Remote Sensing: Principles and Applications
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Lecture – 08
Radiometry – Part 2

Hello everyone, welcome to today's lecture on the course Remote Sensing: Principles and Applications. This lecture, we are going to continue with the concepts of radiometry what we have started in the last class. Just as a recap on the last class, we defined what plane angle is and what a solid angle is.

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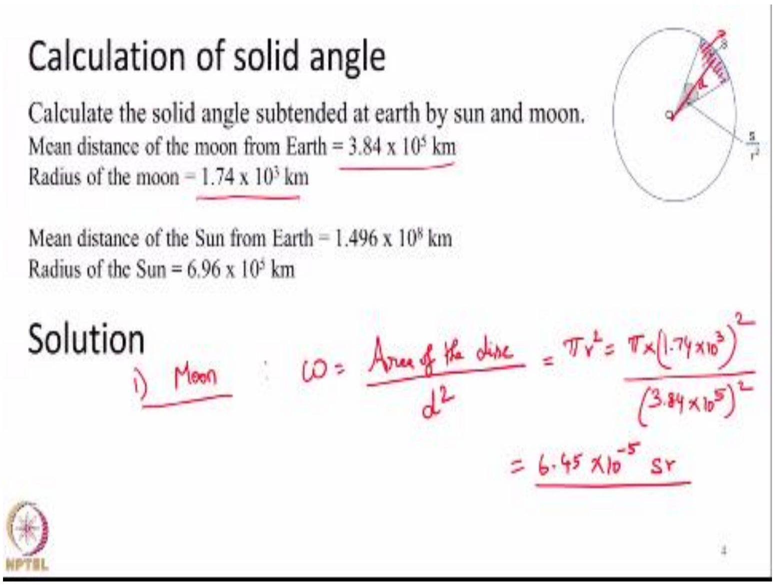

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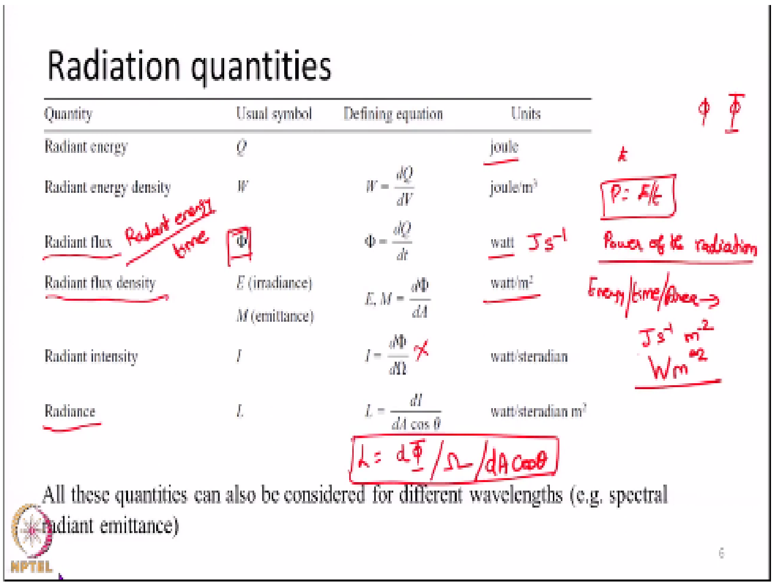

We have done a small problem in calculating the solid angle subtended by sun and moon on earth's surface. Then we defined various radiometric quantities, we defined what radiant energy is, we defined what radiant flux is that is energy per unit time, then we define radiant flux density that is energy per unit time per unit area, if that radiant flux density is coming towards an object, we call it as irradiance or if the object is emitting energy, we call it as emittance.

And also we defined one important property we often use in remote sensing that is radiance. Radiance is defined as the energy emitted in unit time in a given direction per unit solid angle, that is the amount of energy per unit time radiant flux ϕ divided by unit solid angle so, divided by the total solid angle ω will come in that is divided by the projected area in a given direction.

So, the area what you are going to consider we have to project it in the direction in which we are looking at. Why this projection is occurring? Because, as our viewing angle changes, as we look different objects in a 3 dimensional space based on the angle in which we look, we will perceive the object in different sizes and different shapes.

As I said as an example in the last class, if you are flying in an aeroplane and looking at whatever is there on the landscape, based on the angle in which we look, the area on the landscape, be it an agricultural area or like some huge circular surfaces whatever based on our viewing angle, it may appear as a circle; may appear as an ellipse with different, different area depending on in which angle we are looking at, which height we are flying and so on.

So, in remote sensing, normally our remote sensing sensor which is there in space will perceive objects differently based on the angle in which we are looking and that is why this projected area is really important and also in a small solid angle because all objects whenever we see or whenever a sensor sees an object on earth's surface, it is going to subtend a small solid angle.

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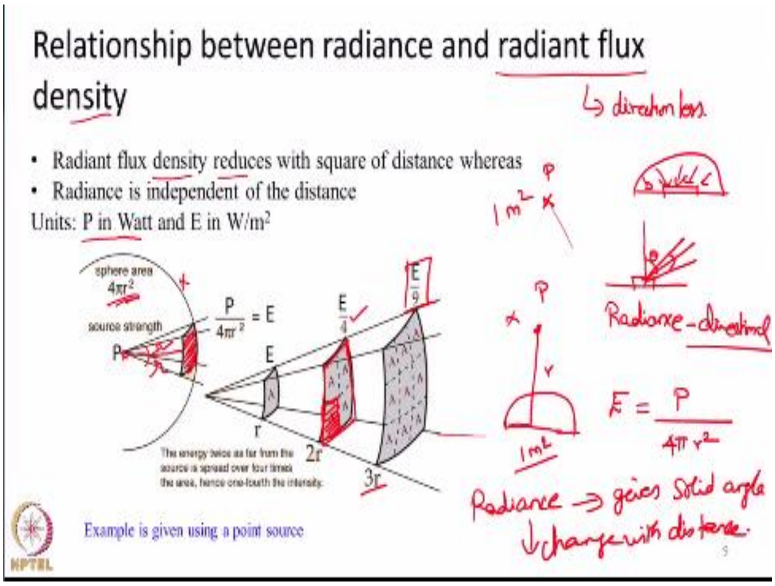

So, what we are going to continue today is, we are going to continue today by looking at the relationship between irradiance and our radiance and radiant flux density. So, before actually seeing this, we will make one thing clear that is radiant flux density is actually directionless. Directionless in the sense when we define we said, if this is the area, I am going to put an hemisphere around it and whatever energy that is either coming in or going out within the hemisphere is radiant flux density.

So, we are not caring about in which direction energy is coming. Whatever energy within the hemisphere, we are taking it. Whereas in radiance, if this is the area, if the energy is going in this direction, within a given solid angle, I am going to calculate in this particular direction. I am going to project this, project the surface area with respect to the normal and calculatw the energy.

If the direction changes, this is further going to change. So, radiance is directional. One more important property is radiant flux density reduces with the square of distance. That is let us assume, there is like a small source of energy is here. Let us say this is point P. I have kept a small source of energy there. Let us say, it is emitting P watts of power that is some energy in unit time we call it as P, P watts that is happening.

Let us imagine, I have placed a square of one metre square area. Unit area, I place it at a distance of r from this point source. So, this is a source of energy, I am placing one metre square area of some object and I am going to calculate what is the power from this particular object is going to fall on this surface area. So, if I see it from the perspective of this particular object, see now, what I am doing, I am going back to the definition of radiant flux density.

This surface area, I am laying it flat on a plane, the point source is now here point P. it is at a distance of R. So, this is one metre squared area. Let us imagine, only this is the power source. If I put a hemisphere around it, only this energy is coming in. No energy source from other directions are coming. Only this is the energy coming and falling over it.

So, I am going to calculate what is the irradiance E received by this unit square metre of area. So, what essentially happens? $E = P/4\pi r^2$. That is the power emitted by this particular point source P is now spreading uniformly in all directions, because it is there in a 3 dimensional space. I said, it is like a small point source and it is emitting energy in all directions.

So, I am placing an object in one particular small area on the sphere. So, what is happening for this particular point P? I put a huge sphere around it of area $4\pi r^2$. Within this, I am placing an one m^2 object and I am just going to calculate the irradiance on this particular area. So, this drawing and this drawing are kind of analogy for you to understand.

Now, what I am going to do is, I am going to move this particular small surface area to a distance of 2r. So, what essentially is happening now? If I move this to a distance of 2r, that is, I move it to a farther point. This point is now moved to like a farther point something here. So, essentially, whatever this point P is emitting, whatever power it is emitting.

Now, it is distributing itself over extremely large area like the area is increased 4 folds because of the sphere surrounding this point is going to double in area. Thus, what to say, the energy emitted by this object is going to now expanded over a area that is four times the initial. So, even if I keep the same A square here, the same area of one unit metre square here, the energy I am going to receive is going to reduce by four times.

Similarly, if I move the distance by 3 times, the energy is going to reduce by 9 times and so on. So, conceptually in order to understand as the distance between the source and the receiver increases, the energy received by the receiver is going to decrease by square of a distance that is the radiant flux density. It is very simple analogy is like, let us say, there is like a big burner that is burning, we are standing at some distance from it.

Say, if we move closer and closer, we are going to feel much hotter. If we move away and away, we are going to feel little bit cooler, not that much of heat will be there because the energy emitted by that particular source is going to decrease as the distance between the source and receiver increases. How this thing decreases by a distance square law that is by square of distance.

As the distance double, the radiant flux density received will become 4 times less. As the distance becomes thrice, the radiant flux density received is going to reduce by a factor of 9 and so on. So, this is the major thing with radiant flux density. The energy received by an object of unit area will keep on decreasing as the object moves away from the source. On the other hand, let us take radiance.

Now, in the same figure, what we are going to do is radiance is defined as in concept of like a solid angle. Now, this is again like a point source. What I am interested is, what is the energy emitted in the solid angle here. Let us call it as like ϕ or ω or whatever it may be. Within the solid angle, what is energy emitted? Even as the distance increases, even when distance increases, we are returning the same solid angle.

As per the definition, we have to measure over the entire solid angle itself. So, we are going to increase the size of our receiver in order to collect all the area basically. That is as the distance increases, if we want to maintain the same solid angle, the surface area is going to increase. So, if I want to place a receiver there at a distance of r, I need to have a smaller receiver to collect all the energy.

At a distance of 2 r, I need to have a larger surface area and so on, same thing. But, if I keep a larger surface area based on distance, I am still going to collect the same amount of energy. So, radiance if you look at the definition of like energy in a given solid angle, this will not change with distance. That is just to tell as small analogy.

Say, for example, I am standing near a small burners burning in front of me, if I go closer to it, I will feel hotter. If I go back, I will feel a little bit cooler. Why? The energy from the burner is spreading across in all directions. If we go closer, most of my surface area of my body is encountering that energy. If I go back, the energy is now spread across a larger area and my surface area is smaller in compared with the total area.

So, the energy I receive is only a fraction of what the object emits. This is radiant flux density. Now, compared with radiance, same burners in front of me, the burner if I treat it as a small source or something, it will have a small solid angled towards me, basically. As I move back and back, I need to maintain the solid angle which the burner had initially then only the radiance definition will be satisfied.

Radiance by definition, it is the energy in that particular solid angle. So, radiance is here, I have to maintain the solid angle. So, what essentially it means? As I move back and move back, it is again to myself has to grow bigger and bigger on surface area, so that I capture all the energy coming in that particular solid angle. So, essentially what is happening? I am maintaining the solid angle and hence, I will be collecting all the energy coming out of the burner which will remain the same.

As I move back, the surface area in which I am collecting the energy is going to increase because I need to maintain the constraint of solid angle definition. That is why with respect to radiance, energy is not going to change as long as a solid angle is held constant, same amount of radiance, we will receive. But by definition of radiant flux density, surface area has to remain constant.

So, even the solid angle is kept same, the surface area will become smaller and smaller, as the distance between the source and object increases. This is the burner. This is me. As I move farther and farther, the total proportional area getting this energy is going to decrease based on distance. And that is why we say, radiant flux density will decrease with distance. Whereas by definition of radiance, it will not decrease with distance.

So, you need to understand this difference between them. It may appear a little confusing in the beginning, but as you try to sit and think deeper about it, you will be able to understand it much clearer.

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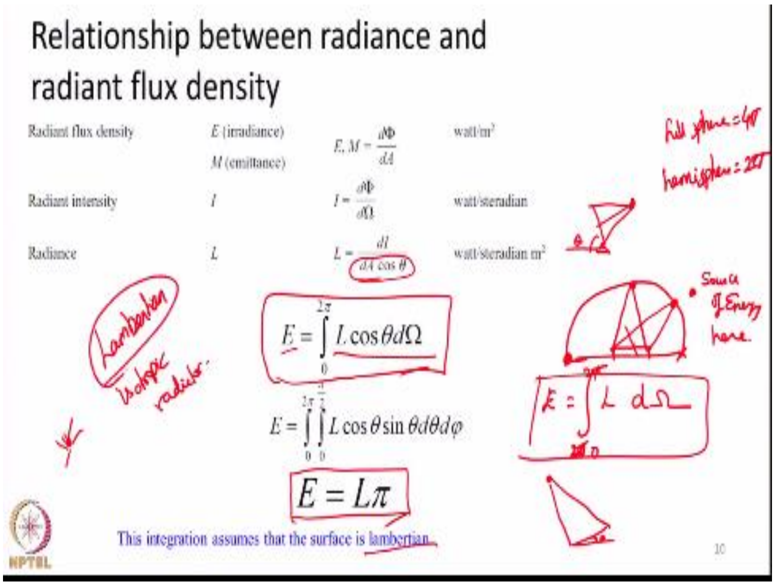

Next, what we are going to see is the relationship between radiance and radiant flux density. Now, let us say again, we have like a small surface area, a small source of energy here. We will take it as an example. The source of energy I placed it on a hemisphere surrounding it, say rather than placing here, I place it here. Now, this is a source of energy.

Let us imagine, the source of energy is going to emit some energy starting from this particular point and it is going to move all along this hemisphere line and come here and stop. This source of energy is going to see this particular surface area continuously. And it is going to move from it. So, essentially, what is the energy emitting in this particular direction we are interested upon.

So, as it is moving here, it is going to see like this, something of this sort. So, after it has moved one full round, I say, I want to calculate the total irradiance received by an object. So, initially, I measured radiant energy from one direction that is radiance, now the source has moved in the entire hemisphere. Now, I say, I need to calculate the total irradiance. So, what essentially the concept wise is?

I am going to integrate the energy that came from these different points on the entire hemisphere. Because by definition of irradiance, it is nothing but the total energy within the hemisphere surrounding the object of interest. So, what I want to do? I want to do integration of the radiance over the entire hemisphere. If I integrate it over the entire hemisphere, I am going to get the total irradiance.

$$E = \int_0^{2\pi} L \, d\Omega$$

Like for full sphere, we had solid angle 4 π steradian and for hemisphere, solid angle is 2 π, half of it. But, here comes one more tricky issue, just go back to the definition of radiance. In definition of radiance, we spoke about projected area that is whatever be the energy coming in or going out, the area which is receiving the energy or emitting the energy should be perpendicular to the direction of motion.

So, coming back to this particular example that we had. So, if this area is here, when the object is here, the area should be rotated like this in a direction perpendicular that is, this is the original position by angle of θ, I should rotate it in order for the area to receive the energy. Similarly, if this receiver goes somewhere here to another position, instead of being horizontal, I should now rotate it to another angle to receive the energy. This is how we have defined radiance.

The area should be projected in the direction of radiation. And hence, there will always be A cos θ term involved in the definition of radiance. If you look at the definition of radiance in this particular slide, is defined as in denominator we have, dA cos θ. Because of this, in the integration of radiance, rather than just using L, I should use L cos θ. Because of the definition difference in irradiance, we do not care about directionality, but in radiance definition, we care about direction.

And hence, due to this projected angle difference, I need to introduce a concept of cos θ into it. So, rather than just integrating L over 2 π steradian, I have to integrate L cos θ. I am going to consider the entire direction between the vertical and the horizontal starting from 0 here, the vertical all the way up to 90 in this direction. Similarly, 0 here, all the way up to 90 in this direction, I have to integrate it. Then only I will be getting the exact irradiance. Okay.

So, just coming back to the derivation. So, the irradiance E is given by L cos θ into the solid angle, whatever the solid angle, integrated over 0 to 2 π solid angles.

$$E = \int_0^{2\pi} L \cos\theta \, d\Omega$$

Now, I am not going to go into the detail of this derivation, a single solid angle, we can divide it into two planar angles. One is called zenith angle and azimuth angle and all. I am not going to go deeper into it.

But the final result, I am giving out here. The final result is,

$$E = L\pi$$

that is, the irradiance received by an object will be equal to π times the L or if an object is emitting energy in different, different directions and the total emittance from an object is π times L. So, this is the relationship between radiant flux density and radiance.

This, we will be using repeatedly in various points in the course. So, you need to remember this relationship always. This relationship has a caveat or like a strict condition. What is it? This relationship holds good only if the area is lambertian. So, lambertian means, say, I said an object is there. Now, I am going to take an aeroplane, fly around an object and look at and measure the energy coming out of an object. So, I have a sensor fitted in the aeroplane. I go in like a circle. I move in different, different directions. I move in different angles and so on.

I fly from this angle and then I come closer whatever. All possibilities, I cover within that hemisphere. If that is the case, in whatever direction I look, if the object is giving out same amount of energy like that particular land surface around which I am flying, if it is giving the same amount of energy in whichever direction I look at it, that particular surface is called lambertian.

It will look exactly the same like. Same means it will emit the same amount of energy, physically the area will not remain same like as the area will project itself, but the amount of energy coming out of it will be the same whichever direction I look, that is called lambertian. So, lambertian surfaces, it will have a uniform reflectance properties. It will appear more or less equal from all directions.

So, only for such lambertian surfaces, $E = \pi L$. The relationship will hold good because the derivation I have not shown you. If we want to take the radiance term L out of the derivation, we need to treat it as a constant. Normally, we take only constants out of integrals. So, out of the integral if we want to take, it has to be a constant. When it will be like a constant?

When L will be constant is only when the surface is lambertian that is why I said, there is a condition. This is like a little bit strict condition because most of earth surface features are non lambertian in nature. We will see in detail in the coming slides and coming classes. But remember this equation as we will be using it often. E = L π.

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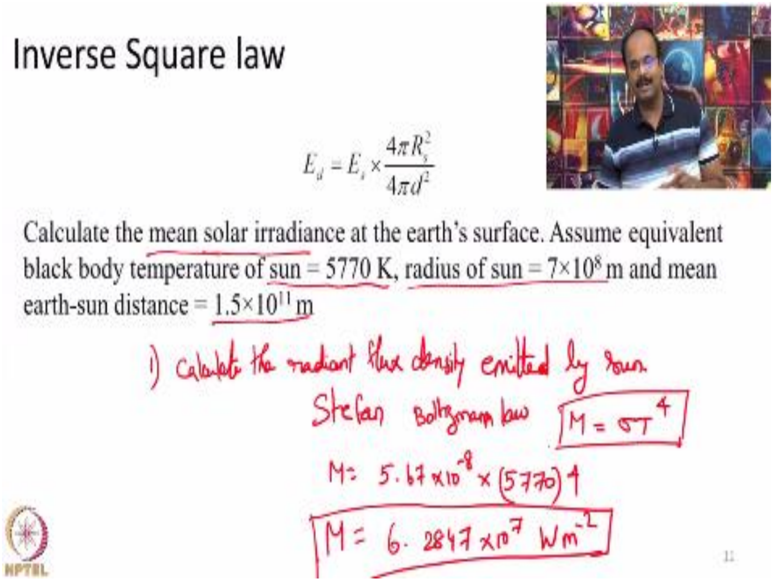

I said Inverse Square Law. We are going to now see an example of how inverse square law works. Again, we are going to take an example of sun and work around it. So, the question given here is, calculate the mean solar irradiance at the earth's surface. Assume equivalent black body temperature of sun is 5770 Kelvin, radius of sun 7×10^8 metres and mean earth sun distance 1.5×10^{11} metres.

It is said, sun is emitting some amount of energy. Calculate what is the energy that will reach the earth surface per unit metre^2 of area. Let us see how we are going to calculate. First step. We have to calculate the radiant flux density emitted by sun. We have learnt a law called the Stefan Boltzmann law which says, the radiant flux density M = σT^4 where σ is Stefan Boltzmann constant.

I gave the value in the initial lectures. So, I am just going to substitute the values. So, $M=5.67\times10^{-8}$. This is the value of σ multiplied by temperature of sun $(5770)^4$. If you solve this, we will get 6.2847×10^7 and the unit is watt/metre^2. So, Stefan Boltzmann law will give us radiant flux density. What is the amount of energy per unit time per unit area that is radiant flux density.

But, we all know sun is sphere, big sphere. It is not like a one point. It has a certain area and the shape of the sun in 3D space is a full sphere. So, this energy that we have just calculated is per unit metre^2 area of sun. So, what now we have to do? We need to calculate the total power emitted by the sun. Okay. So, now, we have calculated the power emitted by sun per unit metre^2 of sun.

Next, we are going to calculate the total power emitted by sun. How are we going to do it?

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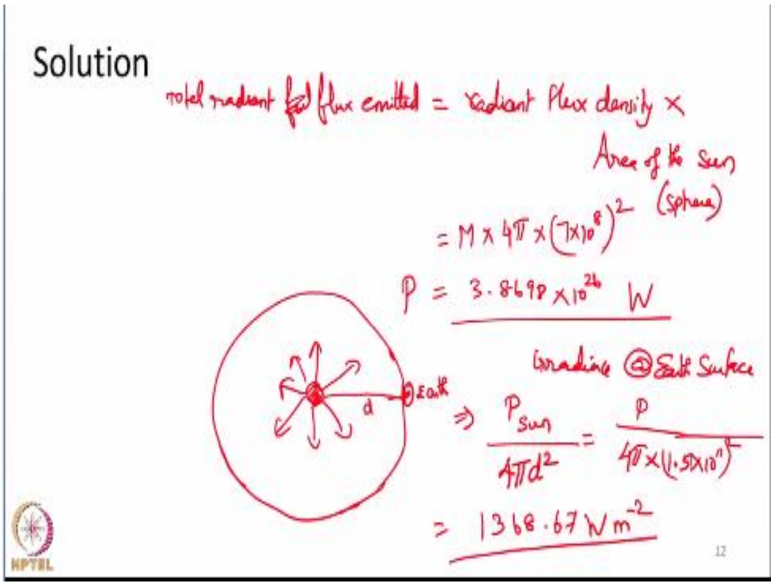

The total power emitted by sun is equal to, I use the correct technical term,

the radiant flux emitted by sun = radiant flux density × area of the sun

So, here sun is like a 3 dimensional figure. There in the initial problem of calculation of solid angles in the last lecture, we treated sun as a circle because when you look at sun, we are seeing it as a 2 dimensional circle alone, but in reality sun is like a sphere.

So, for calculating the total power emitted by sun, we should treat it as a sphere and take the total value like the total surface area of the sphere. Okay. So, here radiant flux density is the M, we calculated in the previous slide, multiplied by surface area of sphere is $4 \pi r^2$. So, $4 \pi \times$ the radius of sun that is given there roughly. If you do this, the total power radiated by sun will be 3.8698×10^{26}. Units is watts. Because, now what happened?

I have converted radiant flux density into radiant flux. So, I have multiplied the power emitted per unit area with area and hence, area terms get cancel out, a unit will remain as watts. Always have the units written beside your quantities, it will help you to cross check whether your calculation is correct or not. So, now, we have calculated the total power emitted by sun.

Now, let us look at the geometry. This is sun as a sphere. I have calculated the total power emitted by sun. Earth is somewhere here at a distance of d. Now, sun is not only emitting energy towards earth. It is emitting energy in all direction, because sun is there in space, nothing there to capture it. It is emitting energy in all directions. If I draw a full sphere around it with the radius of d. Sorry, my drawing is not that great.

I draw a sphere around it with radius of d and this particular power emitted by sun is actually distributed itself equally in the entire sphere around it because sun is an isotropic radiator. It uniformly radiates energy in all directions. So, if I put a huge sphere surrounding the sun with the radius of d, the distance between earth and sun, it will emit energy equally in all directions within that particular sphere.

So, one metre square on the earth's surfaces is actually located somewhere here. So, what I want to do? I want to calculate the irradiance at the earth's surface. That is, I have to divide the power within the centre hemisphere for one metre square of area that is, earth can be anywhere. Here earth is. What is the energy received by one metre square area on the earth surface?

So, essentially I am calculating what is the power remaining at one metre square area on this particular huge sphere that is the total power P emitted by sun divided by the surface area of this big hemisphere, will give me the final answer. So, if we can do this, if you divide this particular power $P/4\pi d^2$. So, $P/4\pi\times(1.5\times10^{11})^2$.

If you do this, we will get 1368.67 watt/metre^2. Again, the unit will become watt/metre^2, because, again we are converting the total power into unit area for that particular distance. So, this is known as solar constant roughly. This value 1368 watt/metre^2, is what we call solar constant that is, on an average, taken for all seasons, in all places together, this will be the average amount of energy that we will receive from sun per unit metre^2 of area per unit time that is solar constant.

So, to summarise, in this lecture, we have learnt about the relationship between irradiance and radiant flux density and radiance. Radiant flux density will vary, decreased with distance whereas radiance will remain constant as long as the solid angle is kept constant.

And the irradiance $E = \pi L$, the relationship we have seen and also, we saw one example of how to use the inverse distance relationship to calculate the amount of energy received on the earth's surface. So, with this we end this lecture, thank you very much.