Groundwater Hydrology and Management
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Lecture 25
Limitations of Groundwater equations

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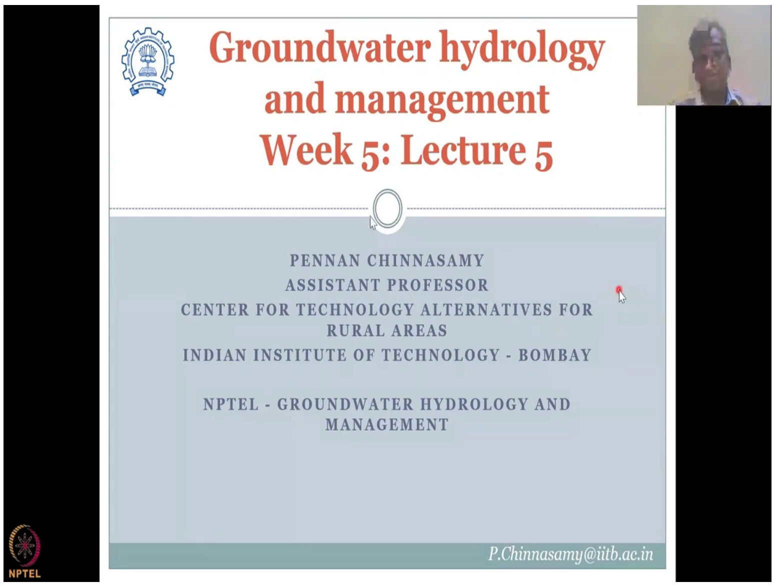

Hello everyone, welcome to Groundwater Hydrology and Management. This is week 5, lecture 5 of the NPTEL series. In this week, we have been looking at the groundwater equations in two different systems for aquifers, which is confined unconfined. We also defined some parameters which were useful for assessing the groundwater equations. The last class we looked at Darcy's application, the strength of Darcy's law, which is mostly for saturated system, we will continue with the discussion on what are the limitations.

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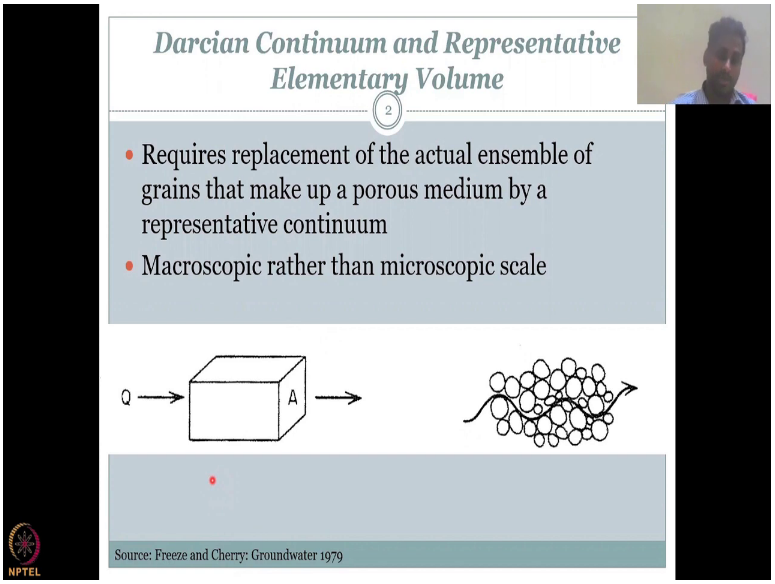

Before that, let us define the volume in which Darcy's law holds good, what are the assumptions. So, Darcy's continuum and representative elementary volume is very important aspect consider by using Darcy's law. What does it require? It requires a replacement of actual ensemble of grains that make a porous medium by Representative continuum.

So, which means, instead of having particles and spaces inside the particles, it is being replaced by a continuous continuous medium is called Darcian continuum. It is not a porous space, it is not like a sponge with different holes and then water flows through it, it is a continuum volume, where water can flow through the volume and come out in a more linear fashion. It is like driving through a tunnel. Again please understand that this was made by Darcy to understand the flow through pipes and fountains, so his assumptions were valid.

However, they have been also valid for groundwater systems at a particular representative elementary volume REV. So, we are going to define this REV, which means at what volume does darcian continuum approach be valid. We also would like to understand that the reality is we have solid grains and inside there, there is pore spaces. So, water would move not in a continuum, but it goes down up and then sometimes gets connected sometimes it does not.

In a darcian volume, it is a continuous volume, where connections are always made. And that is why the saturated condition is made during Darcy's law which means this the medium has to be saturated, once the medium is fully saturated, then the continuum can easily take place because eventually the water will be connected and it will flow through the pipe or in our case it is a solid medium.

The other very important assumption which is needed for darcian to work along with the darcian continuum and Representative elementary volume is the macroscopic view rather than microscopic scale. So if you look at it, once you have a small, very, very microscopic scale of analysis of the soil, you see the pore spaces and the solid grains. But once you go to a macroscopic view, you do not see the individual pore spaces, but you see a continuum and that is all these assumptions are intertwined into one, which is the macroscopic view.

Let us have a look at it. We have already defined this in the earlier classes, so in a normal soil, water particle would move up and down a tortuous path, it will flow through the least resistant path, it will find where the least resistant path is and normally it is not a straight line. It has to go around and it takes time and then comes out.

This is the microscopic view ever in a macroscopic view, I do not care about the individual course. I do not care about the individual solid particles and their shape and how they are arranged. However, I take a volume and inside the volume water goes and comes up. That is the macroscopic view All I need is the cross section of the macroscopic area and a average hydraulic conductivity to estimate Darcy's law which is Q goes in, what is the Q that comes out.

So, this microscopic versus macroscopic view is very very important to define early in your groundwater equation. And that actually makes you pick and choose which model you are going to use. So, in a Darcy, it is mostly macroscopic and once you do the macroscopic also, you need to define the element of volume of analysis, which is this one, because, if it is too small then other forces also contribute to throw darcian out. We will get into those aspects now.

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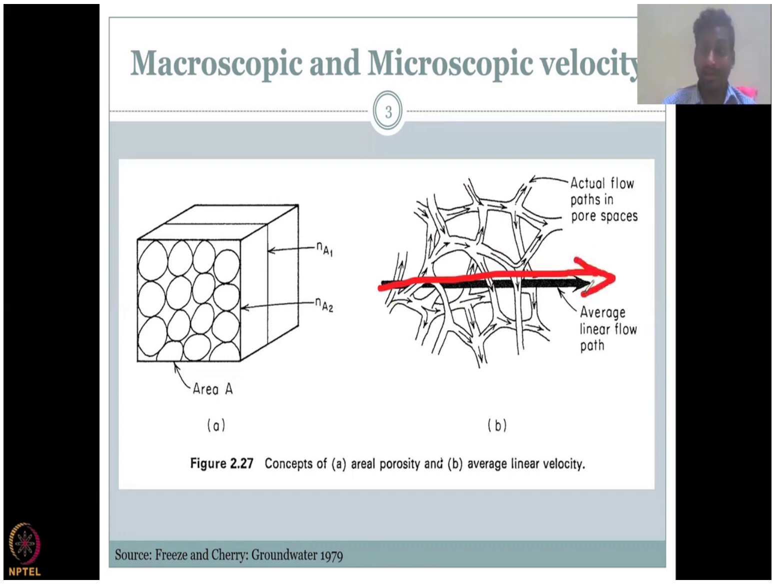

The other thing which is also important in the microscopic versus macroscopic is the actual flow path of the particle as you see is through different different channels rather than one single continuous channel. So, water can go up and then come down and then go into branches and then come out eventually, however, Darcy's approach assumes a average linear flow path. Which means, the actual flow path can be very torturous, as it goes up and down loses velocity gains velocity, however, the average of it is taken as a linear velocity in Darcy.

The other aspect in Darcy is a uniform porosity, which is assumed in Darcy, which means, you have a solid material, let us say a block, if you cut slices along the block of the soil material, you have one slice taken out which is nA1 and nA1 is not the same porosity as nA2 because it could be compaction there could be some other roots growing so, for example, a plant is growing the soil underneath the plant will have more pore spaces because there is roots which go ahead.

However, if you take a sample outside of the plant area, the root zone your pore space will be less. So, in reality in real world, the pore space is not the same across a sample it differs. So, how do you assume that it is the same is what the question against Darcy's but as I said in the previous lecture, you can keep on probing these inaccuracies and assumptions. But however, we have to stop at one point.

Let us say I take a soil, I have plants, this is the top surface I have plants here and after this side, I have no plans, how many samples can I take to assess the porosity? It is time consuming and by the time you do it, your groundwater flow equation is no longer needed. And there is a lot of soil disturbance also when you take the soil you are actually disturbing the soil. So, it is very important to understand that macroscopic view is justified. However, the justification comes with the accurate assumption of the soil particles.

Let us take an example. I have a silty loam soil in one area for surveying, it in one area I have barron's silty loam, and then the other area the silty loam has construction on top of it and the fourth I have silty loam on rice. So, the trees would have high roots big big roots going in, so you have high porosity. Then you have the rice field where it has the roots of the paddy and the rice plants and so you have good porosity so water can move.

In the next version where I have a barren land, the porosity is the natural porosity, you do not have trees you do not have plants, there is some porosity which is intrinsic property of the soil. Whereas if you go to the constructed land, there is compaction on top of it. So the pore spaces are brought low, the lowest. So here is where the as I said the assumption comes in. So you have already seen a range in freeze and cherry groundwater book.

So you need to take the lowest, the lowest range for the salty silty loam for your area. Otherwise, if you take the highest it will be a wrong assumption. So what I am trying to give you is there are uncertainties. There are issues when you do a macroscopic view, but you could justify it using correct assumptions from literature. Otherwise you cannot defend it by going on taking samples and samples to assess a porosity.

Let us take this example the porosity of this is much can be much smaller or bigger of the porosity of this layer it is the same soil I just cut it into two Why should both of them be the same? As is the question because if for example, if there is a trees going on this side on this first slice, which is nA2 the porosity will be much higher than no trees on nA1.

This is where Darcy was very simple, he said no, I know the soil I look at the soil and I see no trees, no plants, no construction. So, there is a middle range of soil porosity I will assume and that assumption he just kept on checking with the model and it worked. Same thing here I do not know how many how many paths are active, how do you know how many paths are active it is very very difficult to estimate and as the name says it is microscopic which means you have to look in a microscope to find all these flow paths.

People use CT scans which means they take the soil sample into a scan and it slices this soil to see these flow paths. It is important for certain applications but not for normal groundwater flow, you do not have to do it. You can get away with this average, but the average should be a very calculated assumption that is what Darcy is good about.

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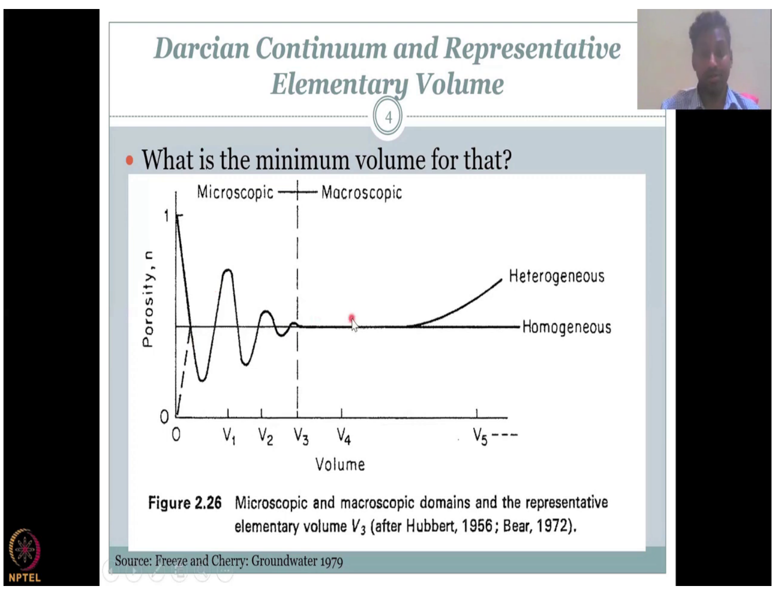

Moving on, so, as I said, there is a Darcian continuum, where it is not regularly connected flow paths there is a continuous flow path and it is an macroscopic view, for the macroscopic view there is a volume for each sample beyond which it become macroscopic. So, if you look at it, you have the porosity on one axis and you have the volume on the x axis. So, when you have the smallest of the smallest volume of the sample, the porosity fluctuates a lot.

This is the microscopic view and when the porosity fluctuates a lot you have these preferential flow paths you have the actual velocities, flow paths and pore spaces which you see here and you have changes in the aerial porosity that is in the microscopic region as you will see, the same material allows the pore space can fluctuate between the volumes between the samples inside the volume. Slowly when you increase the volume the noise comes down, because you are averaging it out the noise is being averaged.

So, v3 is that critical volume threshold above which the systems become macroscopic which means all this undulations and noisiness in the porosity between the samples are now able to be averaged and that is the volume which makes the sample a macroscopic view and it depends on the soil type the type of use of the soil land use land cover or soil for example plants trees or etc. And how it is managed is a tilt it is it cloud is it just barren land those kinds of things. So, once you go on increasing the volume it is still macroscopic.

This is the same for a homogeneous system in a homogeneous system your porosity fluctuates very less whereas in a heterogeneous system it fluctuates a lot and there is a volume above which again the porosity is very important, which means it starts to fluctuate think about big big rocks you cannot have porous space between the rocks model well like the ones you put in the along the beach to stop the water from going in and reduce the waves energy.

So, those cannot be taken as a big macroscopic volume and then you can model the groundwater you have to have a limit also to the macroscopic. So, this is the volume this is the key volume that is very, very important anywhere from v3 to v4, v4 and a half after v5 the heterogeneity in the porosity would continue whereas the homogeneous system is always homogeneous it is always the same. So if you look at Darcy's groundwater equation, it assumes homogeneity soil particles are well arranged or saturated and homogeneous, which means the porosity will be the same across the system whereas in reality it is not the same.

For example, earthworm might have gone in into a soil and that gives a flow path which is which is suitable for groundwater to flow. The idea is it is when you go in a very small scale these issues would go up and your groundwater flow model would crash your your accuracy is lost however, when you take this volume, a high volume for a groundwater sample, then your variations are all averaged out because this and this average would become down right.

So, this and this would can come up here and then it goes down and here it is almost less. So, this is the porosity I can say is the accurate porosity which is the average of the noise after a particular body. So this volume is very important, which is called the representative elementary volume. In a microscopic the volumes are very small, and that is why it is called micro. Macro, which is big the volume is big and all the prosody noises are averaged out.

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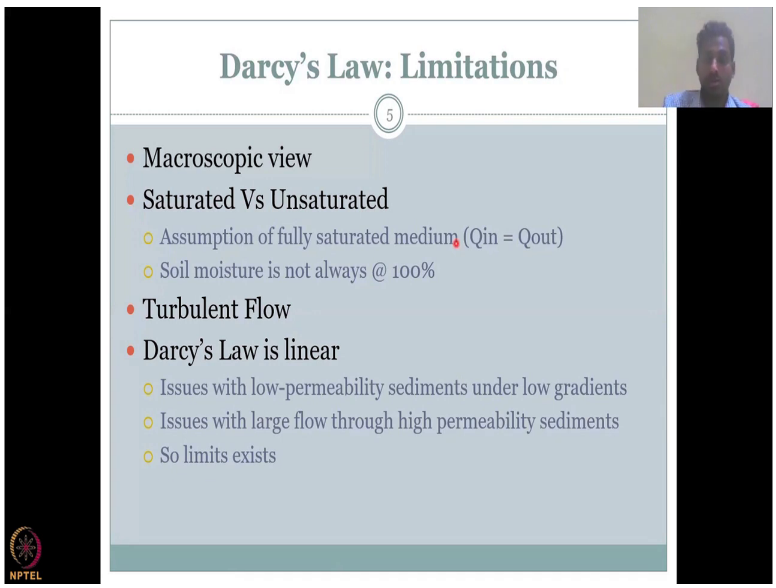

So, this is even though Darcy's law has been widely used, there are some concerns and limitations let me go through them. So, to teach you all these basics of science and models and equations, please understand that you need to share also the limitations when are we doing this kind of work, because if you say that no, I amt going to model nature accurately then it is wrong because nature is very complex and you are using a mathematical equation here. It can be a physically driven equation like the Darcy's law or it can be an empirical equation like your et calculations either way, you have to give the limitations so, that it shows that you have good understanding of the equation.

So, what is the limitations in Darcy's law it is macroscopic view. So, in real life scenarios, it is troublesome because, groundwater flow occurs also at the microscopic level and at the microscopic level. So, the microscopic level assumptions are made for Darcy's law which is a limitation. So, it is a macroscopic view is a limitation. Saturated versus unsaturated the assumption of fully saturated and the medium is always held tight in Darcy's law.

Any experimental setup for Darcy you see would have the soil medium or the rock or whatever the pore space fully saturated with water and then the equations applied as I said, it can be good for a pipe network it can be good for fountains and in lab assessments and it has been good in the real life scenarios, but the reality is that it is not always fully saturated. So, as I clearly has mentioned here, soil moisture is not always 100 percent and it is not always steady state.

Steady state can be in a system where the disturbances are low. Once you disturb the system, the groundwater will fluctuate and then come to a new steady state, then you disturb the system it fluctuates that reaches pumping etc. And then it comes back to a new steady state. Let us say can be the same or a new equilibrium new steady state reached and we know that groundwater has been pumped so much across the world. So, the steady state will not be reached it is a transient in nature.

Also, groundwater flow can be turbulent in nature cannot be very smooth and going in a particular velocity it has multiple velocities depending on the connectivity and turbulent flow causes Darcy's law to crash. I will show you in the next slide how Darcy's law does not hold good during turbulent flow. Turbulent is where the vertex of the water vectors will not only go in the same direction, but it can go in in circles. edges depending on the interaction of the soil media.

In Darcy's, it is actually flowing through a pipe it is an average velocity it is a linear velocity It assumes porosity is averaged out. So what happens is it is like flowing through a pipe. However, there is interaction of the soil particles or the rock particles on the water, which will cause the turbulence. If you go slow, turbulence is less, if you go fast, the turbulence is high, it is like biking through the air.

Or let us say, you boating, you boating on a river. If you go very slow, you do not see the waves behind you forming any patterns. But if you go very fast, then you leave turbulences. The same, like how planes go, birds do not create much turbulence, but if you see a plane it will go and then turbulences are formed. So that is a function of the velocity.

Darcy’s law is assumption is linear. Linear movement and linear velocity. This is has big issues with in real life scenarios where the low permeability sediments are present under low gradients, so for example, you have a very slowly allowing water medium or low permeable soil is solid. And then if there is already a low gradient, low gradient pulling the water then the linearity will not hurt.

Darcy's law is very lenient right it is a function of your hydraulic conductivity and hydraulic gradient assumptions is linear and it does not hold good when the low permeability sediments and low gradients are present, because water can not only keep on moving, it will stop the low gradient does not pull enough there is no not much activity. So water can go in a lateral position or you can just stay there for some time.

The same applies for larger velocities also. So either if it is too slow, the linearity is gone. If it is too fast, the linearity is gone. So that is a second assumption here. Issues with large flow through high permeability sediments. For example, the rocks I said, when you pour the water on the rocks, under the ground water, they are just gushes through and it takes its own path, it is not a linear path, and picks his own path it gets stored in one place and if the high permeability is there, high specific yield is that just water just cashes out.

So, therefore, there are limits. So, you should understand when you use these in your system. And when you report this as a value using Darcy's law you need to understand these limitations.

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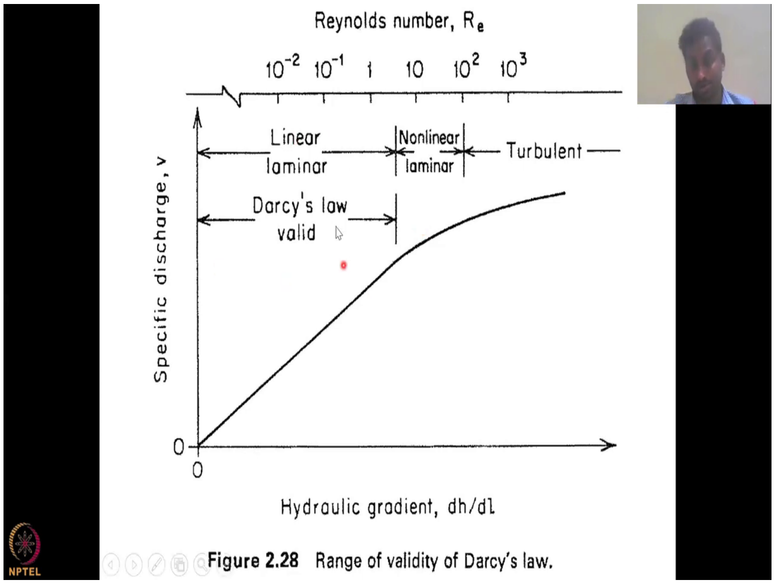

Figure 2.28 Range of validity of Darcy's law.

For example, if you are applying water in a field that is not an turbulent flow, it is a non laminar flow, and the sediments are not low permeable or too high permeable. But as I said, if you are pushing a big dam in canal, just breaking the canal and setting the water into the field, then the equation will not help.

So look at here, we have specific discharge on the y axis and we have your hydraulic gradient, which is d h by dl on the x axis. So, as the specific discharge and hydraulic gradient go up, which is slowly the velocity is going up and the hydraulic gradient which is pulling the water is also going up Darcy's law is good, it is linear and laminar flow, slow well movement of water, but then there is a non linearity and non laminar flow, but also there is a turbulent flow which is given by the Reynolds number.

So, Reynolds number is given to understand the turbulence of the fluid and for water it picks up around 10 power 2. So, once you have this turbulence, which is a function of this fluid material, viscosity and also the surface tension. So, you could see that how this discharge, when at high discharges and that hydraulic gradient your turbulence kicks in and that is where your Darcy's law fails. So, at this region, your Darcy's law fail. Once the flow the discharge becomes non laminar, which is turbulent and nonlinear, the Darcy's law application should stop. So, you should not be promoting Darcy's law in high turbulent groundwater flows, but you do not get that.

So, that is the reality is, you do not get it in much groundwater systems only in Karst geology, where you have limestone and inside that the water flows that is very turbulent, you would have seen caves and underneath water flows, if you take a macroscopic view that is also groundwater because it is under the ground and water just takes away the sediments and the solid materials is dissolved the solid material which is limestone and flows through it, if you apply groundwater flow there, it will not work at that point you should treat the flow as a surface water discharge, and you apply the Mannings coefficient and other hydrologic parameters to, you treat it like a river, you will learn to sample it up.

So, that turbulence is very very important and it happens only in very very specific term. So, in our class where we are looking at groundwater for India and agriculture use, Darcy's law and this range still holds good the only place where it does not hold good is the unsaturated situation and in transient flow situations.

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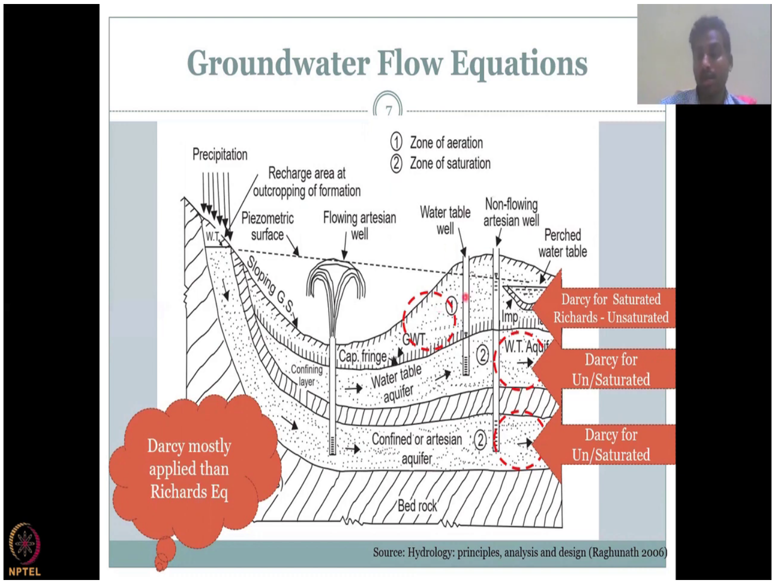

So, to wrap up the groundwater flow equations have been discussed in class, we saw two equations Darcy's equation and Richard's equation for which we have first divided the study area into a unconfined zone of aeration and unsaturated aquifer. And then we had a saturated aquifer in an unconfined aquifer. So, this is an unconfined zone unconfined aquifer with a unsaturated setting and we had a confined aquifer with a saturated setting.

So, we also defined zone of aeration and zone of saturation. So, to club it what did we do? We said the Darcy's can be used on the top for unconfined aquifers for saturated and which is equation is best suited for the unsaturated condition where Richards equation brings in hydraulic conductivity as a function of the degree of saturation.

Whereas, Darcy assumes 100 percent saturation. So, the idea is when you have 100 percent saturation, your Richards's equation boils down to Darcy's. So that happens in the unconfined unsaturated area. So, the unsaturated can also be saturated because when water flows and recharges it becomes unsaturated. So, that zone the zone of aeration if it is saturated please use Darcy if it is unsaturated you can use Richardson.

Then we come to the unconfined region under the water table. Darcy is mostly used for both the conditions are unsaturated and saturated, because it is always saturated at some instances the water level comes down becomes unsaturated and then picks up again. So, Darcy's law has been used widely for that.

In the confined aquifer also Darcy's law hosts most capable because Darcy's equation can capture both the unsaturated and saturated within the confined assuming that some layers if are unsaturated, you do not model those layers, you only model the layers or use the equation for the layers where the Darcy law holds good. Darcy's mostly applied then, Richard's equation there is no issues with that, you just have to be careful in telling that yes I use Darcy, yes I have these assumptions. But still I will get this results from which I could understand what is going on.

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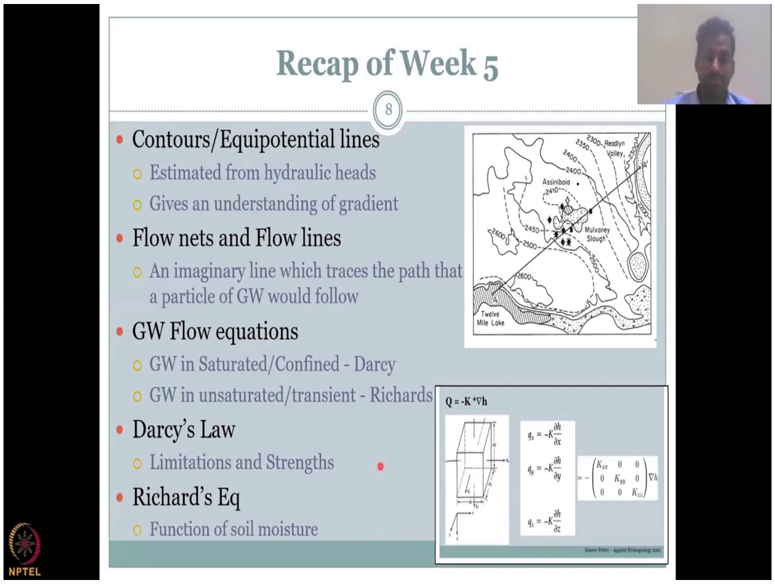

So, to recap, the week five we look at contour lines or equipotential lines, which are basically constructed from the hydraulic heads to place some across the field and then from the field you have hydraulic heads placed across the field, you connect these hydraulic heads of same levels to get contour lines estimated from hydraulic heads and gives an understanding of the gradient the direction in which nor water flow is occurring. And once you have the contour lines and the gradient, you can next estimate the flow nets our flow lines which are imaginary lines that traces the path of a particular groundwater, how it flows from one point to the other.

Then we saw some rules for homogeneous systems and heterogeneous systems. Then we also looked at groundwater equations and mostly groundwater saturated, confined can be given by Darcy's equation and groundwater in unsaturated and transient is given by Richards equation. On the whole we saw that Darcy can be used for most of the systems and even in systems where Richards equation can be used, for example, as I said before Richards Darcy's law was used for everything. Richard's just added more efficiency, more accuracy, but before that, it was only Darcy's law an 1800s to 1900s, So Richard's came only 1900s.

So, across the years still Darcy's law was used and well documented and all the results are still okay. So, Darcy's law is widely used many studies done, however, there are a lot of limitations and strengths, which you should understand before deploying it for your study here. Richard's equation the fundamentally fundamentals are very good and it has a function of soil moisture, the degree of saturation which is not present in Darcy.

However, the data requirement is less than Darcy's law compared to Richardson's law. So, for that Darcy's law has been used widely across the world.

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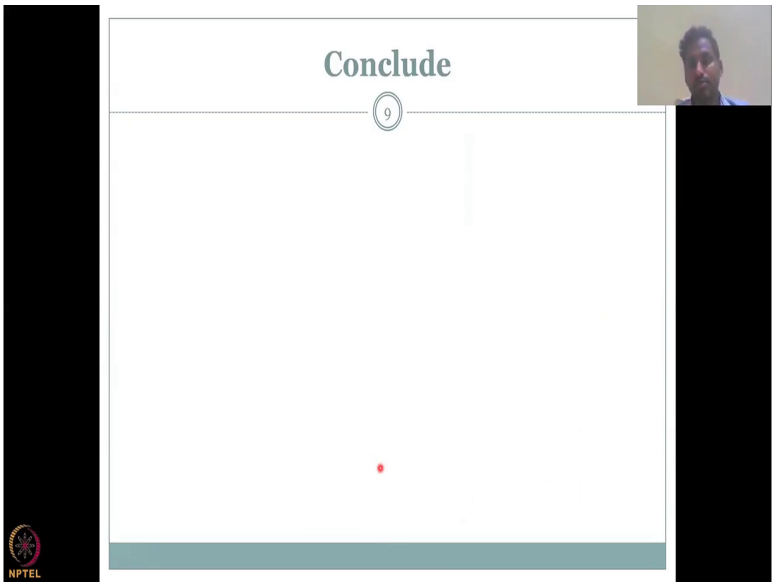

With this, I would like to conclude the fifth week of lecture on groundwater equations and setting up these parameters for the equation. I will see you in week six. Thank you