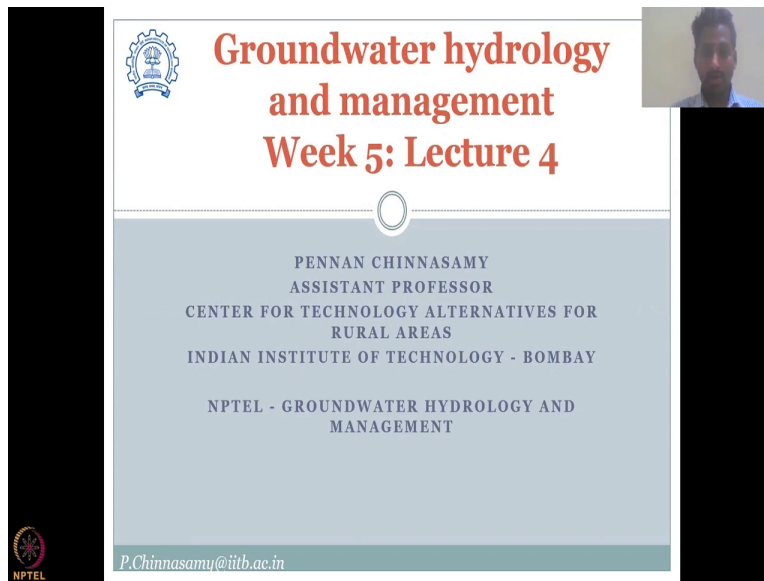


**Groundwater Hydrology and Management**  
**Professor Pennan Chinnasamy**  
**Centre for Technology Alternatives for Rural Areas**  
**Indian Institute of Technology, Bombay**  
**Lecture 24**  
**Groundwater equation in unconfined Aquifers**

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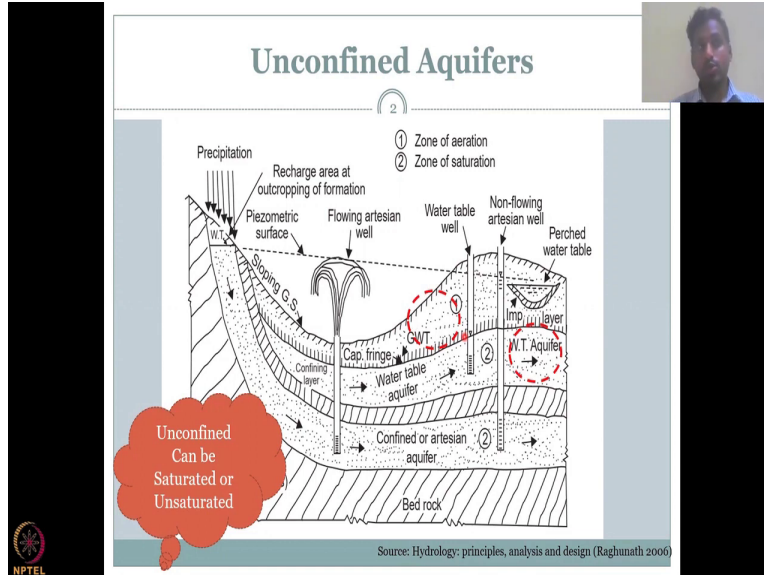


The slide features a central white box with a blue border. At the top left of this box is the IITB logo. The main title 'Groundwater hydrology and management' is in red, with 'Week 5: Lecture 4' below it. A small circular icon is centered below the title. The speaker's name and affiliation are listed in blue text: 'PENNAN CHINNASAMY, ASSISTANT PROFESSOR, CENTER FOR TECHNOLOGY ALTERNATIVES FOR RURAL AREAS, INDIAN INSTITUTE OF TECHNOLOGY - BOMBAY'. Below this, it says 'NPTEL - GROUNDWATER HYDROLOGY AND MANAGEMENT'. At the bottom left of the slide is the NPTEL logo, and at the bottom center is the email address 'P.Chinnasamy@iitb.ac.in'. A small video inset of the professor is visible in the top right corner of the slide area.

Hello everyone, welcome to NPTEL course on Groundwater Hydrology and Management. This is week 5 lecture 4. This week, we are looking at the parameters to convert hydraulic head to different parameters that can be used in the groundwater equation. We also looked at how the groundwater aquifer can be divided into sub-components. And given the merits of the parameters that are available, we chose the equations to model the groundwater equation flow.

In the last lecture, which is lecture 3, we looked at the confined saturated region and we looked at how Darcy's law can be applied and only data needed was the hydraulic head difference between point A and point B or between two wells and then the distance between the wells followed by the hydraulic conductivity of the system. In today's lecture, we will look at the remaining components.

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This is the unconfined aquifer we looked at the top in the unconfined aquifer, we saw that there is two regions there is a zone of saturation which is under the imaginary water table line. Which is this one and also there is a zone of aeration where the porous space has air plus water or just air. And we understood that in the unconfined aquifer which is the aquifer above the permeable layer below the permeable it is called confined above the permeable it is called unconfined aquifer that can also be a saturated zone which is full of water the pore space and no air present or negligible air and that part is characterized by an imaginary water table line.

We also looked at that the Darcy's equation which was well occurs accepted in the confined aquifer can also be used in the saturated zone of the unconfined aquifer. So, we have seen section two and section two B which is in the confined layer. So, what is remaining? What is remaining is, how water moves if the water table goes up, which is the unsaturated zone and in the unsaturated zone, still there is some water present and the water if connected between the pore spaces can move. Also, we should understand that this water table line which is imaginary can be recharged and go up or can be discharged and go down.

Let us say if it is recharged and goes up, then what happens is this distance, this distance where you see all the thickness of the aquifer which was unsaturated becomes saturated, here we have

extra saturation because the water table has gone up. So, there is no definite region in the unconfined aquifer. There is no definite region where we can clearly say above this or below this it is saturated. We need a model that can capture this dynamical movement of the water table.

Basically, we need an equation that accounts for the Unsaturated Zone also remember that Darcy's equation only holds in a saturated system if the experimental setup by Darcy. Darcy initially saturated the soil and then applied a flow into the soil. That is why the applied volume which is  $q$  is equal to the exit volume which is  $q$ . So,  $q$  in is equal to  $q$  out. Masses consult which is one of the key conservation laws for Darcy's equation.

Moving on, this cannot be true in real life especially in India where the fluctuation of the water table is there and we are no longer pumping only from the saturated fully saturated zone we are also bringing in the unsaturated water that can also be applied to the confined aquifers because our pumps are now into the unconfined aquifers which is this one and also into the confined aquifer. So, the wells which are in the unconfined are called shallow wells or dug wells and the wells which go deep into the aquifers and into the confined aquifers are called deep bore wells or more wells.

They are not normally the wells that you see which is big and you can swim and stuff it is like in the villages some wells you can swim and you can see the water levels the deep or the confined aquifers you normally have a bore well which was in deep seen here. So, once you start extracting the water this aquifer is no longer fully saturated there are regions where it becomes unsaturated also. So, there is a need for a unsaturated groundwater equation.

So, let us look at it just for the explanation we will just look at it in this region which is the unconfined aquifer which can be moved up and down depending on the water table the water table if it recharges goes up the water table if it is discharged or pumped goes down. So, that is where your unsaturated zone boundary can be pushed up and down. Moving on let us look at the equation which is going to be applied for the confined zone which is already shown in the Darcy, we are looking at the unconfined zone and especially the zone of unsaturation, we will also look at how the same equation can be used in the saturated.

So, what is the difference the differences the porosity is there and the water occupied is equal to the porosity, 100 percent in the saturation whereas, it is a function of water availability in the Unsaturated Zone It is neither full 100 percent nor 0 percent It is in between somewhere and it fluctuates depending on the water taken up by the plants pumping and also recharge. So, when recharge happens more saturation happens and what was pulled out there is more discharge happening.

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### Transient Unsaturated Flow

3

- What happens in the unsaturated media?
  - The degree of saturation is not 100%
  - So the flow is not only a function of the volume but also the degree of saturation
  - This is absent in the Darcy's approach
- Richards Equation uses this in the flow eq:
  - $\Psi$  is a function of the porous space saturation
  - Computationally more difficult
  - Needs soil moisture data and change with time

$$\frac{\partial}{\partial x} \left[ K(\psi) \frac{\partial \psi}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K(\psi) \frac{\partial \psi}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K(\psi) \left( \frac{\partial \psi}{\partial z} + 1 \right) \right] = C(\psi) \frac{\partial \psi}{\partial t}$$

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Let us look at it. We are going to very focusly, look at transient unsaturated flow as I mentioned, the steady state is only when you have a fully saturated zone and also water comes in and goes out, because you are maintaining the head differences in both location suppose you do not maintain the head it becomes a transient situation and the transient situation is normally occurring in the Unsaturated Zone because the saturation is different.

And so, the water volume may not be the same across time the flow magnitude and the flow velocity it may not be the same across time because there is a time when water gets absorbed in the soil and there is a time when water connects with the other soil pours and then starts to move. So it is not constant in time.

Whereas what translators and unsaturated zone it happens naturally. And why we are looking at this in detail is because most real life scenarios are unsaturated because we pump and then recharge and then pump and then transient in nature because the head is not maintained because you constantly disturb the system. So whenever there is a disturbance it is transient. So it is naturally flowing like in a forest in a dam area where the head is always maintained and always you have the same pressure that the system has stabilized then it is a steady state where there is not much disturbance happening.

But since the core of this course is going to be looking at the disturbances in the groundwater aquifer and how to manage, this is going to be a very, very important situation. Moving on We have to define what happens in the unsaturated media, the degree of saturation is not 100 percent which means the pore space inside the pore space there could be some air volume present and air or water volume percent. So, it is not 100 percent When it becomes 100 percent it becomes saturate the flow is not only a function of volume, but also the degree of saturation.

So, if the soil moisture is fully saturated, then you have Darcy's equations that can apply because whatever water comes in can be pushed can push the volume inside the soil and another volume can come up. However, if the soil moisture is not fully 100 percent The saturation is not 100 percent Then some of the water would first align itself and start filling the pores some of the water would be used for wetting the soil we call it.

So, in that case you will be losing on some flow because you have to compensate for saturation. So, the flow is not only a function of volume and time which is  $t$  zero to  $t$  one and also the volume applied in the top if you see Darcy's setup you have  $Q$  coming in and that  $Q$  drives the outflow also but here it is not only the  $q$ , but also the inside tube soil materials saturation is an important factor.

So, what do you mean degree of saturation as I said zero means zero saturation mean it is totally dry unsaturated, fully unsaturated and then if it is 100 percent is called saturated for normal real life scenarios it is normally in between both and we call it partially saturated and that is a zone of aeration etcetera. So, this is absent in the Darcy's approach, we will not call it a limitation.

Because Darcy initially did this equation for a pipe supply for fountains and in the pipe and wherever the pipe was made, it was fully saturated because they had enough water it was unlimited water which was supplied so that the fountains work you had the fountains coming. That is not the system in the groundwater. So, it was an accidental discovery Darcy did for groundwater equation however, we it has been widely used and it has been successful in dealing with the saturated flow both in the unconfined and confined. So, right now, we have to take a note of it and say thank you, but there are some issues in applying it for unsaturated flow.

Let us look at it as I said there is nowhere in the equation  $q$  is equal to minus  $k$  del  $h$  by del  $l$  there is nowhere the saturation of the soil comes into picture you have the hydraulic conductivity which gives you the ease of which water can flow through the porous medium, but there is nothing related to the actual saturation of the soil. So, writing it down here, So,  $Q$  was equal to your minus  $k$  and the del  $h$ . So, here there is no way you have a saturation attached because your  $K$  was only a function of how easily the material allows the water to flow or the fluid to flow. So, it is a function of the pore space and also the fluid, like here it is water.

Therefore, there was another equation which was built, this was also built drawing some influences from Darcy. It was made by the soil scientists Richards and because he did so much work on it, the equations named after him as Richards equation. So, Richards equation has a

function another parameter called  $\psi$  which is a function of the porous space saturation. It is introduced into the previous equations derived from Darcy.

So to understand how the degree of saturation can play a role in your groundwater equation. It is computationally more difficult just look at the equation how it has been spread out. The derivation of the equation will not be discussed here, but if you can look at it initially, initially the  $K$  was only a function of  $x$  and  $z$  which is the plane, but here the  $K$  is also a function, because here you can see that it is within the  $x$  domain, this is the  $y$  domain, this is the  $z$  domain. So, this is basically  $K_x$ ,  $K_y$ ,  $K_z$ , but you also have a  $\psi$  function, the hydraulic head is also introduced into it.

So, the  $\psi$  here gives you the degree of saturation and also it is a function of the saturation in the porous media. In other words, you could look at it flow as if the saturation is high there will flow then the flow occurring. Suppose, the saturation is zero  $\psi$  goes to zero, and all this collapses, they saying that  $Q$  is zero. So, this is the basic setup of groundwater models, where in you have some complex models, which uses Richards equation and some very basic models, which is using only Darcy's equation.

There are takers for both these equations, because, computationally, we will see how difficult it is to use Richards equation because you need to give the data, how  $\psi$  varies with  $x$ , you need to know, and also how  $\psi$  varies with time. Also, we need to know, which means there is a lot of data. In the Darcy's equation, there is only one variation, which is the hydraulic head, how the hydraulic head between A and B varies with time is all you need to understand the flow here, not only the hydraulic head, but you should also know the in between soil media and how the saturation is present.

For example, you have a well there is water inside, then you have another well, there is another water inside. However, if there is no saturation between them, how will water flow from A to B? It will not right because most of the water will be used to saturate the system and then flow so that degree of saturation is captured in Richards equation. However, Darcy requires that there is a

medium present between A and B, the wells A and B, which is fully saturated. Moving on, most studies still use Darcy due to simplicity.

So, in the real life scenario, where you are using models, please understand that you can make it as complex as you want, you can throw in more numbers, you can throw in more equations, ever, the computational difficulty goes up, the requirement of data goes up. And sometimes the model crashes because you do not have the data supporting the model. In other times, if you do not have enough data, the model will try to assume a lot or you force the model to fit the hydraulic head which is also wrong.

So, most of the real life studies where you have water for groundwater recharge and other things or check dams and monsoon recharges rainfall recharge or summer extraction, if you see that most of these studies still use Darcy and if I if you know Darcy's was done in the 18th century very, very old and still the laws use because of its simplicity and yes, it has some issues, but overall computationally and data intensive it is less and it captures the groundwater flow to a particular extent.

So, as I said, in Darcy, there is no function for psi, where it is a degree of saturation whereas here you do see that psi is present throughout the equation and in your hydraulic conductivity is a function of psi. Because here you can have high hydraulic conductivity, but if there is no saturation there, then how will water flow is the question raised by Richards equation. All are valid points, however, the degree and change of psi of k and psi came and or how it varies across time you can see  $d\psi/dt$  is does take a lot of data.

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## Darcy's Law: Strengths

4

- Darcy's law provides an accurate description of the flow of groundwater in almost all hydrogeological environments.
- Darcy's law holds:
  - saturated flow
  - steady-state flow
  - transient flow
  - for flow in aquifers and for flow in aquitards
  - flow in homogeneous systems and heterogeneous systems
  - flow in isotropic media and for flow in anisotropic media,
  - flow in both rocks and granular media.

Source: Hydrology: principles, analysis and design (Raghunath 2006)

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But what is the assumption?

Source: Hydrology: principles, analysis and design (Raghunath 2006)

So in that note, and as I said that there are many studies which still use and prefer Darcy's law, let us look at Darcy's laws strength. Darcy's law provides a very accurate description of the groundwater flow in almost all hydrogeological environments. Why? Because the only data that it requires other than the groundwater data is  $k$ , which is your hydraulic conductivity. And as I have already shown in class, if you do not, for example, I do not have to go to the field to find  $k$ , what I do is I quickly understand what is the geology present, what is the type of matrix which is a soil or the type of geology present.

And then I go to my book, which is the groundwater book by Freeze and Sherry where all the key values for any material in the planet is given there is no these these rocks and soil do not emerge, which means they do not evolve it is the same thing it only evolution is weathers. It, weathers from the parent rock into soil. So all this has been documented very clearly in Freeze and Sherry's groundwater book, and that data is still used for assuming or estimating the hydraulic conductivity and even though there is a range, you always get away with having the average or the middle of the range for most of the materials.

So, Darcy's law is simple, less data intensive, and all the accompanying data is already well studied in the literature and backed up with data. So, Darcy's law has provided an accurate description in all hydrogeological settings, where it is mostly saturated, where does it law hold good? Darcy's law hold good in saturated flow as any state flow where the hydraulic heads are kept constant, so that there is always continuous movement of water and the magnitude and velocity of the flow at a single point does not change over time.

It also law holds good in the transient flow, where you change the hydraulic head and the transient there is a transient flow occurring wherein the flow magnitude and velocity changes over time. Please understand that it changes over time because of the changing hydraulic head conditions and pumping and recharge regimes not because of the unsaturated here in this in this example I am saying so, in transient flow also Darcy's law holds good as long as the medium is saturated. So, saturation is the key it has to be saturated and then there is steady flow and then there is transient flow.

For flow and aquifers and for flow in aquitards. So, aquifers as I said it can hold good and unconfined aquifers it can hold good in confined aquifers, because both have saturated conditions and it can also hold good in the impervious layer, which is the actividades please understand that recharge also has to go through these aquitards to get into the confined layer it is very small very small amount of water that goes in.

However, Darcy's law can still hold good in that, because hydraulic conductivity you know, you know the hydraulic head up and below. So, for example, if water moves from here to put down,

you know the water level at A water level at B. The water fall from some high potential to low potential. The material is the aquitards which has a particular hydraulic conductivity can check it on the Freeze and Sherry's groundwater book and you can estimate groundwater flow.

It was good in Darcy's law holds, good in flowing homogeneous system and both heterogeneous system also homogeneous wherein the material is homogeneous so that the flow conditions are homogeneous across and also good in heterogeneous systems where you have different types of materials, thereby affecting the groundwater flow rates. And these can be captured by your changes in hydraulic conductivity. A homogeneous system let us say it is a sandy aquifer across the two regions A to B there is sand and I have one value of k it is fine. So you can estimate Darcy's law in ground water flow using Darcy's law.

Suppose the material is heterogeneous, which means I have sand and then clay and sand still have Darcy's law holds good because you can have different K value. So, it will be first compartment and then the clay compartment and then another Sandy compartment. So, you can just add them all together to get the flow in the heterogeneous system we can also do flow an isotropic media and flow anisotropic media please remember what is the difference in isotropic k x is equal to k y is equal to k z in does not change.

However, in an anisotropic way, in a medium, the  $k_x$   $k_y$  and  $k_z$  may not be the same and the changes between two points may not be the same. So, in such a system you can still hold Darcy's law good because all you have to do is dissect the equation one equation  $q$  is equals to minus  $k$  del  $h$  into three equations. Basically, because you have  $k_x$   $k_y$  and  $k_z$  we have seen this in the matrix kind of solution for Darcy's law.

It also holds good in both rocks and granular media. What is rocks? Rocks are the deep deep aquifers where you have the porous space still present but it is mostly fractures and also well less connected porosity. So, that is the rock medium the granular media is above the rock where the rock has weathered and soil has formed. So, both in the granular media and the rock media the Darcy's law holds good.

This is also an example of a homogeneous and heterogeneous system. A homogeneous is purely rock for example, in a heterogeneous system you can have rocks and your granular media also. So, Darcy's law has well proved to be very successful groundwater flow equation across centuries now, remember it was done in 1800s and it has been used till date even I use it till now the Darcy's law even for lab experiment and for your real life field expert experiments, Darcy's law holds well. The only very very important assumption or condition is saturation, it has to be saturated. Still some people have used it for saturated systems.

For example, modflow is a very well developed model. Initially it was only doing Darcy's equation even for unsaturated flow, it just turns off and on some layers, however, it has given good values. So, this is where you can push the signs to higher limits. However, at one point, it has to stop because you just keep on adding equations, it does not improve the efficiency of the model high higher, it just makes it more cumbersome in terms of media.

So, we have seen all the assumptions and we have seen all the strengths of Darcy in the next class I will also go into the limitations of Darcy's law and the wrap up of week five. So, this week we have seen two equations for groundwater flow estimation one is Darcy's and the other one is the Richards both can be used in saturated system.

Richards can also be used in saturated system, but where Richards becomes more beneficial as it can model the Unsaturated Zone Well. Richards equation is also old, it is not a very new equation. So, it has taken time it has proved itself to be a very good equation. And if you have the data and computing power, Richards equation is the best in terms of accuracy. For simplicity, Darcy's law is the best and worldwide acceptance studies how many studies have been done Darcy's law holds good even till date. I will see you in the next class on the limitations and future directions for groundwater equations.

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## *Darcian Continuum and Representative Elementary Volume*

5

- Requires replacement of the actual ensemble of grains that make up a porous medium by a representative continuum



Source: Freeze and Cherry: Groundwater 1979

With this I would stop just showing what is going to come for the next lecture basically the representation of Darcy's in other space. I will see you in the next class. Thank you