile test is report to be less for the Mohr Coulomb failure theory, so the strength in the tensile test is report to be less for the Mohr Coulomb failure theory.

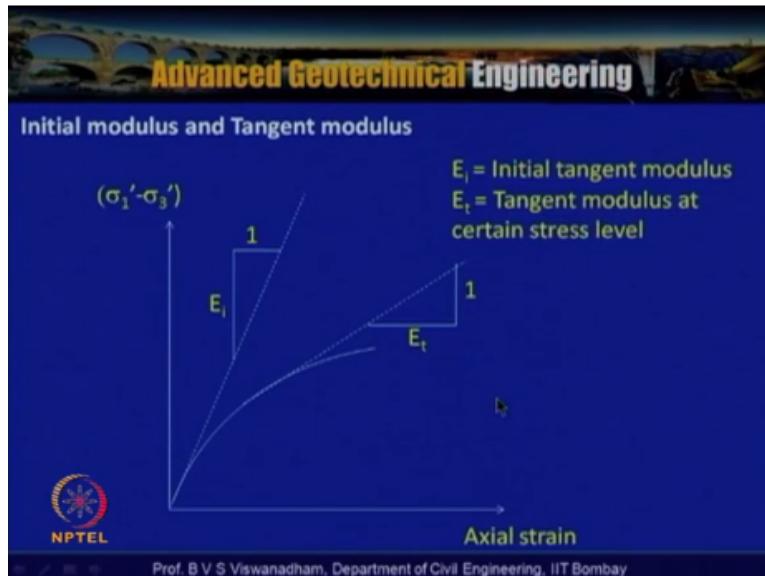
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Here in this particular slide the Scott 1963 plotted various you know Mohr Coulomb envelopes plot in octahedral plane for values of  $\phi$  values of 30 to 40 and 50, so here the Scott 1963 after Yong and Warkentin 1975 you know the various values of Mohr Coulomb analog where been plotted for different values of friction angles for 30, 40 and 50 and these were the based on the test on sand the results of the test on sand for varying normal, varying stress conditions obtained by Kirkpatrick and Kjellman also plotted here.

And these points are actually were reported by Kirkpatrick and Kjellman and as can we noted here the Mohr Coulomb failure analogous plotted by Scott and the measured values which are actually plotted by the or obtained by Kirkpatrick and Kjellman are found to be in good agreement for the values which are actually shown like  $\phi=37^\circ$  and  $39^\circ$  and this is for the  $\phi=37^\circ$  and then this is for the  $\phi=39^\circ$ .

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So after having discussed about the octahedral plane then we have actually have discussed about you know the result of the you know these values of the triaxial test or diaxial test so in case of triaxial test in case of directional test we actually get shear stress verses shear strain variation for different normal stresses so there also we can actually get the initial modulus and tangent modulus then there can be possibility that we can also get a see ken modulus interpretation we can actually obtain from the test data.

So in the initial modulus which is actually drawn for the initial portion of the curve where the soil stiffness is high and then the tangent modulus is actually drawn which is actually portion where which is actually shown here which actually represents you know this particular value here  $E_t$  in the horizontal 1 vertical, so at certain stress level we actually draw and then make this the initial tangent modulus.

The initial tangent modulus is nothing but for the initial portion where when the tangent is actually drawn that is actually shown as initial tangent modulus and let us say that if we are actually drawing let us say at a point here and then a line which is actually joining this point and this and that it actually, let us say that we are having a strain value of 2% and 2% strain or 50% of axial strain, then we can actually get the  $E_{50}$  with the value the slope of that line joining you know the strain of at which is meeting at that particular deviated stress or particular normal stress for the shear stress we can actually get the you know the Young's modulus values. And so with this you know in this particular slide which is actually shown the initial tangent modulus and

tangent modulus at certain stress level computation and in addition to that there is also a secant modulus which can actually be interpreted.

In case of triaxial test when we are actually have got unconfined compression test or unconsolidated undrained test we actually get for you know the based on the, in case of unconfined compression test with  $\sigma_3=0$  we get  $\sigma_1$  verses  $\epsilon$  so from there we can actually interpret to some extent what is the initial tangent modulus and secant modulus you know up to a certain stress level.

So the second modulus which is actually defined as that the modulus which is actually referred up to that particular stress level the slope of that line is actually valid. So it is very important for determining this you know this initial the soil stiffness correctly. Suppose if you are actually trying to determine these you know the soil stresses the stiffnesses in the initial portion then in the prevalence stresses in like as we have discussed in one of the modulus the physical model stress when the soil stresses are very low.

And if and then if you are actually having the you know with, we are dealing with the higher stresses higher stiffness values then resulting you know strains or stiffnesses will be very, very low. But in reality when we actually subjected to the real stress conditions the soil stiffness is low so the soil settlements will be very, very high. So in this particular slide we are actually trying to discuss about the interpretation of the initial modulus and tangent modulus from the triaxial test data.

A typical triaxial stress data for a given  $\sigma_3$  is actually shown here where  $\sigma_1 - \sigma_3$  is the deviated stress and  $\epsilon_1$  is the axial strain. Then we also have some empirical equations where  $E_i$  initial modulus which is also given as.

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Advanced Geotechnical Engineering

Janbu (1963) empirical equation

$$E_i = K p_a \left\{ \frac{\sigma_3'}{p_a} \right\}^n$$

Where,

- $\sigma_3'$  = minor effective principal stress
- $p_a$  = Atmospheric pressure
- $K$  = Modulus number
- $n$  = Exponent determining the rate of variation of  $E_i$  with  $\sigma_3'$

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$K p_a (\sigma_3' / p_a)^n$  where  $\sigma_3'$  = minor effective principal stress and  $p_a$  is atmospheric pressure and  $K$  modulus number and the  $n$  = exponent determining the rate of variation of you know  $E_i$  with  $\sigma_3'$ . So  $n$  basically indicates that the exponent which actually determines the rate of variation of  $E_i$  with  $\sigma_3'$ .

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**Advanced Geotechnical Engineering**

Janbu (1963) empirical equation

$$E_i = K p_o \left\{ \frac{\sigma_3'}{p_o} \right\}^n$$

The value of 'K' and 'n' for a particular soil can be found by number of triaxial testing and plotting  $E_i$  vs  $\sigma_3'$  on log-log scale

Ranges,  $K = 300$  to  $2000$  and,  $n = 0.3$  to  $0.6$

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And then  $E_i$  as we have given the values of  $K$  and  $n$  for a particular soil can be found by number of triaxial testing and plotting  $E_i$  versus  $\sigma_3'$  on the logarithmic scale and the ranges  $K=300$  to  $2000$  and  $n=0.3$  to  $0.6$  so the value of the  $K$  ranges from  $300$  to  $2000$  and  $n=0.3$  to  $0.6$ .

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Duncan and Chang (1970)

$$E_t = \frac{\partial(\sigma_1' - \sigma_3')}{\partial \varepsilon}$$

Duncan and Chang shown that,

$$E_t = \left[ 1 - \frac{R_f(1 - \sin \phi)(\sigma_1' - \sigma_3')}{2c \cos \phi + 2\sigma_3' \sin \phi} \right]^2 K P_a \left\{ \frac{\sigma_3'}{P_a} \right\}^n$$

Where,  
 $R_f$  = Failure ratio, (range 0.75 to 1)

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And according to Duncan and Chang 1970 where  $E_t$  that is nothing but  $\frac{\partial(\sigma_1 - \sigma_3)}{\partial \varepsilon}$ , so Duncan actually have shown that the  $E$  value that is  $E_t = \left[ 1 - \frac{R_f(1 - \sin \phi)(\sigma_1' - \sigma_3')}{2c \cos \phi + 2\sigma_3' \sin \phi} \right]^2 K P_a \left( \frac{\sigma_3'}{P_a} \right)^n$ .

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Janbu (1963) empirical equation

$$E_s = K p_s \left\{ \frac{\sigma_3'}{p_s} \right\}^n$$

The value of 'K' and 'n' for a particular soil can be found by number of triaxial testing and plotting  $E_s$  vs  $\sigma_3'$  on log-log scale

Ranges,  $K = 300$  to  $2000$  and,  $n = 0.3$  to  $0.6$

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So if you look into this the Duncan and Chang actually modified the Janbu 1963 empirical equation, where in to this empirical equation the Duncan and Chang actually have added this particular term which is  $[1-R_f(1-\sin\phi)(\sigma_1'-\sigma_3')]^2 [1-R_f(1-\sin\phi)(\sigma_1'-\sigma_3')/2c \cos\phi+2\sigma_3'\sin\phi]^2$  when where  $R_f$  is nothing but the failure ratio then generally the ratio is equal to 0.75 to 1. So the Duncan and Chang is actually nothing but the modification of Janbu's 1963 empirical equation.

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**Advanced Geotechnical Eng**

Poisson's ratio ( $\mu$ ):

$$v = \frac{\Delta \epsilon_v - \Delta \epsilon_s}{2 \Delta \epsilon_s}$$

Where,

- $\Delta \epsilon_s$  = Increase in axial strain,
- $\Delta \epsilon_v$  = Increase in volumetric strain =  $\Delta \epsilon_s + 2 \Delta \epsilon_r$ ,
- $\Delta \epsilon_r$  = Lateral strain

$$v = \frac{\Delta \epsilon_s - (\Delta \epsilon_s + 2 \Delta \epsilon_r)}{2 \Delta \epsilon_s} = -\frac{\Delta \epsilon_r}{\Delta \epsilon_s}$$


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So for that the Poisson's ratio can be actually obtained by  $v = \frac{\Delta \epsilon_v - \Delta \epsilon_s}{2 \Delta \epsilon_s}$  that is  $-\frac{\Delta \epsilon_r}{\Delta \epsilon_s}$  and where  $\Delta \epsilon_s$ ,  $\Delta \epsilon_v$  axial is nothing but increase in the axial strain and  $\Delta \epsilon_v$  is nothing but increase in the volumetric strain which is nothing but  $\Delta \epsilon_s + 2 \Delta \epsilon_r$  for the axis symmetric triaxial test and  $\Delta \epsilon_r$  is nothing but the lateral strain, so with this when you substitute this we get  $v = \frac{\Delta \epsilon_s - (\Delta \epsilon_s + 2 \Delta \epsilon_r)}{2 \Delta \epsilon_s}$  or the symbol which is actually shown here which is equal to  $-\frac{\Delta \epsilon_r}{\Delta \epsilon_s}$  and  $-\frac{\Delta \epsilon_r}{\Delta \epsilon_s}$ .

So this can be also determined by measuring or by pasting strain gauges where you are actually having you know unconfined compression test and with that we will be able to get the Poisson's ratio of a soil particularly with the ratio of  $\Delta \epsilon_r$  to  $\Delta \epsilon_s$ , and so the  $v = \frac{\Delta \epsilon_s - \Delta \epsilon_v}{2 \Delta \epsilon_s}$  where the  $\Delta \epsilon_v = \Delta \epsilon_s + 2 \Delta \epsilon_r$  but substituting this what we have got is that you know this one  $-\frac{\Delta \epsilon_r}{\Delta \epsilon_s}$ .

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Advanced Geotechnical Engineering

Typical values of Young's modulus of granular material (in MPa)

USCS	Description	Loose		
		Loose	Medium	Dense
GW, SW	Gravels/Sand well-graded	30-80	80-160	160-320
SP	Sand, uniform	10-30	30-50	50-80
GM, SM	Sand/Gravel silty	7-12	12-20	20-30

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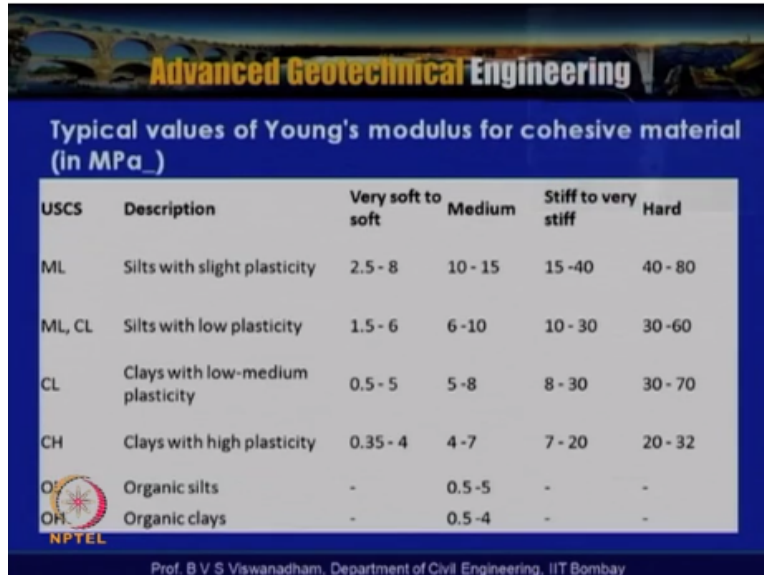
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And they are the typical values of Young's modulus for granular material particularly here we have you know if the use unified a soil classification system according to that if you have a granular materials which the Young's modulus which are actually shown for Mega Pascal's the values are shown in all these values are actually reported in MPa where GW and SW that is well graded gravel and well graded sand, gravels and sand well graded when the loose state they actually have got 30 to 80 MPa and medium and then say in dense state you can see that the E values of the higher side that is 160 to 320 MPa.

When you have you know the sand which is uniform that is called poorly graded sand where you actually have got all uniform size particles then the loose state we actually have got only 10 to 30 MPa, and in the dense state we actually have got 50 to 80 you know MPa. Similarly we have you know slity soil gravel and slity gravel when we are actually have got GM and SM type of soils we can see that the Young's modulus values so typically arranged from 7 to 12 MPa and the dense state or you know which is actually represented as 20 to 30 MPa.

So you can see that depending up on the groups the types even in the case of granular materials you know the ranges of the you know in the different stress states the loose medium and dense configuration so is a function of density or the packing of the particles and with that we can also you can see that you know the, how the values particularly the stiffness soils, stiffness are Young's modulus value changes with the soil type particularly we have got well graded gravel or poorly graded sand or sand and gravel in slity nature or sand gravel in slit ion nature.

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The slide is titled "Advanced Geotechnical Engineering" and "Typical values of Young's modulus for cohesive material (In MPa)". It contains a table with 6 columns: USCS, Description, Very soft to soft, Medium, Stiff to very stiff, and Hard. The rows list soil types: ML (Silts with slight plasticity), ML, CL (Silts with low plasticity), CL (Clays with low-medium plasticity), CH (Clays with high plasticity), OL (Organic silts), and OH (Organic clays). The Young's modulus values are given in MPa for each soil type across the five consistency states.

USCS	Description	Very soft to soft	Medium	Stiff to very stiff	Hard
ML	Silts with slight plasticity	2.5 - 8	10 - 15	15 - 40	40 - 80
ML, CL	Silts with low plasticity	1.5 - 6	6 - 10	10 - 30	30 - 60
CL	Clays with low-medium plasticity	0.5 - 5	5 - 8	8 - 30	30 - 70
CH	Clays with high plasticity	0.35 - 4	4 - 7	7 - 20	20 - 32
OL	Organic silts	-	0.5 - 5	-	-
OH	Organic clays	-	0.5 - 4	-	-

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Now let us consider for the cohesive materials particularly well graded soils where you have got a silts with low plasticity so we have got low plastic silts that is ML type of soils in this the consistency is actually represented as very soft to medium and stiff to very stiff to very hard so in this case the E values range from 2.5 to 8MPa to 40 to 80 MPa, and similarly we have the silts with low plasticity they vary from 1.5 to 6MPa to 30 to 60 MPa.

And CL that is clay with low medium clays with low medium plasticity they can actually have in a very, very soft state the E value can be as do as 0.5MPa and in hot state the CL deposit soil can actually have 30 to 70 MPa, and this CH that clay with high plasticity CH type of soils can have in very soft state very low you know the Young's modulus values and the deposit ratios for this type of soils under saturated conditions are definitely can range from 0.45 to 0.5 and the clays with high plasticity CH will have the very soft state will actually has got 0.35 to 4MPa.

To in the hot state it can actually have as high as 20 to 30 MPa and organic silts OL which is actually medium consistency have 0.5 to 5 very low organic clays also have actually got very low you know the very low E values even under the medium consistency so in these two slides what actually have seen the distinct difference actually what we have for the different soils where you have got you know the values which are actually for gravel soils and very high values are shown for depending up on the dense condition or loose conditions.

Where in case of fine grain soils are cohesive soils where actually have got low values when they are in the very soft to soft state and the values are on the high state for higher order for the, I mean the same soils particularly in the hard state. So in this particular lecture we try to understand about you know the octahedral plane and octahedral shear stresses and based on that three failure criteria's namely Von Mises and Tresca and Mohr Coulomb failure surface.

So the Mohr Coulomb failure surface we have seen as a hexagon and then on the you know these because it actually has got, it has got the capability of having different for different  $\theta$  angles we are actually have got the different hexagonal cylinders are possible. But in case of the Tresca it is also indicated as you know the hexagonal cylinder so but we have seen for as far as the compressive the soil in compression is concerned.

That both you know all the three Tresca and Von Mises and you know Mohr Coulomb criterion where found to coincide. But as far as tension is concerned the value which is actually predicted from the Mohr Coulomb criteria was found to be on the lower side. So in this particular module we have try to understand about the stress strain relationship for the soil and then we try to discuss about you know different stress paths particularly we have discussed about MIT based stress paths and as well as the Cambridge based stress paths.

And we have referenced the stress paths with reference to unconfined compression test and unconsolidated, undrained triaxial test and consolidated undrained and consolidated drain triaxial test and then in the case of consolidated undrained triaxial test during the shear we do not allow the pore water pressure to dissipate so because of that there can be a possibility that you will be able to measure the pore water pressure.

So in that case when we have you know normally consolidated soil or a loose sand then there is a possibility that the entire pore water pressure is actually positive and the sample undergoes you know the volumetric compression. When we have got very dense sand or a stiff clay or highly over consolidated clay then there is a possibility that initially to undergoes compression and there after with increase in the axial strain.

There is a possibility that the soil undergoes a desiccation where in the riding of the soil particles on each other actually happens and because of this increase in volume up on the strain there is you know a phenomenon which is actually called the negative dilation from which actual results in the negative pore water pressures.



And the relevant stress pass actually what discuss then there after we connected ourselves to the you know the failure criteria particularly with the principal stress space with Mohr Coulomb failure criterion and Tresca and Von Mises we have discussed and then finally in this lecture we discussed about how to interpret you know elastic modulus from the triaxial test data.

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