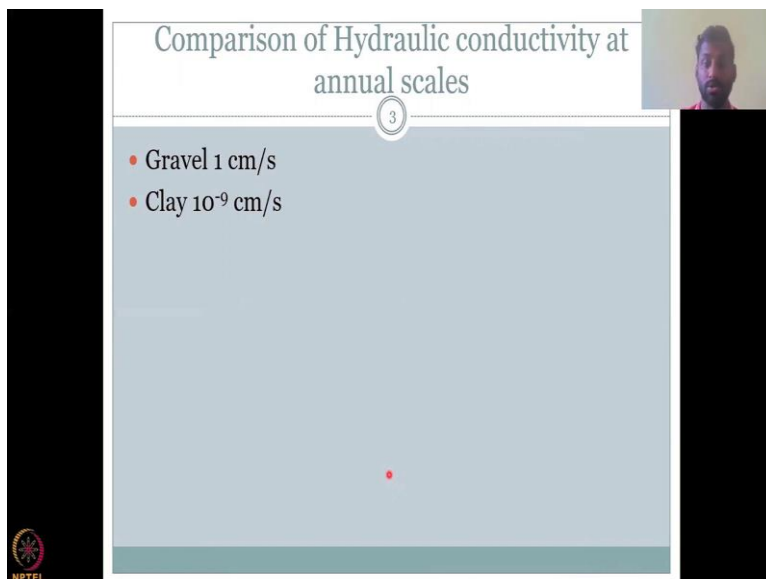
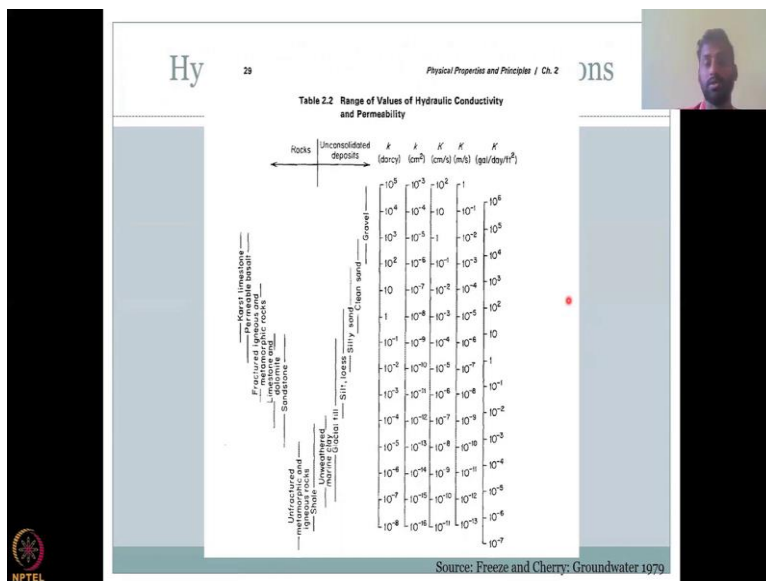


Groundwater Hydrology and Management
Professor Pennan Chinnasamy
Indian Institute of Technology, Bombay
Lecture 20
Heterogeneity and Anisotropy

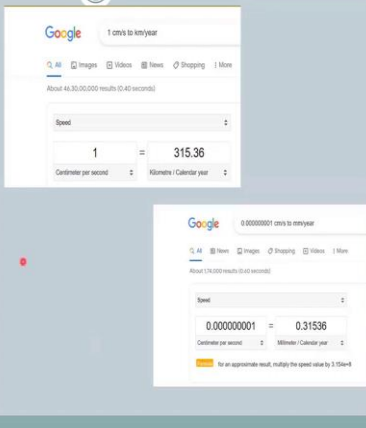
Hello, everyone. Welcome to NPTEL course on Groundwater Hydrology and Management. This is week 4, lecture 5. We're coming to the close of discussing the hydrological components for groundwater analysis. We understood the major components in this lecture, this week's lecture.

(Refer Slide Time: 01:02)



Comparison of Hydraulic conductivity at annual scales

- Gravel 1 cm/s
- Clay 10^{-9} cm/s



3

NPTEL

And we will look at some more important aspects of the component that we saw, especially hydraulic conductivity. So, hydraulic conductivity has a lot of variations, which means a range was expressed in the detailed data set that we share from the book. And most of these books have a similar range and so, if you check with other aspects also, the ranges will be similar, most of these data are collected in the lab and measured. So, more or less we can use it across the world for any system.

So, I as I promised we will do some comparison for gravel of hydraulic conductivity at annual scales so, we have a gravel at around 10^{-1} meters per second. And we also compare it with clay with very, very less around 10^{-9} centimeter per second. So, we have two values for hydraulic conductivity and we have taken the mid average for gravel around 1 centimeter per second here. And then 10^{-9} centimeter per second power here.

Let us see what that equates to if we convert it to an hourly as I said, you have to convert it to hourly, then daily, then annual. So, to convert that you could do it on your calculator or you could just quickly run it through Google. So, 1 centimeter per second to kilometer per year please look at how I have typed it, centimeters slash second to kilo meter slash year.

And once I type the automatically Google will give you the calculator and convert it. So, you have the one value I put is just 1 but you can put 10, 5 depending on the value you have in your measurements. So, for now, let us take 1 we have taken it from the table. So, 1 centimeter per second equals to 315 kilo meters per calendar year. So, that is how much groundwater would travel from a particular location to another location.

So, h_A with h_B , the two wells it would travel around estimating the velocity between the wells we could estimate it to be around 315.36 the hydraulic conductivity per year. So, let us compare that to clay. We had 1 centimeter second power 10 power minus 9. So, that is you have eight 0's and 1 to millimeters per year. If you convert that, that is not even in kilometers so, we forget the kilometers we are not going to go to kilometers we will just go to millimeters.

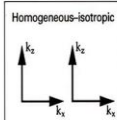
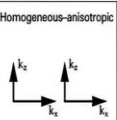
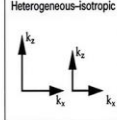
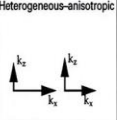
So, not even 1 millimeter per calendar year the water will move. Millimeter is very small. But even that distance it does not move. So, that is how the differences in nature and so, when you go to a soil and a location with clay structures and soil, please understand the water moves very, very slow. So, if you are depleting the aquifer, the recharge will happen very slow. And so, you need to warn the farmers that you are not just going to get water every year the same rate. It will come low whereas in a gravelly system, it travels fast.

So, that is why the mountain water you could reach in the springs and waterfall because water enters these gravel and it quickly comes to the springs and waterfalls. Just think about 315 kilometers per calendar year is the hydraulic conductivity as a rate. So, this if you have a unit for example, Q is the velocity is equal to K times $\frac{dh}{dl}$ and the $\frac{dh}{dl}$ is a unit difference you have 1.

So, if you take a unit difference and the distance between the locations was 1. So, that 1 will go off and all your Q is absolutely equal to your K . And that is how you can estimate the velocity of a unit in your soil. So, that unit is very, very important and it gives you around 315 kilometers per year. So, that is how much faster water can flow through in a year, in a gravel system, but in a clayey system it is very, very slow.

(Refer Slide Time: 05:50)

Heterogeneity and Anisotropy: Rocks are heterogeneous and anisotropic

Homogeneous-isotropic	Homogeneous-anisotropic
	
Heterogeneous-isotropic	Heterogeneous-anisotropic
	

Ingebritsen & Sanford (1998)

Anisotropic—direction dependent property

Heterogeneous—property that varies from one point to another (REV scale)

Therefore need to solve Darcy's law at different points and take into account directionality

NPTEL

4 Source: Freeze and Cherry: Groundwater 1979

Not only this, so, why was the range present and also what are the differences in the hydraulic conductivity? Let us look at a typical case. So, we have a high heterogeneity in the system and anisotropy in the system. When I say system it is the soil or the rock matrix. Rocks are heterogeneous and anisotropic same with soils. Let us see what does that mean? What does the, what do you mean by heterogeneity and anisotropy.

So, let us take a soil sample, we are taking a soil sample at a particular location and the k it can be permeability or hydraulic conductivity, let us take hydraulic conductivity here is k_x in the x direction and k_z in the z direction. Why am I not taking y because x and y are almost same. So, this is how the plane is, this is your z plane and this is your x plane, your y plane will go here.

So, your x and y can be represented by $2 \times I$ am just showing you in a tilted fashion. So, this is your z plane and this is your xy plane, it is lateral in direction but those who want I could just draw it quickly so, that you could see. So, this is your xy plane, which is horizontal or lateral in groundwater flow, whereas your z is on the vertical.

So, z is very important for groundwater hydrology, because gravity acts on it. xy is when your gravity pulls the water, but then more lateral movement happens due to slope and other reasons. So, just to simplify your x can be equal to k_y . So, your x and y could be the same velocity or hydraulic conductivity or permeability, So, we will just take two planes one is the z plane and then the xy plane.

Moving on, let us take one example. One soil taken at $x_1 y_1$ location and that is k_x in the hydraulic conductivity x plane and in the z plane it is k_z . In another location, location $x_2 y_2$ is here, you have k_x and k_z and at the two locations the k_x is the same as k_x , k_{x1} is equal to k_{x2} , same k_{z1} is equal to k_{z2} . So, which means the medium is homogeneous and isotropic which means at the same location the values are same.

And when you move to another location also the values are same because k_x is equal to k_z . So, in one location, the hydraulic conductivity is the same in x plane and your yz plane which is isotropic same in the direction and when you move to a different location homogeneous it is the same k_{x1} is equal to k_{x2} and k_{x1} is also equal to k_{z1} because of isotropy.

So, now moving to homogeneous and anisotropy. So, here at one location let us say $x_1 y_1$ you have k_x , which is not equal to k_z , k_x is not equal to k_z . So, it is anisotropic. So, in different planes the k value is different. So, that is not concerning the isotropy. So, it is anisotropic. But then when you go to a different location, your k_x are the same, k_x in the location 2 is the same as location 1.

So, k_x , k_x are the same, but same k_z is also present k_{z2} is equal to k_{z1} . So, both k_z values are the same as k_x values. However, k_x is not equal to k_z which means it is homogeneous. So, between locations it is the same, but within the location, within the location it is not the same. So, k_x is not equal to k_z within the location so, it is anisotropy. However, if you take the same sample in a different location, it is the same values in both the x and y direction or z direction. So, that is homogeneous.

Now, come to the next example. So, these are cases in the real world. So, you can have a homogeneous isotropic, you can have a homogeneous anisotropic. And now, we are going to heterogeneous isotropic. Which means in a particular location the k_x and k_z are same. So, in location 1, k_x is equal to k_z in location 2, k_x is equal to k_z .

However, your k_{x1} is not equal to k_{x2} . In different locations the magnitude differs in your values. However, within the location the values are same. So, within the location k_x is equal to k_z . However, in a different location k_{x1} is not equal to k_{x2} . So, that is heterogeneity. But within the location is the same, so, it is isotropic.

Now, we move to the more realistic version the heterogeneous anisotropic. So, which means within the location your hydraulic conductivity is different in different planes. So, k_x is not

equal to k_z . So, you have to have two measurements at least. And say when you go to another location your k_x is not equal to k_z .

So, basically it is anisotropic and moreover the k_{x1} the measured value in k_x at location 1 is not equal to k_{x2} . So, that is a heterogeneous. So, in a real world this is the most abundant property it is heterogeneous and I know anisotropic which actually complicates the groundwater estimation, it complicates the estimation of your hydrology. And that is why you need more and more data for groundwater flow.

Anisotropic is direction dependent property so it is direction it is k_x or k_z and how it differs is the isotropic and anisotropic properties. Heterogeneity properties that varies from one point to another. So, it is this spatial property and it is a relevant scale. So, you need to understand that one is at the directions for a same location how within the direction it changes and homogeneous and heterogeneous are part of the space, how spatially it differs.

Moving on. So, as I said therefore, there is a need to solve Darcy's equation law at different points and take into account directionality. You cannot assume that k_x is equal to k_z and also it is going to be the same as two locations. But how much can you do is dependent on your time and the cost to do these estimates. So, you need to balance your cost and time with the heterogeneity and anisotropic conditions.

But in the real world, please understand that anisotropy and heterogeneity is much, much high than homogeneous isotropic et cetera. So, only the beach maybe you could find homogeneous and isotropic conditions. But even then, you are going to study groundwater in the beach locations. So, you need to be very careful in where you are going to do these applications. And understand the soil and hydraulic conductivity values. Do they differ or not?

(Refer Slide Time: 14:38)


Darcy's law expressed in vector shorthand

$$\mathbf{q} = -\mathbf{K}\nabla h$$

Expanding vector shorthand to represent flow in one dimension (e.g. x):

$$q_x = -K \frac{dh}{dx}$$

Expanding vector shorthand to represent flow in two dimensions:



$$\begin{pmatrix} q_x \\ q_z \end{pmatrix} = - \begin{pmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{pmatrix} \begin{pmatrix} \partial h / \partial x \\ \partial h / \partial z \end{pmatrix}$$

or

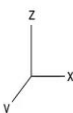
$$q_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xz} \frac{\partial h}{\partial z}$$

$$q_z = -K_{zx} \frac{\partial h}{\partial x} - K_{zz} \frac{\partial h}{\partial z}$$

Source: Freeze and Cherry: Groundwater 1979

Expanding vector notation to represent flow in three dimensions:

Source: Freeze and Cherry: Groundwater



$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = - \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \begin{pmatrix} \partial h / \partial x \\ \partial h / \partial y \\ \partial h / \partial z \end{pmatrix}$$

or

$$q_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z}$$

$$q_y = -K_{yx} \frac{\partial h}{\partial x} - K_{yy} \frac{\partial h}{\partial y} - K_{yz} \frac{\partial h}{\partial z}$$

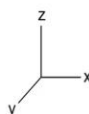
$$q_z = -K_{zx} \frac{\partial h}{\partial x} - K_{zy} \frac{\partial h}{\partial y} - K_{zz} \frac{\partial h}{\partial z}$$

Hydraulic conductivity & permeability are *tensors*

How do we evaluate the components K_{ij} of these tensors?

Expanding vector notation to represent flow in three dimensions:

Source: Freeze and Cherry: Groundwater



$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = - \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \begin{pmatrix} \partial h / \partial x \\ \partial h / \partial y \\ \partial h / \partial z \end{pmatrix}$$

or

$$q_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z}$$

$$q_y = -K_{yx} \frac{\partial h}{\partial x} - K_{yy} \frac{\partial h}{\partial y} - K_{yz} \frac{\partial h}{\partial z}$$

$$q_z = -K_{zx} \frac{\partial h}{\partial x} - K_{zy} \frac{\partial h}{\partial y} - K_{zz} \frac{\partial h}{\partial z}$$

Hydraulic conductivity & permeability are *tensors*

How do we evaluate the components K_{ij} of these tensors?

Advanced Groundwater modeling

Moving on, so we are going to see the property that is more relevant, more important is the heterogeneous anisotropy, how do you solve it? So, in the Darcy equation, when we discussed last class, we saw q is equal to minus K del h which is the gradient, hydraulic gradient. And if the hydraulic gradient is 1 your q is equal to your hydraulic conductivity magnitude the direction is minus goes in the lowering direction.

So, let us write it in terms of vector shorthand to represent flow in one direction. So, the flow in one direction is given as q_x is equal to minus k dh by dx , we are just expanding your gradient. However, we saw in the previous slide, that it is heterogeneous and anisotropic which means from one location to the other location, the values can change. But within the location it differs as per the plane x , y , and z .

So, in the normal very, very simplistic version, we say x and y are same. So, but however, there is difference in the z . So, your xy plane, you could take but z is definitely different. Because z is gravity and the process how water moves is different than the process groundwater moves on the horizontal version.

So, then you have to expand in shorthand q_x q_z so q becomes q_x , now we are expanding it into two dimensions q_x q_z that is equal to minus k_{xx} k_{xz} k_{zx} k_{zz} and then you do the partial differential equations. It can be written in short form as q_x is equal to minus q_{xx} del h by del x minus k_{xz} dh by dx dz . So, all these are partial differentiators and you would know how to do this in the mathematical classes.

But more importantly, you can have another one for q_z and the similar equations can be written. But do we stop here? No, because it is a 3-dimensional problem. So, in a 3-dimensional world, if you express it, it becomes a huge matrix. And you know how to solve it is by using matrix and differential equations.

Hydraulic conductivity and permeability are tensors. So, you would have to calculate the component of K_{yz} of these tensors. And that is a advanced groundwater hydrology class. So, since we are not going into the advanced level in this course, it is a perspective and introduction course on groundwater and groundwater hydrology. I am just going to introduce the concept of what is the equation and how it varies in the different planes but solving it will not be part of the class.

If you think you are going to solve this by pen and paper is going to be really really difficult and time consuming. And how many times can you do because you if you want to study one

hydrology groundwater equation, per location per time, it is okay. But then you want to do it as a time series and you want to combine one location to the other and from there to the other.

And so, this cannot be done by hand. And that is why we do have groundwater models. The most widely used model is MODFLOW, which is open source, we will be introducing the model in the future lectures.

(Refer Slide Time: 18:26)

Recap of Week 4:

7

- Groundwater Hydrology components
- Porosity
- Specific yield and Specific Retention
- Permeability and Hydraulic Conductivity
- Hydraulic Conductivity 3 Dimensions

The diagram illustrates the subsurface hydrology. It shows precipitation on the surface, which infiltrates into the ground. Key features include: a phreatic surface (water table) in an unconfined aquifer; a capillary fringe above the water table; a confined or artesian aquifer below a confining layer; and a perched water table above a local aquifer. Two types of wells are shown: a flowing artesian well tapping into the confined aquifer and a non-flowing artesian well tapping into the unconfined aquifer. The diagram also labels the zone of aeration (above the water table) and the zone of saturation (below the water table). Other labels include 'Recharge area of outcropping of formation', 'GWT' (Groundwater Table), 'NIT Aquifer', and 'Bed rock' at the base.

Source: Hydrology: principles, analysis and design (Raghunath 2006)

So, with this, I have covered most of the important hydrology components for the course. Let us do a quick recap so, that we tie all the lectures from both week 3 and week 4 to a common understanding of what we did. So, we introduced the hydrology components in terms of groundwater. We studied how water enters the system and goes into infiltration and percolation into different compartments.

We looked at the zone of aeration and zone of saturation. And then we differentiated the aquifers based on is it open the surface to recharge as unconfined aquifers and then if there is a confining layer, we set the aquifer within the confining layer to be a confined aquifer. We also noted that the porosity is the most important factor to determine the groundwater storage and flow because that is the space which water occupies.

And water would relocate itself between the pores to move. So, first infiltration gets the water inside and after that it starts to move through percolation and if the pore spaces are connected, then more groundwater flow will occur. Porosity is a function of the soil or the rock material and we call it the matrix and how the soil particles are structured and in between how the pore space is there.

We also noted that the pore space can have air, water, combination of both or none. So, none is in the lab conditions, but most of the time it will be air and water. In a dry situation it will be only air, in a wet flooded situation it will be only water. When it is dry it is called dry soil where only air is present in the pore space and when it is full of water we call it saturated soil.

Here we call it zone of aeration where air is there and water can come in by pushing the air out and then zone of saturation is two which is under the imaginary water table where water occupies the pore space. Then, once we established that we wanted to see how the pore space can hold on to the water and what are the forces and the indicators of establishing this. So, we looked at specific yield wherein water enters a soil profile and gravity starts to act on it.

So, how much is the drainable porosity was given by specific heat. Why is this important? This is important because a plant can know how much pressure to exert to take the water a, farmer can know how much he or she has to put a pump to exert the water and how much gravity actually keeps pulling down the water. So, in a gravel field, you saw the specific yield to be very high, which means water would just flush through the gravel beds and not much water is going to remain.

Specific retention is the opposite of specific yield, where it is a property of the soil to hold on to the water. And this holding on to the water actually benefits the plants in a particular aspect if this specific retention is not too much. So, because this is the property of holding the water in your soil. To visualize it, we took the sponge in the kitchen washing dish example so, you have a sponge and you squish it all the water is out, it is a dry matrix.

Now when you put water and soak it in water and lift it up the water will start to drip and that is specific yield acting on the drowning of the water out of the sponge through gravity. And then, what we did is when after gravity has exerted its force then some water is still present in the sponge which is called the specific retention.

Specific retention should not be too much like in clay, clay can hold on to water long, but that holding potential can also limit how much the plant can exert. The plant can have some water that can be taken easily, but if it is too tightly bound to the soil like a clay, then the plant will not be able to grow.

Then we looked at permeability and hydraulic conductivity. Permeability was also a function of the material of how water is allowed to permit to go through and it is also a measure of the porosity at connected pores how well is the pore connected. We looked at effective porosity

and we looked at water velocity as a function of these permeability effective porosity. Because the pore space can be at different spaces big, big spaces, but if they are not connected, then the water will not flow, water will just get stored and be there. And we wanted groundwater flow to occur.

So, from permeability, we looked at another property which is also a function of the fluid and not only the soil which is a solid particle, we also looked at hydraulic conductivity wherein the medium or the soil or rock matrix allows or conducts the water to flow as based on the properties of the fluid. Here the fluid is water. So, we used the viscosity and density of water to estimate hydraulic conductivity in solids.

So, hydraulic conductivity is both a function of the soil and the liquid that is passing through the solid and for our rural applications and agricultural applications. We took water as a fluid. And based on that we had different values for hydraulic conductivity of water in these different soil parameters.

We looked at hydraulic conductivity in a 3-dimensional space wherein you have your z, x and y, the x and y could be almost equal in the velocities and the magnitude. However, z can be different. We looked at anisotropy and heterogeneity to be the major factors of making groundwater very complex. Just look at this in a surface hydrology perspective.

In surface hydrology, you can quickly estimate is it going to be homogeneous or heterogeneous, because the parameters are very less. The stream bed, you can look at water is going to flow in the river, it is almost the same soil on the side. So, it is okay. And is it isotropic, there is no big causes to stop the water like check dams and the large taps.

Whereas in groundwater, there are a lot of parameters that can impede the flow, which can stop the flow and the impede is not the same at different points it is heterogeneous and anisotropic. The basic properties that actually allow the water to flow is complex in nature and we have looked at how it can be very, very complex.

So, the best way is to get a good fundamental understanding of these properties and the limitations or challenges in applying them in the real world. At one point, we also argued that we cannot afford both cost and time to take multiple samples along every location. However, we need to estimate and assume some things. So, most of our assumption would be at a range. So, yes, we agree that it is heterogeneous, but it is within that range.

And then we also said anisotropy yes, we have different values for k_z and k_x . However, we said between x and y it is almost the same. So, we said isotropic conditions for x and y . Homogeneous was dealt with very cautiously. So, you need to understand the soil type in different locations to estimate if it is homogeneous or heterogeneous.

So, in a groundwater model, you will actually dissect the differences in the layers and say is it homogeneous or heterogeneous and the model will calculate it. We also looked at solving these equations, once we established the variations in hydraulic conductivity, we said okay it is not going to be a single 1-dimensional problem or a 2-dimensional problem, it is a 3-dimensional problem. And we looked at the equations that will govern the 3 dimensionality in Darcy's law.

We also found that it is not easy to solve the equation on pen and paper maybe on one equation, one location we can but for groundwater, you need to estimate what is the hydraulic conductivity here and the Q here Darcy's Q . And then what is it here, what is it here to actually monitor the flow and for that you need a connectivity and a feedback from each of the Q 's. And that is mostly done by models.

We also looked at water levels and hydraulic head how to estimate the hydraulic head and water, from water levels and groundwater depth and from the hydraulic head and water levels you jump into the Darcy's law. So, with this, we have almost finished most of the important components for groundwater hydrology.

In the coming weeks we will see how to use them and also we will go in deep for particular aquifers, what are the parameters that are important? So, we will revisit some of these, but the basic introductions have been done. I will see you in the next class. Thank you.