#### NPTEL

## NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

#### **IIT BOMBAY**

### CDEEP IIT BOMBAY

### ADVANCED GEOTECHNICAL ENGINEERING

Prof. B. V. S. Viswanadham Department of Civil Engineering IIT Bombay

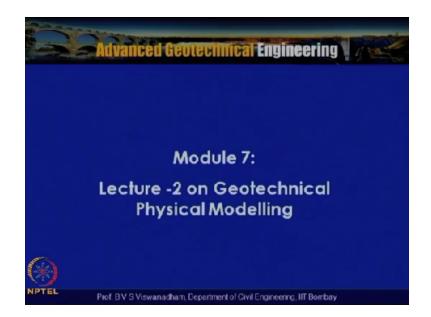
Lecture No. 51

Module - 7

## Lecture – 2 on Geotechnical Physical Modelling

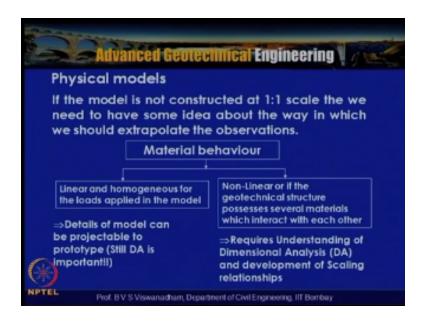
Welcome to course on advanced geotechnical engineering in the yesterdays lecture we had deduced ourselves to different modeling techniques in geotechnical engineering and we narrowed down to a technique which is called as physical modeling or geotechnical physical modelling. This lecture is a continuation of geotechnical physical modelling topic.

(Refer Slide Time: 00:51)



So this comes under module 7 lecture 2 on geotechnical physical modeling. So yesterday we have discussed that if a physical model can be model.

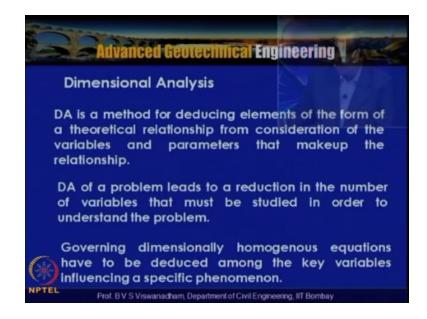
(Refer Slide Time: 01:05)



At full scale then we did not have to worry about the, you know if the scale factors or scaling relationships. But if the model is not constructed at 1:1 scale then w need to have some idea about the way in which we should extrapolate the observations, that means if you look from the material behavior point of view linear and homogeneous for the loads applied in the model if the material is linear or if the material behavior is linear and homogeneous for the loads applied in the model in the model.

Then details of the model can be projected to prototype without much of you know worries that means that still with the help of dimensional analysis we can do that, but if the material behavior is non linear or if the geotechnical structure has several materials with interact, which interact with each other then it requires understanding about more into dimensional analysis and simulate relationships between parameters influencing model an prototype.

(Refer Slide Time: 02:21)



So this dimensional analysis basically it is method or deducing elements of the form of a theoretical relationship from consideration of the variables and parameters that make up the relationship. The dimensional analysis is now a method for deducing the relationship among variables which are influencing a particular phenomenon. And these variables can be you know dependent variables and independent variables.

And dependent variable is a one where which grounds the entire phenomenon and independent variables are the one which influences the phenomenon. So if any change in the you know independent variable then there is an influence on the dependent variable. So, dimensional analysis of a problem leads to a reduction in the number of variables that much be studied in order to understand the problem.

So by doing dimensional analysis we will be able to lead to a reduction in the number of variables that must be studied in order to understand the problem. So the governing dimensionally homogenous equation have to be deduced among the key variables influence in the special phenomenon, so for this purpose we have two methods one is Rally's method or we also call as method of product of powers.

The other one is buckingham's pi theorem, in the rally's method if we are having say variables like q1, q2, q3, q4 so qn if they are influencing a particular phenomenon by expressing as a method of product of powers and expanding the infinite series of tans and by satisfying the condition of dimensional homogeneity we can prove that the dimension of q1 is to be equal to dimension of  $q2^{a1}$ ,  $q2^{a2}$  so on to  $qn^{an}$ .

Where in n is nothing but here the number of variables, so by writing the indicial equations we can actually get you know the values of a1, a2, a3 so on to an. And by getting this values we will be able to you know get a relationship among the dependent variable and independent variable. But this method fails if you are actually having more number of variables, let us say more than seven variables then this method the operation involves TDS and converting one solution into another solution involves the algebraic adjustments.

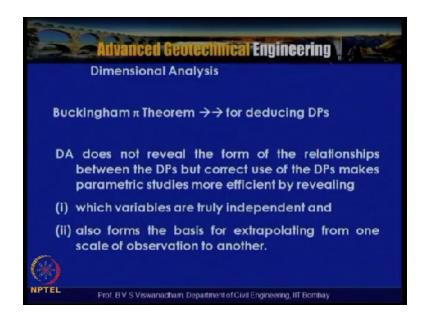
In such situations the buckingham's pi theorem is a viable option and where in if you have got let us say q1 this on to q2, q3 so on to qn then here and r' and r'' and r''' if there are you know these dimensionless ratios then we can actually deduce in the term of pi terms where is  $\pi 1$  is a function of  $\pi 2$  so on to  $\pi 3$  so on to  $\pi n$ , and r' and r'''. So here what it implies is that when from the similar procedure we will be able to get this.

But in order to get the number of dimensionless product what we need to do is that in buckingham's pi theorem we have to arrange these variables in such a way that you get the non singular matrix towards the right hand side of the matrix of the dimensional matrix. Once we solve this dimensional matrix and determine the rank of a matrix then we can actually ascertain the number of pi terms.

Let us say the number of variables are say 10 and we have got say rank of that matrix as 2 then number of dimensionless products possible for that particular phenomenon the consideration is equal to n-r that is 10-2, 8. So after having obtained the ascertain the number of pi terms and writing, by writing linear algebraic equation and by putting  $\pi 1$  to  $\pi 8$  in the matrix of solutions and satisfying those equations which are deduced based on the you know dimensional matrix coefficients.

We can actually write get the  $\pi 1 \pi 2$  so on to  $\pi 8$  in the given example, so buckingham's pi theorem is used for deducing dimensionless products and dimensionless and you know dimensional analysis does not reveal the form of relationship between dimensionless products.

(Refer Slide Time: 06:55)

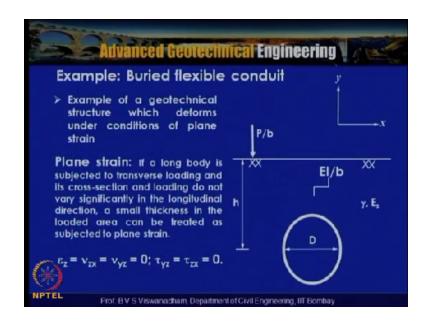


But correct use of dimensional products make parametric studies more efficient by revealing which variables are truly independent which variables are true you know in significant and also basis for the also forms the basis for extrapolating from one scale of observation to other scale of observation. So after once we get the pi terms let us say, we have got say set of 5 pi terms, so our similarity between model and prototype what we say is that.

The particular phenomenon which actually has been modeled is influenced by  $\pi 1$ ,  $\pi 2$ ,  $\pi 3$ ,  $\pi 4$ ,  $\pi 5$  this implies that  $\pi 1$  is a function of  $\pi 2$ ,  $\pi 3$ ,  $\pi 4$ ,  $\pi 5$  for similarity between model and prototype if you are actually not modelling at 1:1 scale then for 1:n model small scale model for  $\pi 1$  to be equal to, for  $\pi 1$  to be equal to be equal to model and prototype that is  $\pi 1$  and model to be equal to  $\pi 1$  in prototype.

We have to satisfy  $\pi 5$  in model is equal to  $\pi 5$  in prototype,  $\pi 4$  in model is equal to  $\pi 4$  in prototype so on up to  $\pi 2$  in model equal to  $\pi 2$  in prototype. If you are not able to achieve the complete similarity in satisfying all the  $\pi$  items then we say that the partial similarity is achieved. Then we need to investigate that particular effect of not modelling not satisfying this particular this thing have to be verified by experimental investigations.

(Refer Slide Time: 08:40)



So let us take an example, of buried flexible conduit so this is a example of a geotechnical structure which deforms under conditions of plane strain. So plane strain in the scene that if a long body is subjected to transverse loading and its cross section and loading do not vary significantly in the longitudinal direction.

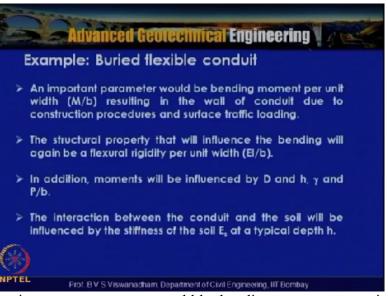
A small thickness in the loaded area can be treated as subjected to plane strain, so here in this particular slide what we are seeing is underground conduit having diameter D and h is the you know the embedded depth and EI/b is nothing but flexible rigidity of this conduit per unit width that for per b units of bits. And  $\gamma$  is nothing but the unit weight of the soil and Es is nothing but the soil stiffness or elastic modulus of the soil.

And P/b is the line load which is at a certain distance, if it is at this particular point it is right on top of that and P/b is at a certain distance and x and y are the coordinates in the x direction and y direction and perpendicular to this is say is the z axis. So the plane strain condition when we say that these conditions are satisfied for this type for example like retaining wall or slope of a high way embankment or railway embankment, embankment dam or levy action.

They are the examples of plane strain or a stiff foundation of a footing of a stiff footing of a typical foundation which extending for number of columns in one line and these are on a examples of plane strain. So  $\varepsilon z$ = the shear strain in zx direction, shear strain in yz direction will be equal to 0, that means that the strain in this direction that is zx plane and yz plane will be 0 and similarly the shear stress in yz direction shear stress in the zx direction will be 0.

So, only it will have in xy direction and you know deformations and as well as the strains will be there in this directions. So this particular structure is an example of plane strain structure so what we see is that the load P/b which actually causes you know in a bending moment per unit width let us say, so the depending up on the you know the loading you know the bending moment per unit width will per b unit widths will increase or decrease.

(Refer Slide Time: 11:22)



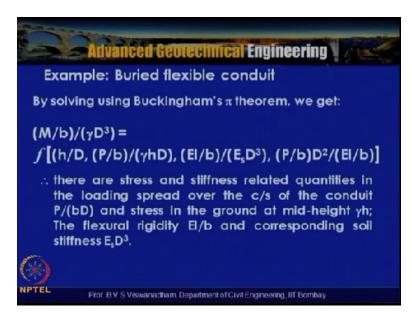
So let us say that an important parameter would be bending moment per unit width that is M/b resulting in the wall of the conduit due to construction procedures and surface traffic loading. So the example of say line load what we have considered is let us say a boundary wall and or a railway track. The structural property that will influence the bending will again be a flexible rigidity per unit width that is EI/b.

In addition moments will be influenced by diameter D that is the internal diameter, h that is embedded depth of a conduit  $\gamma$  that is the density surrounding the conduit, and P/b that is the loading which is applied. So the interaction between the conduit and the soil will be influenced by the stiffness of the soil that is Es at a typical depth h that is the interaction between the conduit and the soil will b influenced by the stiffness of the soil Es at a typical depth.

So what we have got is that bending moment that is M/b and we have parameters like EI/b it is a conduit parameter and loading parameters are nothing but P/b and again conduit parameter D and

soil parameters are h, that is the embedded depth and that is the geometric configuration and  $\gamma$  is nothing but the unit weight of soil and Es is nothing but the stiffness of the soil.

(Refer Slide Time: 12:49)



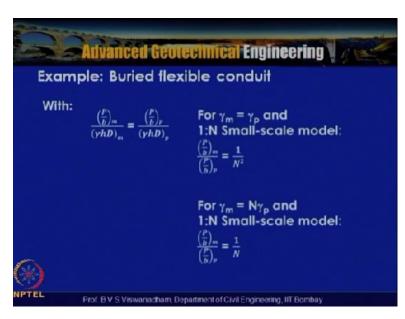
So by solving by using the buckingham's pi theorem and by writing dimensional matrix by influencing the variables and solving the matrix of writing the matrix of solutions we get M/b,  $\gamma D^3 = f [(h/D, (P/b)/\gamma hD), (EI/b)/(EsD^3), (P/b)D^2/(EI/b)]$ , so if you look into this, this particular arrangement has been made such a way that we can lead to some important discussion by using this problem.

There can be different arrangements are possible depending up on the type of repeating variables which we have chosen. But one solution to other solution can be transformed by doing the algebraic adjustments for the pi terms because if you are having a  $\pi 1$ ,  $\pi 2$ ,  $\pi 3$ ,  $\pi 4$  influencing the phenomenon. If these are  $\pi$  terms which are say dimensionless if  $\pi 1$  is  $\pi 10$ ,  $\pi 1/\pi 3$  is also dimensionless.

So depending up on the requirement by without changing the number of  $\pi$  terms with deduced originally we can transform and get the new  $\pi$  terms of choice. If you are having a particular variable and if that variable leads to be more than say if it is appearing in more than three  $\pi$  terms, if that variable leads to be eliminated can be eliminated by doing the adjustments what has been said.

But it has to be seen that the first pi term are the principal pi term used only once, so  $M/b/\gamma D^3$  which is function of this things, so here there are stress and stiffness related quantities in the loading spread over the cross section of the conduit P/bD and the stress in the ground at the mid height. For example, here this is  $\gamma$ h represents the vertical stress in the ground at the mid height of the, this thing.

And here this is the conduit stiffness and soil stiffness you know you can see that the conduit stiffness and soil stiffness are related here, and so that flexible rigidity EI/b and corresponding soil stiffness which is actually are related. So let u discuss further you know how we can actually deduce you know different relationships.



(Refer Slide Time: 15:16)

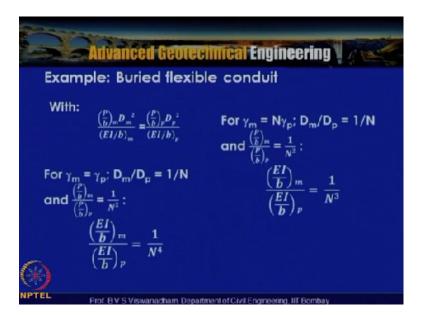
So here further the continuation of the solution for the example for buried flexible conduit now what we do is that we have said is that, when you have set of  $\pi$  terms and by for similarity you know so by  $\pi$  term let us say  $\pi$ 3 in model has to be equal to  $\pi$ 3 in prototype so if all these are satisfied then only the primary or principal  $\pi$  term that is where involving bending moment will be satisfied.

So with  $(P/b)m/(\pi hD)m=(P/b)p/(\pi hD)p$  in prototype so here we can actually get two situations what we said is that if we are not modeling at full scale you know then at 1:1 the (P/b)m=

 $(\pi hD)m$  both are same, you know in model and prototype are same. But when you are actually modelling at a scale different then 1:1 that is a small scale model 1:N, let us say the environment what we have is that same soil is used as that in the prototype with that we can say that  $\gamma m=\gamma p$  by with for  $\gamma m=\gamma p$  and 1:N small scale model.

What we get is that (P/b)m to (P/b)p=1/N<sup>2</sup> so that is nothing but what we have done is that by bringing this term here and bringing this term there  $\gamma m$  and  $\gamma p$  1, hm/hp=1/N Pm/Pp=1/N by substituting that in this we get (P/b)m/(P/b)p=1/N<sup>2</sup>. So m suffix indicates model and p indicates prototype which is nothing but  $1/N^2$ , let us assume that we have got a situation where  $\gamma m=N\gamma p$  is possible that means that there is a let us say there is a requirement which actually has come and where we are able to in a physical model we able to enhanced the unit weight of the soil by N times. So that means that if you are having you know  $20kN/m^3$  of a soil at by enhanced, with enhanced N we get  $\gamma m=20N$  and for 1/N scale model we get by substituting in a similar way with  $\gamma m/\gamma p=N$  we get (P/b)m=(P/b)p=1/N. So we can see that if you are having  $\gamma m=N \gamma p$  we are actually getting P/b)m/(P/b)p is 1/N.

(Refer Slide Time: 18:14)

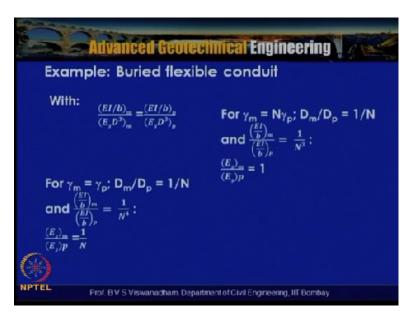


Now with P may considering another pi term so what we do is that once after having obtain the line load scale factor in model and prototype in with  $\gamma m=\gamma p$  for 1/N small scale model  $\gamma m=N\gamma p$  for 1/N small scale model. By considering another pi term which actually has got P/b and EI/b

terms by using this particular pi term and this relationship and for  $\gamma m=\gamma p$  and Dm/Dp=1/N and we have deduced in the previous slide.

(P/b)m=1/N<sup>2</sup>(P/b)p by substituting this we get (EI/b)m/(EI/b)p is  $1/N^4$  so please note that this is  $1/N^4$  that means that whatever we are having you know the value of flexible rigidity per b widths will be  $1/N^4$  that of in the  $1/N^4$  times of that so that means that if you are having EI/p value which has to be  $1/N^4$  times smaller. Similarly with  $\gamma$ m=N $\gamma$ p and Dm/Dp=1/N and Pm, (P/b)m P/b=1/N this is 1/N with that we will be able to get (EI/b)m= $1/N^3$  (EI/b)p, so now we have understood that flexible rigidity with (P/b)m/(P/b)p=1/N with that we will be able to get this particular scale factor.

(Refer Slide Time: 20:07)



Now what we do is that by using this you know term which involve flexible rigidity and flexible the soil stiffness term (EI/b)m/(EsD<sup>3</sup>)m=(EI/b)p/(EsD<sup>3</sup>)p by with  $\gamma$ m= $\gamma$ p and Dm/Dp=1/N and (EI/b)m/(EI/p)p=1/N<sup>4</sup> if you put this in the substitute in this what we get is that with  $\gamma$ m=  $\gamma$ p and small scale model and with the deduction what we deduced here for EI/b scale factor for 1:N scale model with  $\gamma$ m=  $\gamma$ p, we get the soil stiffness in model is 1/N times smaller than the soil stiffness in the prototype.

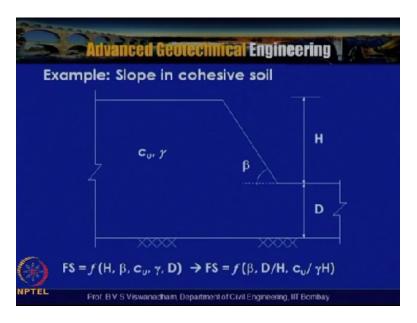
Whereas when you have like  $\gamma m=N \gamma p$  involvement with Dm/Dp=1/N we can actually get (EI/b)m /(EI/b)p as 1/N<sup>3</sup> and by using this we get the soil stiffness in model prototype=1. So this actually has got you know very high strong relevance that means that in modeling the particular

prototype behavior if you are actually having a stimulation of identical stiffness as that in the prototype.

The response of a particular structure for example in this case a buried flexible conduit in model so this implies that if you are actually having an environment like  $\gamma m = \gamma p$  and even for a small scale model we can actually maintain you know the same stiffness as that in the prototype. If you are actually having the same stiffness at that in the prototype that implies that the identical stress strain behavior of a you know in the prototype can be captured very well in a model which is actually tested in a small scale model which is reduced by 1/N tested at  $\gamma m = n \gamma p$ .

So this is very important as far as you know the physical model due to physical modeling particularly with the  $\gamma m=N\gamma p$  consideration point of view. So let us now consider an example like slope in cohesive soil.

(Refer Slide Time: 22:22)



And assume that we actually have got a saturated clay under untrained conditions so it actually has got undrained coefficient and  $\gamma$  also the unit weight of the soil and D is the depth below the base that is, this is base layer depth and H is the height of the slope and  $\beta$  is the slope inclination. So if you look into it the stability of a slope it actually depends up on the parameters like Cu,  $\gamma$ , $\beta$ h and d. So when you list out the variables we actually have got factor of safety is a function of h that is nothing but the slope height  $\beta$  that is the slope inclination Cu undrained coefficient  $\gamma$  and D. So by again by using either this can be done by using Rally's methods r by using buckingham pi theorem and while writing dimensional matrix if you are having terms like this we can actually use four cell length approach.

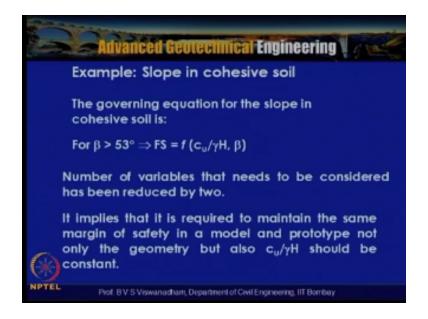
That means that force is expressed as MLT-2 the dimensions of force MLT-2 and L as L and the stress is nothing but  $F/l^2$  in unit weight is nothing but  $F/L^3$  and you know by using this you will get two algebraic equations and by solving them and we by writing matrix or solutions we can actually get relationship among the variables. Now here 1,2,3,4,5,6 6 variables are there and we get the rank of the matrix of the dimensional matrix will be 2.

So 6-2, 4 so 1,2,3,4, pi terms so in this out of this two are already dimensionless terms one is factor safety another one is  $\beta$  that is the angle or slope inclination. So here what it actually says is that the factor safety is a function of  $\beta$  D/h and Cu/ $\gamma$ H, now from the similarity point of view what we say is that factor of safety you know for factor of safety in model and prototype to be same if at all we are modeling this particular situation of a slope in cohesive soil.

The factor of safety in model and prototype to be same what we say or what we have to do is that  $\beta$  in model and prototype to be same, that means that the slope inclination model and prototype to be same. And D/H that is the ratio between the D that is the depth below the you know toe of the slope to the height of the slope, D/H ratio in model and prototype to be same.

If you and then  $Cu/\gamma h$  in model and prototype to be same, if all these pi terms are same in model and prototype for 1/N scale model then we can say that the factor of safety in model and prototype will be same. So if at all you know let us see by looking into the you know the different combinations we said that how this is actually possible.

(Refer Slide Time: 25:23)

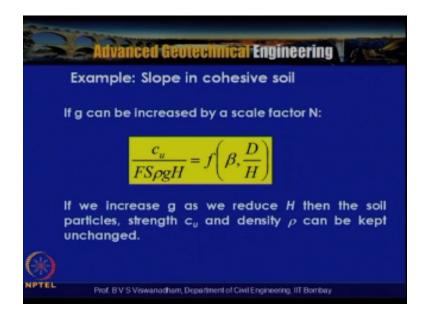


Now one thing we can actually look is that according to Taylor's 1948 theory the Taylor's actually has given for slope inclination greater than 53° we can say that it is independent of D/H factor depth of factor then you can write factor of safety=  $f(Cu/\gamma H)$  and  $\beta$ . So here the number of variables that need to be considered only two it implies that it is required to maintain the same margin of safety and model and prototype not only the geometry but also Cu/ $\gamma$ H should be say constant.

So this implies that for similarity between model and prototype for a we actually have to you know for maintain the same margin of safety in model prototype not only the geometry that is the slope inclination but also  $Cu/\gamma H$  should be constant. The  $Cu/\gamma H$  should be constant means how that is possible means let us say that we are actually have constructed a small scale model H is reduced by H/N.

If you look into its somehow let us say that the slope inclination is achieved then the Cu/ $\gamma$ h in model and prototype to be constant what it implies is that the for a slope which is reduced by 1/N times the Cu/ $\gamma$ H you know to be same what it says that the term to be same the Cu/ $\gamma$  has to be reduced by 1/N times. If that is reduced by 1/N times then only it is possible that you know it will be able to make you know the similarity possible.

(Refer Slide Time: 26:56)



So if you look into this if the g can be you know one of the alternatives if you see that you know by reducing the equation in this particular term for similarity to be achieved what we can say is that by maintaining identical unit weight as that in prototype like  $\gamma m = \gamma p$  and Cu in model and prototype let us say that is reduced by 1/n times then we can actually say by maintaining identical  $\gamma$  in model and prototype we can reduce Cu in the model and prototype 1/N times.

That means that the coefficient of a soil is reduced by 1/N times that in the prototype if you are able to do that when you reduce you know H/N and Cu/N then they get cancelled when possibility that they will be same. But you know by, you know this is you know topic to be discussed the stress strain behavior of a soil which is actually having a coefficient of 1/N times the coefficient of that in the prototype.

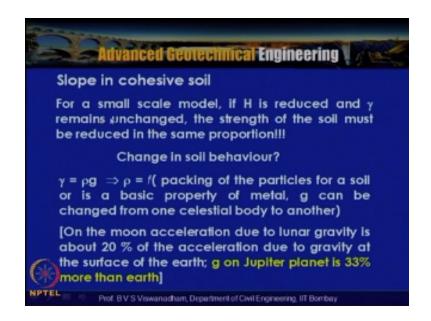
And actual coefficient of say Cu the both you know the stress strain behavior is drastically different for a, this can be explained through an example let us consider that in a prototype we are actually having a 50kPa of coefficient and we are actually trying to reduce this by say 5 times that means that a coefficient of a soil in the model is 10kPa so the stress strain behavior of a soil on a undrained conditions for you know yielding a 10kPa of coefficient and 50kPa of coefficient is different. So this implies that the change in soil behavior which says that, this implies that the change in soil behavior and the response of a model will not be you know as similar as that in the prototype. So the change in soil behavior cannot be you know accepted, but one thing you know

in this particular you know approach the scaling down of Ca implies that the change in soil behavior.

But another option is that we have discussed in buried conduit example is that making  $\gamma$  N times that means that the soil the self rate of the soil N times heavier that means that  $\gamma m=N\gamma p$  and maintaining identical coefficient as that in the prototype. If we are able to achieve a situation where in  $\gamma m=N \gamma p$  and Cu is identical in 100 we were with that also what we are saying is that the Cu/ $\gamma$  termed is 1/N times that of the Cu/ $\gamma$  term is reduce.

So with that we can actually say that Cu/  $\gamma$ H and model prototype will be satisfied, in that situations what is actually we are doing is that if we are not able to reduce the coefficient for satisfies the similarity point of view then another option another viable option it appears to be as that by enhancing the unit weight of a soil  $\gamma$ m=  $\gamma$ p with that we can actually maintain Cu/  $\gamma$ H in model prototype identical.

(Refer Slide Time: 30:07)



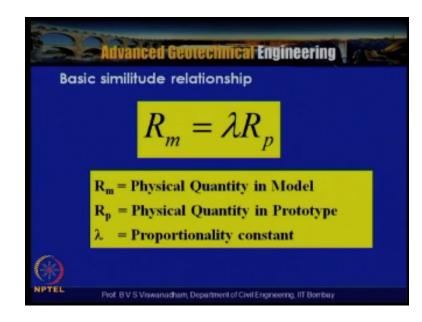
So this is explained in this particular slide so what we said is that for a small scale model if H is reduced and  $\gamma$  remains unchanged the strength of the soil must be reduced in the same proportion. So this implies that what we are actually discussing the change in soil behavior, another option what we actually said is that the enhancing  $\gamma$ ,  $\gamma$  is nothing but if you define you know this you know term  $\gamma$  unit weight of the soil as  $\rho g$ .

Where  $\rho$  is nothing but the mass density let us say in this case of soil mass density of the soil and g is nothing but acceleration due to gravity. So  $\rho$  which is mass density is a function of packing of particles for a soil or is the basic property of a metal and g can be changed from one sustainable body to another body. So by using this particular concept and assuming that you know the variation of g is possible.

We have an examples like on the moon acceleration due to lunar gravity is about 20% of the acceleration due to gravity and the surface of the earth. Similarly the Jupiter g on the Jupiter planet is 33% more than the earth, so by with the allied thinking we say that this  $\gamma m=N \gamma p$  is possible, where  $\gamma=\rho g$  whereby maintaining identical mass density as that in the prototype that is  $\rho m=\rho p$  and gm is actually say increased by N times we can say that gm=Ngp with that  $\gamma$  model and prototype the scale factor for unit weight  $\gamma m=N \gamma p$  is possible.

That means that provided the g sustained g is actually reduced to the model which is in this example a slope in cohesive soil with makes actually is possible for us to achieve that  $\gamma m=N \gamma p$ 

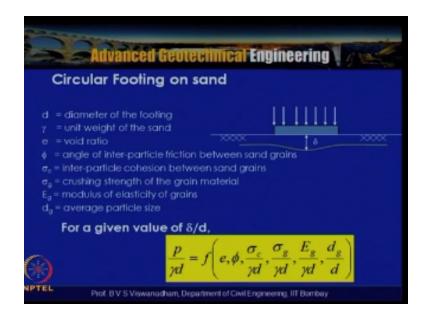
condition, with that what we said is that we the dimensionless term Cu/  $\gamma$ H in model and prototype will be identical without changing the soil properties.



(Refer Slide Time: 32:14)

So that means that when you are actually having you know any parameter which is actually influencing in the model and prototype, this you know it is governed by a relationship call Rm, Rm is nothing but physical quantity in model Rp is nothing the physical quantity in prototype,  $\lambda$  is the proportionality constant. So the relationship so the physical quantity can be time velocity, acceleration or force or it can be stress it can be strain.

So this established by you know the relationship need to be established for each and every variable so that for similitude to achieve between model and prototype which is not scaled which is not tested at 1:1. (Refer Slide Time: 33:03)



Now let us consider another example which is you know let us say circular footing on sand, so if you look into this we have got a footing of diameter D trusting on the sand having you know dry sand and  $\gamma$  is the unit weight of the sand and E is the void ratio and  $\phi$  is the angle of inter-particle friction between sand grains that is the angle of internal friction is  $\pi$ ,  $\sigma c$  is nothing but interparticle coefficient between the sand grains.

When the sand grains interact with each other at the inter-particle coefficient generated between the sand grains is indicated by  $\sigma c$ ,  $\sigma g$  is nothing but the crushing strength of the grain material that means that you depending up on the composition of the grain they have different crushing strengths so  $\sigma g$  is nothing but the crushing strength of the grain material.

Similarly depending up on the type of grain material we have different modulus of elasticity so eg is nothing but the modulus of elasticity of grains. Now by taking  $\sigma c$ ,  $\sigma g$  and eg as the material properties of the you know the grains which are actually involved we can, and then dg is something but the average particle size. So, now if you look into it if you are actually having a situation of model in this at 1:1 scale and by for a given value of  $\Delta/d$ .

We and by using the buckingham pi theorem we can get  $p/\gamma d = f(e, \phi, \sigma c/\gamma d, \sigma g/\gamma d, eg/\gamma d$  and dg/d) so here this is the particle size and this is particle size to the diameter and  $eg/\gamma d$  and that is nothing but the you know modulus of elasticity and unit weight of the soil multiplied by d diameter of the footing. Similarly we have got this terms like this, now for  $p/\gamma d$  to be model and prototype to be same what it says is that Dg/d in model to be equal to dg/d in prototype similarly

this  $\phi$  term has to be same in model and prototype and this  $\pi$  term has to be same in model and prototype.

And this pi term that is  $\sigma c/\gamma d$  in model to be same prototype and friction angle in model and prototype and void ratio the particular arrangement to be same in model and prototype. Now we can actually say that if you are a modeling the situation in 1:1 and all these pi terms will be identical. But if you are having a 1:N model and with  $\gamma m=\gamma p$  condition then what we have is that let us see what will happen when you wanted to compare this pi terms.

Let us assume that we are actually could able to achieve brought the same soil and void ratio and friction angle are achieved then E in model and prototype and  $\phi$  model and prototype are same. But being these are material properties and by identical  $\gamma$ .But diameter of the footing is reduced by d/ n so this term is not equal model  $\sigma c / \gamma d$  and model  $\sigma c / \gamma d$  model is not equivalent to  $\sigma c / \gamma d$  prototype which is nothing but  $\sigma c / \gamma d / n$ .

So this is different from you know what we are actually need to be obtained then  $\sigma c / \gamma d$  similarly Eg /  $\gamma d$  similarly dg/ d so if you look into is for similarity for look the deviation 1, 2, 3, 4, 5 terms are you know deviating from the you know similarity that means that you know we are actually having very well similarity as for as similarity is concern for with  $\gamma m = \gamma p$  and 1: for 1: n scale model.

If the similar situation let us say that by you know enhancing the gravity if you are able to impose a condition for the similar problem  $\gamma m = n \gamma p$  if your able to do  $\gamma n = \gamma p$  for a small scale physical model which is 1:n reduced with the 1/n times then what we can see that again with the same assumption of so with the  $\sigma c \sigma g$  and Eg as material properties we can say that  $\gamma$  is increased by n time and decreased by 1/n times  $\sigma c / n \gamma / d/ n$ .

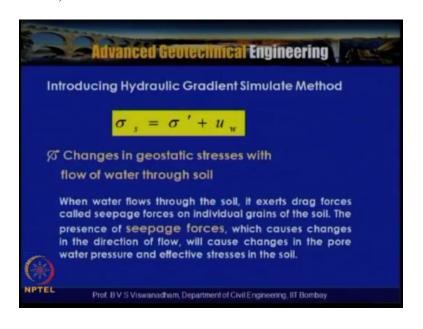
So they get cancelled  $\sigma c/\gamma d$  in model and prototype will be identical and  $\sigma g/\gamma d$  model prototype will be identical similarly Eg in model prototype in Eg /  $\gamma d$  in modern prototype will be equivalent but here being you know within a way or going into assumption that identical soil as that in the prototype that means identical soil as that in the prototype that means the particle sizes are constant and we are reduced in this case also d/l so here what we have is that 1, 2, 3, 4, 5 terms are identical.

So the strong you know similarity is achieved except one pipe term that is dg/ d/ l in case of  $\gamma m$  = m $\gamma$  p also we are you know we are not able to deal a capability not able to you know scale

down the particles that lead to you know an effect modern we call and we are going to discuss is called particle size effect so what we need to do is that how this particle size effect can be eliminated that is what within the diameter also if any parameter are any variable or dimension is product.

Deviates from the similarity then we have to see by maintaining you know as by satisfying certain conditions how that particular you know parameter is in significant in fleecing the you know particular phenomena let us say or making p/ $\gamma$ d in modern and prototype identical so with those you know this discussion we understood that circular footing on sand and slope soil and you know buried flexible conduit examples what we said that by for a small scale model which is not estimate 1:1 but we just tested 1:n it implies that you know  $\gamma m = m\gamma$  condition full fills and as actually got this superiority over  $\gamma m = \gamma p$  and 1:n scale physical model.

Now in order to enhance  $\gamma m = n \gamma p$  there also you know some attempts late 1960's by so this was actually you know introduced based on the you know total stress = effective stress + pore water pressure so if you look into this.

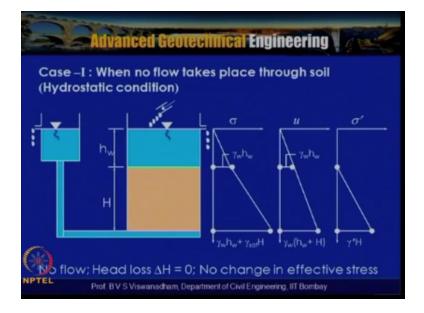


(Refer Slide Time: 40:40)

You know the change in the geo static stress with flow water through the soils at possible so introducing the hydraulic variance simulated method so here let us consider if water is actually flowing through the soil we know that it exists drag forces called see pages force on individual grains of the soil and the presence of the seepage force which causes changes in the direction of the flow will cause you know changes in the pore water pressure and effective stress in the soil.

So by using this concept 1969 as come out with a method called hydraulic gradient simulated method which actually has got you know as a possibility that  $\gamma m$  can be maintained as n  $\gamma p$  by doing that what actually we get is the stress and you know the stress strain behavior of the soil can be maintained identical as that in the prototype.

(Refer Slide Time: 41:03)



So let us consider the two cases, the one is that you know hydrostatic condition when no flow takes place in this particular condition as there is no head of lose that is  $\delta h = 0$  because the no flow condition then we actually have this is called hydrostatic condition where you have got you know this the total stress and this is the pore water pressure and this is the effective stress. Where

effective stress is nothing but  $\gamma$ ' h at this particular point and this particular point it is 0 so here also in this level it is 0 and then it is 0 at this point and here at this point the alternatives  $\gamma$ ' h.

So no flow head lose  $\delta h = 0$  and no change in the effective stress, so this is you know the case one. Case two, you consider where this limb is actually brought down by h and if you look in to this here because of this the water flows from you know downward direction. So the water flow is actually shown here in this case and h w is the height of water column and h is the thickness of the soil sample.

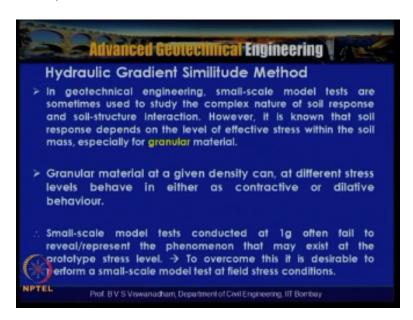
So we can actually write the total stress as you know nothing but  $\sigma$  which is here the down stress then the depth here we can write  $\gamma$  w h w and  $\gamma$  w hw +  $\gamma$  sat h but here what we can actually write is that this is  $\gamma$  w hw because water flows from this point to this point there is a head lost which actually takes place the pore water pressure drops by  $\gamma$  if there no flow condition what we have got is  $\gamma$  w = h w + h.

Now but it is dropped by a term which is nothing but -h that is nothing but  $\gamma \le x + h = h$ . now by taking total stress by computing for a effective stress that is total stress minus pore water pressure what we get is that  $\gamma' h + h \gamma \le x$ , so at this particular point we can actually write  $\gamma' h + h \gamma \le y$  and where the h is nothing but the head drop that is over a length h so hydro tic gradients which is actually you know it will be something like a triangle which is actually having a ordinate h here and over a height of h the slope of that triangle hypotenuse is I.

I is nothing but h / h the h / H, so by for h = substituting I time H we can write  $\gamma$ ' h + I h  $\gamma$  w, so if you see the downward flow of a water increases effective stress in soil. similarly in case three let us say if this limb is taken upwards then what we see is that effective stress decreases, so we by this in the hydro tic gradient symmetry method this particular that increase in the effective stress in the direction of flow is taken as you know as a concept and then particular method is developed.

So further developing on this method so we know that in the hydro tic in geo technical engineering a small scale model test are sometimes used to study the complex in nature of soil.

(Refer Slide Time: 44:18)



Response and soil structure inter action but is well known that now the soil response depends upon the level of effective stress within the soil especially for granular materials. If you look into that for granular materials the soil response depends on the level of effective stress within the soil mass. So granular materials at a given density can at different stress levels behaviors either has a contractive or dilative behavior.

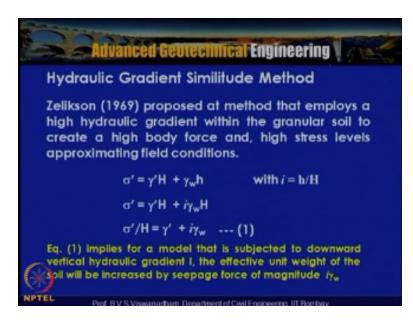
So they can actually have contractive behavior or dilative true behavior, so the small scale model test conducted one g often fail to reveal or represent the phenomenon that may exist at the prototype stress level, so that is what actually we have discuss from the problems like buried fluctual conduit or from the footing resting on the sand we actually have said the small scale model test conduct at one g often fail to reveal or represent the phenomenon that may exist at the proto type stress level.

To work on this it is desirable to perform small scale model test at field stress conditions and for that what we said is that  $\gamma m = n \gamma p$  is requires, say let us say that if you are actually having at a

depth h we are having say unit weight of soil say  $\gamma$  then the prototype we are having vertical stress at a depth h is total stress is nothing but  $\sigma v = \gamma h$ . now if the same depth which is actually model in 1:n scale model with  $\gamma m = \gamma p$  and h m / hp = 1/n the stress there is  $\sigma v$  model =  $\gamma h / n$  that is 1/n times smaller than the stress which is actually there in the prototype.

Now the same situation when it is say modeled with  $\gamma m = n \gamma p$  or nothing but  $\gamma = \rho m g$  condition with that what we can actually get is that with  $\gamma m = \gamma p$  and h m / hp = 1/ n we get  $\gamma$  is  $\sigma v \text{ model} =$  that is  $\gamma m$  that is n  $\gamma p x h / n$  which is nothing but  $\gamma h$ , so what we actually say is that the stress in model prototype will be identical if you are actually having you know the  $\gamma$  which is actually n times the  $\gamma$  in prototype. Now we will see how that is actually simulated by Zelikson.

(Refer Slide Time: 46:51)

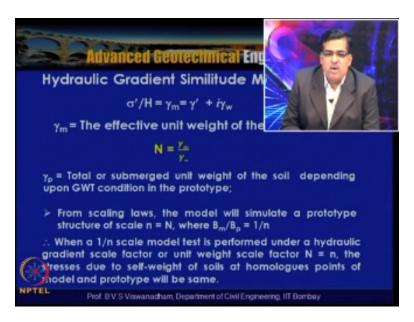


So Zelikson propose method that employs a high hydraulic gradient within the granular soil to create a high body force and high stress levels approximating the field conditions, so we have discussed that self weight forces and the cps forces they are treated as body forces and the based on that concept and you know the direction what we actually made  $\sigma' = \gamma' h + \gamma w h$  with I = h / h what we can say is that  $\sigma' = \gamma' h + I \gamma w h$ .

So here with  $\sigma' = \gamma' h + \gamma w h$  and by substituting I = h / h what we can write is that  $\gamma' h + I \gamma w h$ / dividing throughout by h we can write  $\sigma' / h = \gamma' + i \gamma w$  if you look in to it the effective unit weight the  $\sigma' / h$  is called as effective unit weight is increased by term I  $\gamma$  w is nothing but the cps force magnitude it is increased by the term which is equivalent to the cps force magnitude in the direction of the flow.

So this equation one implies for a model that is subjected to downward vertical gradient I the effective unit weight of the soil will be increased by cps force of the magnitude I  $\gamma$  w, so this is from the basis what we actually discuss we what we have reduce, we said that this  $\gamma' \sigma' / h$  is written as  $\gamma$  m that is the effective unit weight of the soil which is increased by the cps force term that is I  $\gamma$  w.

(Refer Slide Time: 48:36)

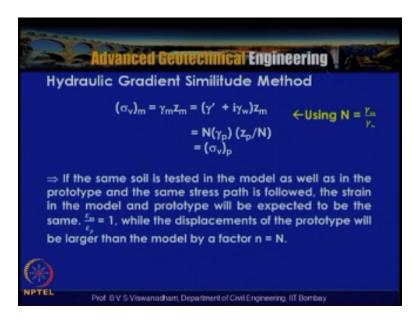


Now by writing  $\sigma' / h = \gamma m = \gamma' + I \gamma$  w and now we can write the unit weight scale factor at hydraulic or hydraulic gradient scale factor as  $\gamma m / \gamma p$  this is  $\gamma p$ , so  $\gamma p =$  total or submerged unit weight of the soil depending upon the ground water condition of the in the prototype. So  $\gamma p$  is nothing but the total or submerges unit weight of the soil depending upon the ground water conditions in prototype.

And for saturated condition it can be  $\gamma p = \gamma' p$  so from scaling laws the model will simulate a prototype structure scale n: n where bm / bp = 1 / n, for so from scaling laws we can say that the model will simulate a prototype structure for scale n = n where bm / bp = 1/n, when a 1/n scale model test is performed and a hydraulic gradient scale factor or unit weight scale factor n capital N = n the stress to self weight of soils and homologues points of model prototype will be same.

So when a 1/n scale model test is performed and a hydraulic gradient scale factor or unit weigh scale factor capital N = n the stress due to self weight of soils and homologues points of model in prototype will be same.

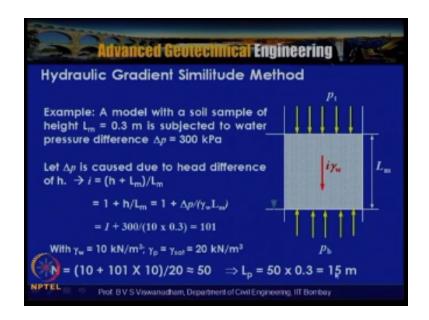
(Refer Slide Time: 50:03)



So that means that  $\sigma$  v model =  $\gamma$  m z m where when we substitute the  $\gamma$  m that is the effective unit weight of soil as  $\gamma' + I \gamma$  w z m we can write as using  $\gamma$  m /  $\gamma$  p = n we can write that n  $\gamma$  p x z pbn that is nothing but  $\sigma$  v in model =  $\sigma$  v in prototype. So with the same soil is tested in the model as well as in the prototype and the same stress path is followed the strain in the model prototype will be expect to be same that is  $\varepsilon$  m /  $\varepsilon$  p = 1 while the displacement of the prototype will be larger than the model by a factor n = N.

So with that what implies is that by maintaining a hydraulic gradient which is you know my higher with differential pressures between top and bottom of the soil, we can actually ensure that identical stress has that in the prototype. So in this slide we can actually see that a model with the soil sample of height lm say 0.3 m.

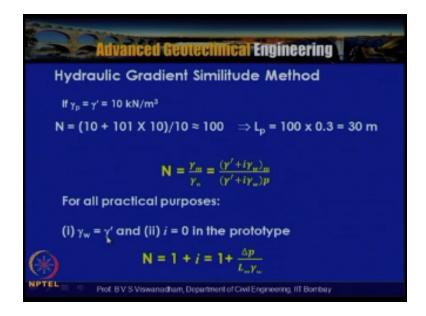
(Refer Slide Time: 51:06)



Is subjected to water pressure difference of say  $\Delta p = 300$ kPa, pt is the pressure on the top of the soil pb is the pressure on the bottom of the soil so pt – pb  $\Delta p = 300$ kPa so if  $\Delta p$  is caused due to the head difference of h then I = we can write that let us say h is the head which is you know equivalent to that 300kPa of differential pressure, and h + lm / lmk is nothing but the hydraulic gradient.

So what we can write I = 1 + h / Im and we can write for  $h \Delta p / \gamma$  w that is the differential pressure by  $\gamma$  w is the unit weight of water in to multiplied by 1 m is nothing but the length of the sample, so we can get that  $I = 1 + 300 / 10 \times 0.3$  where  $\gamma w = 10^3$  and it is 101 that I hydraulic gradient is about 101 with  $\gamma w = 10^3$  and  $\gamma p = \gamma$  saturated is say 20km<sup>3</sup> we can write and we can get n as n + 101 x 10 / 20 with that we will be able to get 50 scale factor there is implies that if this conditions are maintain if the differential pressure of pt – pbs 300kPa is maintain for a with  $\gamma m = \gamma p = 20km3$  we can say that  $lp = 50 \times 0.3$  that is 15m of you know the length of the soil which is actually represented in the field. So this is you know interesting you know technique wherein this is possible.

(Refer Slide Time: 52:44)



And similarly when with  $\gamma p = \gamma$ ; = 10km3 with n = you know  $\gamma p = \gamma'$  that is with 10km3 under submerged unit weight conditions it actually says that it is 100 = 100 that means it is about 30m, so but the relation n =  $\gamma$  m /  $\gamma$  p if you write that  $\gamma' + I \gamma$  w in model  $\gamma' + I \gamma$  w in prototype for all practical purposes  $\gamma w = \gamma'$  that is you know the submerged unit weight and unit weight of water to almost identical and I = 0 in the prototype then in that case we can write n = 1 + I = 1 +  $\Delta p$  / lm x  $\gamma$  w so this is the expression what we actually use for reducing the hydraulic gradient method.

So in the hydraulic gradient similitude method which can be applied for granular soils particularly here you know the conditions are that you know here we are actually do the application like testing of footings on sand and particularly testing you know footing on sand with either concentric load or eccentric loads or recently it has been applied for testing of piles in calculating the uplift capacity of the pile in the but in sandy soil saturated soil prop vile wherein you know so here one of the demerits is that you know the control of this particular  $\Delta$  p differential pressure need to be maintain and second thing that the surface has to be horizontal under those conditions this possible that if you are able to do that it actually simulates and satisfies the condition that  $\gamma$  m = l  $\gamma$  p and identical stress in model prototype can be achieved.

So the applications involve you know can be applied for piles in imbed in sandy soils or footing resting on sand or anchors embed in sand particularly to the vertical or incline pull out, so these are the possible applications of this technique, and this technique actually has got merits and

demerits in this application. So this you know technique with this particular limitations not extended further but the technique what we have discusses is that by manipulating the change in the g that enhancing the g and reducing for a small scale model and that technique what we have defined or what we have named as the centrifuges' physical modeling.

So the centrifuges based physical modeling technique evolved as very powerful tool for geo technical engineers for understanding the geo technical behavior of structures. So centrifuges based physical modeling technique is a physical modeling technique in which a small scale model that is reduces by 1: n subjected to a rotation about a vertical access in a horizontal plane. By doing this then identical stress is model I prototype can be maintain by with that the stress field can be maintain as those in the prototype this actually is possible to understand and you know the behavior of the number of geo technical structures.

# NPTEL NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LERNING

## NPTEL Principal Investigator IIT Bombay

Prof. R. K. Shevgaonkar Prof. A. N. Chandorkar

> HEAD CDEEP Prof.V.M.Gadre

> **Producer** Arun Kalwankar

**Project Manager** M. Sangeeta Shrivastava

Online Editor/Digital Video Editor Tushar Deshpande

> Digital Video Cameraman Amin Shaikh

> > Jr.Technical Assistants Vijay Kedare

**Project Attendant** Ravi Paswan Vinayak Raut

Copyright CDEEP IIT Bombay