Dynamics of Structures
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Module - 02
Seismic Response Spectra
Lecture - 17
Numerical Response Evaluations and Earthquake Response Spectra

Welcome back everyone.

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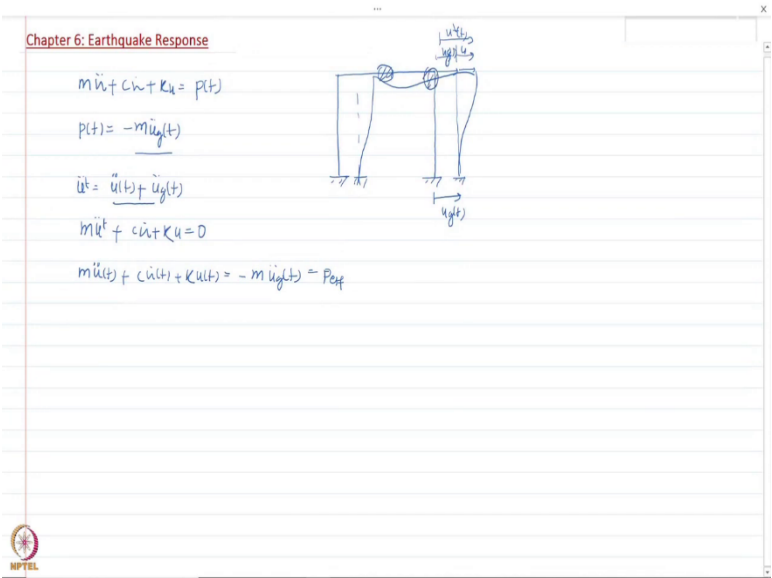

We have seen so far in this course how to obtain response of a system subject to either well-defined analytical forces like harmonic forces, and we also discussed if there is an arbitrary excitation how to get the solution. And in the previous lecture, we saw that if the response or if the loading cannot be expressed as a closed form representation like p(t) equal to some function, then how to employ numerical methods to obtain the solution.

Now, once we are equipped with this tool, we are in a position to analyze a system or a single degree of freedom system subject to earthquake excitation. And there would we would present a just we will discuss in briefly there is a whole field of earthquake engineering which we may which you may do in an advanced course, but a in this basically course of structural dynamics, I will just give you the introduction the concept of how to obtain response of single degree freedom system subject to arbitrary or random excitation like earthquake excitation.

And also discuss the concept of peak response and basically response spectra which is very famously used for analysis and design of the structure subject to earthquake loading. So, let us get started. Earthquake response of single degree of freedom system, and as we have previously done we would be focusing on linearly elastic system.

So, let me write down the equation of motion that we have been using till now for linear elastic single degree of freedom system. Now, finding response to seismic excitation or earthquake excitation is one of the major field where you can see the application of structural dynamics. And basically the problem becomes in that case as we have previously discussed.

So, if I consider this frame representation of a single degree of freedom system, so let us say initially the system was there and then there was a ground movement let us say this is $u_g(t)$, and then the relative deformation of a structure which let us say. So, this is the relative deformation u, so that my total deformation is $u_t(t)$ which is sum of $u_g(t)$ and then u(t). And this is how it looks like alright. So, basically the problem statement for these cases becomes the in this case p(t) is effectively represented using mass times $u_g(t)$. So, I can substitute it here alright.

So, remember that this basically this expression basically comes from assuming that the total acceleration is nothing the relative acceleration of the mass plus the ground acceleration. And then we substitute in here ok the equation of motion is actually written in terms of total acceleration, however, with relative velocity and displacement equal to 0. Considering that there is no external force.

$$\dot{u}^t = \ddot{u}(t) + \ddot{u}_g(t)$$

$$m\ddot{u}^t + c\dot{u} + ku = 0$$

However, when I substitute this expression here, then I can rearrange this and I can get it ok, get this expression right here. So, again you can write this one in terms of function of t as well.

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=-m\ddot{u}_g(t)=p_{eff}$$

And this basically is the effective force that we say is acting on this mass here, in this case alright. So, let us do one more thing. Let us divide this whole equation here by m.

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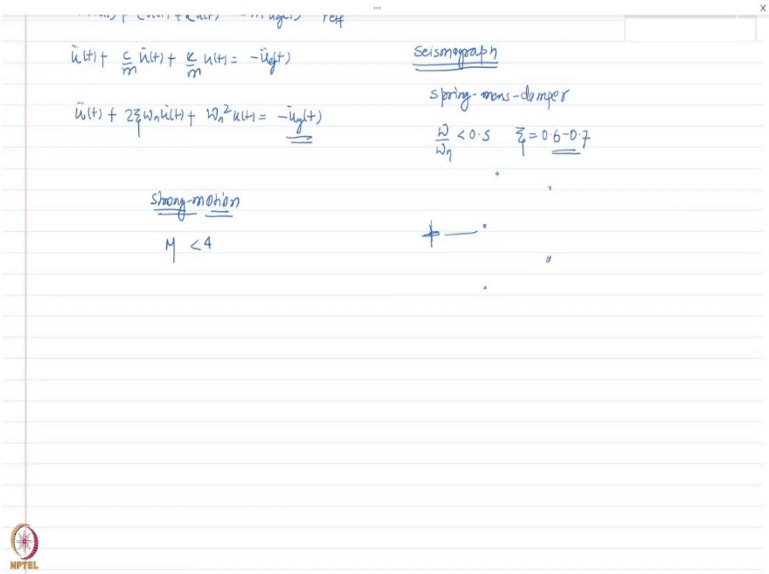

So, that I can get my expression as

$$\dot{u}(t)+\frac{c}{m}\dot{u}(t)+\frac{k}{m}u(t)=-\ddot{u}_g(t)$$

And this can be further written as

$$\dot{u}(t)+2\xi w_n\dot{u}(t)+w_n^2u(t)=-\ddot{u}_g(t)$$

And this is my expression here. Now, this ground motions are basically measured using seismograph. And we have discussed this the working principle of a seismograph in a previous chapter. A seismograph is nothing but in it is very elementary form, it is a spring mass damper system. So, it's a single degree of freedom system ok, the spring mass damper elements.

And the way the seismograph actually works is that we utilize frequencies for the seismograph which are quite high such that w/w_n where w is the excitation frequency; and w_n is the natural frequency of the seismograph alright, this should be equal to 0.5. And we introduce very high damping ok 60 to 70 percent in the system. If that happens, then my seismograph can be calibrated to measure various type of ground motion with different frequencies.

So, this seismograph actually measure the ground acceleration which is also many a times you would see being referred as strong motion. We are only concerned about the strong motion because unless the ground acceleration is above a certain threshold ok, it is not going to do any damage to a structure.

So, if you consider magnitude of earthquake let us say less than 4, typically it does not lead to any kind of damage in the structure. So, in terms of strong motion, what we do this seismographs are actually located at various distances from the source of an earthquake. And these seismographs are lying there for long period of time or 20 years, 30 years like that. And you can imagine these earthquakes did not to be very frequent. It might come once in a lifetime.

Let us say once in a 50 year or 100 year. And the idea is that you cannot continuously be measuring the acceleration due to very small earthquakes like less than 4. So, these earthquakes these accelerographs are actually triggered when the acceleration is ever a certain threshold or when it is calibrated to bit triggered with some other mechanism.

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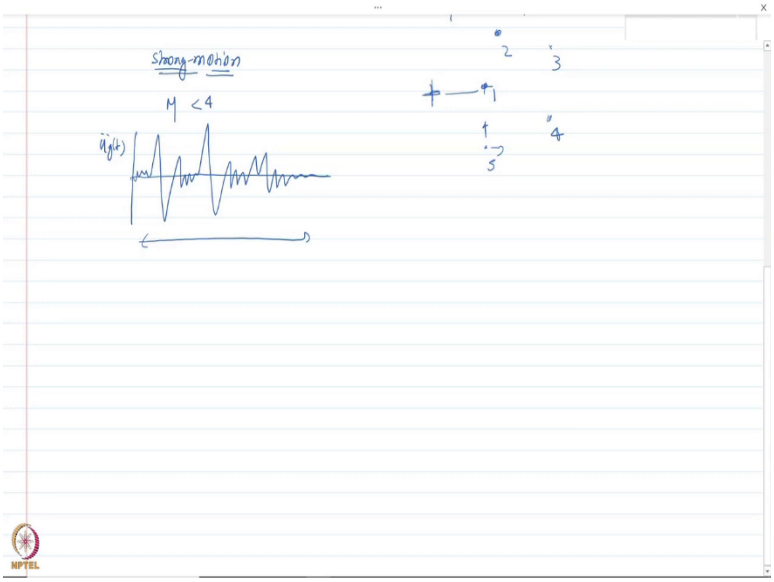

So, what do we do? These accelerograph would give you a strong motion. And then it could also report you the strong motion duration of an earthquake ground shaking. Again it is based on the same principle that you are only bothered about the ground motions, which are going to have any realistic effect on the structure. So, a strong duration, there are different way to characterize the duration of a ground strong of a strong motion.

There is area's intensity, there is like you know threshold at the max with respect to maximum PGA, certain kind of thing. But basically, just remember that these do not continuously measure, but it measures above a certain threshold.

And each of these, each of these accelerograph which are located at different location, they can measure depending upon what kind of accelerograph they in one direction, two direction or three directions. So, ok two horizontal directions, and one vertical direction. So, typically any earthquake would have three components. Although some of the old accelerograph would give you used to be only two components. So, usually any earthquake would have three components.

Of course, there are additional you know research which are now characterizes six component earthquake including rotational, but we have a not bothered about that. This is beyond the scope of this course. So, remember that accelerograph would provide you basically the acceleration history which would be in terms of ok either time acceleration.

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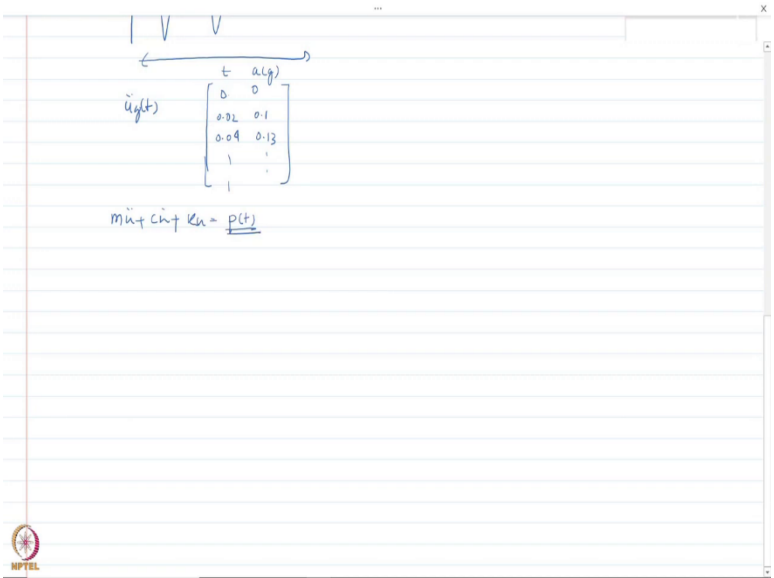

So, time and then acceleration which might be expressed let us say in terms of units g. So, something like let us say like that if it is in interval of let us say 0.02 second uniform interval, it would be something like this. So, this is just the fictitious value. So, it would give you basically discretized value of the acceleration at different time instance.

Now, in the previous chapter, we have seen that given any arbitrary excitation how to find out the response of a system to any arbitrary excitation p(t). And what we do we do? Basically, we use numerical methods ok. So, depending upon the whether the system is linear or non-linear ok, we basically discussed three methods.

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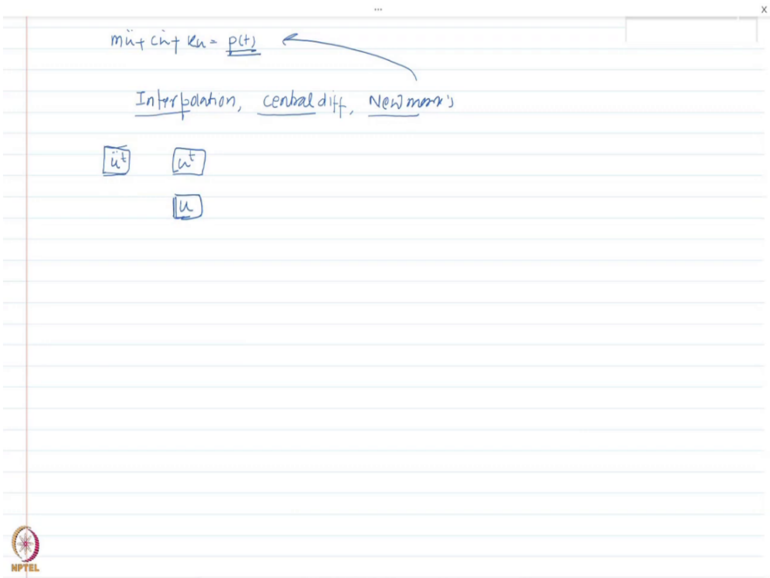

One was interpolation, excitation interpolation method. Other one was central difference method. And you also discussed a Newmark's methods. So, any of these methods can be utilized to get the response of the single degree of freedom subject system subject to this ground excitation. Now, depending upon, depending upon what is structure, there could be different response quantities of our interest alright.

So, for example, it could be of absolute values of the response quantity like the total acceleration in the system, or it could perhaps be total velocity let us say or total displacement, or it could be relative acceleration, relative velocity, relative velocity, and relative displacement.

Now, usually what happens for a structural engineer, the total velocity is not of concern to us. What is concerned to us is basically the total acceleration, and I will talk about where do we actually utilize that, but also total displacement is also concerned to us. And of course, the relative displacement which is basically the most important parameter if you want to analyze a structure ok to find out the internal forces in the system.

Now, the idea is how do we apply the knowledge of a structural dynamics to earthquake engineering. So, let us say somehow, I am able to find out these response quantities. Once these response quantities are found out, then we can utilize this to get the forces and moments in a structural member.

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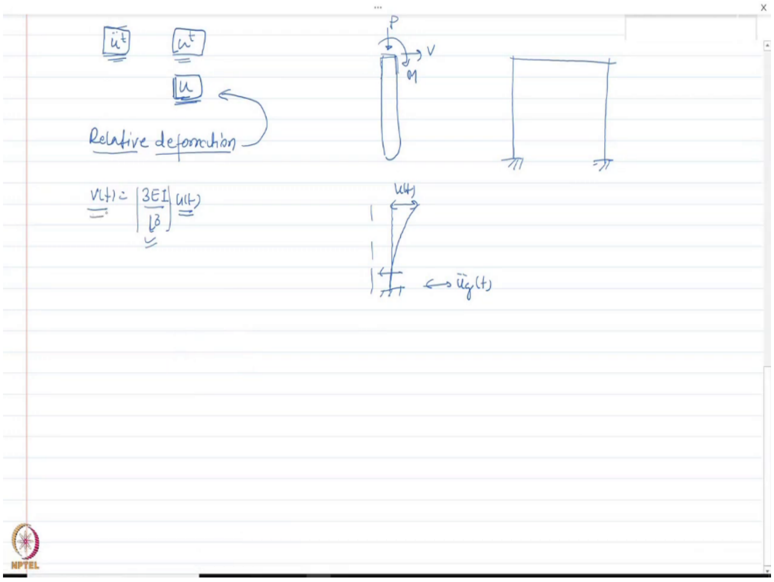

Because in the end remember as a structural engineer your goal is to find out basically forces on a beam or a column. So, depending upon let us say this is axial load, this is shear force, and this is m. Because once you have these forces in the members you can perhaps go head and design the structural member for those forces. So, utilizing the knowledge of a structural dynamics, we are going to find out the values of these parameters. And then from that we are going to find out the internal forces.

Now, if you remember the internal force in any a structure, it depends only on relative deformation. It only depends on relative deformation. It does not depend on velocity or it does not depend on the acceleration ok, it only depends on the relative deformation. So, if we somehow able to find out this parameter u(t) using some sort of numerical methods or any other method, then I would be able to find out the forces in the member.

And one of the simplest examples would be let us say have a cantilever column here and under the action of let us say $u_g(t)$, it has certain deformation of course, it would be some value like this but let us say relative deformation is u(t). Now, you know that if the deformation of u(t) the force let us say v here would be nothing but, I can write it as

$$A = w_n^2 D$$

So, if you know the displacement knowing the structural property and geometric and material properties of the structural member, you can find out or relate it to the forces in the member. And then you can design the member to sustain those forces. So, that is what we are going to learn in this chapter.

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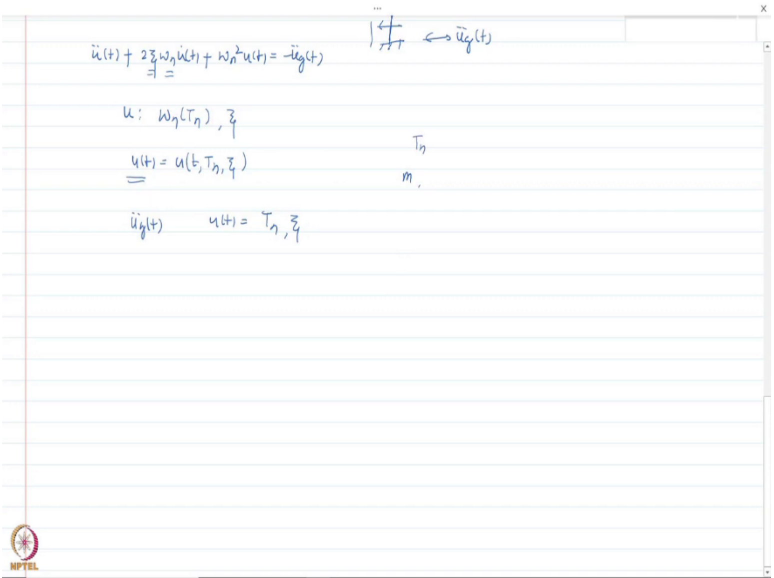

So, let us see how do we do that? Now, I have already written down an alternative form of the equation of motion. So, we said that our equation of motion can be written as like this

$$V_{bo} = \frac{A}{g} w$$

Now, if you look at carefully at this equation, can I say my displacement u depends only on two factors here?

Which are those factors parameters? Basically, ξ and w_n because if there are no I mean the if you look at this equation, they except these two parameter there are no other parameters that are involved in this equation. So, basically u or the displacement depends only on w_n, or you can also call this like a T_n basically the time period and the damping in the system, only these two parameters.

So, I can write down my u t as a function of course the time t, and then the time period of the system and ξ which is the damping in the system. And similarly, the acceleration which is related to u and velocity also depends on only these three parameters. So, given any ground motion given any ground motion $u_g(t)$ alright, my response u(t) only depends on the time period and damping in the system.

And for this case, remember it does not matter whether the system is heavy or lighter, or whether it is flexible or rigid, two different systems with different masses and stiffness would have similar displacement if the ratio are such that it gives you the same value of the time period T_n ok. So, yet you have to keep in mind.

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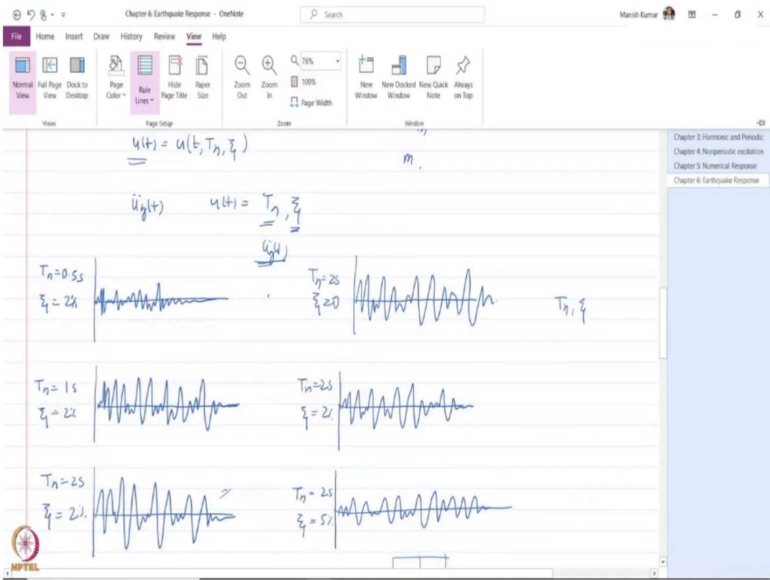

So, depending upon this two parameter T_n ok, the response to a ground motion might differ ok. So, basically if you consider, let me consider first two cases or like you know two series of cases here. So, in the first case, I am going to consider systems in which the time period is actually varying, but the damping is same. And in the second case, I am going to consider systems in which time period is same, but damping is actually varying alright.

So, let us see what happens. So, for any and I am just drawing like in an approximate representation. So, for any ground motion $u_g(t)$, let us say the first system is T_n is equal to 0.5 second, ξ is equal to 2 percent. Second one is T_n is equal to 1 second, ξ is equal to again 2 percent. And third one is T_n is equal to 2 second, and ξ is equal to 2 percent.

Now, in the second case let us say T_n is same at 2 second. However, ξ is first is equal to 0. In the second case, again T_n is same, ξ is 2 percent. And in the third case, let us say T_n is again I mean it is same, but let us say ξ is equal to 5 percent. So, what happens, typically if you give in a ground motion, this would look like something like this ok alright. As you increase the time period, typically it would look like something like this.

Now if you further increase the time period, it would look like something like this. And this is the direct consequence of increasing the time period which is evident from the response. If you consider the response here, first the amplitude is smaller, and the second basically the time period of the response is smaller. But as you go to 1 second and 2 second, you can see that the time period of the response is also increasing as well as the amplitude is also increasing.

Now, let us go to the second case in which it is somehow somewhat similar to this case. So, let me just draw it like this. As I increase the damping, so this was with the zero damping. As I increase the damping, what you will see, while the this remains same, but the amplitude actually decreases. And with the further increase in damping, these further decreases.

So, these are the typical characteristic. And you know you can go ahead and perhaps obtain the response of n single degree of freedom system subject to a selected ground motion with different time period and damping. And this type of characteristics would be obtained alright.

So, two things are to be noted here. First that the response of a ground motion or response of a linear elastic single degree of freedom system can be completely characterized using the time period of that structure at the damping of the structure. So, this you have to keep in mind.

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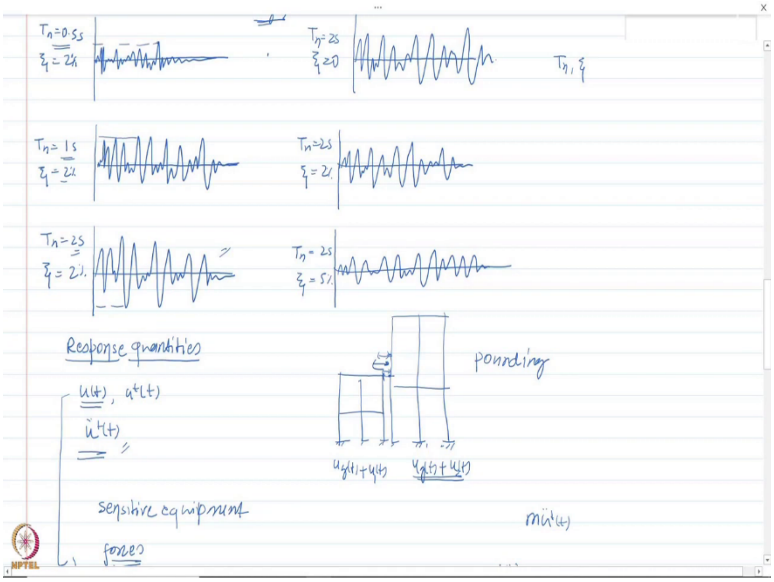

Now, let us come to the response quantities that are of interest to us. And we said that it is the relative displacement, or it could be total displacement, and the total acceleration. Now, relative displacement as I have previously mentioned leads to the internal stresses of basically forces and internal stresses in the structural number, in a structural number of a structure.

Then the total displacement might be important for cases where you have buildings that are adjacent to each other, so that you need to find out what is the separation between these buildings that T to provided. So, there is no pounding damage because what happens when you have structures of different time period shaking with respect to shaking due to the ground excitation.

At this point due to the displacement of the structure at the top of it, this structure might leads to might go ahead and like you know basically impact this structure at the point of contact it might lead to the damage to the structure. So, here the relative displacement is not the important factor, but for pounding you need to find out what is the effect of the ground displacement plus the relative displacement because that is what would be the response the responsible displacement here that might lead to making decision regarding the separation of this building.

Similarly, for this one, it would be $u_g(t) + u(t)$. So, let us saw this is $u_1(t)$ and $u_2(t)$. So, you need to find out the total basically displacement here in this case if you want to assess the pounding damage. So, the relative displacement is important in cases where you want to find out internal forces and stresses. Total displacement becomes important if you have to consider the pounding damage.

And total acceleration becomes important if you have some sensitive equipment, sensitive equipment inside a structure. So, whether if there is sensitive way whether it is kept on floor or whether it is connected to the ceiling, these equipments are actually sensitive to the acceleration or total acceleration. So, there it becomes important.

So, and of course, like you know apart from that the forces in the members which of course is related to the relative deformation. So, basically these are the typical response quantities that are of interest to us.

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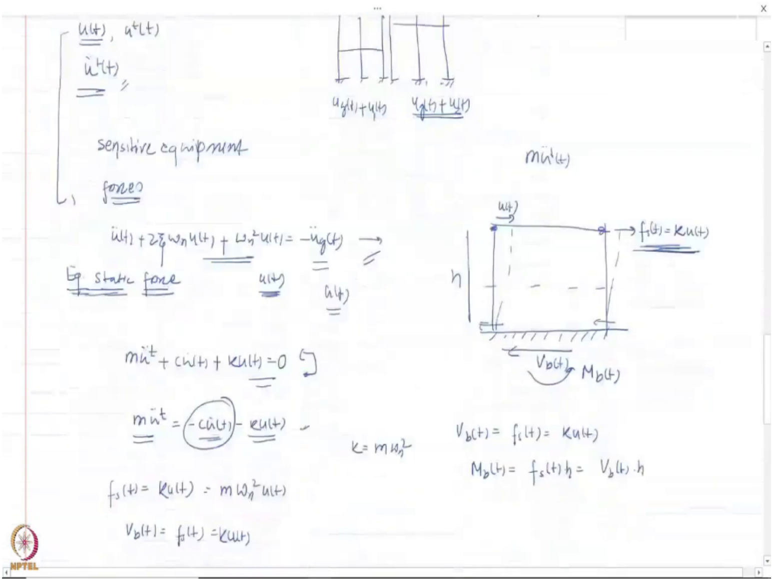

So, the question becomes the question becomes I have this equation of motion let us say, how do I go from this equation of motion to the forces finding out the internal forces in the structure? And what we are going to do here is basically introduce the concept of equivalent static force.

Let us see what is an equivalent static force. Now, as I have told you there is internal forces or stresses in a structure depends only on the displacement, not on the acceleration. So, let us say I am able to solve this equation, and find out the value of u(t) ok with any method any numerical method. Let us say this is my structure and I am able to find out the value of u (t).

Once that is found out, the internal forces in the structure can be determined by considering the response of this let us say single degree of freedom system subject to an equivalent static force which is equal to the lateral stiffness K of the systems times u (t).

If you apply this load ok and find out subject to this load, of course, it would be an static analysis that is why is called equivalent static analysis that you first do the dynamic analysis, and then you find out at each whatever time and time instant of interest to you is you find out what is the displacement ok or at each every at every time step you find out the u(t) value.

And then the response to a system can be found out considering the force under a action of a statically applied force which is equal to K times u(t). And you apply that what you would find out the response in the structure is similar if you had considered the dynamic analysis and found out the same thing.

And this can be observed from the equation of motion like this. If you consider that in terms of the total value of the absolute value of the acceleration, remember we had this equation here plus K u(t). So, somebody might ask the question why is this external force is K times u(t) and not $m\ddot{u}(t)$? Because in the end the external force is basically the total acceleration that the experience that the structure is experiencing right mass times the total acceleration.

So, why not this force? If you consider this equation of motion here that question can be rephrased if I write it like this,

$$m\ddot{u}(t) = -c\dot{u}(t) - ku(t)$$

So, the question becomes why I am neglecting this? Because if I am considering K u t it is effectively that I am considering the mass times total acceleration if I neglect this damping term here.

Now, the reason that I am considering only K u(t) as an equivalent static force is because any type of damping mechanism let us say it is due to viscous damping mechanism if I am assuming the nature of the energy dissipation to be viscous damping. So, if there is a some energy dissipation at the joints or any other mechanism, those kind of forces actually do not lead to any kind of internal stresses in the member mostly you know but I mean there are like you know if you go into non-linear analysis. Of course, non-linear systems are equated as equivalent linear system, and in that case the damping becomes important.

However, for this case where equivalent static analysis, we can say that the response u(t) ok, if I apply this force Ku(t), how much would be the response in the system? Let us find that out. I have done the dynamic analysis using this equation these equations. So, basically I have solved this and find the found out the u(t) in the system.

Now, let us say if I apply the f_{st} equal to Ku(t), how much is the displacement in the system ok? And this is a static analysis. So, for this case, can I say that if I apply this force the displacement in the system, this displacement here would be the lateral stiffness of the system total force divided by the lateral stiffness of the system which is u(t).

So, whether I do the dynamic analysis and find out u(t) in the system, or I apply a static equivalent static force of Ku(t), and find out the response in the system the response in terms of deformation would still be u(t) ok? So, that is why we consider an equivalent static force. And this is very central this concept of equivalent static force is central to earthquake resistant design.

And it is basically adopted in many of the design course earthquake design course around the world. So, I hope that concept is clear to you.

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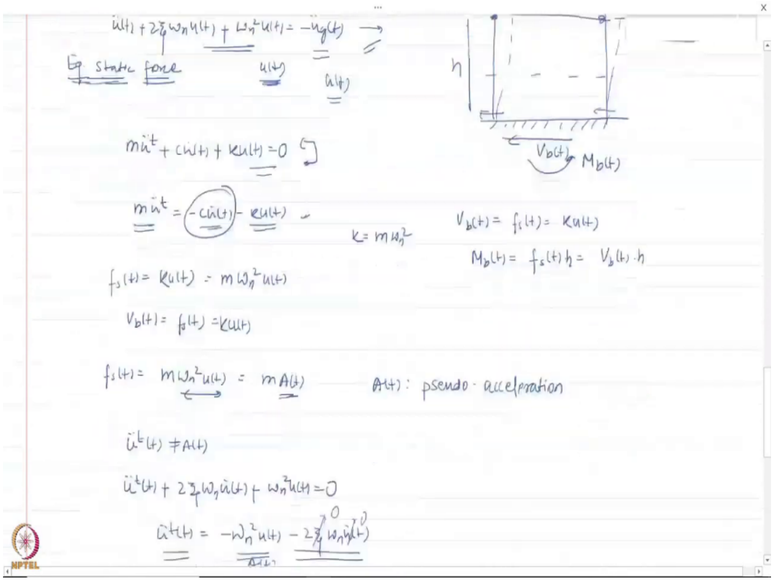

So, let us see once I have the Ku(t), what would be the base shear in this structure ok? So, base shear if I consider due to this applied force Ku(t),, and now remember that once I apply once I find out u t ok, I apply the equivalent to static force Ku(t),, then I am again now I am doing basically static analysis. So, I am interested in finding out what is the base shear and basically what is the base moment. Because once these quantities are known, then I can distribute the total base shear ok in the structure or the moment and the structure can be designed for that.

So, base shear in this case if you consider the free body diagram subject to this equivalent static force, can I say my base shear would be nothing but whatever the equivalent static forces is, and that is equal to Ku(t),. And my moment would be nothing but whatever the f_{st} times the height of the structure, which I can also write as base shear times the height of the structure.

So, let me write it again. Let me write this equation for the base shear and moment again ok, and then see what do we get. So, basically saying subject to an equivalent static force of Ku(t), I finding out my base shear as same as f_{st} which is equal to Ku(t). Now, one more thing can be done here if you look at this expression here, K can be written as mw_n^2 Remember K is nothing but mw_n^2 . So, I can substitute it here and write it as mw_n^2 square u(t) ok.

Now, if you consider this expression here

$$f_{st}(t) = mw_n^2 u(t)$$

this quantity here has units of acceleration. And basically, we write this expression as mA(t), where A(t) is referred to as pseudo acceleration. Now, this acceleration is actually not the actual acceleration of the system. It has units of the acceleration. But remember the actual acceleration of the system is. And this A t is not equal to $\dot{u}_t^t$ ok, and that is evident from this equation here the equation that we have been considering.

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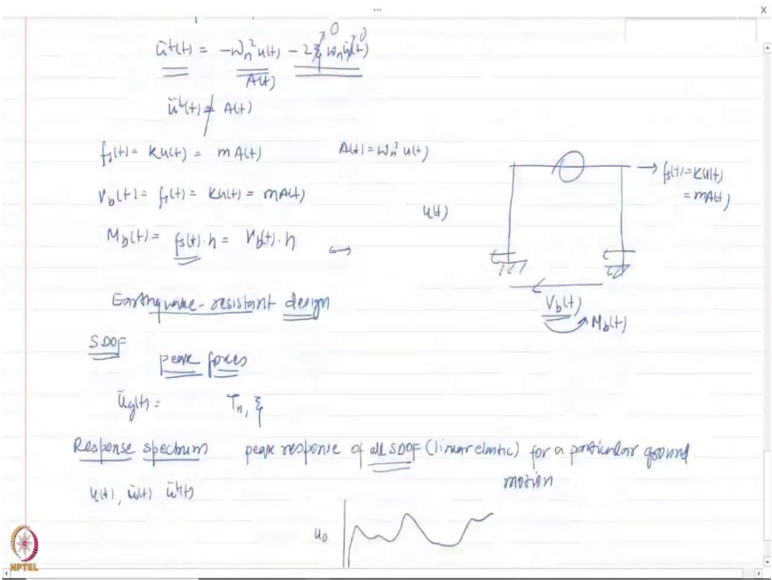

So, if I write down in terms of total acceleration, remember my equation of motion becomes

$$\dot{u}^t(t) + 2\xi w_n \dot{u}(t) + w_n^2 u(t) = 0$$

$$\dot{u}^t(t) = -2\xi w_n \dot{u}(t) - w_n^2 u(t)$$

So, I have this term here which differentiates the total acceleration from the pseudo acceleration here.

And if a system has zero damping or when the velocity is 0, so when this is 0 or when the velocity is 0, then of course, my total acceleration or the absolute acceleration is the pseudo acceleration but in general that is not the case. So, you have to keep in mind that distinction ok alright.

If that is the case, then I can say I can either apply K times u(t) as equivalent static force or I can also apply mass times acceleration the pseudo acceleration where A(t) is basically $w_n^2 u(t)$. Now, if that is known, then my base shear can be again written as

$$v_b(t) = f_s(t) = ku(t) = mA(t)$$

And my base shear is

$$M_b(t) = f_s(t)h = v_b(t)h$$

Now, remember what I have done here for this single degree of freedom system, I found out at any time instant what is the u(t), then I applied the equivalent force f_{st} equal to Ku(t) or mA(t) and subject to this force I found out what are the; what are the internal forces, for example, in this case the total base shear and the total base moment.

So, from the dynamic analysis for this seismic excitation, I am, I found out what is the base shear end moment. And once that is known I can go ahead and do the seismic resistant design or earthquake resistance design, earthquake resistant design of these structural members alright.

Once that is known, it becomes typically for a designer, it becomes very difficult. And remember here we are only considering single degree of freedom system, but in reality any structure is multiple degree of freedom system. And you know I mean it is computationally expensive to do response or too like you know numerically analyze the system and find out the response at each and every time step.

So, what do we do? And also you know that if a structure is experiencing several magnitudes of forces, typically we design a structural member for peak forces. So, if somehow, I can find out the peak values of the response quantities, I do not have to go ahead and do the numerical analysis every time. And I have told you previously that if a ground motion is there, then the response of a system to this ground motion depends only on these two parameters T_n and ξ ok.

So, what we are going to do now, we are going to introduce concept of response spectrum, and then see how this concept of response spectrum can be utilized to come up with a method to efficiently find out the response of a of any structure. So, the response spectrum is nothing but peak response and this definition is basically we are defining with respect to seismic excitation. So, peak response of a single degree of freedom system.

And of course, we are considering linear elastic here, because there are also non-linear response spectrum, but we are only considering linear response spectrum. So, this is a peak response of all single degree of freedom system for a particular ground motion, this for a particular ground motion. And let us see how is that useful.

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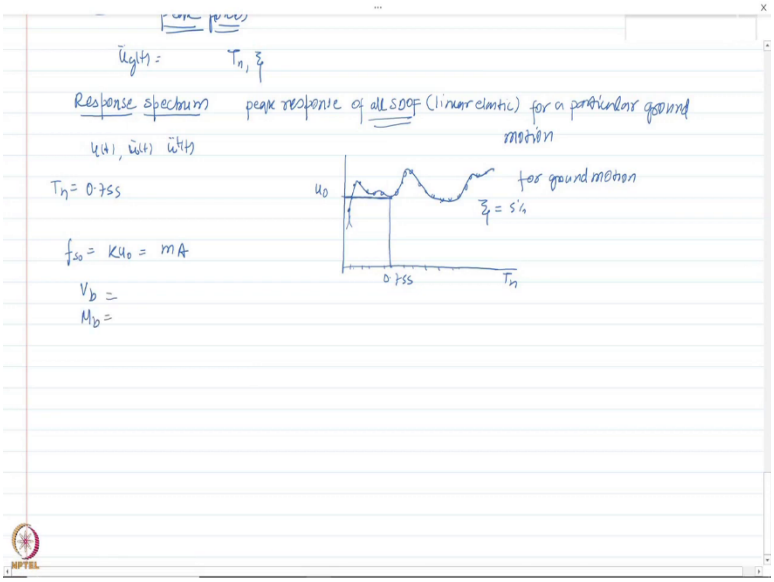

So, let us say that peek response could be anything. It could be displacement; it could be velocity; or it could be let us say the acceleration. Now, let us say if I have a plot of peak response let us say this is u_0 with some arbitrary function. I do not know what that is I do not know the nature of it let us say it is any arbitrary function. And let us say this is T_n here. All single degree of freedom system means that systems with different values of T_n here.

So, basically what do we do, if you go back to the graph that we had considered here, this one just go here yeah. So, remember depending upon the value of T_n is 0.5, or 1 second, or 2 second, I basically got different value of the peak response is not it ok? And similarly different value of damping also gives me different value of peak response.

So, now, let us consider a single value of damping and different value of time period. I can perhaps plot the peak responses on this system here for all possible values of T_n. And this is specific for a ground motion. So, each ground motion would have a response spectrum for particular value of ξ. Let us say it is 5 percent.

Now, if that is derived and I am designing a certain system for which let us say T_n is let us say 0.75 second. Then I can come here ok, I can come here and I can find out subject to 0.75

second what is the response of the system. So, what is the value of u_0. So, I could be basically finding out what is the value of u_0 or the peak displacement.

Once the peak displacement is known, I can again basically adopt the same procedure where I am going to apply the peak value of the equivalent static force which is Ku_0 or m times let us say peak value of pseudo force which is mA. And subject to this force, I can find out what is the base shear and what is the base moment.

So, if the peak values are known that is what our goal is for design of any structure of member. So, this is how we connect the idea of structural dynamics using response spectrum to design a system for any seismic excitation using the peak responses. So, we are considering, remember we are considering peak responses.

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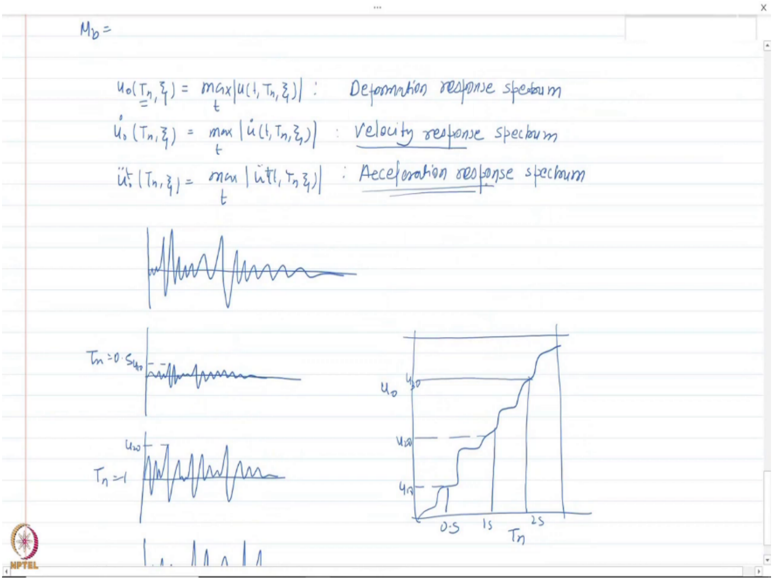

So, I can write down that my peak response for any system. So, peak response in time u_0 T_n and ξ would be basically I would find out what is the maximum response of this quantity here is. So, this is for a particular T_n. Similarly, I am going to find out the velocity ok. And then total acceleration here ok again this is this.

So, using this, we can vary the value of T_n, and we can find out different response spectrum. For example, this would give me, this would give me if I do for displacement a deformation response spectrum, deformation response spectrum. This would give me velocity response spectrum. And the third one would give me acceleration response spectrum.

So, these three-response spectrum are of interest to us. And given these three-response spectrum, I can go ahead and I can find out the response. Now, let us see for any ground motion let us say if I have a ground motion which has a very random response something like this.

Now, depending upon the value of time period of the structure as I have shown you before ok, so let us say this is time period 0.5 second, let us say this is time period 1 second. So, then it is little bit spaced apart then the above, and the amplitude is also more. And then I have the third one, which is T_n equal to 1 second.

Now, each of these would give me a certain peak response. Let us say this is u_{10}, let us say this is u_{20}, and let us say this is u_{30} ok. So, I can if I plot this, the displacement response spectrum what will happen, if this is my displacement u_0 and this is time period, these would correspond to T_n remember this is 0.5 second here. And this is u_{10} value, then this would correspond to 1 second.

And let us say this is 1 here not actually this is 1 second, and this is 2 seconds here. And this is actually 2 second. And each of these correspond to peak responses that we have obtained ok u_{20} and u_{30}. So, this is how the displacement response looks like ok. And similarly, we can go ahead and we can find out the basically the velocity response spectrum and the acceleration in small spectrum.

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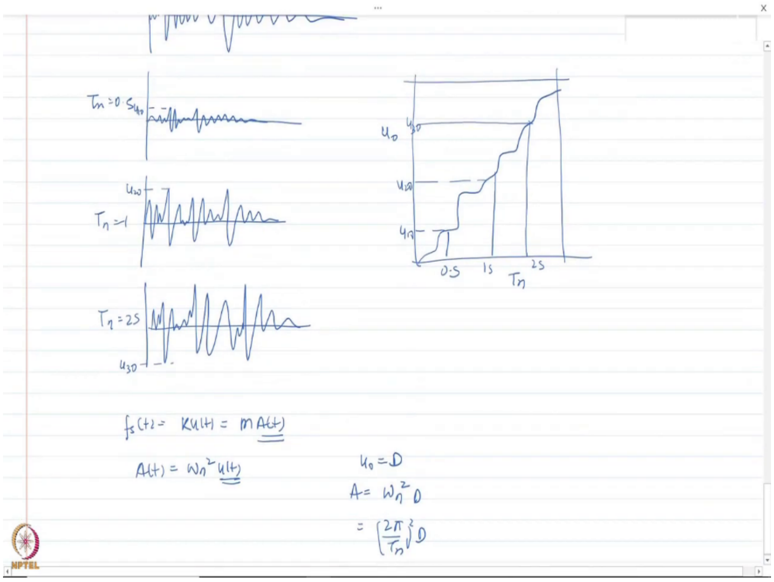

However, if you remember when I wrote down the equivalent static force is actually K times the relative velocity which is equal to mass time pseudo acceleration here. This is not the total acceleration which I have written here. So, it would be useful for me if I can get the response spectrum for pseudo acceleration. So, if I can somehow get the response spectrum for A(t), and remember A(t) is related to u(t) by how $mw_n^2 u(t)$.

So, if I can get the value of u_0 let us say now I am start to represent this as D, the peak value as D, I can find out the peak value of the pseudo acceleration as

$$A = w_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

So, if my displacement response is obtained, then I can go ahead and I can obtain the acceleration response. Similarly, I can also define a new parameter called pseudo velocity.

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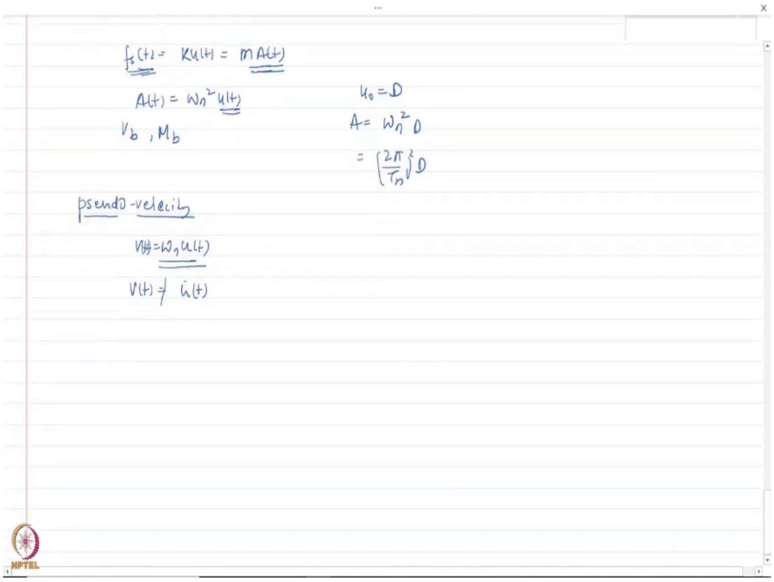

I can define pseudo velocity as

$$v(t) = w_n u(t)$$

Now, if you look at it, this quantity here has units of velocity. However, my v(t) is not actually the relative velocity in the system. So, do not again get confused. So, the, that is why again I refer to this as pseudo velocity and not the absolute or the relative velocity.

So, I can also define pseudo velocity. So, I can get the pseudo velocity spectrum as well. So, instead of finding out velocity response spectrum and acceleration response spectrum, I am more interested in finding out the pseudo acceleration response spectrum because my equivalent static force is actually a function of pseudo acceleration not the absolute acceleration. And if the equivalent static force is function of pseudo acceleration, then of course my base shear and moment would also be a function of that.

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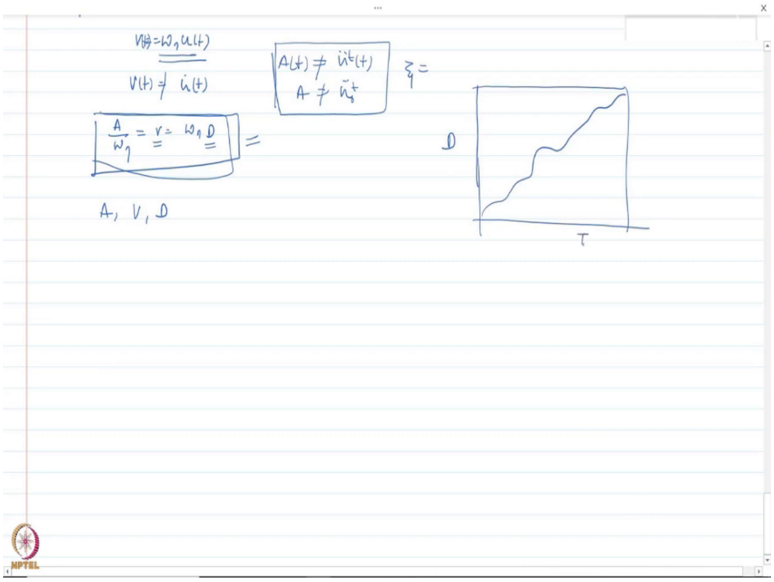

And you know because of this relationship, I can write down the relationship between basically

$$\frac{A}{w_n} = v = w_n D$$

And this describes the relationship between relative displacement. Remember displacement is relative, it is not pseudo. It is the actual relative displacement in the system. And velocity is pseudo velocity, and acceleration is pseudo acceleration. These are related by these expressions here.

Now, you as I have previously told you that for any system the pseudo acceleration is actually not equal to the total acceleration in the system. So, the peak value is also not equal to the peak value of the acceleration ok in the system ok. However, if ξ is very small ok, let us say 2 percent, 1 percent, or 5 percent, these values are quite close actually. And we can go ahead and plot this for different value of damping for different value of T_n as well. For our realistic purposes, these are quite close ok alright.

So, now, I know that if I know one of either A, V, or D, I can find out the other two because these are ok related by this expression here ok alright. Now, knowing that, that I have acceleration response spectrum which typically something looks like this. So, let us say this is D here, and this is T_n.

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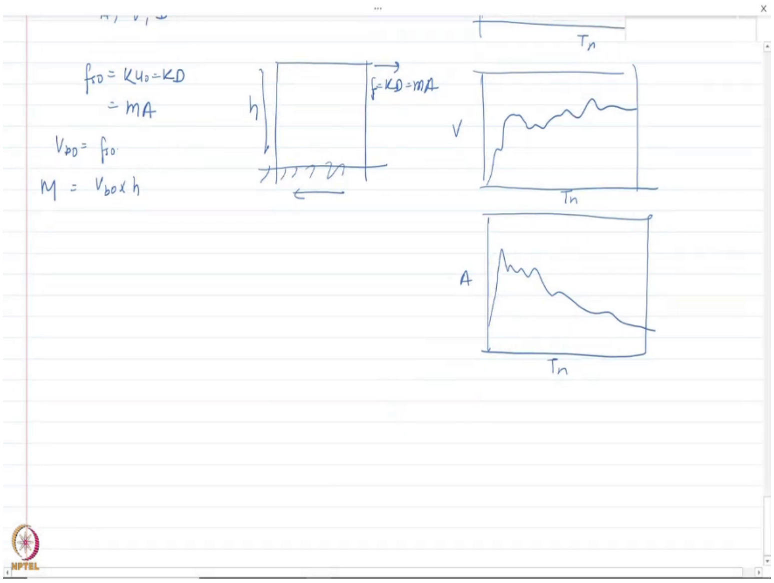

I can also plot the pseudo velocity response spectrum which would ok look like something like this. And then pseudo acceleration spectrum which looks like this. This is V, and this is A, the horizontal axis is T_n. Once one of these are obtained, then it becomes very easy for me to find out the peak response in the system, is not it?

So, look I if I look at this here, remember the peak response is basically would be obtained subject to the equivalent static force which is either Ku_0 or let us write this KD, or I can also write this as mass times pseudo acceleration, so either K times D or mass time pseudo acceleration. Subject to that I can find out the peak value of the base shear which is f_{so} times the height of this structure.

And mass as whatever the base shear times the height of the structure. There is no height here. This is simply base shear is the applied equivalent static force. And we utilize this knowledge.

$$f_{so} = ku_0 = KD = mA$$
$$V_{bo} = f_{so}$$
$$M = V_{bo}h$$

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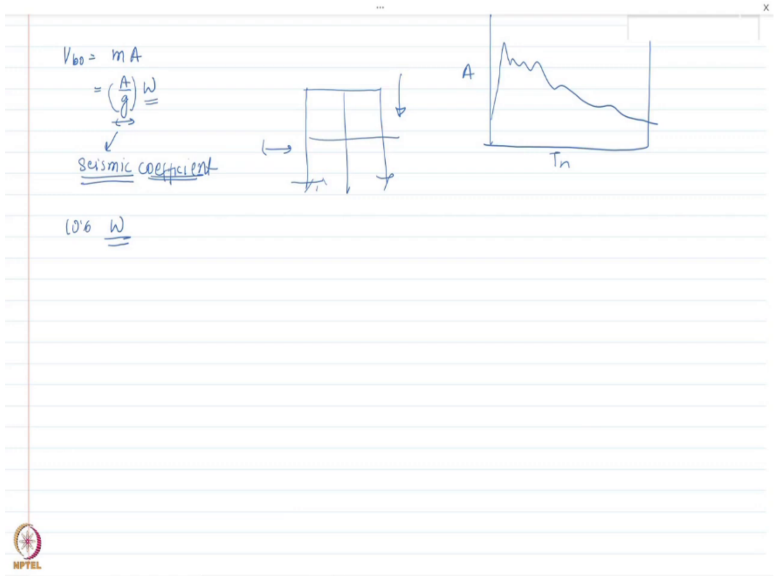

Now if you look at the expression for the base shear here , basically we get as mass times the peak acceleration. And if I write down mass as weight divided by g, this can also be write down as A divided by g. Now, this quantity here which is the peak pseudo acceleration divided by g times the weight of the structure. This is also many a times referred as seismic coefficient.

$$V_{bo} = \frac{A}{g}w$$

So, the initial seismic resistant design of the structure when the practice of earthquake engineering was not yet matured had not matured, at that time what people simply used to do remember initially all the structure were used to design for vertical loads. And earthquake is actually a lateral load. So, everybody knew that you have to design this structure for lateral loads to sustain earthquakes, but nobody knew how much.

So, basically initially they started the design as such that they are going to take certain percentage, certain percentage of the seismic weight, let us say 10 percent, and applied laterally and design the structure for that. And even to this state that method is reasonably accurate even if you design a structure so let us say 10 percent or 20 percent, even if you do sophisticated seismic analysis your basically results come out to be somewhat closer ok alright.

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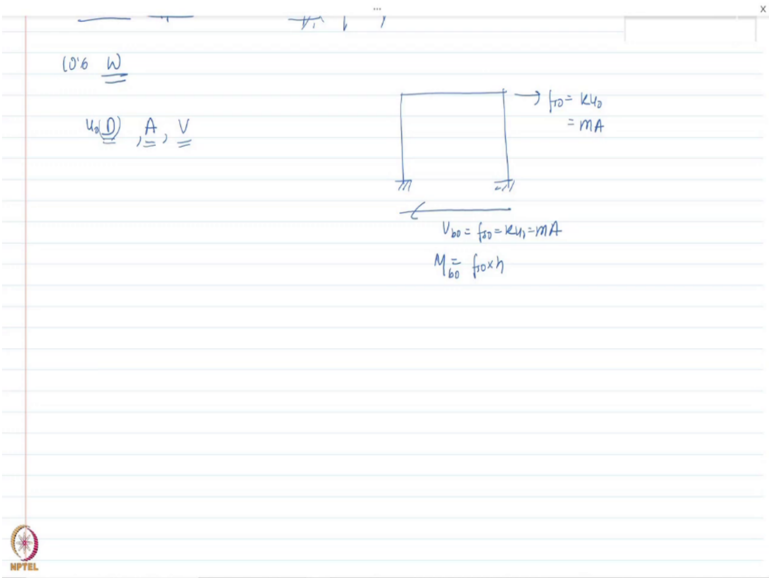

Now, let us get into finding out the peak responses back to peak responses. And we have already discussed that if somehow, I can get my D, or let us say pseudo acceleration, or let us say when V then I can go ahead and do the equivalent static analysis. So, let us say this is f_{so} is equal to Ku or KD also equal to mass times acceleration, where total base shear would be as I have previously indicated f s o naught equal to Ku_0 times acceleration. And basically, the moment, the base moment would be f_{so} times h.

And we would be following this procedure. And depending upon you know a what response is a spectrum is available to you, you can directly get D, or even if A is given let us say the pseudo acceleration is given or V is given, you can find out the other parameter ok, alright. Now, let us get on to now we know how to find out the peak response is there is one challenge here.

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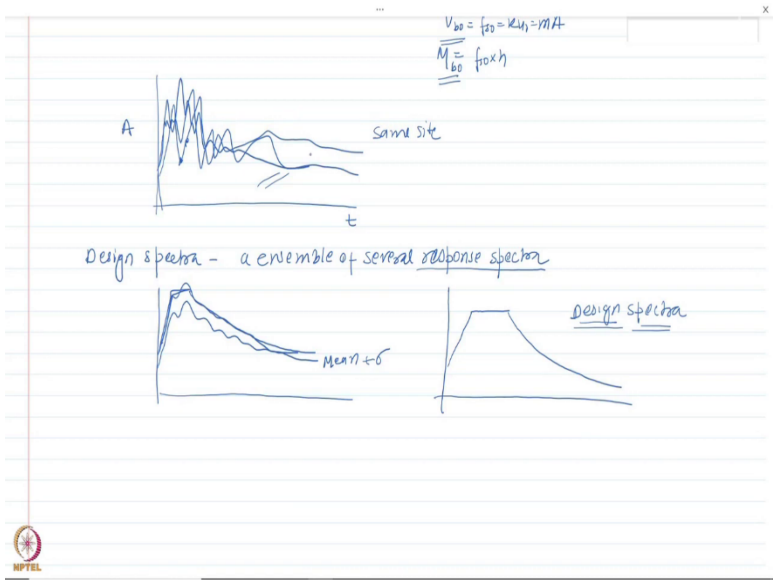

Challenge is I have shown you that an acceleration response pseudo acceleration or response spectrum looks like this ok. So, there are lot of non-uniformity if you look at a it, lot of jagged lines. This is still like in a very smooth, but actual response spectrum would look like something like this. Now, if you measure, if you measure the acceleration at the same site, after certain number of years the same thing would now look like this. And again, after certain year, it could again change.

So, the question becomes if I have to design the structure, so basically to design the structure, I need to find out these peak forces and peak moments. Which one to choose ok? Remember I have told you this is at the same site only that it is measured at different instances of time. And the response spectra would be quite different in terms of the peaks that you see here. So, the question becomes which response spectra should I use?

So, actually what happens in these cases that just for if would select any response spectra just for a small change in the time period, your forces would increase substantially, and that is not realistic from the design perspective. So, what do we basically do we define or we develop something called design spectra from a set or let us say an ensemble of ensemble of several response spectrum?

And how do we do that? Remember if you take the average of these response spectra here, how would they look like? If you look at this the average would look like little bit smoother compared to individual response spectra. So, let us let us say this is the mean value of different response spectra at the same site. Now, typically what do we design? There is different like in very sophisticated design procedures ok where you do seismic hazard analysis, you know do a statistical analysis to come up with.

But in very simple or laymen term if we consider let us say mean plus standard deviation, it would look like something like this. So, that it would consider ok most of these peaks were to be captured under mean plus standard deviation. And then approximate that with set of straight lines straight lines or curves, so that my response or my design spectra in reality looks like a much smoother curve which is the idealization of the or which is the idealization derived from the statistical analysis of several response spectra.

So, this is my design spectra. And this is what we use in design of structure, not a single ground motion like this. So, design spectra basically obtain doing a statistical analysis of several ground motions or several response spectra. And then come up with some conservative estimate of the acceleration at different time period. And then approximate that using ok curved lines or smoother straight lines.

So, these lines basically would represent the response spectra at the site. And this is what is utilized in the design of the structure ok alright. So, these are basically the concepts that are required to understand once you know what is the value. So, from here let us say this is the pseudo acceleration response spectra. From here for a given structure of given value of T_n, you can get what is the value of the peak acceleration.

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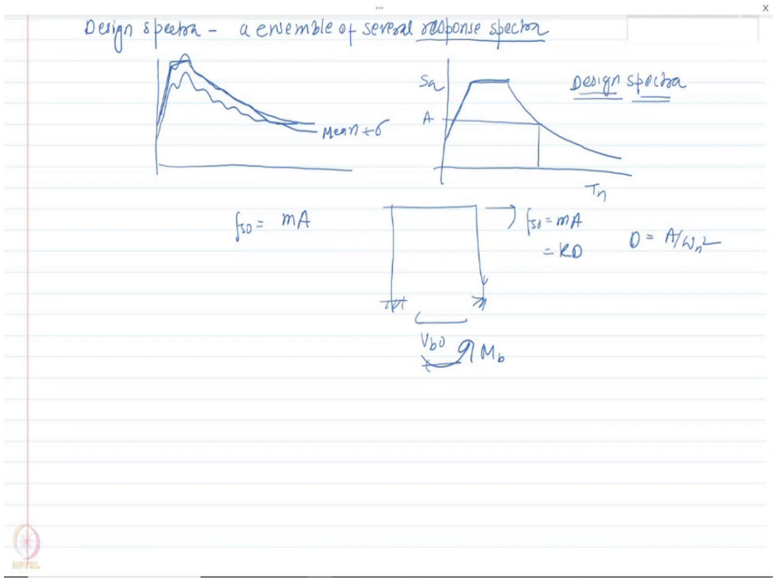

Once you know the peak acceleration from here, you can go ahead, and you can find out the equivalent static force that you need to apply on this structure ok. And subject to that or you can also write it as KD, you can find out the peak base shear and peak moment. And this is the typically the procedure that we use. So, this is a very simplified presentation of the concept of earthquake resistance analysis and design using response spectra.

Of course, there are like you know much more detailed discussion of this in that would be presented in an earthquake engineering course, but for this case this is how we relate or apply a concept of structural dynamics to earthquake resistance analysis and design of structure alright ok. But this I would like to conclude this lecture.

Thank you.