Groundwater Hydrology and Management
Professor Pennan Chinnasamy
Center for Technology Alternatives for Rural Areas
Indian Institute of Technology, Bombay
Lecture 23
Groundwater equations in Confined Aquifer

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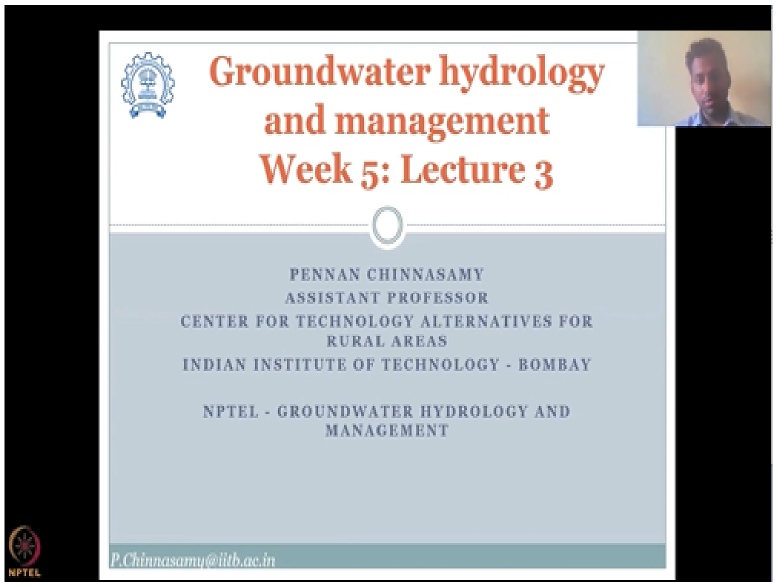

Hello everyone, welcome to Groundwater Hydrology and Management NPTEL course, this is week 5, lecture 3. In this week's lecture, we are looking at establishing the groundwater equations for groundwater flow in different media. So, it is very important to understand the differences between the media and where the groundwater equation can be valid and what are the limitations and challenges. The past two lectures we looked at converting the data into a groundwater equation ready asset thereby looking at hydraulic head converted to contour lines, then to flow lines, gradients, and then we also discussed about travel time.

In today's lecture, we will look at the differences in different compartments of groundwater storage which is the unconfined, confined saturated versus unsaturated. In the last lecture, we also looked at transient flow versus saturated flow, these are very, very important bifurcations that are needed to understand groundwater equations, because the equation might be simple, but if you do not know where the differences are then you could easily misuse the equation.

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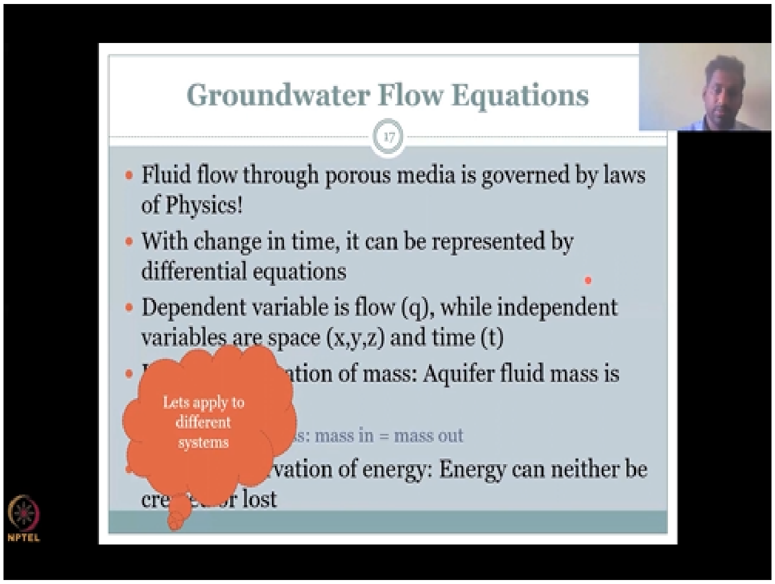

That is why in the last class, we went through all the equations, let us go through the physics of the groundwater equation. So, groundwater flow equations have a very strong basic relationship with physics. So, fluid flow through porous media is always governed by laws of physics. So, when I said it flows from high potential to low potential, the potential energy is high in water at a higher level and then it has to come down to the lowest stable state by giving off the energy what would it do it would convert the potential into kinetic, kinetic energy and then move with the velocity and bring down the potential energy.

So here energy is conserved. So, this conservation of energy or masses all are included in the laws of physics, which are highly governed by groundwater flow. So, when something is governed by physics laws, there is always a change in time. For example, at time equal to zero the water was at well A at a height of 100 meters, at time equal to 10 minutes the water has moved from A to B, where the potential is 90 meters so from 100 it came to 90 and so there is a change of time.

So, with change in time, it can be represented as a differential equation. So, this is where groundwater flow occurs. So, when at water at A time equal to zero, it was stationary, it did not move, but when you start the time, time equal to zero to time equal to 10 minutes, when water starts to slowly move, and that movement is a function of time and it can be represented as differential equations, the dependent variable is always flow, which is Q, while independent variables are space x, y, z and time t.

For example, you have in the space let us do a two space continuum X Y axis and from here at time equal to zero it moves to time equal to 10 at this location, so, you could see that the independent variables are space, which is XY and Z and time at time equal to zero to time equal to 10 whereas the Q is what we are trying to understand what was the Q from here A to B.

The other important laws in physics that are being conserved or governed is the law of conservation of mass, which means aquifer fluid mass is constant. So when it moves, it does lose some water in storage and, and wetting etc. But in a saturated medium, the aquifer fluid mass is constant if you remember the Darcy's setup, we said we said a volume Q and we want Volume Q to come out in the tube.

So, that is conservation of mass no change in mass which means mass in is equals to mass out when the Q is the same, which goes in the volume or discharge per unit time, when it goes in the same and comes out the same value, then that means the mass has been conserved. The other important thing is law of conservation of energy; here energy can neither be created or lost. So, this is where I stressed on the fact that at A the water would have had high potential energy and zero kinetic energy because it was standing there, but then it started to move and while it moves, the potential energy is converted to kinetic energy and the kinetic energy is taking it down into potential energy.

So, you have a lower potential energy at B but the kinetic energy is compensating for the loss of energy of potential. So, a law of conservation of energy which says that energy can neither be created or lost is what is conserved, it can only be transformed from one to the other. For example, here we are transforming potential energy to kinetic energy by motion, and your energy is conserved. So two things: no conservation of mass and law of conservation of energy.

And it is also important that these variables do change in time and how it changes in time is governed by the flow equations. For example, at time equal to zero there is no flow just water stands at well A and when you start the time, then the water starts to move and you can capture the movement through groundwater flow equations. So, that is where it is saying that it can be represented by differential equations, what is a differential equation it actually is presenting a DX by DT how the change in X happens at a change in time.

Moving on let us now apply this groundwater flow equation to different systems as I said, it is not the same and it cannot be the same in all settings. In the last class, we looked at stay steady state and transient flow; we understood that water can in natural state mostly be in transient flow. However, the equations are different for different settings, because the laws that govern each system is different and how we do assumptions to arrive at a particular groundwater flow is different.

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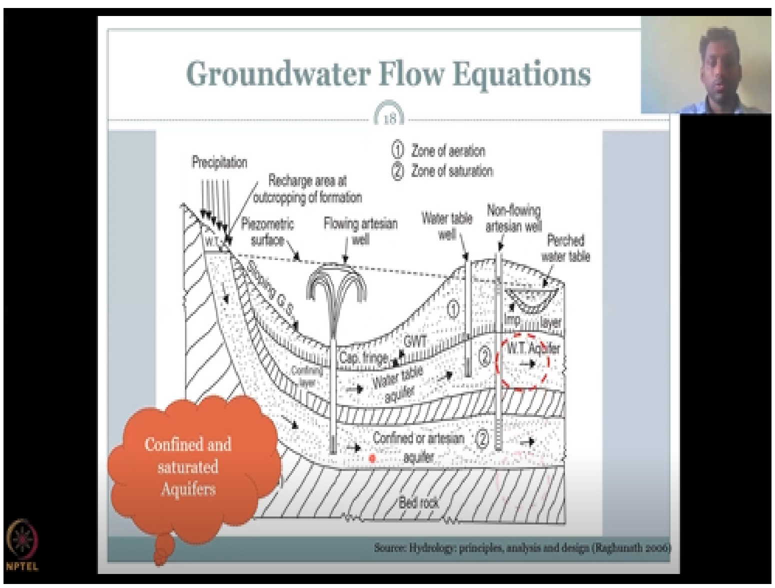

Let us take some examples. Here we look at the precipitation coming in and we have some different zones of porous space where water is there and water is not there, we have gone through slide a lot. So, let me quickly explain what it is? In detail you have a confined aquifer here and a confined aquifer here and you have saturated conditions where water is fully occupying the porous space in the confined layers and in the unsaturated layer which is a zone of aeration, there is less groundwater saturation.

So, the confined layer is here where you can see below the confining unit and on top you have an unconfined aquifer. And this unconfined aquifer can be further divided into saturated and unsaturated for ease because this is just a basics of groundwater hydrology class, we will look at the groundwater which is in the saturated condition in the confined aquifer which means, here the water is there present the aquifer is confined aquifer and the water is present 100 percent of the porous space fully saturated.

We are not going to look at partially saturated when we come here yes we have to look because it is an unconfined aquifer. And we also have a water table that differentiates that dissects the unconfined aquifer into a unsaturated zone and a saturated zone. So, we are sure we cannot apply that there. But here since in most cases you are confined aquifer as an impervious layer on the top and bottom, so in between you have a saturated water front.

Let us not think about pumping now because we are just looking at groundwater flow when we look at recharge and discharge, we will be discussing about these cases in general here we are only going to divided the whole system into two aquifers confined and unconfined and within the confines it is only going to be saturated steady state flow and in the unconfined it can be a saturated steady state flow or an unsaturated transient flow.

So, transient means I explained in the previous the velocity and magnitude would change and that would change only when you have a change in the saturation content or the groundwater head which is falling since we are not going to pump it is going to be the same head. So, we are going to look at only this condition where we have a saturated aquifer and it can be divided into confined and unconfined.

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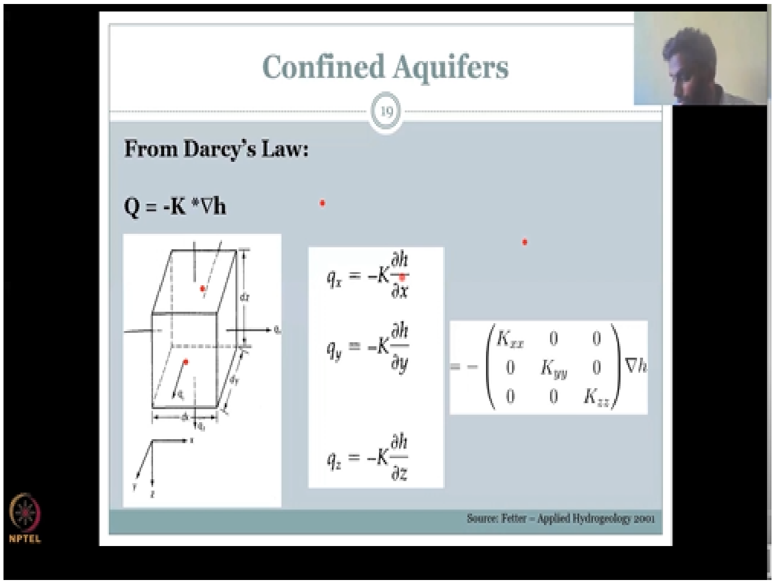

What is the law? So, we have already looked at this law the governing equation for groundwater flow it is called Darcy's law. So, Q which is the discharge is given as minus K times Del h which is the hydraulic gradient. Del h can also be further differentiated as the change in head by the change in length between the two points.

So, please, understand these equations are for assessing the groundwater flow between two points and then you convert it into a flow. So, for example A to B your first objective is to find Q between A to B then you go to B to C then you go to C to D. So, this is where differential equations have helped. So, this is first differentiated into a particular limit and then this is done and then this is done.

So, this continuum at have occurs through differential equations. so moving on we have two points, and in between the two points, we estimate the normal water flow using Q, which is minus K times Del h which is including your hydraulic head difference by your length difference between these two points, the area of cross section is taken away, because we are looking only at flow the not the discharge volume with the flow velocity, let us say the flow velocity just dividing by area cross sectional area. So, let us first take this point, so we have this point, the first point where the water enters is given as here on the side and then this on the backside of the cube and on the front side of the cube.

So there is two points, we are going to see how we estimate Darcy's law through these two points in a three dimensional plane, so, you have change in so, the Q is coming out and the DX, DY, DZ are the change or the thickness of the material through which it comes and you also have the QY in the QY plane, QZ in the QZ plane and QX in the QX plane axis. So, what does this happen? So, each one would have its own Darcy's law. So, each plane each access is now going to take its own Darcy's law to first estimate how it flows in the x axis, y axis and then z axis all these will be different entities that are going to be assessed.

So, for example, as I said A to B is there, but A to B happens in a three dimensional plane, it is not A to B a straight road, right, it is a three dimensional plane. So, how from A the water particle most to B depends on QX, QY and QZ and the combination would come as net Q and that can be represented as a vector times your Del h which is your change in gradient because the change in gradient could be the same in HXY.

So, you can take that out what is going to be changing is your hydraulic conductivity, because hydraulic conductivity is an isotropic and inhomogeneous so, what happens is it is not the same if it is a same well and good, you can use the same values, but most of the time the KXX is not equal to KYY is not equal to KZZ. So, you will have to have this in the matrix format to solve the tensors.

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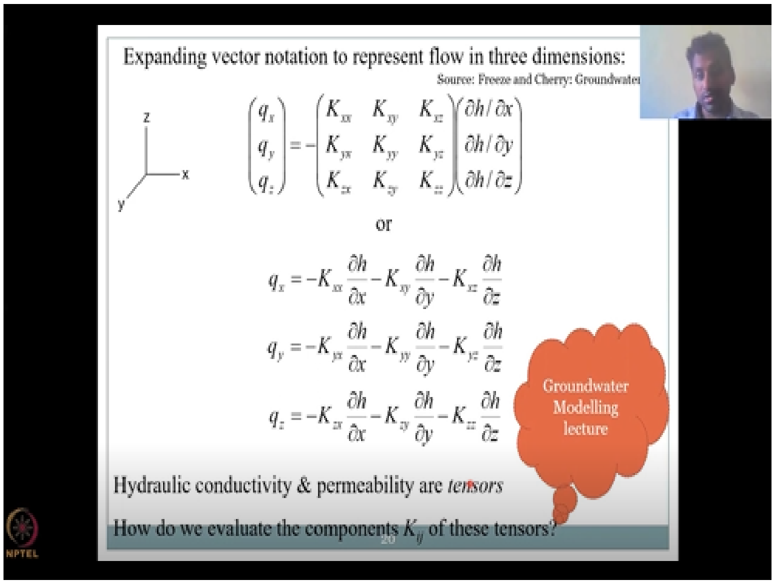

And then when you make it big, this is how it looks like. So, expanding vector notation to represent flow in three dimensions you have QX, QY and QZ you also have to understand that KXX is the hydraulic conductivity of the water particle seeing the hydraulic conductivity the XX direction and then there is an XY plane an XZ plane. Similarly, you have KYZ, KYZ on the YY plane AZX and KZY on the ZZ plane most of the times these are zero, which is what you saw on the previous slide by Fetter.

So, Fetter's book has given it as zero for the all the other matrices just keeping the identity matrix same. However, it can also be different as for Freeze and Cherry, but this is kept out as one Del h as you can see here, this, this Del h value. So, expanding the vector the matrix notation you get each equation separated out. So, you have a QXX which is a function of your AXX values DH by DX, DH by DY and DH by DZ. Why are these terms important because there is still a hydraulic conductivity term on the XY plane here it is on the XZ plane.

So, we have to see how it differentiates in the DHDY same thing can be done for the other Q values also QX is done, QY is done and QZ is done. So, what happens here is the hydraulic conductivity and permeability are tensors in these equations. So, these are solved using manually by the tensor notation in matrix however, it is very, very difficult and takes time to do for two points example, you want to look at groundwater flow from one land to another land which is 100 meters, are you going to do 100 meters in one equation?

No normally you will have to do segment, segment, segment and then ABCD as I showed you the previous slide, you will have to estimate from A to B what is the flow from B to C, what is the flow C to D etc, etc? And so, that is where it is very important to understand it is not going to be easy doing it by manual calculations, you will have to use groundwater modeling. So, this aspect how these equations are processed in the groundwater modeling environment will be taught in the groundwater modeling lecture, which is going to come in the following weeks.

We will have a very detailed explanation of how these equations are formed and how they are solved in the groundwater modeling software. And also use MODFLOW, which is a open source software, anyone can download it and use it, the user interface is quite different. But still you can get the values up. Most of the others use a proprietary software that can help the automation and visualization of MODFLOW results. We will also see some of the packages, there is a student version that most of you can apply and work on is a trial version that you can use it for a month or two to learn the software and if you like it you can buy it.

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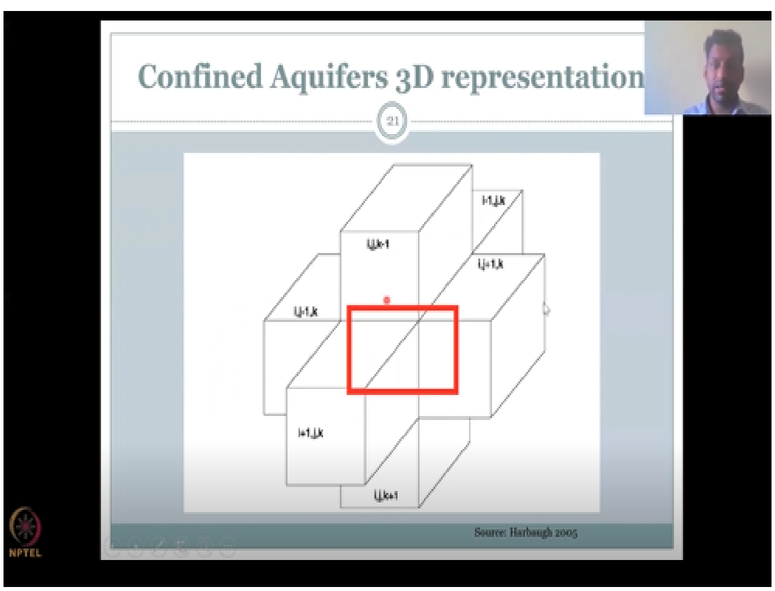

Now, this is the important part what we saw in the previous slide is each point each point from one point to the other the DH DX change from one access point to another. So from A to B you have a distance of the DX. So, DX is there DY is there DZ is there, that is the thickness of your cube thickness and height of your cube. So, all this would differ right and you would have the water moving through these cubes.

So is it one cube, no, it is a cube which is kept, you know in tandem like a chain and through that the groundwater flow. So this is how you can visualize it. Your initial Q is somewhere around inside here, which is IJK. And then on the top it will be IJK minus one because K is on here inside there is a cube where we set up the equations. I will show you the equation. This equation right so, we made this equation based on one particular this equation we made a particular cube. And that is how we formed this equation.

Now, we are going into the other cubes that are around the initial cube. So, there is a top there is a left there is a right and then there is a forward backward and bottom all the three axes will have to at least in front back up down sides. So, now you could understand why you need a three dimensional view of this groundwater.

So it gets more and more complex right. So, you initially started with two points A and B, then you put a skew to it, then the cube has multiple faces, which is shared by other cubes, and then the cube grows bigger. So, this is how system inside a groundwater domain is broken into cubes, and then you estimate the flow between the cubes. so, on the top you have IJK, the bottom is IJK plus one, the minus one is on the top plus one and the back is I minus one here, this I plus one here it is minus J and here this is J plus one.

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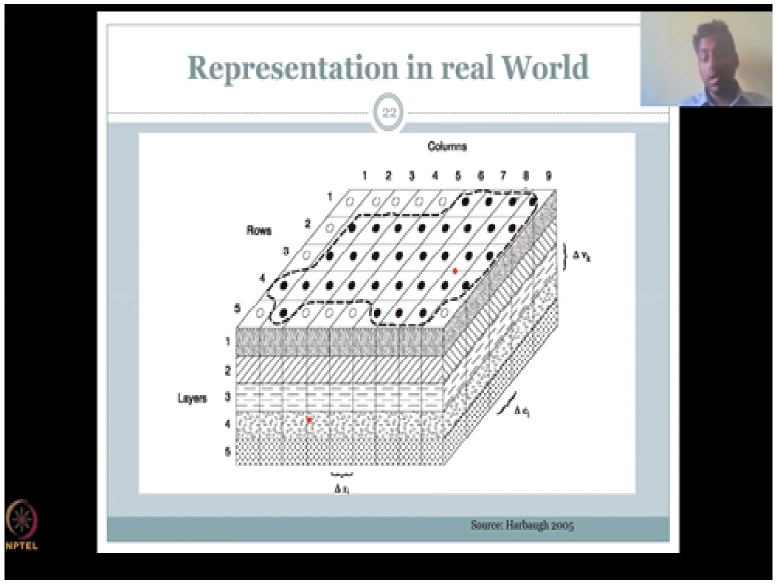

So this is how it is presented in the real world, I will now remove the cube. Now, what you see here somewhere anywhere inside is the first starting point where you have the data of the bore well where you have the well A, ? So, well A is somewhere let us say you put it somewhere here, and B is somewhere here in the fourth layer.

So, now, you have to make the water move through each and every cube to come here in a groundwater equation, that is where it is very, very complex to do it with hand and you will be using the software's for it, but to understand this is how it is done one cube to another cube, you do the cube equation, which is Darcy's law and then from there another Darcy's law, another Darcy's law and then you come on. So, you can also do this as an integration from zero to 10 cubes and how they change.

So, here if you see in a real representation, what they have done, they have dissected the land in equal size cubes, and then they arrange it block wise and from one block it goes to another block. So, this IJK that you saw in the previous slide is where it is here. So I J and K. So, your I layer would be your, your width and then your rows and then the columns. So, this is how you will dissect and apply the Darcy's law at each stage in your model.

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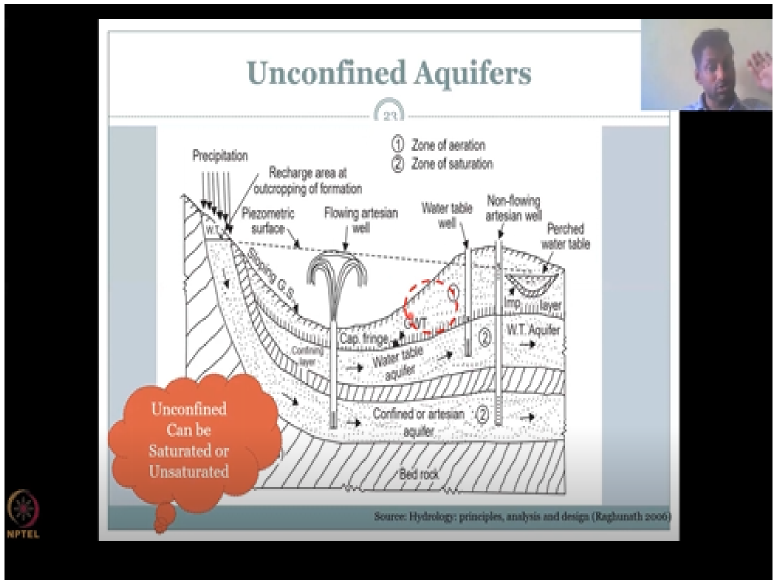

Moving on, you have used it for estimating the cube in a Darcy's law environment we have already looked at what is Darcy's law, Darcy's law states that water moving from A to B in a saturated column would be governed by an equation which is saying it is proportional to the hydraulic conductivity and it is also proportional to the hydraulic gradient the proportionality constant becomes your K and your area is also proportional.

So, the hydraulic conductivity is kind of your proportionality constant where it changes your proportionality symbol into equal to sign we have seen this a while we demonstrated Darcy's law. So, here what we saw is how is Darcy's law applied from one point to the other point through A and B and how you can use these equations in tandem.

So, now, the next step we will be looking at is an unconfined layer is an unconfined layer as I clearly mentioned in the starting you will be using the setup that the unconfined layer can be saturated or unsaturated for using Darcy's law as per Darcy's information we are going to say it is going to be saturated because in the unconfined aquifer you have a water table and below the water table it is saturated and above it is not saturated.

So, now one question comes why am I not looking at groundwater flow here? If there is no water there why is there going to be a flow so most other people would say just leave the unconfined unsaturated layer. I am not interested because there is not fully saturated and unless there is full saturation there is not going to be water movement so I am going to only focus on the unconfined and saturated layer, which is the bottom one.

So the, this, this layer is going to be unsaturated and so there is not much water connections happening and not much flow going to happen. So I am not going to be worried about it in the unconfined layer there is a saturated layer and that is where we are going to be working on. And for that, the same equation can be used for Darcy's law q is equal to minus k times del h, where the Del h is your hydraulic gradient.

And it is also given as a function of your gradient as per the different axis x, y and z and your hydraulic conductivity which is your proportionality constant can also be dissected in the three domains three axis as KXX, KYY, KZZ and then you solve the same equation as I showed earlier.

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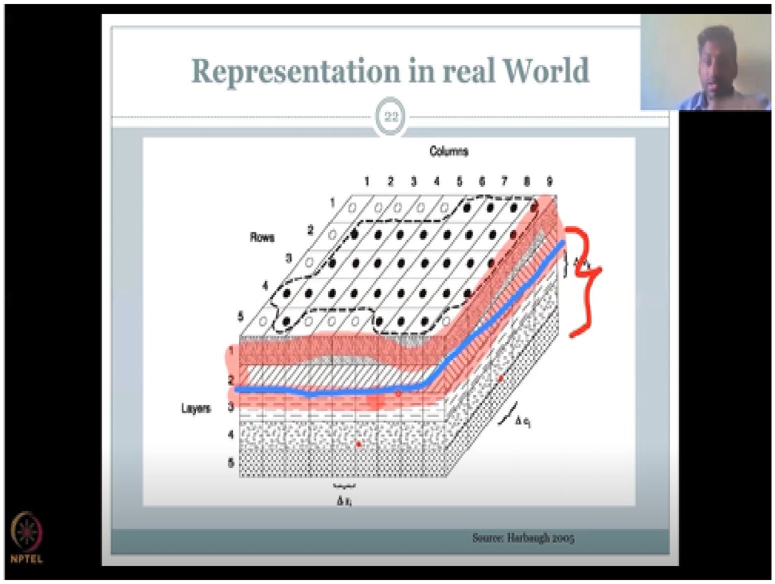

Now, this is the representation in the real world and suppose you have layer one and layer two or unsaturated and on the top it is only be unconfined and all the layers are kind of an aquifer layer. You do not have any impervious layer, the impervious layers only at the bottom. So the total thing for this example is going to be a unconfined aquifer. In the unconfined aquifer, as I said, let us say third layer onwards, there is water on top of that one and two are unsaturated. So I am not going to care about it.

So now what happens is you are going to neglect these layers from your groundwater assessment, groundwater flow equations, and you are going to only look at these places from A to B, how is water going to move and since the understanding that all area under the unsaturated zone or the water table, so, the water table would be somewhere around here, right. So on top of here, let me say this is the imaginary water table and the monitoring points are below the water table, which means it is saturated.

So now you can only use these three layers for your estimation of the groundwater flow, the layer one and two is not needed, because it is not going to happen your, your flow does not go up it has to go down because of gravity unless there is pumping there is water movement of the top and there is capillary fringe. So here is where most of the vertical upward moments are neglected. So no groundwater model will say, oh, I am going to put some water because the amount of water moving up is very, very less.

So, in the real world, there is some movement up, but for estimation purpose it is not going to be very easy. So, we are going to neglect it, most of the groundwater models will neglect it, the key force acting on the water particle is your gravity and the force which is governed by the hydraulic potential is going to push it more down. So you are going to see only water moving from your top surfaces to bottom because of hydraulic conductivity or your lateral movement.

So within the layer, how it moves, those kinds of things so, the unsaturated layer is not going to be captured most of the time in these big, big models. Normally what you do is you inactivate these layers saying that yeah, water will go up, but let us not take of work, but infiltration can come in. Again, soil infiltration, which gives you water into the groundwater is, is acceptable, but the infiltration does not constitute your groundwater flow, or as long as the water comes and hits the groundwater table contributes the groundwater aquifer.

Then it becomes groundwater flow until then it is only governed by infiltration equation. There are many, many infiltration equations Green-Ampt for example, that is what governs the water movement from the top surface into the groundwater aquifer. Once the water moves into the groundwater aquifer. It is governed by groundwater flow equations. With that I think we have given a good understanding about the Darcy's law being used both in the confined aquifer and unconfined aquifer in the unconfined aquifer. I told there it could be possibility of saturated versus unsaturated, the unsaturated is not a big issue for us because we are not going to have water much there for water flow to occur so we are not going to study that we only going to study the saturated part.

Similarly, in the confined aquifer we assume most of the water is saturated the soil material is saturated so the water flow is purely all equated through the Darcy's equation and just look at how simple and beautiful the equation is it has only two values which is your need of two values is your hydraulic gradient and hydraulic conductivity, the hydraulic gradient is also simplified as the change in the head by the distance and the distance is very easily calculated because when I give you I want the groundwater flow from A to B I will give you the distance also right.

So I will say how what is the distance between A and B 100 meters I want how groundwater flows from A to B give me the flow but then I will have to give you only one data which is your hydraulic head in A and hydraulic head in B so indirectly if I tell you the soil type let us say it is a clay medium then you can quickly go back to the books that I referred in the previous class on hydraulic conductivity values and you can apply that to estimate hydraulic conductivity.

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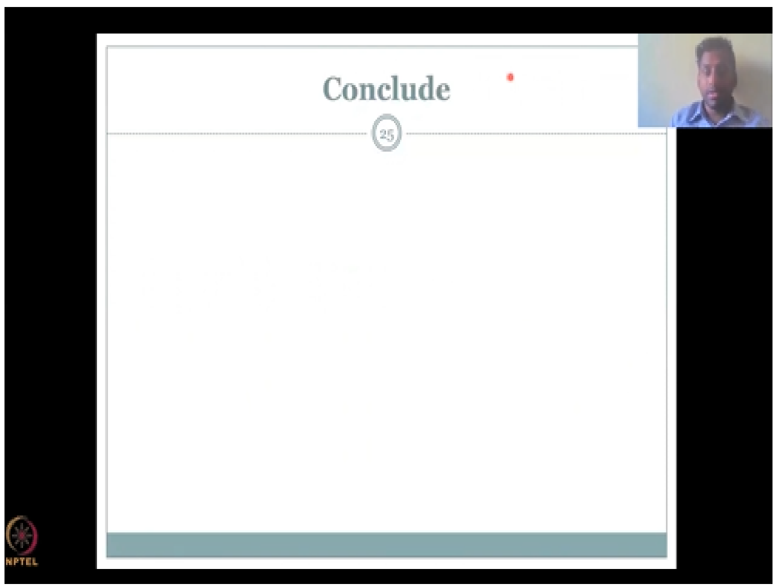

We will give you some of these homework's so that you can also test it. With this I would conclude today's lecture. Thank you.