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NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

CDEEP
IIT BOMBAY

ADVANCED GEOTECHNICAL ENGINEERING

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IIT Bombay
Lecture No. 55

Module-7
Lecture-6 on Geotechnical physical modeling

Welcome to lecture series in advance should geotechnical engineering course we are in module 7 geotechnical physical modeling lecture number six.

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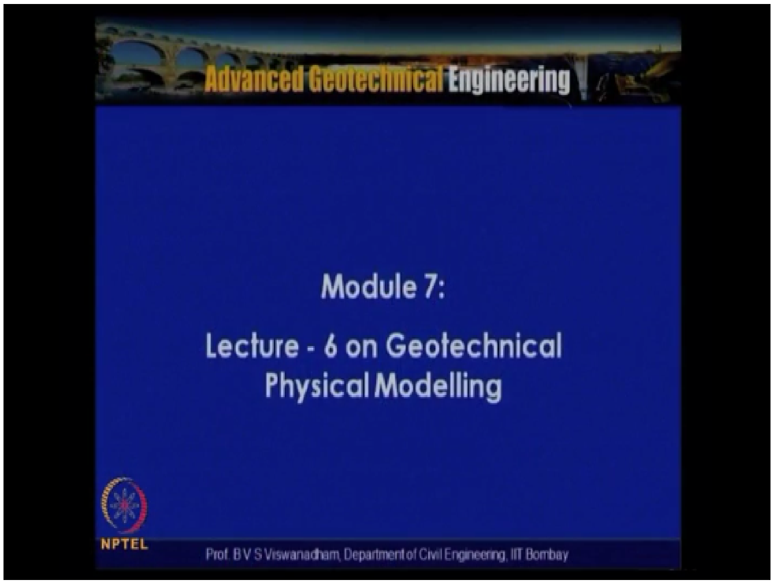

Advanced Geotechnical Engineering

Module 7:

Lecture - 6 on Geotechnical Physical Modelling

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Module seven lecture number six on geotechnical physical modeling so we have been discussing about the scaling loss in centrifuge basis of physical modeling and that we have discussed about the how we can scale down the force work and energy and while the scaling down energy we said that it can be in the form of a potential energy or in the form of kinetic energy so in the in the form of potential energy for that what we said is that considering the definition of potential energy PE normally explicit the energy lost by following a mass through. The height H so this is the P = mg H so for getting the scale factor.

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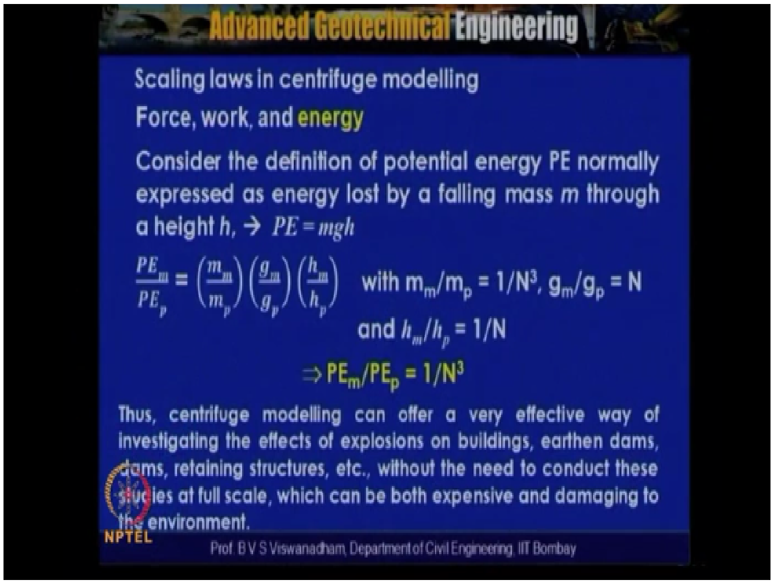

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Scaling laws in centrifuge modelling

Force, work, and energy

Consider the definition of potential energy PE normally expressed as energy lost by a falling mass m through a height h, → $PE = mgh$

$$\frac{PE_m}{PE_p} = \left(\frac{m_m}{m_p}\right)\left(\frac{g_m}{g_p}\right)\left(\frac{h_m}{h_p}\right)$$ with $m_m/m_p = 1/N^3$, $g_m/g_p = N$ and $h_m/h_p = 1/N$

$$\Rightarrow PE_m/PE_p = 1/N^3$$

Thus, centrifuge modelling can offer a very effective way of investigating the effects of explosions on buildings, earthen dams, dams, retaining structures, etc., without the need to conduct these studies at full scale, which can be both expensive and damaging to the environment.

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For centrifuge based physical modeling when we take for compare for model and prototype we get you know PE in model and PE and prototype as 1 by n cube so that means that in this a diffuse modeling basically can offer a very effective way of investigating the effect of explosions on buildings at the dams and dams retaining structures etcetera the third the need to contact these studies in the full scale so if you wanted to conduct this test you know.

In the full scale first of all they are expensive and then also can cause damage to the and non-vented so let us see some examples practical examples with some typical problems so we said that the potential energy model and prototype is1 by n cube.

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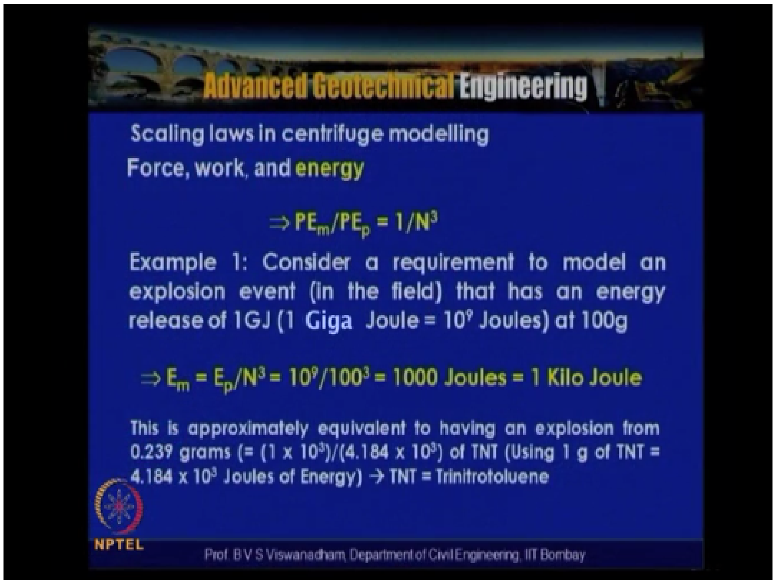

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Scaling laws in centrifuge modelling

Force, work, and energy

$\Rightarrow PE_m/PE_p = 1/N^3$

Example 1: Consider a requirement to model an explosion event (in the field) that has an energy release of 1GJ (1 Giga Joule = 10^9 Joules) at 100g

$\Rightarrow E_m = E_p/N^3 = 10^9/100^3 = 1000$ Joules = 1 Kilo Joule

This is approximately equivalent to having an explosion from 0.239 grams (= $(1 \times 10^3)/(4.184 \times 10^3)$) of TNT (Using 1 g of TNT = 4.184×10^3 Joules of Energy) → TNT = Trinitrotoluene

So in example 1 consider a requirement to model an explosion event in the field that has an energy release of 1 Giga joule is nothing but10 to power of 99 joule at 110 were nine joules and then we need to model this event at 100 gravities now you know the energy which is required to which is planned for you know the release is about 1 Giga joules that is one into 10 to power of 9 joules so the e model is equal to e suffix P divided by n cube using the scaling considerations we have deduced it in the previous slide then 10 to power of 9 / 100 cube we get thousand joules that means that one kilo joules.

So this is an approximately equivalent to having an explosion from 0 . 39grams of TNT the TNT is nothing but try not true to trinitrotoluene so 1 gram of TNT religious an energy equal to 4.1 8 4 into 10 to power of 3joules of energy so one gram of TNT is equivalent to 4.184 kilo joules so by using this relationship we can calculate by using this relationship we can calculate 1into 10 cube / 4.1 4 into 10 raise to 3 so with this what we get is that .2 39 grams of you know TNT which is required and if this is ignited at100 gravities.

The equivalent energy release it will be equivalent to the top1 Giga joules that is equal to 10 to power of -9 10 ever of nine joules in the field.

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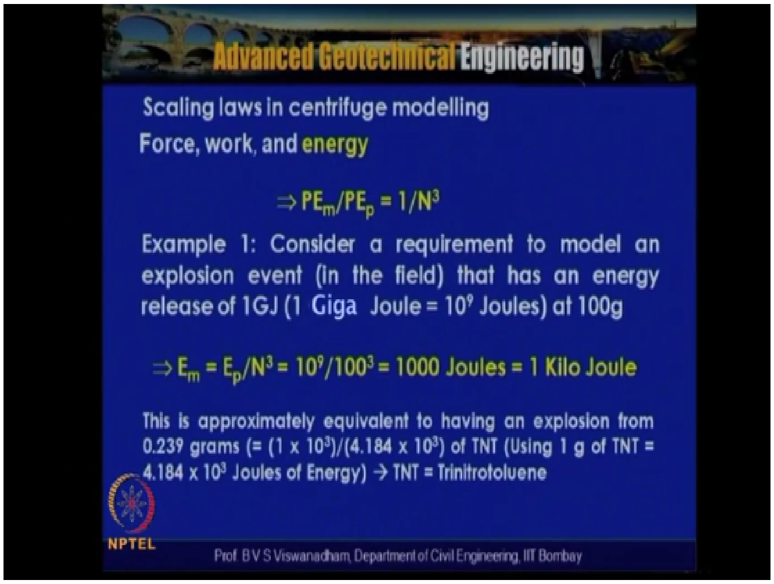

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Scaling laws in centrifuge modelling

Force, work, and energy

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The another example let us say that we consider in a 3.5 grams of TNT including detonator used for modeling an explosion event at 50 gravities so energy released in the centrifuge model is nothing but 3.5 into 4.18 4 into 10 raise to 3 Jones so we are in we have14.6 4 4 into 10 10 raise to 3 joules that means that 14.6 for 4 kilo joules is the energy religion in this interface model but equivalent energy in the field is nothing but 14.6 4 ,4 into 10 raise to3 into 50 raise to 2 3 that is that scale factor 50 cube when you multiply you will get 1.83 intoox9 Joules .

Which is nothing but 1.83 Giga joules so you know this implies that the effect of explosions can be modeled on geotechnical structures using quite small charges in a centrifuge model to simulate the blast with extensive energy release in the reality so before that you know let us also consider in a typical example when you have got some2.5 grams of TNT culinary nitro tale reignited at 200 gravities.

That means that this may not be really possible but as far as the for example is concerned if you look into it then let us look what will be the equivalent energy and equivalent mass. In the field so consider you consider.

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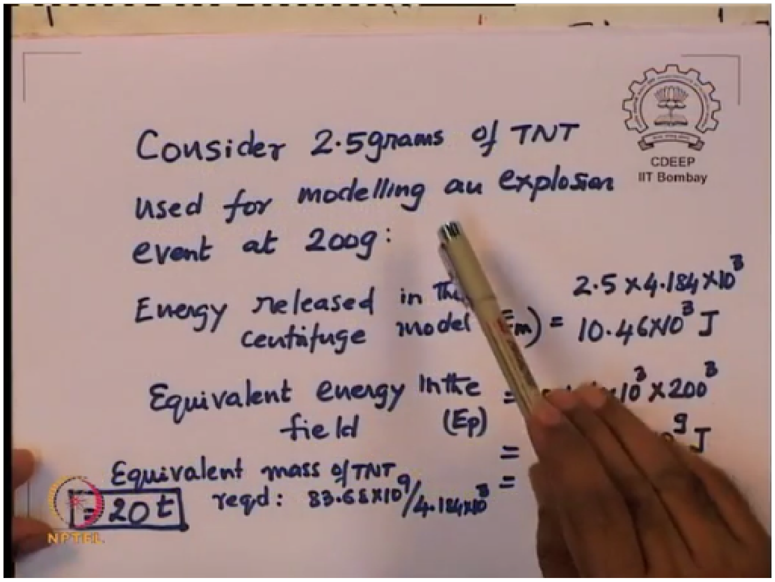

Consider 2.5grams of TNT
used for modelling an explosion
event at 200g:

Energy released in the centrifuge model $(E_m) = 2.5 \times 4.184 \times 10^3 = 10.46 \times 10^3$ J

Equivalent energy in the field (E_p) = $10^3 \times 200^3$
^{9} J

Equivalent mass of TNT reqd: $83.68 \times 10^9 / 4.184 \times 10^3$ =
= $\boxed{20t}$

Let us say 2.5 grams of TNT and this is the example and then we said that we will conduct this experiment at200 gravities the visit 200 gravities energy release it in this interface model which is nothing but 2.5 into 4.1night for into and those two three jobs so which is nothing but n 0 point 4 6kilo joules now equivalent energy in the field is 10 to 10.4 6 into 10 raise to 3into 200 raise to 3 so because the gravity level is very high we have considered here 200 times the gravity with that what will what we get is that 83.6 8 into 10 to power of 9 joules.

Which is nothing but 80 3.68 joules1 k kajal is equal to 10 to power of 9 joules now equivalent mass of TNT if it is whatever is required wanted to estimate in the field that comes out to be 83.6 8into 10 raise to nine divided by 1 gram of TNT is equal into 4.18 4 into 10raise to 3 joules so by using that equal and see what we get is that 20,000 kg are 20 tons that is 20 tons of mass is required to you know in the field of 20 tons of TNT is required to generate an energy which is equivalent to energy release of 80 3.68 Giga joules.

Also so this explains that you know these what are the three examples we have considered these excise that the effect of explosions can be modeled and geotechnical structures using quite small charges in a centrifuge model to simulate blast with an extensive energy release in the reality but however while you know considering these things in the city few basic physical modeling the safety of the equipment and other issues need to be addressed now after having discussed about the force work energy scale factors.

Now let us look into you know different aspects of scaling loss you know what we have said is that with the GM by GP is equal to n that.

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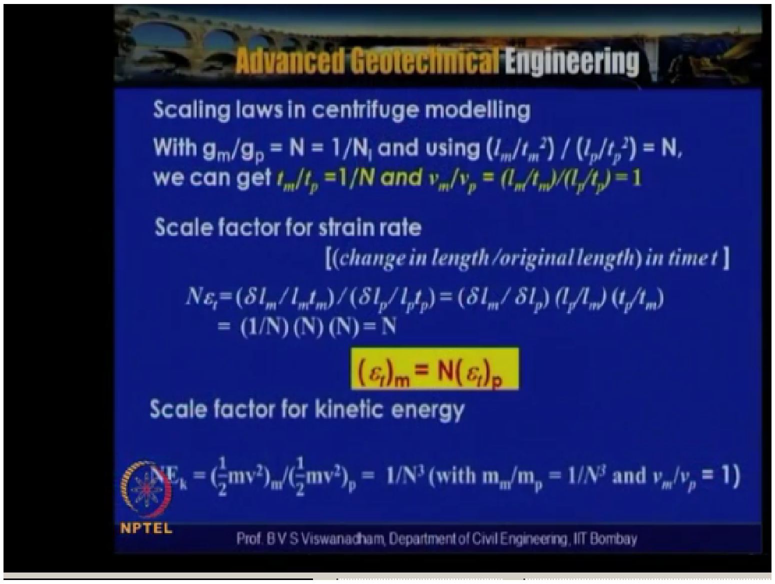

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Scaling laws in centrifuge modelling

With $g_m/g_p = N = 1/N_l$ and using $(l_m/t_m^2) / (l_p/t_p^2) = N$, we can get $t_m/t_p = 1/N$ and $v_m/v_p = (l_m/t_m)/(l_p/t_p) = 1$

Scale factor for strain rate

[(*change in length / original length*) *in time t*]

$$N\varepsilon_t = (\delta l_m / l_m t_m)/(\delta l_p / l_p t_p) = (\delta l_m / \delta l_p)\,(l_p/l_m)\,(t_p/t_m)$$
$$= (1/N)\,(N)\,(N) = N$$

$$(\varepsilon_t)_m = N(\varepsilon_t)_p$$

Scale factor for kinetic energy

$$NE_k = (\tfrac{1}{2}mv^2)_m/(\tfrac{1}{2}mv^2)_p = 1/N^3 \text{ (with } m_m/m_p = 1/N^3 \text{ and } v_m/v_p = 1)$$

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when γ M becomes N γ P that a GM by GP the scale factor is Yin that means that which is nothing but if we call a length scale factor and suffix L as LM by LP = 1 by n then that is nothing but GM by GV is = n = 1 by n L so using you know the fundamental principles of acceleration we can write GM by GP as LM by DM square divided by LP by TP square V = n so with that what we get is that TM by DP = 1 by n suppose.

You know if you are actually looking at physical modeling small scale physical modeling at normal gravity and if GM by GP =1 and if you substitute in LM by DM square divided by LP by TP square =1 we get t TM by TP the scale factor for time is equal to root n that is the square root of n that is 1 by square root of n that TM by TP you will get it as 1 by n power of off now in case of centrifuge based physical modeling with the GM by GV = n = 1 by NL then with that we are you know by equivalence see.

What we get is that LM by LM by TM square by LP by TP square = n with that we can get TM by TP = 1 by N and by using velocity definition fundamental definition where LM by TM by what we are written is that V suffix N we have written as L M by TM and VP LP by TP and if you substitute for TM by TP = 1 by n LM by RP = 1 by n so here what we get is that one that means it the velocities are same as that in the field VM BM = VP and the time is you know it takes 1 by n timeless time and this is for you know external loading is concerned.

That means that a loader a force when it is exerted and this scale factor it will be observed that for the weight forces are external loading the scale factor for centrifuge basic physical modeling for this phenomenon it works out to be to TN by TB = 1 by N now the scale factor for the strain rate for example when we wanted to apply you know certain loading at a certain you know time then you know we can define the strain rate change in length by origin length in time T.

So that means that we can write that is nothing but Δ L by Δ l by l into time T so by for writing the scale factor for strain rate we can write as n epsilon suffix T as Δ L M divided by LM divided by Δ L P by L PTP with that you know by simplifying this we get Δ L M by Δ P into L P by L M into TP by TM now substituting for TP by TM = n LP by L M = Δ L M by Δ p =s when the length is reduced by 1 by n times the small change in length also is reduced by 1 by n times so if that 1 by n into n into n you will get n.

So that means that the strain rate in model is n times the strain rate in the prototype there so the stranger in the model is n times the strain rate in the prototype so the scale factor for kinetic energy so in the previous lecture we have introduced just a parameter kinetic energy suppose this is an example like you know if a ship impact is actually planned in a centrifuge based on physical modeling at a certain gravity then you know we have to subject the block to the certain you know impact is created by some moving mass.

So further to the scale factor for kinetic energy is given as n epsilon any k = half MV square in model to ratio to of MV square to prototype so which = 1 by n cube because mass is actually buy em ,em ,em ,em by em p = by n cube and vm by vp = 1what we have got here so with that the kinetic energy also is scaled as 1 by n cube and so as far as you know the weight force and external loading is concerned what we have noted is that velocities in model and prototype for one and the time scale factor is tame by TP = 1 by n and though this scale factor for the strain rate is working out to be epsilon suffix T in model = n times epsilon sub 60 in prototype.

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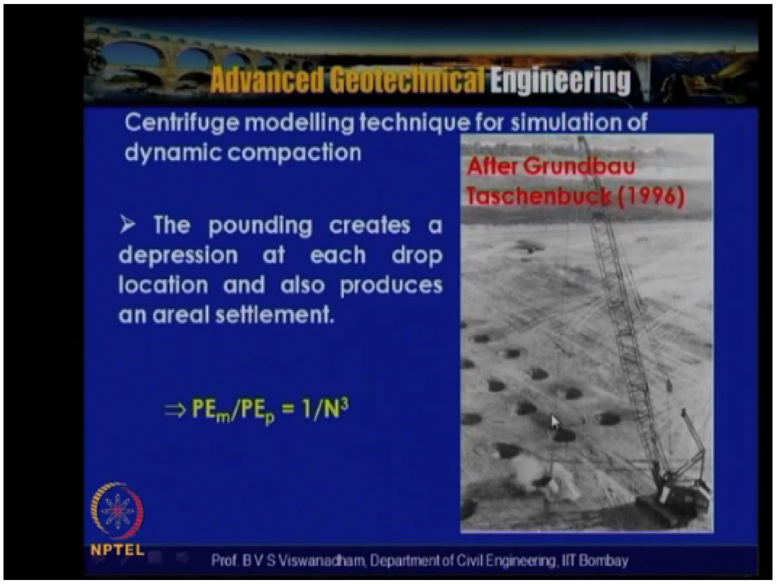

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Centrifuge modelling technique for simulation of dynamic compaction

After Grundbau Taschenbuck (1996)

- The pounding creates a depression at each drop location and also produces an areal settlement.

$\Rightarrow PE_m/PE_p = 1/N^3$

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Now another example for another example for you know the energy scale factor so we are actually looking into this you know we know that the pounding are a dropping of a tamping of a weight onto the loose sandy soil or sand e strata is one of the methods for improving the ground one of the methods for improving the ground so in this slide what we have shown is that you know the application of centrifuge modeling technique for simulation of dynamic compaction so dynamic compaction is a technique.

Which creates a depression at each drop location and also produces an areasettlement so when we do this in a grid fashion as it is shown in this you know photograph we're in a known weight is dropped from a known height so we are in the these weights are in the range of 10tons to 30 tons and the dropping heights are in the range of 10 meters to 30meters because it depending upon the you know the availability of the crane so in this if you wanted to model this let us look how the you know. The modeling can be done and how the impact velocity is compared.

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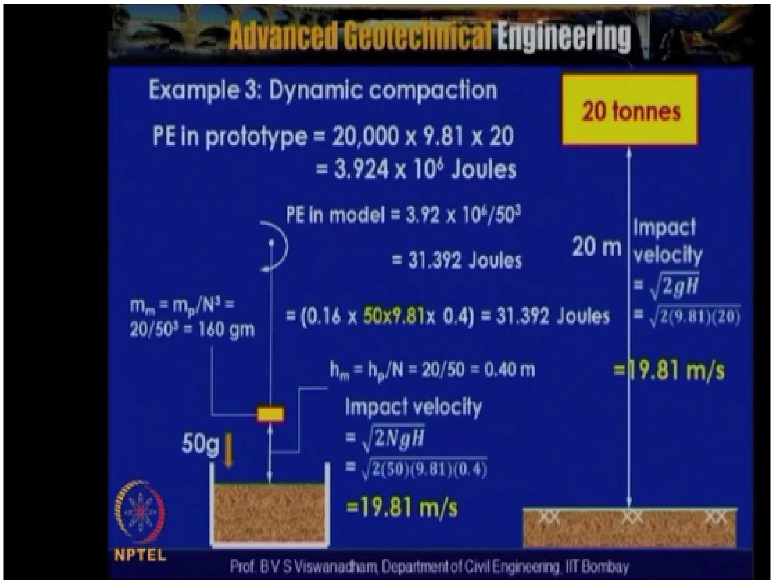

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Example 3: Dynamic compaction

PE in prototype = 20,000 x 9.81 x 20

= 3.924 x 10^6 Joules

PE in model = 3.92 x $10^6/50^3$

= 31.392 Joules

= (0.16 x 50x9.81x 0.4) = 31.392 Joules

$m_m = m_p/N^3 =$
$20/50^3$ = 160 gm

$h_m = h_p/N$ = 20/50 = 0.40 m

Impact velocity
= $\sqrt{2NgH}$
= $\sqrt{2(50)(9.81)(0.4)}$
=19.81 m/s

Impact velocity
= $\sqrt{2gH}$
= $\sqrt{2(9.81)(20)}$
=19.81 m/s

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So in this example the dynamic compaction let us consider in the field a 20 tonnes of weight is actually dropped on to the soil mass which is shown here a 20, 20 tonnes of the weight dropped onto a soil a loose sandy soil and they're dropping height is 20meters so the potential energy in the prototype which is nothing but 20,000that is kg into 9.81 into a height is 20mg H so with that what we get is that 3.92 4 into 10 to power 6 joules and the potential energy in the model which is nothing but p.m. in by in model = PE p by l cube so 3.92 e210 26 / 50 q with that what we get is that 31.3 92 joules.

So this is the potential energy which is actually modeled in the centrifuge so that means that we consider a small model which is reduced by 1 by n times so what we have done is that we have produced the same soil as that in the type and the model is rotating about vertical axis in a horizontal plane what you looking is the top view so this is the 50 gravities you know is subjected to this model so this is the small mass so in order to calculate.

What will be the weight of the mass or weight of the you know tamper which is required to be released at 50 gravities so mass of the amper is nothing but MP by n cube with that 20 x 50 cube what it works out tome 160 grams of tamper if it is released from a height of set of T by 50 is about you know 40 centimeters.

So when you lease from height of 40 centimeter on to this thing it creates you know the so-called energy which is nothing but 0 .16 that is you know the weight in cages 160 grams divided by 1000 into the gravity level is not one gravity it is 50 to 9.81 so 59.8 one in 2.4 is the dropping height so we bet what we get is that 30 1.39 to joules from this scaling laws also we got the same figure.

Now you know if you look into the impact velocity the impact velocity is nothing but how do you obtain is that by equating you know potential energy with the kinetic energy of MV square =mg H we get V = root 2. So in the prototype the impact velocity in the in the field you know when it when we drop 20 tonnes master my height of 20 meters that the impact velocity is nothing but root over 2gh with that 2 into 9.81 into 20 when you get 19.8 1meter per second in the model which is at 50 gravities in the small-scale model at 50 gravities.

The impact velocity is nothing but root over 2 into n into g into h with that what we get is that 19.8 1 meter per second so here in this particular example what we have noticed is that how you know the enhanced gravities can be used for modeling dynamic compaction and we also have seen the application of you know the energy scale factor and the you know merits of using this technique for you know the understanding the phenomena of these structures subjected to this type of energies is discussed.

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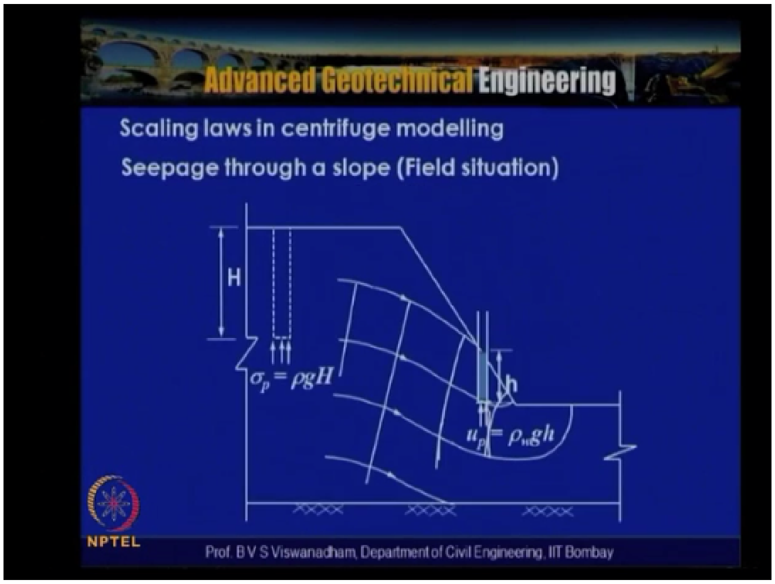

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Scaling laws in centrifuge modelling

Seepage through a slope (Field situation)

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Now after having seen you know the energy scaling loss and as you know salient applications let us try to look into you know so I you know these reducing the scaling loss when CP is actually having to happening three slope or when the consolidation is happening that means this when the sea page and you know these consolidation phenomenon basically they are the diffusion events that means that when a flow occurs in the field in a slope when there is a you know head difference between you know let us say that the flow can actually happen from up streamside.

To downstream side if there is ahead takes place over a length here so consider a typical slope in the field situation wherein we have got certain difference of the head where the water is flowing from you know these are the flow lines which are actually shown here and these are the equip-potential lines which are actually shown here a curvilinear squares and at a point H below the top surface of the slope the σ, P is nothing but ϱ; into g into H because ϱ is nothing but the mass density of the soil.

In the slope and the G is normal gravity that is 9.81into meter per second square into H is the height now similarly at a given point on a typical flow line we can see that the you p is equal to rho w GH the pore water pressure in the prototype is equal to of small height say H if you put a standpipe assume that there is a H which is the H which is actually measured then rode WG h that is the mass density of the water and times the gravity into H so if the similar situation is modeled then there is a possibility that we have to when you retain the same shape and then you know.

This is created by inducing the seepage by putting you know a constant source of water on the upstream side and maintaining that head of the water then it is possible for us to you know reduce these you know the steady-state seepage conditions.

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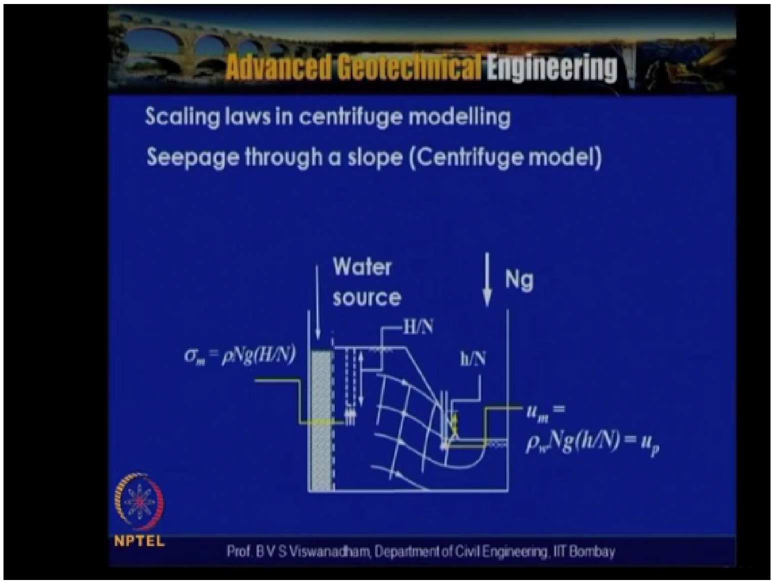

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Scaling laws in centrifuge modelling

Seepage through a slope (Centrifuge model)

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These when we have this steady state seepage conditions which are actually simulated in the model let us assume that this model is subjected to a you know rotation about a vertical axis and I said you know vertical axis in horizontal plane such that you know the n gravities are imposed to a model which is reduced by 1 by n times of the field situation which was actually shown in the previous slide then what we can say is that you know this is the slope with this configuration these flow net which is represented here.

But because of the radial acceleration field there can be you know minor you know influence the influence on the flow rate with curvilinear lines because of the you know the acceleration gravity field that is because of the radial acceleration field so except from that the pressures which are actually there in model and prototype shall be identical so let us look again at the point here in the total stress which is nothing but σ, M = ϱ into NG into H by n.

So we are in this is equivalent to you know the prototype stress similarly the water pressure which is actually measured at the same point ha Malaga's point which is actually shown in the field situation UM = ϱ W into NG into H by n which is also nothing but UP so impact we have to understand we can see that the physical distances are small physical distance are small the pressures are identical and pressures are identical.

So let us see how the time is actually you know factored or how the time scale factor is our arrived in the you know the you know said fuels based physical modeling and also see how we can actually model the you know for seepage pressure and you know also how the time factor x 6 scale factor for the time is actually moderate so in order to do that you know first let us look into you know how what is seepage force and what is seepage pressure and then by applying the you know fundamental definitions.

We can reduce the scaling laws so that in this particular in the slide what we are actually trying interested in that reducing the scale factor for the you know the seepage force so for seepage force.

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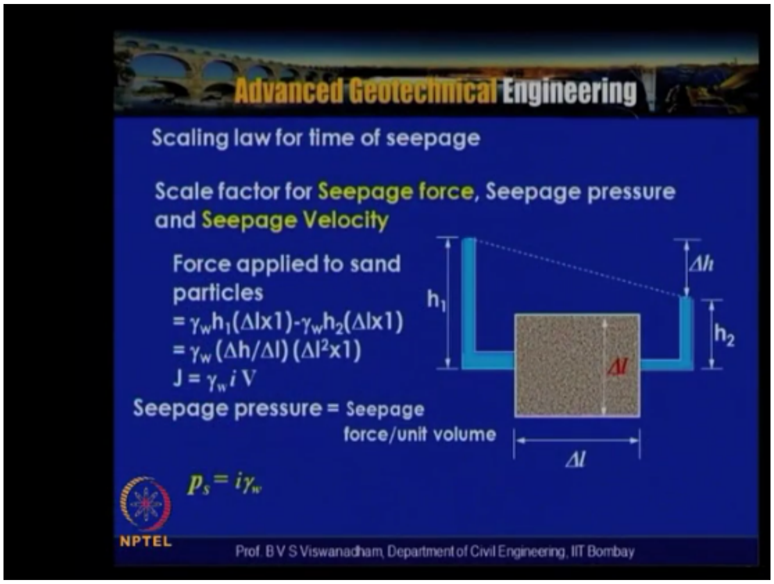

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Scaling law for time of seepage

Scale factor for Seepage force, Seepage pressure and Seepage Velocity

Force applied to sand particles

$= \gamma_w h_1 (\Delta l x 1) - \gamma_w h_2 (\Delta l x 1)$

$= \gamma_w (\Delta h / \Delta l) (\Delta l^2 x 1)$

$J = \gamma_w i V$

Seepage pressure = Seepage force/unit volume

$p_s = i \gamma_w$

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So we also said that you know when we do similar to D modeling we have linear similar to and dynamic similitude and kinematic signature so the dynamics of signature is nothing but you know the force in model and prototype has to have a certain scale factor so for you know in this case of say for the warning loss in centrifuge based physical modeling where the σ, M by σ, P =6 1 σ, dash M by σ, Dash P = and UP = 1.

Then in that case the force will factor that is NF and suffix F has to be you know n ▿ square is equal to 1 by n square so that means that whether it is a seepage force the relatives of eight fours or whether it is an external load or whether it is due to certain you know type of activity thecae factor in order to fulfill the dynamic similitude similarity.

The scale factor the force enough has to be n l square so with better understanding let us try to look into how far in order to maintain that enough is equal to null square for dynamic similitude then other factors will be subjected to variation so considered here a small element of soil where the flow is actually occurring within the flow rate and this is the one flow line this is the another flow line so here perpendicular to this there are the this is one equip-potential line and this is another equip-potential line assume.

That this length is ▵ Land this length is also ▵ L that means that perpendicular to this plane wave the flow is occurring the area is ▵ l into 1 and the volume of the element is nothing but ▵ L into ▵ L into one that is that we have considered per meter length perpendicular to plane of this figure this is point a and this is point B at a the head of water is h1 at point B at of what with h 2 and h 1 greater than h2 so the flow actually occurs from point A to B and h1 - h2 = ▵ H.

Now what we do is that we calculate the force applied to sand particles and that is obtained as γ W into h1 into ▵ l into 1 that means that γ W into h1 is the pressure acting on the area through which the flow is occurring is that ▵ l into 1 that is the γ w into h1 into Δ l into 1 and similarly on the at point B γ W 2 h2 γ W into H 2 into ▵ l into 1now by simplifying this what we do is that γ W into h1 - h2 that is ▵ H into ▵ L.

Now what we do is that we write γ W into ▵ H by ▵ L into ▵ L square into 1 so read of the terms we have done so writing I = ▵ H by ▵ L and v = ▵ L square into 1 so what we have got the expression for C pressure force is that J = γ W into I into V so I into V of γ W into V is called as the weight of the fluid phase that means that then the seepage force is equal to I hydraulic gradient into W F that is the weight of the fluid phase so the seepage pressure is nothing but defined as seepage force per unit volume in that case seepage pressure P suffix s = I γ w what we have derived is that I γ W so this occurs in the direction of the flow.

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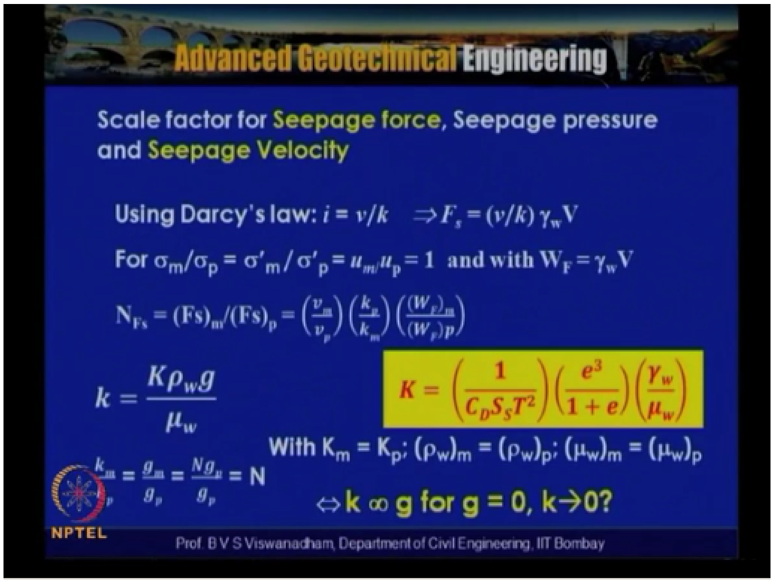

Scale factor for Seepage force, Seepage pressure and Seepage Velocity

Using Darcy's law: $i = v/k \Rightarrow F_s = (v/k)\gamma_w V$

For $\sigma_m/\sigma_p = \sigma'_m/\sigma'_p = u_m/u_p = 1$ and with $W_F = \gamma_w V$

$N_{Fs} = (Fs)_m/(Fs)_p = \left(\frac{v_m}{v_p}\right)\left(\frac{k_p}{k_m}\right)\left(\frac{(W_F)_m}{(W_F)p}\right)$

$$k = \frac{K\rho_w g}{\mu_w}$$

$$K = \left(\frac{1}{C_D S_S T^2}\right)\left(\frac{e^3}{1+e}\right)\left(\frac{\gamma_w}{\mu_w}\right)$$

With $K_m = K_p$; $(\rho_w)_m = (\rho_w)_p$; $(\mu_w)_m = (\mu_w)_p$

$$\frac{k_m}{k_p} = \frac{g_m}{g_p} = \frac{Ng_p}{g_p} = N$$

$\Leftrightarrow k \infty g$ for $g = 0, k \rightarrow 0$?

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Now what we do is that by adopting considering the Darcy's law you know the Darcy's law states that V=KI and the another condition is that for the Rangers of the you know the hydraulic gradient whatever you applied the flow has to be in the laminar regime so we know that the flow can be in the laminar regime or it can when the velocities are very high it can go into transient and then turbulent zone even when in the normal practice when the flow is taking place through coarse-grained soils or gravel the laminar regime cannot be guaranteed.

So using the Darcy's law I = V by K you know what we can do is that we substituted fact that a seepage force is equal to γ w v so by writing v by ka into γ W into V and by using this for σ M by σ P = σ dash m by σ Dash P = UM by AP = 1 and with γ W into B V= WF there is weight force then we can write you know the scale factor for sea page force as seepage force in model and prototype is equal to V M by K VP KP by km.

Because it is the this at this point and then WF in model WPF in prototype so this is what actually be how cut so by using the you know the previous discussions the force weight force can be scaled on by 1 by L square but we have two terms which is actually not known to us and that is nothing buttoned is a pyramid of the soil so prebuilt of the soil is actually defined as the property of the soil and which is actually he's with which the water can flow through the soil.

So but we found that the K which is nothing but coefficient of permeability our Darcy's quotient of permeability is home to every function of K that is called absolute permeability or intrinsic permeability row w that is the mass dash of the fluid or permeate and G is nothing but the gravity at which this flow is occurring and mu that is the new w that is the flow of that is the dynamic viscosity of the permanent or the water so now k absolute permeability is found to be you know according to forcedly Carmen equation.

We can say that k is nothing but a function of soil Skelton and wherein k is nothing but 1 by c d so series the thing the shape factor and SS is nothing but specific surface area and t square is nothing but the tortilla city so this 0 is nothing but when water is flowing through the solids this is nothing but the length are the actual path taken by the water to the imaginary length that is length of the sample so that is nothing but the tortillas path is you know is about 1.4 14 times you know the length.

What we assume L for you know for bran for granular soils so what we have seen is that k which is actually function of you know these shape factor whether it is angular rosebay angular or specific surface area that means that whether it is actually having large specific surface area are you know for example for it many have got place actually have got higher specific area so they have the low permeability and T that is the total velocity and also the void ratio eq by 1 + e and then γ W that is the mass density of the unit weight of the fluid and mu mu W that is the dynamic viscosity fluid now.

When we have K which is actually intrinsic permeability or absolute permeability which is a function of soil skeleton when you are actually representing the same soils skeleton as that in the prototype in this interface model then with km is equal to KB we can we can say that km is equal to KP and mass density of the water in model and prototype identical and then we kiss trustee of the water and a model and what would happen identical so with that when you say that when you substitute what you get is that k is proportional to G that is when the G increased by M been ten times in a small scale model in 1 by n times model then you know k = KN by KP =

GM by GP = GP by GP = n so this indicates that the permeability you know n times that of the prototype but it actually leads to you know a question.

That when you say that k is proportional to G that indicates that at zero gravity does it mean that the soil is impervious that is that means that k 10 20 so this is a you know some you know the conflict or a what you can say they are see which is you know comes out with this you know particular you know understanding is that when you use take by this expression k = K ϱWG by GW it implies that when other factors are same that K proportion to G so that the permeability scale factor is box out to be km = n KP that means that the soil.

Which is having a probability of 1 into 10 to power of- 9 meter per second and it appears to have a permeability of n times you know 10 to power of minus 10 meter per second if n is equal to 100 it is something like 10 to power of -7meter per second that means that a clay soil will send tend to have permeability as that of silty soil so this you know need to be understood and how this conflict can be addressed we shall look into it so in order to do this you know let us look once again you know the cut the definition of hydraulic render.

Itself hydraulic gradient itself so if you are actually saying you know this V=K I when I is equal to H by L so with that what we can write is that k is nothing but K ϱ WG x UWE to H by L .

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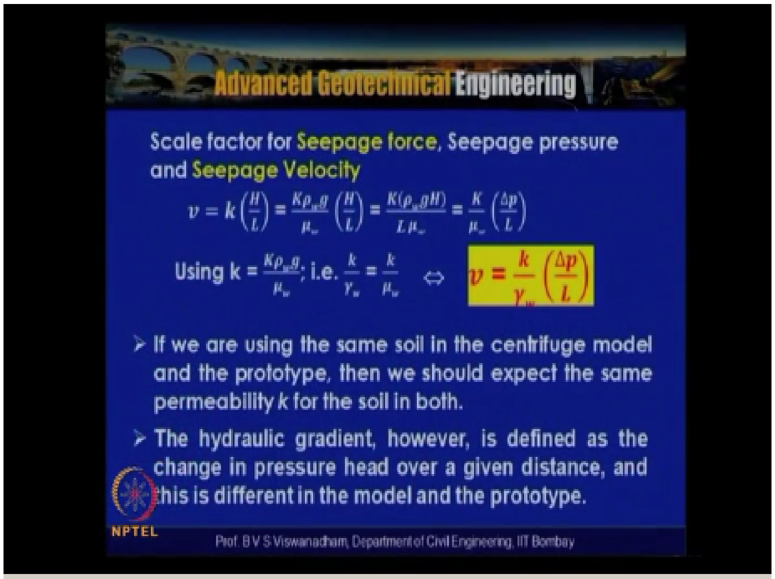

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Scale factor for Seepage force, Seepage pressure and Seepage Velocity

$$v = k\left(\frac{H}{L}\right) = \frac{K\rho_w g}{\mu_w}\left(\frac{H}{L}\right) = \frac{K(\rho_w g H)}{L\mu_w} = \frac{K}{\mu_w}\left(\frac{\Delta p}{L}\right)$$

Using $k = \frac{K\rho_w g}{\mu_w}$; i.e. $\frac{k}{\gamma_w} = \frac{k}{\mu_w}$ $\Leftrightarrow$ $\boxed{v = \frac{k}{\gamma_w}\left(\frac{\Delta p}{L}\right)}$

- If we are using the same soil in the centrifuge model and the prototype, then we should expect the same permeability *k* for the soil in both.
- The hydraulic gradient, however, is defined as the change in pressure head over a given distance, and this is different in the model and the prototype.

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So here by rearranging these terms what we do is that this ϱ WG h u v when you put it within the parentheses as shown here and K ϱ WG h divided by l mu w so by writing k by mu w and ϱ w GH is nothing but γ W into Hand is nothing but you know H is the pressure the head drop so we can write like it k by mu w into Δ p by 1 Δ p by l is that pressure drop our preference pressure drop or a great length l that is nothing but K by mu W into there is nothing.

But a pressure gradient or pressure drop over a length L where L is the length of the sample through which the flow is actually taking place now so with this you know it actually comes out that this also you know lead to one interesting you know direction if you look into it we can write k = K ϱ WG x mu W now by registering the terms k is nothing but the coefficient of permeability of the soil in the laboratory divided by G.

When we take K cave by ϱ w g as γ w here and this k capital k that is nothing but the absolute permeability divided by mu w so k by γ W divided by capital K by BW with that by using this and by you know because absolute ability is known it is measured in Darcy's and it is a function of soil skeleton and which actually has got a units of meter square but you know to in order to you know operate you know determine the permeability or a velocity at the you know seepage velocity in the velocities of the models what we can do is that we can actually use this replace this with K by γ W so when we sub substitute for K by γ W for K by capital K by UW with K by γ W we can write V Darcy's you know discharge velocity is equal to small K that is the coefficient of permeability divided by γ W into Δ P by n.

So in a given centrifuge model if you are able to measure a pressure difference between two points and the over a length L then we can actually calculate what is the no discharge velocity by knowing thecoefficient of permeability of the soil in the laboratory so this K is nothing but coefficient of permeability of the soil determined in normal gravity in the laboratory γ W is nothing but is unit weight of the soil that is a unit weight of the water k is nothing.

But the coefficient of permeability in the laboratory which is determined either by falling head test or constant a test and γ w is nothing but the unit weight of water so by knowing Δ P by L we can actually calculate you know discharge velocity and we know that this charge velocity and seepage velocity is you know this you know seepage velocity is the actual velocity which is actually passing through the grains so that VS= to V by n very end is nothing but the porosity so for a given soil skeleton when the porosity identical.

Then we can say that the seepage velocity also can be obtained by using this expression so if we are using the same soil in the center face model and prototype then we should expect that same permeability K for soils in both so by enabling the modifications and by defining the hydraulic gradient as Δ by L and this you know the conflict which was you know you know which was introduced k is proportional to g does it mean that k = 0 at you know the you know at zero gravity is can be arrested like this so by defining the hydraulic gradient.

You know as you know some change in pressure head or a given distance and this is different in the model intro tonight if we say that the hydraulic gradient is different in modern prototype because here the lengths reduced by length is reduced by 1 by n times that is that physical distance is reduced by 1 by n times but the pressure change in pressure head between model prototype are identical so the hydraulic gradient however is actually defined.

Now if you define this hydraulic gradient as you know as a change in pressure over a given distance and this difference is actually and if it is treated as different in modern prototype this can be addressed so let us look know this can be looked into that means that I am by IP which is what we say that Δ P by L in model and prototype.

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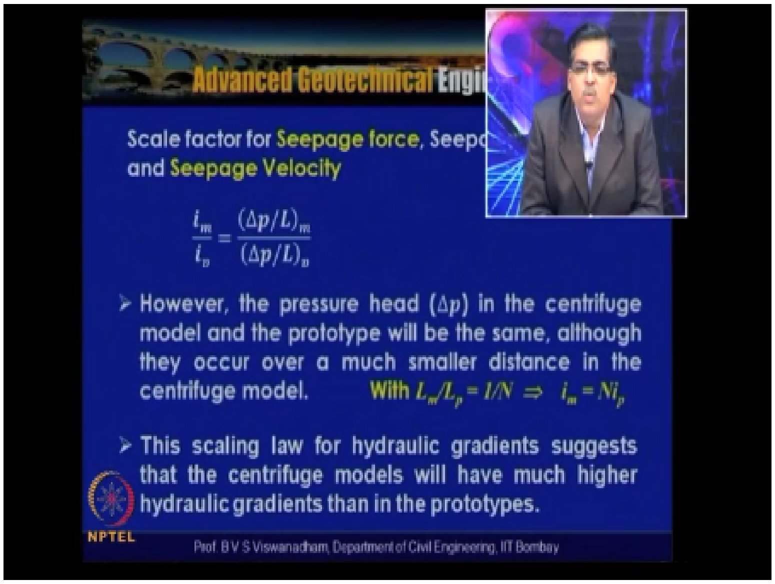

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Scale factor for Seepage force, Seepa
and Seepage Velocity

$$\frac{i_m}{i_p} = \frac{(\Delta p/L)_m}{(\Delta p/L)_p}$$

- However, the pressure head (Δp) in the centrifuge model and the prototype will be the same, although they occur over a much smaller distance in the centrifuge model. With $L_m/L_p = 1/N \Rightarrow i_m = Ni_p$
- This scaling law for hydraulic gradients suggests that the centrifuge models will have much higher hydraulic gradients than in the prototypes.

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Now if you look into if pressure head Δ P in this interviews model and prototype will be the same then although the curve or a much smaller distance in this edifice model and with the LM by LP = 1 by n so what we can write is that I m is equal to NI p when we say that I m = NI p when we say that v = k I that is the K I be that I when we define as a change in pressure over a length L so when we that you can say that I M = n IP then we can actually have the scaling law of k.

Which is you know maintained as identical as that in the protein that is that km = KP that means that the only the pressure over a length L you know is n times that in the prototype so that means that the scaling law for the hydraulic gradient actually suggests that the synthesis models will have much higher hydraulic gradients than the prototype so if you are having a hydraulic gradient of one then it can be you know ten at you know ten times gravity.

So with this definition if you look into that for a the grout I doubt gradients are high they say this caring law whatever we have actually discussed over hydraulic gradient with the discussion what we had in the previous slide this suggests that the scaling law for hydraulic gradient suggest that the centrifuge models will have much higher hydraulic gradients than in the prototypes.

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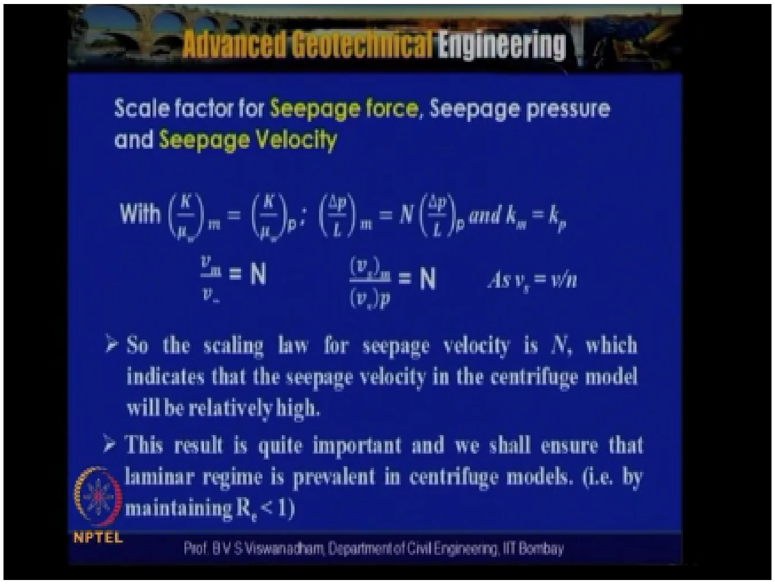

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Scale factor for Seepage force, Seepage pressure and Seepage Velocity

With $\left(\frac{K}{\mu_w}\right)_m = \left(\frac{K}{\mu_w}\right)_p ; \left(\frac{\Delta p}{L}\right)_m = N\left(\frac{\Delta p}{L}\right)_p$ and $k_m = k_p$

$$\frac{v_m}{v_-} = N \qquad \frac{(v_s)_m}{(v_s)p} = N \qquad As\ v_s = v/n$$

- So the scaling law for seepage velocity is N, which indicates that the seepage velocity in the centrifuge model will be relatively high.
- This result is quite important and we shall ensure that laminar regime is prevalent in centrifuge models. (i.e. by maintaining $R_e < 1$)

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Now with the definitions whatever we have discussed we can write that k by mu W in model = K by U W in prototype and Δ BL model is equal to n times Δ P by L prototype that means that Δ P by L is nothing but I m = NIP and k m is equal to KP so here we are actually maintaining the km = KP and with that you know what we get is that the velocities the discharge velocity is V M = n V P that is there n times in the model so as the physical distance are small.

And the velocities n times you know VP and similarly the seepage velocity as VS = V by N and with the processes are identical for a given soil skeleton in model in prototype so VS in model is equal to NVSP so the scaling law Percy Paris velocity is yen which is indicated the seepage velocity incentives model will be relativity I so the seepage velocity in this edifice models will be relatively high n times that of in the prototype this result is actually quite important you know we need to see that you know.

We shall ensure that laminar regime is prevalent in service models so for that you know when you say that the Reynolds number when it is you know when we define Reynolds number the Reynolds number can be defined as ϱ VD by mu where ϱ is the mass density of the permanent or fluid V is the velocity and d is nothing but you know the effective particle size.

Let us say let us say TD 10 I am UW is nothing but the dynamic viscosity so when we set this you know when we calculate this threshold values and except when you actually have got you know the very coarse-grained particles then what it says is that for if you say that RE less than 1 according to 1979 you know V act for flow of water through soils it says that you know for flow.

To be laminar the Reynolds number of in the in soil in row an Reynolds number have to be less than or equal to1 so when we take you know RE = 1 then we can calculate the threshold you know the g levels then we found that the g levels are actually for when you have got some coarser particles only it says that it should not have you know you know the G level more than 45 or so but when you have got the range of the particle model soils.

Which are actually used in field basically passing that G tables which are actually required for going you from laminar for crossing the laminar regime is you know very high that means that the G levels are very high they are actually beyond the Scopes of the scope of the centrifuge equipments which are actually available in the world now so that means that by all means it is possible for us to ensure the laminar regime ʌ regime for the type of velocities what we actually occur because of the virtue of the IM =NI P with that a BM = VP is you know occurs.

So because of that you know this so-called issue is important but you know when you look into this case factor for you know Reynolds number four same soil in modern prototype when we have this the Reynolds number scale factor works out to be n times Reynolds number in the prototype Reno's number in model = n times Reynolds number in the prototype so this Reynolds number with what we have done is that because VVM = n VN VP so because of that what will happen is that you know the soil d 10 model is equal to D 10 in prototype.

And when we have got identical pore fluid in model and prototype we can say that the Reynolds number is equal to n times you know summer in prototype but the question is that if you want the Reynolds number identical as that in the prototype then you know two ways one we can think of one is to you know scale down the particles that means that d 10 in the prototype in the model is written in the prototype bio n that means that when he was actually scaled down.

The particles that means the scaling down of the gradation is one option the second option is that it to increase the you know the dynamic viscosity of the fluid that means that if you are able to increase the dynamic viscosity fluid then by say let us say in a n 1 by n times reduce to a model we can say that the dynamic viscosity is increasing by n times so that means that we have if you have a higher viscous fluid there is a possibility.

That you know Reynolds number and model prototype can be maintained identical so but however the interaction of the fluids particularly with the high viscous fluids with saturation is all types of fluids all types of soils like you know silty sand clay soils is very difficult so and also from the scaling discussion and with the requirement of threshold gravity levels it is you know not required are not mandatory to change the pore fluid as far as the seepage phenomenon is concerned particularly.

When we are actually investigating seepage of water seepage of water through embankment dams or you know certain geotechnical structure so what though the velocities are n types this result is actually important and we also have to see that this was the we what we saying is that when we say a Reynolds number is equal to one less than one or so we say that the laminar regime is you know to them more or less is you know satisfied now after having scaled down or the seepage force and see pays you no pressure and then.

Let us look into you know deduce what is the stein scale factor now we said that from the definition whatever we have that velocity in modern that is the Darcy's discharge velocity in model is n time to decide velocity in prototype so from the fundamental definition of velocity we can write TM by TP = L M by VE BM = NP by VP.

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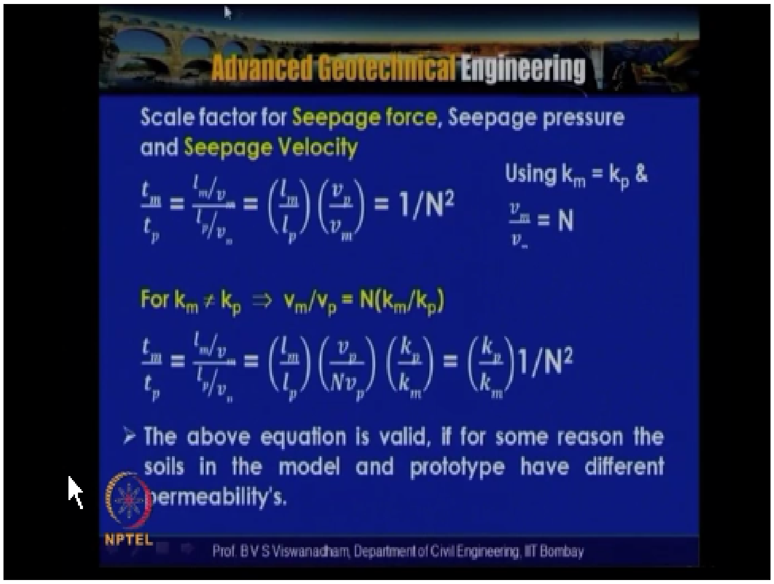

Advanced Geotechnical Engineering

Scale factor for Seepage force, Seepage pressure and Seepage Velocity

$$\frac{t_m}{t_p} = \frac{l_m/v_m}{l_p/v_p} = \left(\frac{l_m}{l_p}\right)\left(\frac{v_p}{v_m}\right) = 1/N^2$$

Using $k_m = k_p$ &

$$\frac{v_m}{v_p} = N$$

For $k_m \neq k_p \Rightarrow v_m/v_p = N(k_m/k_p)$

$$\frac{t_m}{t_p} = \frac{l_m/v_m}{l_p/v_p} = \left(\frac{l_m}{l_p}\right)\left(\frac{v_p}{Nv_p}\right)\left(\frac{k_p}{k_m}\right) = \left(\frac{k_p}{k_m}\right)1/N^2$$

- The above equation is valid, if for some reason the soils in the model and prototype have different permeability's.

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So LM by BM =L P by VP this VM and VPV is nothing but the Darcy's velocity and with that we can write LM by LP = BVM = 1 by n square so that means that here if you look into it time which is actually the velocity is n times as that in prototype the hydraulic gradient is n times that of the prototype so if you are having a soil with the km = KP and VM by VP= n so for came not with it says that TM by TP is 1 by n square so TN by TP = 1 by n square that indicates that the time required for true for flow to takes place is you know1 by n square times.

That of the prototype that means let us consider you know a 50 gravities and the model is you know the model is actually occurring subjected to 50 gravities and let us say that you know if you wanted to have a certain see page time wanted to see let us say 50 TM by TP = 1 by 50square so let us say that if it takes you know one year so one year time 52days time in the prototype / 1 by 50square so what we get is that you know if you look into the after simplification you will get an often.

Over that means that the 52 ,52days of prototype time is equivalent to of an hour at 50 gravities' this indicates that you know the so-called seepage phenomenon is actually is very rapid and there is a possibility that no this can be done and it can be used for a number of applications like wherever the like contaminant you know contaminant transport let us say that an example of contaminant transport where it can actually have a recur in 1 by n square times that of the time takes in the prototype.

So by some chance actually the cave is not equal to KP that is that the prototype soil and mod model soil they have a difference in permeability then we can actually also account for that you know in the expression whatever the logic we have discussed it with IM is =NIP so we can write their vm xvb = n into K M by K P so by using again the same definition and some shooting here what we get is that TM by TP = KP by km into an by n square so this KP by km is you know for example they are 1 then they get cancelled.

Then what we get is the TM by TPC 1 by n square so you know this TM by TP when there is km is not equal to KP we can actually account for this we are in the ratio into 1 by n square so the above equation is valid but if some reason the soils in modern prototype have different permeability if the soils in model prototype have different variability's then the time scale factor for the CPH is TF by TP = KP by km into 1 by n square.

So now after having discussed you know the seepage scale factors then we also have you know phenomenon like consolidation a consolidation of soil say for example we know that the consolidation of a soil takes place or a long period time depending upon the type of the clay.

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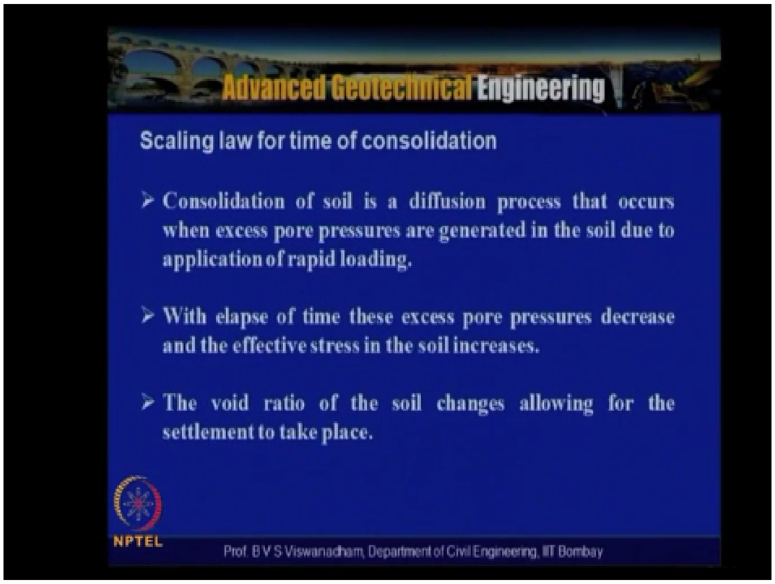

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Scaling law for time of consolidation

- Consolidation of soil is a diffusion process that occurs when excess pore pressures are generated in the soil due to application of rapid loading.
- With elapse of time these excess pore pressures decrease and the effective stress in the soil increases.
- The void ratio of the soil changes allowing for the settlement to take place.

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And it is a permeability so the consolidation of soil is a diffusion process and basically that occurs when excess pore-water pressures are generated in the soil due to application of rapid loading so when you have you know rapid loading then there is a possibility that the dissipation of pore water pressure takes place excess pore-water pressures that is the excess pore-water pressures landing.

But the pore water pressures above the hydrostatic pressures and this takes for a period of time so with elapsed of the time these excess pore-water pressures decrease and Excel effective stress in the soil increases so the void ratio the choice oil changes allowing for the settlement to takes place so when we have this the you know the void ratio of the soil changes and that allows the settlement. To take place so let us see how you know the scaling laws for time for consolidation.

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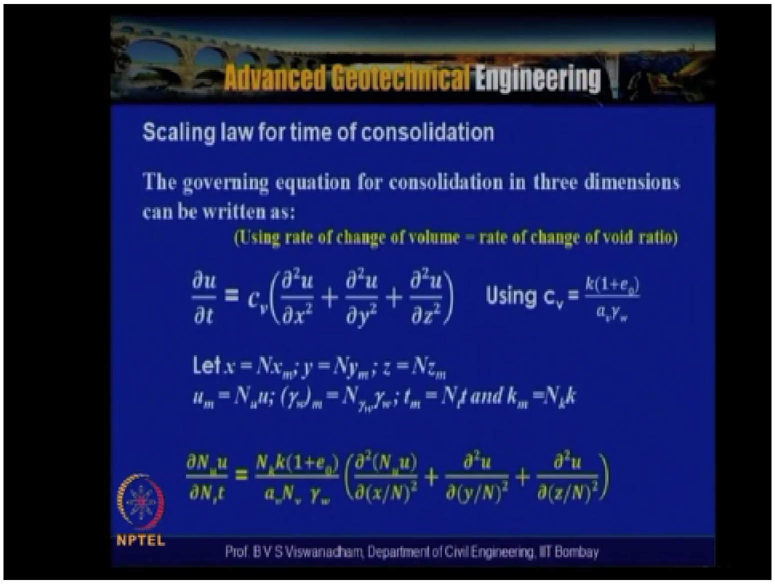

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Scaling law for time of consolidation

The governing equation for consolidation in three dimensions can be written as:

(Using rate of change of volume = rate of change of void ratio)

$$\frac{\partial u}{\partial t} = c_v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad \text{Using } c_v = \frac{k(1+e_0)}{a_v \gamma_w}$$

Let $x = Nx_m; y = Ny_m; z = Nz_m$

$u_m = N_u u; (\gamma_w)_m = N_{\gamma_w} \gamma_w; t_m = N_t t \text{ and } k_m = N_k k$

$$\frac{\partial N_u u}{\partial N_t t} = \frac{N_k k(1+e_0)}{a_v N_v \gamma_w}\left(\frac{\partial^2 (N_u u)}{\partial (x/N)^2} + \frac{\partial^2 u}{\partial (y/N)^2} + \frac{\partial^2 u}{\partial (z/N)^2}\right)$$

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Can be reduced so the governing equation for consolidation three-dimensional three dimensions can be written and this is actually reduced by using the rate of change of volume to rate of change of void ratio so we can write δ that x u by δ T = CB into δ square u by δ X square plus δ square u by δ x square plus δ square u by δ square so using CV = k into 1+ u 0by a V γ W very AV is a coefficient of compressibility which is nothing but Δ e by Δ so CV is equal to question of consolidation k is the coefficient of permeability he not is the initial void ratio γ W is the unit weight of the water so let X = n XM so x and y and Jade are the prototype dimensions.

So X is written as n times X M that means that when we take = 1 by n there is it is nothing but X my XP = 1 by n so we have for simplicity and convenience PR 0 10 x = n XM and y = n ym and j = n ZM and with that um also we write it as n you that is the scale factor for the pore water pressure and γ W = n γ W γ W this n γ W is the scale factor for the unit weight of the you know pore fluid TM is equal to entity, see the interest is that how much time it will take for reckons when the consolidation is taking place in a enhanced gravity models then km = n k.

So by substituting this in this you know governing equation of consolidation then what we get is that δ u by δ NT t = n k ,k that is nothing but4k what we substituted is that m k k into1 + V naught and for a same soil skeleton for same degree of consolidation the void ratios are assumed to be seen so NK k+ 1+ u0 into a V that is also assume it to be same and V into γ W so this is n and this is nothing but n γ W again that this is the scale factor for the unit weight of water plus into the δ square nu u by δ X by n whole square what we have done is therefore X we have written X by n whole square plus.

Because this is we are writing for model in the center face model plus δ square u by δ into FY by n whole square plus δ square u by δ into jet by n whole square so by simplification readjusting the terms.

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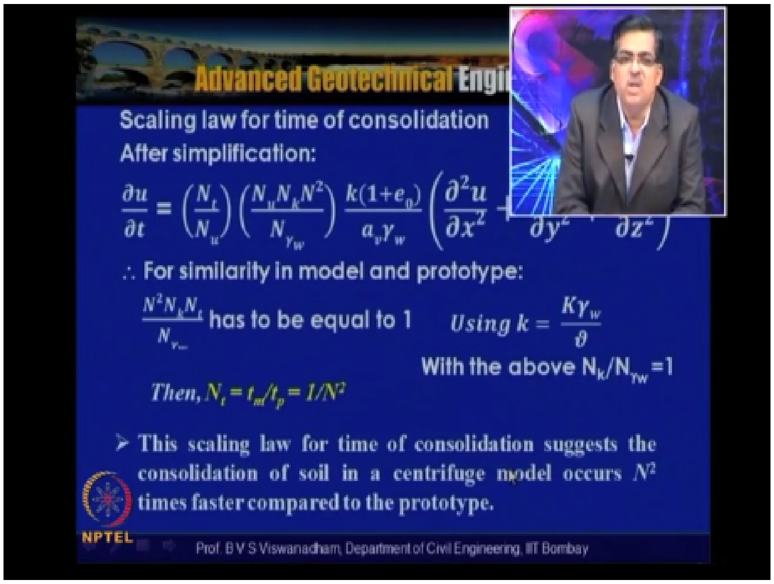

Scaling law for time of consolidation

After simplification:

$$\frac{\partial u}{\partial t} = \left(\frac{N_t}{N_u}\right)\left(\frac{N_u N_k N^2}{N_{\gamma_w}}\right)\frac{k(1+e_0)}{a_v \gamma_w}\left(\frac{\partial^2 u}{\partial x^2} + \frac{}{\partial y^2} + \frac{}{\partial z^2}\right)$$

∴ For similarity in model and prototype:

$\frac{N^2 N_k N_t}{N_{\gamma_w}}$ has to be equal to 1 $\quad$ $Using\ k = \frac{K\gamma_w}{\vartheta}$

With the above $N_k/N_{\gamma_w} = 1$

$Then, N_t = t_m/t_p = 1/N^2$

- This scaling law for time of consolidation suggests the consolidation of soil in a centrifuge model occurs N^2 times faster compared to the prototype.

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What we get is that y by δ t = n t by nu into nu + k +square divided by n γ W into K into1+ e NR a V γ W into δ square u by δ X square +δ square u by δ x square +δ square u by δ Z square so if you look into this accept these terms you know this term actually obtained from here by simplifying this but except this term and this is actually appeared to be that equivalent to then the prototype.

So for simple so we need to see for similarity in model prototype how to have say identical differential equation differential equation in Marshall difference mode land prototype that n square n kk because here the Nu and then you will get cancelled so n square n square is actually nothing but you know which is by simplification of these terms we will get δ square u by δ X by n whole square those that n square is actually by taking common we have got here n square n kk NT and γ W has to be equal to 1 so again using k ,k that is absolute permeability times γ W by kinetic viscosity and by using this we can say that n K by n γ W = 1.

So that is that because these are identical and then K increases en times g is γ W increases n time so n K by n γ W = 1 with that by substituting here what for the n square y MK and by n γ W = 1 that m t = t m by has to be=1 by l square that means that this is scaling for law for the time of consolidation basically suggests that the consolidation of a soil in a centrifuge model occurs n square times faster compared to the prototype the base there if you consider a you know if30 minutes are often our run of centrifuge at 50 gravities is equivalent to 52 days of consolidation in the field that is nothing but 52 two days / you know the 50 square.

When we convert it into minutes you will get as 30 minutes so this is you know the scale factor implies that the time of the consolidation you know the suggest that the consolidation of soil in a sentence model occurs n square times faster compared to the prototype so most of the scale factors like time permeability and all these scale factors several investigators for a period of time they were edited this by using again the experimental evidence.

Which is actually the centrifuge basic physical modeling so in this particular lecture what we have understood is that we are tried to do some examples on the energy scale factors and then we have tried to see how a seepage phenomenon can be modeled in a self use and how we can actually compute velocities in a city filled based physical model by knowing or by measuring the pressure or a length L.

And then also be discussed about how a consolidation phenomenon can be modeled and a how the Time Square fact so for the time scale factor for consolidation as well as the time scale for the diffusion is works out to be 1 by n square that in the prototype that is that time in model is 1 by n square times of the that in the prototype.

In the next lecture we will like to look into the capillary flow when it is actually happening what will be the rate of rise of capillarity and what will be the time of capillarity in this interval based on physical modeling and then subsequently we look into how a dynamic event like an earthquake can be modeled and what will be the scale factors you.

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