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**ADVANCED GEOTECHNICAL
ENGINEERING**

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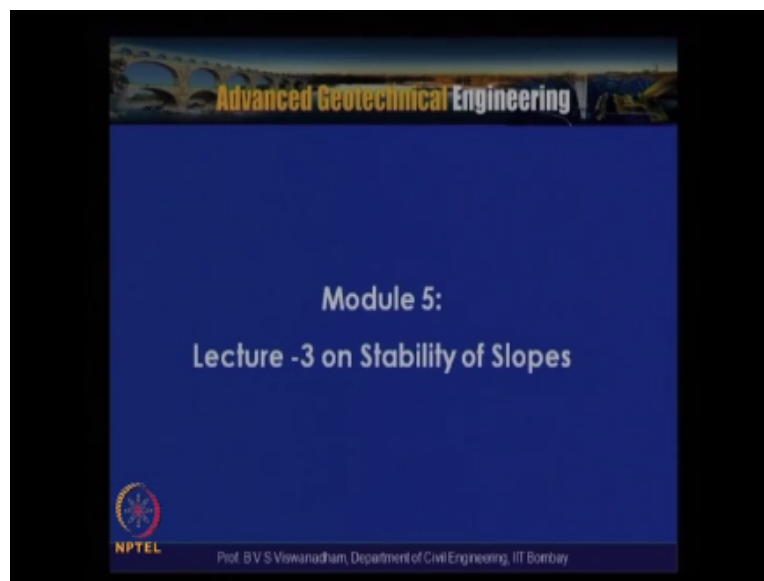
Lecture No. 42

Module – 5

Lecture – 3 on Stability of Slopes

Welcome to course on advanced geotechnical engineering we are discussing about module 5 that is the slope stability analysis.

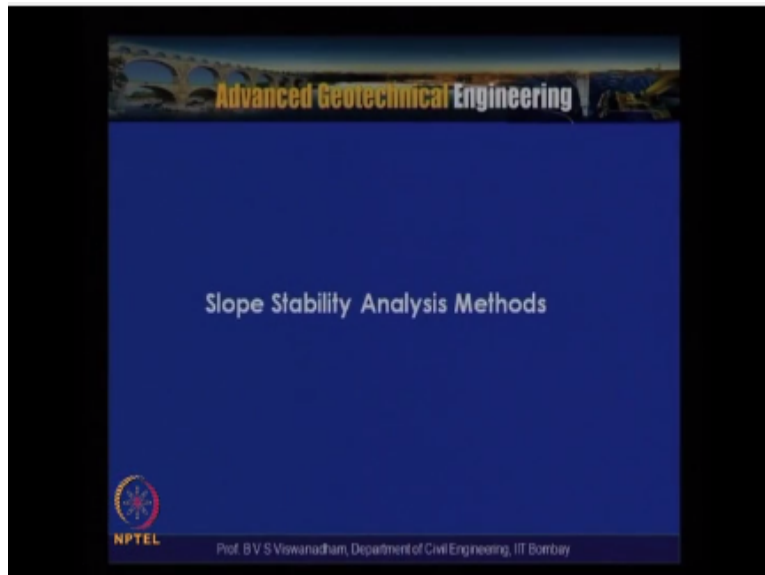
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So this is the lecture number 3 on stability of slopes, in this lecture or in this lecture 3 we are going to discuss about the slope stability analysis methods comparison of different slope stability

analysis methods and some examples for evaluating at factor of safety. So in the previous lecture we introduce ourselves to different slope stability.

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And we said that there are methods like ordinary methods of slices and followed by the bishop methods of slices and there is Morgan strength rise method and Janbu method so coming to the ordinary methods of slices.

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The ordinary method of slices

- In this method, the potential failure surface is assumed to be a circular arc with centre O and radius r.
- The soil mass (ABCD) above a trial surface (AC) is divided by vertical planes into a series of slices of width b .
- The base of each slice is assumed to be a straight line.
- The factor of safety (FS) is defined as the ratio of the available shear strength τ_f to the shear strength τ_m which must be mobilized to maintain a condition of limiting equilibrium.

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Let us recollect once again in this method the potential failure surfaces is assumed to circular in nature with centre at O and having the radius r. and the soil mass which is between the circular or joule from the slope surfaces and the crust joule it is divide into vertical planes into series of slices o width b. differential preferably these slices has to be of identical width, the base of each slice is assumed to be a straight line or purposes.

The factor of safety is defined as the ratio of the available shear strength τ_f to the shear strength τ_m which must be mobilized to maintain a condition of limiting equilibrium. So the factor safety in this method is defined as the ratio of available shear strength τ_f , to the shear strength τ_m which must be mobilized to maintain a condition of limiting equilibrium.

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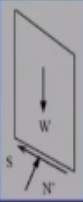
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The ordinary method of slices

◊ The Ordinary method (OM) satisfies the moment equilibrium for a circular slip surface, but neglects both the interslice normal and shear forces. The advantage of this method is its simplicity in solving the FOS, since the equation does not require an iteration process.

In summary, OM

- satisfies moment equilibrium condition,
- neglects the interslice normal and shear forces,
- gives the most conservative FOS, and
- is useful only for demonstrations.

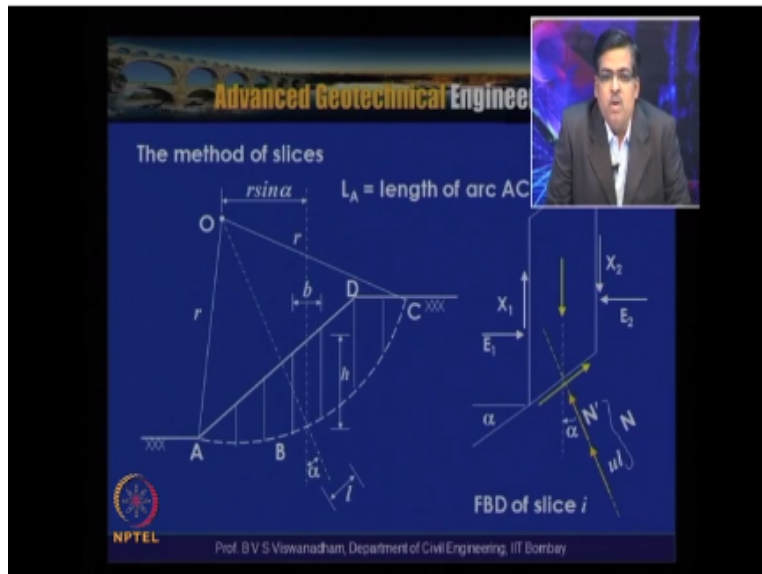


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So in this slide on the right bottom corner a free body diagram of a typical slice is shown, the weight of the particular slice is assumed to be active at the center of the slice and the normal reaction that n- and s is the tangency force which is actually shown in the typical slice. The ordinary method of slices satisfies the moment equilibrium for circular surface but neglect both inter slice forces, that is normal as well as shear forces which are actually there between the slices.

They are assumed to be, the forces which are acting here and here and along this surface we are actually assumed to be neglected. So advantage of this method is it is simplicity solving the factors of safety this is the equation require any process. So in summary the ordinary method of slices satisfies the equilibrium conditions and neglects inter slice normal and shear forces and used the conservative factor safety and useful for demonstration are estimation. So the detailed explanation of this method is showed here.

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In this the cross section of a slope which is A D C which is shown and with the O at the center of the rotation and R is the radius and the surface A,B C is assumed to be one of the failure surfaces and if a typical size of width b is assumed that the soil mass within A, B , C , D is divided into some equal number of slices having width B in the horizontal direction and if it is considered the true free body diagram which is actually shown free body of the slices I which if this is the slice free body diagram which is actually shown here, wherein E1 and E2 are the normal forces.

X1 and x2 are the interstice transition forces and this is the normal reaction which is actually acting to with gets oriented throughout the centre of rotation and this is the transition force which is actually acting opposite to the moment of the soil. So that is actually basically the resistances offered by the soil.

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The method of slices

$$FS = \frac{\tau_f}{\tau_m}$$

∞ The FS is taken to be slice, implying that there must be mutual support between slides. I.e. forces must act between slices.

1. Total weight of slice $W = \gamma b h$
2. Total normal force $N = \sigma l$ (includes $N' = \sigma' l$ and $U = u l$)
 u = PWP at the centre of the base and l is the length of the base.
3. The shear force on the base, $T = \tau_m l$
4. Total normal forces on sides E_1 and E_2
5. The shear forces on the slides, X_1 and X_2

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Further simplifying on this we can actually get now the factor of safety we have defined already and the total weight of the slice we can give it as $\gamma \times h$, the h is the height of the centre of the slice $\times B$ the width of the slice $\times 1$ unit, that is the because that is the 2 dimensional analysis we are doing per meter analysis. And then the total number of force $n = \sigma \times l$ and which includes $n - = \sigma - \times l$ and $u = u \times l$.

Depending upon the water table location we can actually calculate what is the u pour water pressure \times length of that particular straight line portion of the slice we can actually get u and \times effective stress $\times l$ we get $n-$. so u is the pour water pressure at the centre of the base of the slice and L is the length of the space.

The shear force on the base can be given as tm is the shear strength which must be mobilized to keep the slope in the equilibrium or you know ensure adequate factor of safety $tm \times l$ and the total normal forces slices $E1$ and $E2$ shear forces are $x1$ and $x2$.

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The method of slices

For an analysis in terms of effective stress

$$FS = \frac{\sum (c' + \sigma' \tan \phi')}{\sum W \sin \alpha}$$

$$FS = \frac{c' L_a + \tan \phi' \sum N'}{\sum W \sin \alpha} \quad (1)$$

Equation (1) is exact but approximations are introduced in determining the forces N' .


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Now further considering the moment about O the some of the moments about the shear forces t on the failure R = moment of the weight of the soil A, B , C , D, so if you can actually take the moments like $\sigma t \times R = \sigma w R \sin \alpha$. So here what we done is that we have taken the weight component into R radius. Now by using $T = tm \times L$, by writing $TF /$ factor of safety $\times L$ and substituting for σtr .

We can write $tf / \times L = \sigma w R \sin \alpha$, so by bringing the factor of safety term on the left hand side ew can write factor safety = $\sigma tf \times l / \sigma w \sin \alpha$. So further this can be worked out like this that tf we can actually write it has $c- + \sigma - \tan \phi - \times L / \sigma w \sin \alpha$, so for analysis in terms of effective stress we can write safety = $\sigma \sum c'+ \tan \phi \times L / \sigma w \sin \alpha$ and this can be further simplified by writing $\sigma' \times n'$ we can write factors safety = $c' \times L_a$ where $L a$ is the if you look into the \sum which is removed for the equation term.

Because $L1, L2, L3, L4$ at the \sum of that is L_a , so $c' L_a + \tan \phi' \times \sigma \phi n'$, so $\sigma \times n' / w \sin \alpha$ within \sum , that is the equation 1 is exact what approximation are actually introduce in terms of the forces n' , so this expression gives the factor of safety, in this lecture we are also going to solve some typical problems for arriving at a factor of safety of a typical slope by using manual calculation methods or also by using some software packages. The further the fellenius or Swedish solution it is assumed that.

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The Fellenius (or Swedish) Solution

It is assumed that for each slice the resultant of the interslice forces is zero.

The solution involves resolving the forces on each slice normal to the base i.e. $N' = W \cos \alpha - ul$

Rewriting Equation (1):

$$FS = \frac{c' L_a + \tan \phi' \sum (W \cos \alpha - ul)}{\sum W \sin \alpha}$$

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For each slice the resultants of the interslice forces is 0, in the Swedish method what is assumed that for each slice the resultants of the interslice forces is 0. In the ordinary methods of slices the interslices forces have been neglected but in the fellenius method it is more or less same as ordinary methods of slices only difference is that each slices the resultants of the interslice forces is 0. So the solution involves resolving the forces on each slice normal to the bears.

So $n' = w \cos \alpha - Ul$ because ul is nothing but the pore water pressure acting on a particular length of the slices that is l , so rewriting the equation one we can write $c' L_a + \tan \phi' \times \sigma$ that n' we write it has now $w \cos \alpha - Ul / \sigma w \sin \alpha$. So this is the revised expression for the computing factors safety by using the fellenius or Swedish methods of slices.

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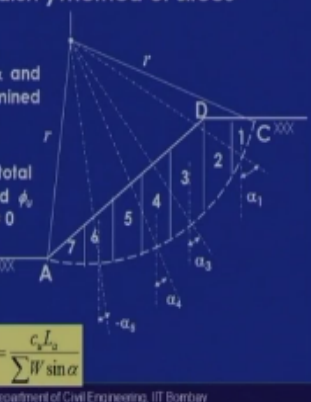
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The Fellenius (or Swedish) method of slices


→ The components of $W \cos \alpha$ and $W \sin \alpha$ can be determined graphically for each slice.

→ For an analysis in terms of total stress the parameters c_u and ϕ_u are used and the value of $r = 0$

$$FS = \frac{c_u L_u + \tan \phi_u \sum (W \cos \alpha)}{\sum W \sin \alpha}$$



For $\phi_u = 0$ →
$$FS = \frac{c_u L_u}{\sum W \sin \alpha}$$

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The explanation for the procedure for calculating the factor safety by using Fellenius or Swedish methods of slices is given here, let us assume a slope which is actually having a cross section A, D, C which is shown here and the slope of A, D which is the question because if you know the adequate factor safety is ensured then there is the possibility that if the material is actually having adequate strength characteristics, the slope of the AD line can be maintained steeper if the material is not having adequate strength characteristics and there is the need for you know adopting a flat inclination for the slopes.

Generally the slopes which are normally adopted for highway amendments or railway amendments the range from one vertical 1.5 horizontal to in some urban areas there is the need for the steeping of the slopes in such situation one need to adopt some strengthening options for these slopes so that the areas can maintained steeper, that means that whenever there is the land equalization problems the slope of AD can also be maintained vertical and in order to make it to stand in appropriate strengthen solution need to be designed.

So in this module we are going to discuss about these options for these things, now let us assume that this particular example here the procedure we divide this slices into the entire soil mass A, C, D within the assumed failure surface, the failure surface which is circular arc having R radius and it is assumed to be divided into let us say 7 number of slices. So this is the one slice which is the portion of the triangle and this is looking like a trapezium and this is having a certain dimensions here and so there are different slices and this is the centre of the rotation.

So first what we want to do is that you calculate what is the area and into L which actually gives the volume $\times \gamma$ weight of the slice 1, w_1 , w_2 , w_3 , w_4 , w_5 , w_6 , w_7 and all these respective weight for these slices will be acting in the centre of the slices and let us assume that this slices having the width of the small B in horizontal direction, then we assume that this slices is actually acting at the centre.

So if this is the width B, $B/2$ and this is the line joining, so after once the weight is actually indicated here draw a line from to the centre of the slice here where the weight is passing and with vertical this angle need to be recorded from graphically α_1 , α_2 , α_3 , α_4 , α_5 , α_6 which is actually shown, so as we travel from 1 to 7 you can see that the angle changes from positive to negative.

So the components of weights w_1 , w_2 , w_3 need to be find out, like $w \cos \alpha$ and $w \sin \alpha$ can be determined from the α once it is obtained from graphically then the components can be obtained for analysis in term of total stress the parameters C_u are used basically what we said that is for short term stability calculations and the value of the u will be 0 in that case factors of safety = $C_u \times l_a + \tan \phi_u \times \sigma_w \cos \alpha$, that means that let us assume that we have got now 7 slices now.

So $w_1 \cos \alpha_1$, $w_2 \cos \alpha_2$, $w_3 \cos \alpha_3$ the $\sum w_7 \cos \alpha_7 / w_1 \sin \alpha_1$, $w_2 \sin \alpha_2$ upto $w_7 \sin \alpha_7$, so once we have this and then $C_u \times L_a$, suppose it is also possible that we can also take like $C_u \times l_1$, $C_u \times l_2$, it is nothing but L_1 is nothing but the for slice 1 what is the particular length, it is approximated as straight line, so this graphically this length can be obtained once the diagram is drawn to a scale to be obtained.

So let us assume that $\phi_u = 0$ case so that is something like $C_u L_a / \sin \alpha$, that means if you are having a slope with undrained conditions where saturated clay slope, then the factor of safety can be obtained as $C_u \times L_a / w \sin \alpha$. Now bishop simplifies method of slices in this solution it is assumed that the resultant force on the sides of the slices are horizontal and $x_1 - x_2 = 0$. So for equilibrium the shear force on the base on any slice is given as $T = 1 / \text{factor of safety} \times C' \times l + n' \tan \phi'$ so resolving the forces in the vertical direction.

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Bishop simplified Method (BSM)

In this solution it is assumed that the resultant forces on the sides of the slices are horizontal. i.e $X_1 - X_2 = 0$

For equilibrium the shear force on the base of any slice is:

$$T = \frac{1}{FS} (c'l + N' \tan \phi')$$

Resolving forces in the vertical direction:

$$W = N' \cos \alpha + ul \cos \alpha + \frac{c'l}{FS} \sin \alpha + \frac{N'}{FS} \tan \phi' \sin \alpha$$

After some rearrangement and using $l = b \sec \alpha$:

$$FS = \frac{1}{\sum W \sin \alpha} \sum \left[\frac{c'b + (W - mb) \tan \phi'}{1 + (\tan \alpha \tan \phi' / FS)} \frac{\sec \alpha}{1 + (\tan \alpha \tan \phi' / FS)} \right]$$

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We can get $w = \cos \alpha + ul \cos \alpha + C' \times l + n' \text{ factor safety } n + n' \tan \phi' / \text{ factor safety} + n' \tan \phi' \times \sin \alpha$ after some rearrangements of the terms and using $L = B \sin \alpha$, we will get a factor of safety term as $= 1 / \sum \sin \alpha \times \sum$ and $c' B + w - Ub \tan \phi' \times \alpha 1 + \tan \alpha, \tan \phi' \text{ safety}$. So this expression actually has the factor of safety in both left hand side and right hand side, so this involves in a procedure initially the factor of safety is assumed to be a computed from the Swedish method of slices.

And then with the value after setting number of iteration one can calculate what is the factor of safety of a slope by bishop method of slopes and this is one of the versatile method for assessing the factor of safety of the slopes.

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Bishop simplified Method (BSM)

Bishop (1955) also showed how non-zero values of the resultant forces (X_1, X_2) could be introduced into the analysis but refinement has only a marginal effect on the factor of safety.

The pore water pressure can be related to the total fill pressure at any point by means of dimensionless pore pressure ratio $r_u = u/\gamma h$.

For any slice, $r_u = u/W/b$ By rewriting:

$$FS = \frac{1}{\sum W' \sin \alpha} \sum \left[\frac{c' + W'(1 - r_u) \tan \phi'}{1 + (\tan \alpha \tan \phi' / FS)} \right] \frac{\sec \alpha}{1 + (\tan \alpha \tan \phi' / FS)}$$

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So bishop 1955 in his paper he also showed that non zero values of the resultant forces $x_1 - x_2$ could be introduce into analysis by refinement only marginal effect on the factor safety and so bishop 1955 stated that non zero values of the resultant forces $x_1 - x_2$ could be introduce into analysis by refinement only marginal effect on the factor safety.

So in the bishops method the pour water pressure can be related to the total filed pressure at any point by dimensionless pour pressure ratio which is $R_u = u/\gamma h$, suppose if the R_u values = 0 that means the slope is you can say that partially saturated and it is almost dry $R_u = 0.5$ that indicates that the slope is a completely saturated. For any intermediate value between 0 to $u/\gamma h$ it is partially saturated.

And $R_u = 0$ it indicate this the slope is dry for any slope $R_u u / wb$ by rewriting this terms we can write for any slices by rewriting we can write safety of expression by $1/\sum w \sin \alpha, \sum x c' p x w x 1 - R_u \tan \phi' 1 + \tan \alpha, \tan \phi' /$ factor safety. Then $R_u = 0$ then you know we have this.

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Bishop simplified Method (BSM)

Bishop's simplified method (BSM) considers the interslice normal forces but neglects the interslice shear forces. It further satisfies vertical force equilibrium to determine the effective base normal force (N').

In summary, BSM

- satisfies moment equilibrium for FOS,
- satisfies vertical force equilibrium for N ,
- considers interslice normal force,
- more common in practice, and
- applies mostly for circular shear surfaces.

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So here the summary bishop simplifies method considers the interstices normal forces that is x_1 , x_2 , are the tangential forces here the tangential forces and even we do the normal forces so Bishop simplify method considers the interstice number forces but neglects the interest slice shear forces so kindly note here bishop simplify method considers the interest sliced normal forces but neglects the interstice shear forces it further satisfies the vertical force equilibrium to9 determine the effective base normal force N' .

So it satisfies equilibrium for factor of safety soon in summary bishop simplify method satisfies moment equilibrium for factor of safety satisfies vertical force equilibrium for N for determining N or N' considers interstice normal force interest slice normal forces are considered and more commonly used in practices and applies mostly for circular shear forces that means that wherever there is a homogenous soil which is used as embankment construction or for the highway embankment of construction.

When it is actually obtained from identical then we can say that possible failure surface can be circular in nature, then coming to the Janbu simplified method is based on composite surface that is basically non circular in nature and factor of safety is determined by horizontal force equilibrium. So as in bishops simplified method, but neglects the shear forces. So here also in this free body diagram of this slice which is shown for Janbu simplified method, the tangential forces x_1 , x_2 , or the forces which are acting in this direction they are consider to be neglected.

In Janbu method like in bishop it considers the E_1 E_2 that these are the interstices normal forces and it satisfies both force equilibrium that is vertical force equilibrium that is vertical force equilibrium as well as horizontal force equilibrium but it is not satisfied the moment equilibrium and considers this interstices number of forces and it is commonly used for determining factor of safety for composite shear surfaces.

Composite shear surfaces it is actually used, then coming to another method for deterring factors of safety that is the Morgenstern price method.

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Morgenstern-Price method (M-PM)

◊ The Morgenstern-Price method (M-PM) considers both force and moment equilibrium and an interslice force function.

In summary, M-PM

- considers both interslice forces,
- assumes a interslice force function, $f(x)$,
- allows selection for interslice force function,
- computes FOS for both force and moment equilibrium.

The diagram shows a soil slice with forces T_1 , E_1 , W , T_2 , E_2 , and N' acting on it.

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This satisfies both force and moment equilibrium and also assumes that, the interstice function was assumed, so here the interstice forces is assumed in the form of a function and considers both the interstice forces here you can see in this free body diagram of this slice which is actually shown here, where E_1 , E_2 , T_1 , T_2 are forces, they both are consider and they both are related in form of function $f(x)$.

And considers both the interstices forces and they consider assumed as a inter slice forced function $w(x)$ and allows selection of the inter slice force function and computes the factors safety for both force and moment equilibrium, so it computes the factor safety for both force and moment equilibriums.

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Spencer's method

⇨ Spencer's method (SM) is the same as M-PM except the assumption made for interslice forces. A constant inclination is assumed for interslice forces and the FOS is computed for both equilibriums (Spencer 1967)

In summary, SM

- considers both interslice forces,
- assumes a constant interslice force function, and
- satisfies both moment and force equilibrium, and
- computes FOS for force and moment equilibrium.

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Spencer's this is same as the Morgan's strength and price method the assumption made for the inter slices forces but a constant inclination is assumed for inter slices forces and factor safety is computed for both the equilibriums, this is same as the Morgan's strength and price method the assumption made for the inter slices forces assumed for inter slices forces and satisfies both moment and force equilibrium conditions.

And computes factors safety for a force and moment equilibrium and the free body diagram of the typical slice in the Spencer method is shown here, rate is actually is active here this is the moment of the slopes, so opposing that this is the soil resistance which is actually offered from the mobilized shear strength. So this is the after the Spencer method after Spencer 1967.

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The slide features a dark blue background with a title bar at the top that reads "Advanced Geotechnical Engineering" in a stylized font. Below the title, the text "Example 1" is displayed. The main body of the slide contains a paragraph of text describing a geotechnical problem. At the bottom, there is a citation "After Craig (2004)" and the NPTEL logo on the left. The NPTEL logo consists of a circular emblem with a gear and a book, followed by the text "NPTEL".

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Example 1

A 45° slope is excavated to a depth of 8m in a deep layer of saturated clay of unit weight 19 kN/m^3 ; the relevant shear strength parameters are $c_u = 65 \text{ kN/m}^2$ and $\phi_u = 0$. Determine the factor of safety for the trial failure surface specified in Figure. The cross-sectional area ABCD is 70 m^2 .

After Craig (2004)

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So let us after having discussed about the number of methods let us try to solve some typical problems by using manual calculations as well as some computer added methods. So let us consider the example 1, if 45° slope is excavated to a depth of 8m in a deep layer of saturated clay of unit weight 19 k N/m^3 the relevant shear strength parameters are given as 65 k N/m^2 and $\phi_u = 0$.

So determine the factor of safety for the trial failure surface specified in the figure in the next slide and the cross section area ABCD which is within the joule from slope to the failure surface is 70 m^2 . So 45° slope is excavated to a depth of 8m in a deep layer of saturated clay of unit weight 19 k N/m^3 the relevant shear strength parameters are given as 65 k N/m^2 and $\phi_u = 0$ determine the factor of safety for the trial failure surfaces but we are actually calculating for a typical trial failure surfaces.

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Example 1

Figure for Example 1

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So the cross section is actually shown here where in we see that ABC is assumed trial surface and AD is the slope which is incline at 45° and DC is the crust width and the radius is 12.1m and the horizontal distances OD is 4.5m and assumes that w is actually acting right below the D vertically down; the area of ABCD is 70×1 m is the volume which is involved in the in active joule.

So what we call that weight is nothing but $70 \times \gamma$ if the soil is assumed to be uniform here in this case then we get the weight. Then the height of the slopes is 8m and the rotation which is actually taking place at height say 11.5 m from the t level which is actually a.

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Solution for Example 1

Weight of soil mass = $70 \times 19 = 1330 \text{ kN/m}$

The centroid of ABCD is 4.5m from O. The angle AOC is $89\frac{1}{2}^\circ$ and radius OC is 12.1m. The arc length ABC is calculated as 18.9m. The factor of safety is given by

$$F = \frac{c_u L_a r}{W d}$$

$$= \frac{65 \times 18.9 \times 12.1}{1330 \times 4.5} = 2.48$$

This is the factor of safety for the trial failure surface selected and is not necessarily minimum factor of safety.

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So the solution is like this finding out the weight of the soil mass which is nothing but $70 \times 1 \times 90$ that is the unit weight, so we get if it is not consider as 1 not then we calculate the center of ABCD I s4.5m from O and angel which is subtended between AO and OC is 89.5 so hence radius is 12.1m the arc length we can actually calculate once we know the angel AOC.

So the factor of safety can be given by $C_u L_a \times R$, is nothing but the force $\times R$ is the liver arm for force along the arc surface and w is nothing but the weight of soil mass \times horizontal distance t , so $w d$ as the driving moment and $C_u L_a \times R$. that the factor safety whatever we have computed for the trail failure surface did not be that it gives a minimum factor safety.

One need to get what is the potential failure surface what is the minimum factor safety we can actually can be obtained, so minimum factor safety we can actually can be obtained for the similar problem by using stability one need to get the what is the potential failure surface and what is the minimum factor safety we can actually can be obtained. So the the minimum factor safety we can actually can be obtained for the similar problem by using charts also.

So let us look into the how is that tailors stability charts can be used to get a minimum a factor safety.

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Solution for Example 1

The minimum factor of safety can be estimated by using $FS = c_u / N_s \gamma H$.

Using Taylor's chart for N_s vs Slope Inclination β , For $\beta = 45^\circ$ and assuming that D is large, the value of N_s is 0.18.

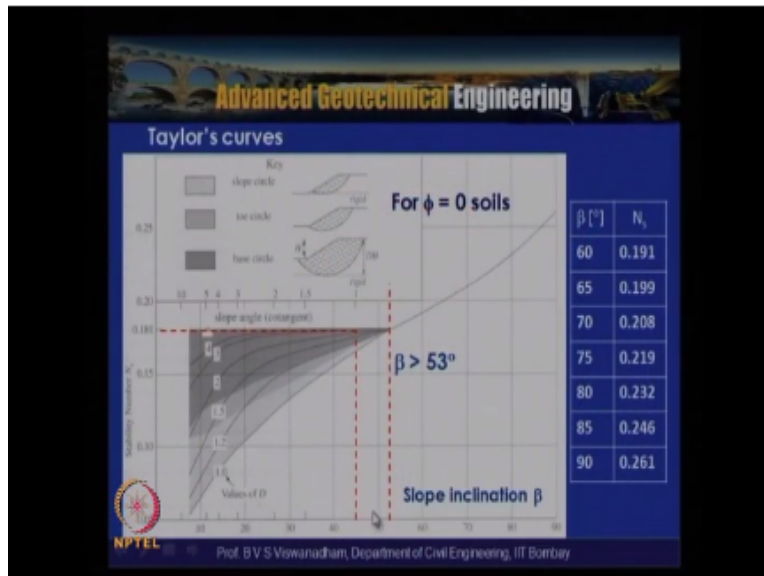
$$\begin{aligned}
 F &= \frac{c_u}{N_s \gamma H} \\
 &= \frac{65}{0.18 \times 19 \times 8} \\
 &= 2.37
 \end{aligned}$$

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So here a minimum a factor safety can be estimated but using factor safety = $C_u \times N_s \times \gamma h$ where N_s is nothing but the Taylor stability number, so using Taylor stability chart N_s + slope so Taylor stability chart that we have discussed in the previous lectures wherein we have seen that $\beta = 45$ the value of the assuming that D is large. So by substituting this values factor safety = C_u which is $65 / N_s \times 0.18 \times 90 \times 8$ we will get the factor safety 2.37.

Now if it is noted that this particular estimation is assumed to give the factor of safety close to the minimum factor safety or it is a minimum factor safety, so how the value of the N_s is obtained from.

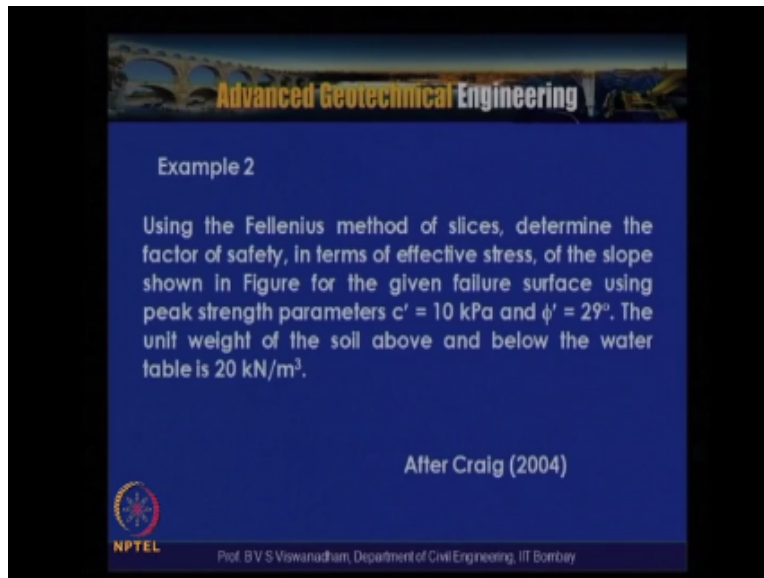
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Wherein we have the sloping on the x axis stability number on the y axis and for the slope 45 where as we are assumed that the extended of the D below the t level is assume to be large, if the B is shallow then we are actually having here only but as we assume that the extended of the D much further below then $D = \infty$ we get stability number of 0.181.

So based on that the factor of the safety is obtained and varying then we can actually calculate what the factor safety of the given slope is. So in this example we try to determine the by using the $\phi = 0$ method and we also used the Taylors chart to get the minimum factor safety for a potential failure surface. Now let us consider another example 2 wherein we are using the fellenius.

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The slide features a header with the text "Advanced Geotechnical Engineering" in a stylized font, set against a background image of a bridge. Below the header, the text "Example 2" is centered. The main body of the slide contains a paragraph of text describing a problem involving the Fellenius method of slices. At the bottom right, it says "After Craig (2004)". In the bottom left corner, there is a circular logo for NPTEL. In the bottom center, there is a line of text identifying the professor and department.

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Example 2

Using the Fellenius method of slices, determine the factor of safety, in terms of effective stress, of the slope shown in Figure for the given failure surface using peak strength parameters $c' = 10 \text{ kPa}$ and $\phi' = 29^\circ$. The unit weight of the soil above and below the water table is 20 kN/m^3 .

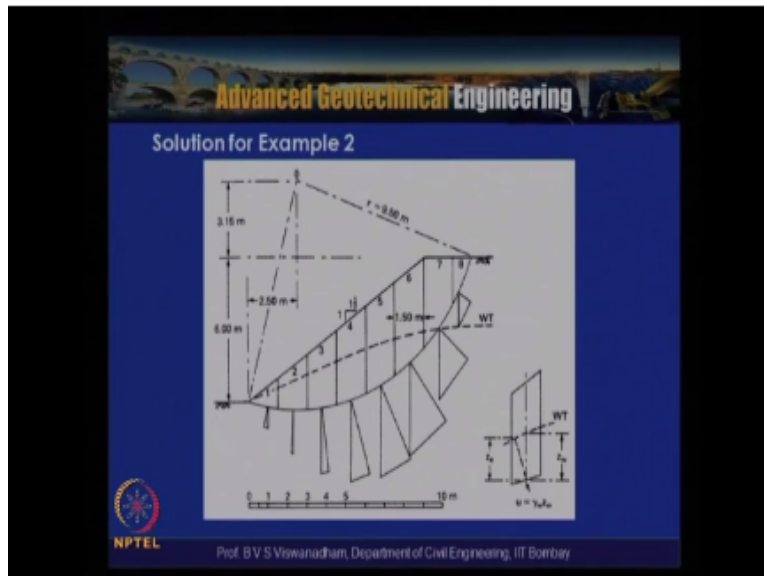
After Craig (2004)

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Determine the factor of safety in terms of a effective stress of the slope shown in the figure for the given failure surface using peak strength parameters $c' = 10\text{kPa}$ and $\phi' = 29^\circ$. So the unit weight of the soil above and the below of the water is given as 20kn/m^3 . So in this method fellenius methods of slices factor of safety determine in terms of a effective stress, so this after 2004.

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So the cross section is given here and if it is noted here the potential given surface is shown here and these are the normal reactions which are actually passing through that and these are the tangential forces and these are the weights this force trying what we have seen and these are the normal reactions these are the shear forces mobilized by the soil and this is the weight of the slice.

And the slope inclination is 1 so what we are interested is that what is the factor safety of slope for the given slope inclination which is here, generally for factor safety = 1 means we say that it is at the range of failure, and the factor safety = 1.5 for some slopes if it is designed then it said as stable and here the $R = 9.5$ meters and vertical distance is about 9.15m. Here in this method is that we calculate the u value with difference to this vertical height.

It is not this height in principle one need to consider this particular height that means that if you consider this reductive distance then z effective is this height, so but in this method we estimate the factors safety at this the centre of the slice where ever the water table surface is here then we take the entire area in this particular height we calculate what is the pore water pressure and multiplied by this length we will get the u .

But in reality it is this particular height so by determining with this Z_w we end upon the conservative slice so it is actually safe for the slope which is being designed. So that is need to be understand while using the method or Swedish method of slices, so the computation works out like this after Craig 2004.

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Table giving computations (After Craig 2004)

Slice No.	$h \cos \alpha$ (m)	$h \sin \alpha$ (m)	u (kN/m ²)	l (m)	ul (kN/m)
1	0.75	-0.15	5.9	1.55	9.1
2	1.80	-0.10	11.8	1.50	17.7
3	2.70	0.40	16.2	1.55	25.1
4	3.25	1.00	18.1	1.60	29.0
5	3.45	1.75	17.1	1.70	29.1
6	3.10	2.35	11.3	1.95	22.0
7	1.90	2.25	0	2.35	0
8	0.55	0.95	0	2.15	0
	17.50	8.45		14.35	132.0

$$F = \frac{c' L_a + \tan \phi' \sum (W \cos \alpha - ul)}{\sum W \sin \alpha}$$

$$= \frac{(10 \times 14.35) + (0.554 \times 393)}{254}$$

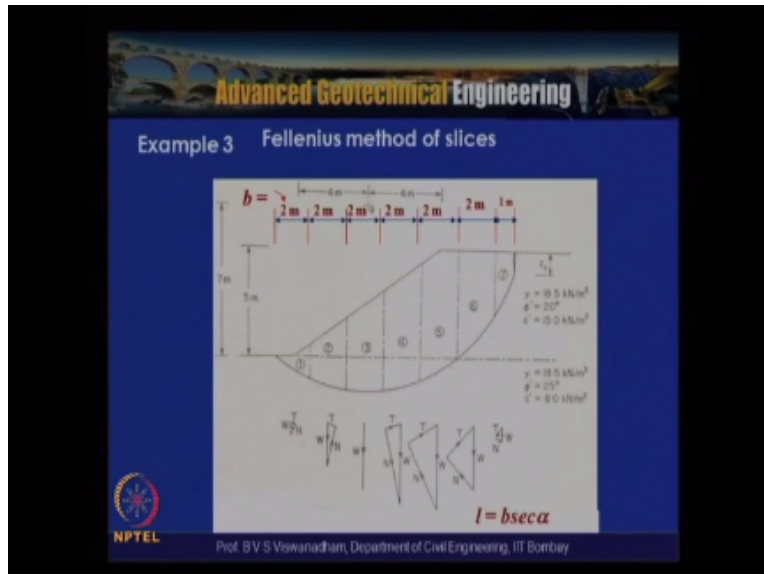
$$= \frac{143.5 + 218}{254} = 1.42$$

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Wherein we divided into 8 number of slices we determined rates of the slices and which are plane as it can be here it shown in term so sin like slice number and weight computation and α that is obtained graphically, then calculating $w \cos \alpha$, $w \sin \alpha$ and also by noting down the h is the height of slice in the center but if you consider let us say that $\frac{3}{4}$ th of the height is water table and then graphically if you measure what is the length $u \times l$ force we can say per meter length.

So by using this expression factors safety in terms of effective stress we can calculate $c' L_a + \tan \phi' \times \sigma w \cos \alpha - ul - w \sin$. So the computation after simplification you get the factor safety as 1.42 so it indicates that the slope which is one vertical $\frac{1}{2}$ horizontal is having the factor of safety of 1.42 if one needs the factor safety more than 1.3 and then it indicates that the slopes needs to be flatten that means that one as to go for one vertical two horizontal then the slopes is tends to be insure the higher value of factor safety.

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So this is again another example of fellenius method of slices in this particular slice what is actually shown is that the slope is actually divided into partial within the failure surface is divided into the 7 number of slices but there is the a tension crack of a certain depth. So one need to estimate the value of the L is suppose this is the entire L because there exist a gap and where because of the virtue of the tension crack we cannot actually what is the estimate what is the form a resistance.

So here the center of the rotation is assumed to be at the height of the 7m and the slope is actually having the such an inclination and one can obtained like here height of the slope is 5m and horizontal is 8m, so it is actually got as an inclination and the soil it is not necessarily that we get homogeneous soil we can also get the laid soils, so in such situation where the portion above this is having a properties $c' = 15 \text{ km}^2$ $\phi' = 20^\circ$ and this can be base soil.

And it can be also situation of water table but in this example no water table is given so here only to consider like here when we are considering these soil properties in this portion we need to consider the soil properties for the shear strength resistance when you are considering here particularly for slice, 1, 2, 3 4 and 5 we need to consider these properties. We need to consider for this portion the unit weight is 18.5.

And calculate the actual entire weight of composite weight of the slice by performing this exercise like identifying the normal reactions and calculating the weights, so in this case we can see here.

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Solution for Example 2
Fellenius method of slices

$$FS = \frac{\sum c'l + \tan \phi' \sum (N - ul)}{\sum T}$$

Slice	W (kN)	c' (kPa)	tan φ'	l (m)	N _s W cos α	T _s W sin α	c'l (kN)	ul (kN)	N-ul (kN)	(N-ul) tan φ'
1	27.7	8	0.466	2.3	24.5	-13	18.3	4.8	19.7	9.2
2	96.5	8	0.466	2.1	93.9	-22.5	16.5	14.8	79.1	36.9
3	148	8	0.466	2	148	0	16	22.2	125.8	58.6
4	188.7	8	0.466	2.1	183	44.4	16.5	29	154	72.8
5	199.8	8	0.466	2.3	176.1	94	18.3	34.2	141.9	61.1
6	148	15	0.364	2.8	105.5	103.9	42	31.4	74.1	27
7	37	15	0.364	2	16.5	32.6	30.4	11.4	5.1	1.9
					$\sum T =$	$\sum c'l =$	$\sum (N-ul) =$			
					239.4	158	267.5			

FS = (158+267.5)/239.4 = 1.78

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So $\alpha = 0$ and then value changing to $-$ because there are coming towards the left side of you know moment of rotation. So the solution for this example 3 which is actually 3 which is actually given as failures methods slices and wherein we get the weight computation we can actually get the unit weights. In this case there are 7 number of slices and c' and $\tan \phi'$ we can see here upto 5th slice these properties will consider.

And then $n = w \cos \alpha$ $t = w \sin \alpha$ these are computed from weight by knowing the angle α then one can calculate $c' \times l$ and $u l$ and for getting the $u l$ measure the height of the let us assume that is the center and if the water table is here in this case the water table is also consider wherein we can actually calculate water table height. So with that you can calculate what is z_w we get the l we get the $U l$ and $n - u l$ we get the.

By multiplying this $n - u l \times \tan \phi'$ we will get $n - \sum$ of this is 267.5 and the $\sum c' l$ is 158, \sum of t is 239.4 so the factors safety expression for by using fellenius method of slices is $\frac{\sum c' l + \sum (N - ul) \tan \phi'}{\sum T}$ this / this is the driving moment. So for this by using this condition with for the type of slope what we consider then factors safety is obtained as 1.78 as it is more than 1.5 for that type of slope configuration which is consider the slope is consider to be stable. Now in this example

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Example 3

Using the Bishop method of slices, determine the factor of safety in terms of effective stress for the slope detailed in Figure for the specified failure surface. The value of r_u is 0.20 and the unit weight of the soil is 20 kN/m^3 and the shear strength parameters are $c' = 0 \text{ kN/m}^2$ and $\phi' = 33^\circ$

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This example basically it is an example 4 this is the using the bishops method of slices, we need to determine the factor of safety in terms of effective stress for the slope detailed in the figure which we are going to see that in the next slide, the value of the r_u which is actually the ratio of $u/\gamma h$ is given as 0.2 nad the unit rate of point is soil is 20 kN/m^3 and the shear $c = 0$ and $\phi = 33$.

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Example 3

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So this is the slope transition which is shown and this is the center o rotation and the factor of safety is the slope inclination is 1 horizontal, and vertical height is 17.5, if 17.5 means this

horizontal distance it will be around 35m and this assumed entry is here and the exit point is at the toe and this height is 48m.

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Solution for Example 3

$$F = \frac{1}{\sum W \sin \alpha} \sum \left[\frac{W(1 - r_u) \tan \phi'}{1 + (\tan \alpha \tan \phi')/F} \right]$$

$\phi' = 33^\circ$
 $r_u = 0.20$
 $W = \gamma b h = 20 \times 5 \times h = 100h \text{ kN/m}$
 $(1 - r_u) \tan \phi' = 0.80 \tan 33^\circ = 0.520$
 Try $F = 1.10$
 $\frac{\tan \phi'}{F} = \frac{\tan 33^\circ}{1.10} = 0.590$

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Now let us see how this can be solved by using the bishop method of slices the expression is given here and which we have discussed in this lecture itself and we divided like any other method into the equal slices. So these are shown here and the $\phi' = 33$ and $\gamma ru = 0.2$, so estimate the rate of the slices, so $\gamma B \times H$ in terms of h it is $100h$ and $1 - r_u \times \tan \phi'$ is estimated as 0.52 and here as it told because of the factor safety term which is actually there in left hand side and right hand side you try with initial value which is say 1.1. So $\tan \phi' / \text{factor safety} = \tan 33 / 1.1$ which is 0.59.

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Solution for Example 3

Slice No.	h (m)	$W = \gamma bh$ (kNm)	α°	$W \sin \alpha$ (kNm)	$W(1 - r_u) \times \tan \alpha'$ (kNm)	$\frac{\sec \alpha}{1 + (\tan \alpha \tan \alpha')/F}$	Product (kNm)
1	1.5	75	4	5	20	0.963	49
2	3.1	310	9	48	161	0.926	149
3	4.5	450	15	120	234	0.892	209
4	5.3	530	21	190	276	0.873	241
5	6.0	600	28	282	312	0.862	269
6	5.0	500	35	287	260	0.864	225
7	3.4	340	43	232	177	0.882	156
8	1.4	28	49	21	3	0.908	3
				1185			1271

$F = \frac{1271}{1185} = 1.07$

The trial value was 1.10, therefore take F to be 1.08.

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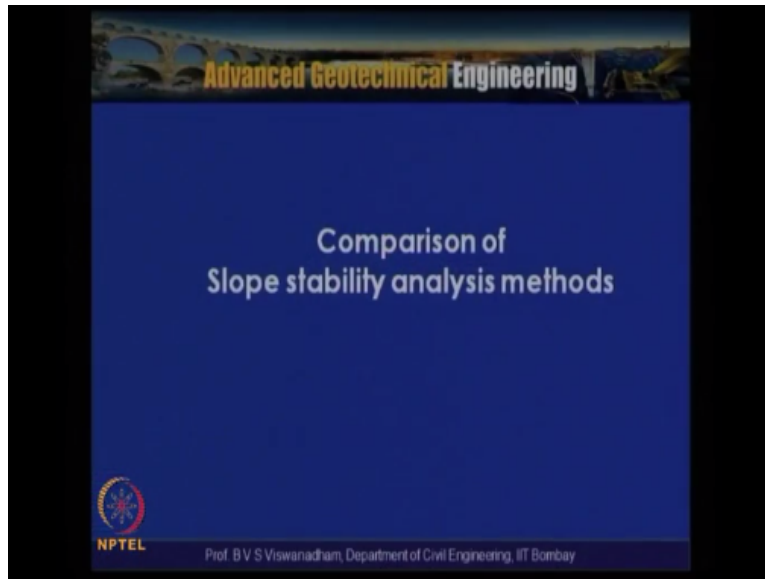
So the solution for the example 4 works out be slice number which is 8 number of slices and the height the centre of slices are given here and weights which are actually given in terms of γBH similarly we calculate what is α and the compute $w \sin \alpha$, so with that we can actually we can get this particular term. And then compute $1 + \tan \alpha + \tan \alpha \times \tan \phi$./ factor safety and the product which is of these two is here.

So this divided by this particular is 1185 because in the previous slide if you look into this we have got this and this in one column and in another column we estimated this term and so if you look into this here the product is this particular term, divided by this $\sum w \sin \alpha$ is 1185, so with that the factor safety is computed as 1.07. so the trail value which we assumed is 1.1, so the factor of safety is 1.07 or 1.08.

So this indicates that the slope is the type of soil parameters which are actually considered the slope configuration is just stable, any you know the fluctuation in the r_u there can be slope can

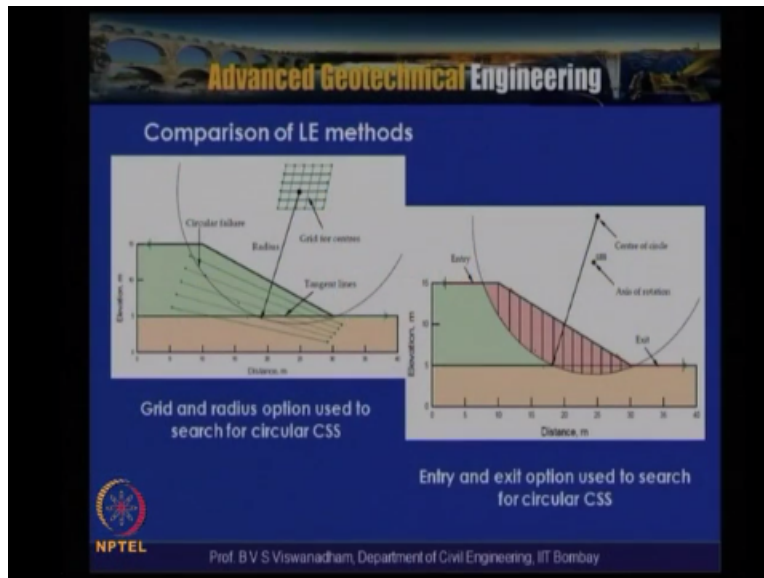
actually increase of r_u there will be the possibility the slope will undergo failure so let us now after having discussed about the.

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The example for a manual effect now let us tried to look in to the comparison slopes stability analysis method and in this particular slide a typical slope is actually shown how you know.

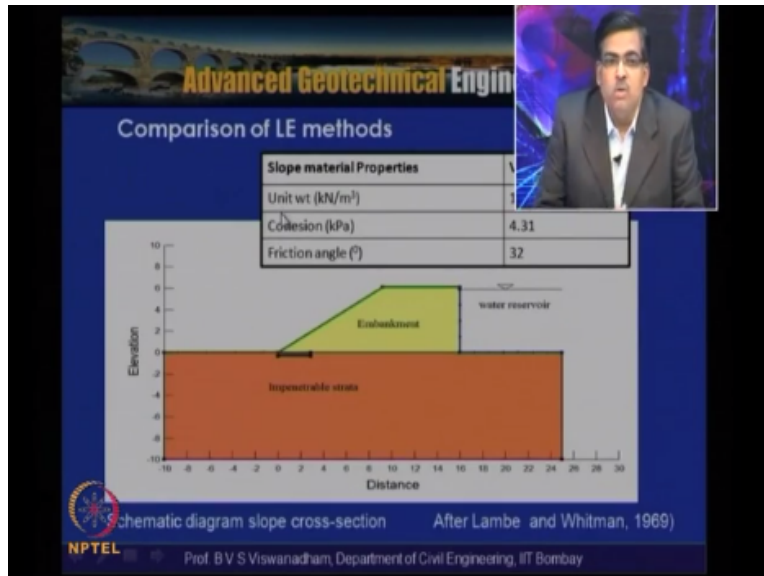
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Failure surface is located so there are you know different options and one option is that you considered a grid of radius super that the perpendicular bisector of this one you the grid of radius so this horizontal and vertical distances can be specified as point 5 meter/point 5 meter are it can range from up to 5 meter can more the grid of centers for the accuracy and also one can specified in the here what is the extension of minimum radius and maximum radius.

Where this circle surface can proceed is based on that we can actually calculate the circle failure surfaces generally what is done is that numerical number of failure surface are tried the one who which actually gives so in this we can actually get the concludes where factor is 1.5 or 1.3 where ever the one the center which gives the list factor affectivity that is actually calculated or regarded critical factor safety in this case you know the center of the circle.

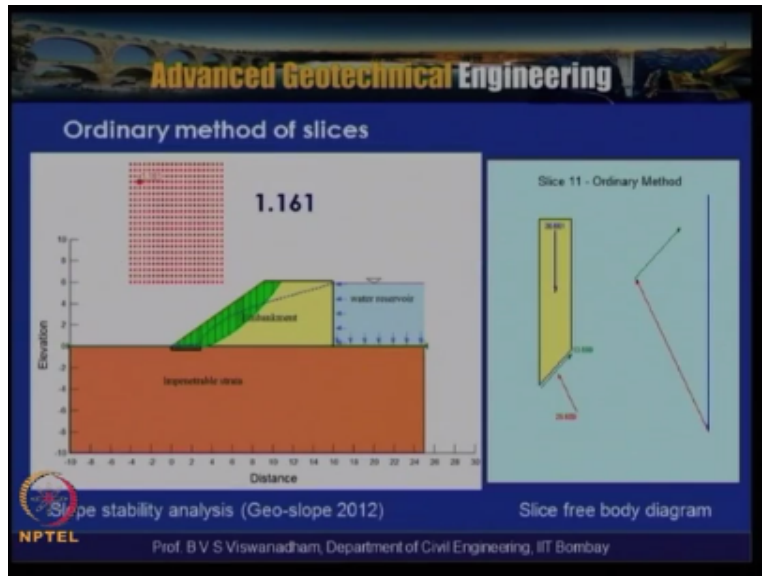
And here the with entry exit option for using searching so different methods are there for searching the critical factors of the safety in some software packages here grid is actually used in that the minimum factor the minimum radius and maximum radius is specified and with that searches for the trying number of circle surfaces it tries to give the center which gives the radius which gives the list factor of safe.
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So a typical problem which is given in lambe and Whitman 1969 was considered this is the embankment which is retriying a water on the rare side that is actually here and the band is constructed with a unit weight of 19.64 KN/m² cohesion is 4.31 (kpa) and fiction angle that it would be decrease there is a drained here which is provided and this is the impermeable strata and the thickness of this considered as 10 meters.

And this is the particular sloping which is actually shown so this is the systemic diagram slope cause session what is been done is that different methods for actually adopted and then tried to compare the factors of slice.

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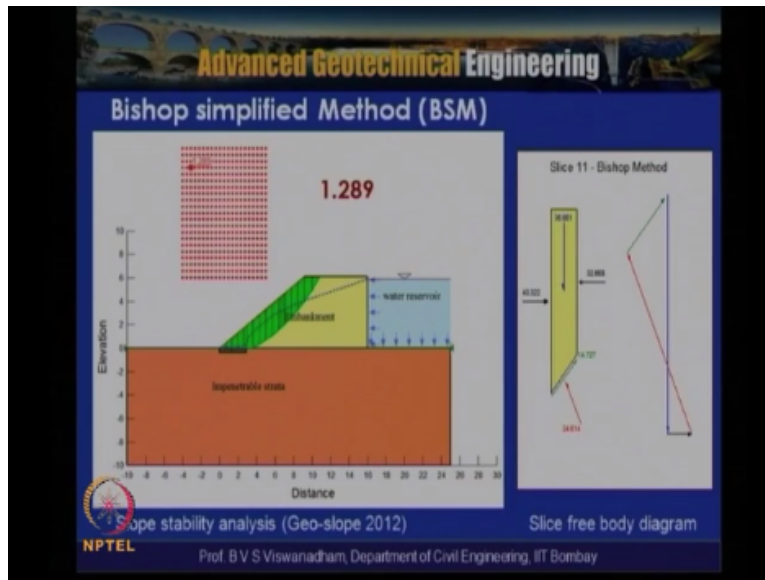


So here first we the ordinary method of slices where used in this Geo-slope 2012 comparison of the varying first the c plus analysis the carry down and using the cw model and varying the dietetic surface of time because there is a drained here and this being the line so this is the flow line wherein you can see that it actually meets at orthonical to be educational line here and this fraction is the react surface and it is considered here that is position is the one of the failure slope is shown here.

For this is regarding as one of the centers which actually gives the reached factors of working for surface ordinary method of slices so here that we free body diagram of the ordinary method of slices one of the eleventh slice that is that particular slices is elaborated here so you can see that here both that surfaces are not considered shear force is not considered and only weight of size is there that is actually achieving downward this force is there actually active direction is normal reaction which is actually shown here.

So this is been fourth triangle for the eleventh slice which is exaggerated here so based on this ordinary method of slices or the type of problem what we considered it is actually giving as 1.161.

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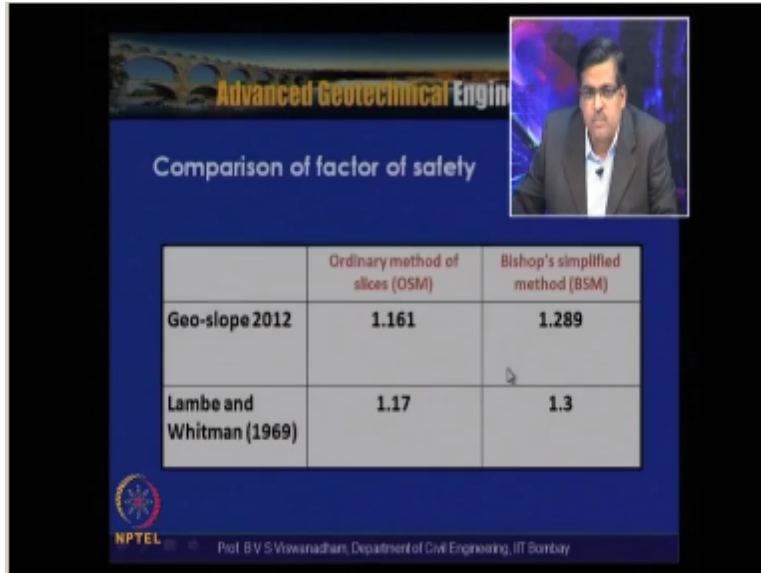


Similarly by adopting for the same problem bishops simplified method wherein similar procedure wherein estimated the erratic surface and now you can see that we have these normal forces shear forces are again considered to be 0 so because of this difference is actually considered here so this is the net force acting in this direction for the again the eleventh slice and this is the soil shear resistances actually shown here.

And this is the weight of this slice so this is here is the factor safety of 1.289 so when you look into the you know given slope the bishop of the higher safety and john method simplify method is giving a factor of safety 0.22 where in here we can see that this is 40.3222 and 32.665 where in you actually have these normal forces and then you actually have this shear resistance which is actually acting at the base of the soils now Morgenstern price method where in the here we have consider.

The horizontal interlude forces that is horizontal forces as well as the you know the horizontal the shear stress on the along the surfaces as well as the normal forces so because of this force diagram changes you can see that this is that at normal tangential forces and these are the normal forces which are actually acting as the slides and this is the bit of the size this is the normal reaction and this is particular method this is the factor of safety.

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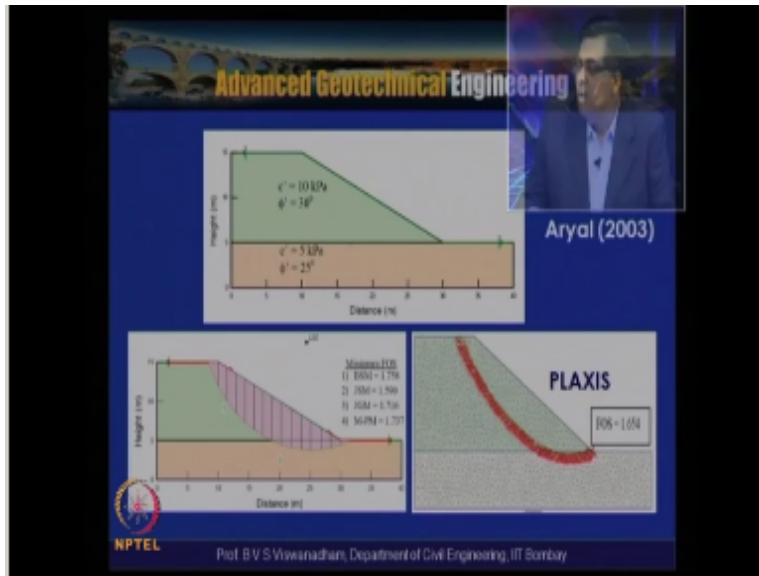


The slide, titled "Comparison of factor of safety", features a table comparing two methods: Geo-slope 2012 and Lambe and Whitman (1969). For each method, the factor of safety is calculated using the Ordinary method of slices (OSM) and Bishop's simplified method (BSM). The Geo-slope 2012 results are 1.161 for OSM and 1.289 for BSM. The Lambe and Whitman (1969) results are 1.17 for OSM and 1.3 for BSM. The slide also includes the NPTEL logo and the name of the professor, B.V.S. Viswanadham, from the Department of Civil Engineering, IIT Bombay.

	Ordinary method of slices (OSM)	Bishop's simplified method (BSM)
Geo-slope 2012	1.161	1.289
Lambe and Whitman (1969)	1.17	1.3

So this when you compare the factor of safety actually given by lambe and Whitman and as well as computer because you can see that geo slope 2012 computes ordinary method is 1.161 and bishops method computes 1.289 this is actually what obtained by using geo slope 2012 and lambe and Whitman gives 1.17 and the same analysis by for the bishops simplified method it gives support to it.

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So here in this particular slide the factor of safety also be obtained by using finite element based methods or finite difference programs like FLAC so here 2003 compare the limited Cuban methods of comparison like bishop simplified method and john brose method and organ simplified method and where as the compared the method by using finite method that is by using flux so where in they adopted this shear stress method.

And here both base soil and soil in the zonal form they have different soil properties and we can see that the finite based method by using flux to did you say the band of way the failure production failure can naturally failure surface can adjust and reduces the factor of safety about 1.65 volt so this is actually a particular missing failure surface which is actually obtained for bishops method and john brose method.

And principle you can see that because of the consolidation of this pores the higher the factor safety that indicates that more you can say that the slope is you can actually go adopt sleep suppose let us say that we compute by using ordinary method of sizes you know 1.1 factor of safety but however by considering that we tend to revise the slope making it sleeper that leads to you know a sleeved adoption a flat slope inclination in such situations.

The adoption of imp propitiate method helps us to arrive at configuration of the factor of safety so in this particular lecture what we try to discuss is that slope stability analysis method different types of slope stability analysis method are reduced and then we try to cover some typical

examples by using ordinary method of sizes and failure number of sizes and bishop simplified solution.

And we compared the typical cross section example which is build by lamp in 1969 by analyzing with base method for the determining factor safety by using bishops method and john brose method and the values are found to be in random and then finally also we discussed the hill station which are given by results we reported or given by RN 2013, 2003 RL 2003.

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