Geotechnical Engineering-II
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Lecture 4
Shear Strength of Soils III

We have been talking about the strength of the materials and this is where I gave you some idea about the state of equilibrium in the soil mass. And why do we require the tools to analyse the state of stress in the soil mass? And what I did is I took a point and I was trying to analyse the state of stress acting at this point which is lying on a plane along which the failure would occur.

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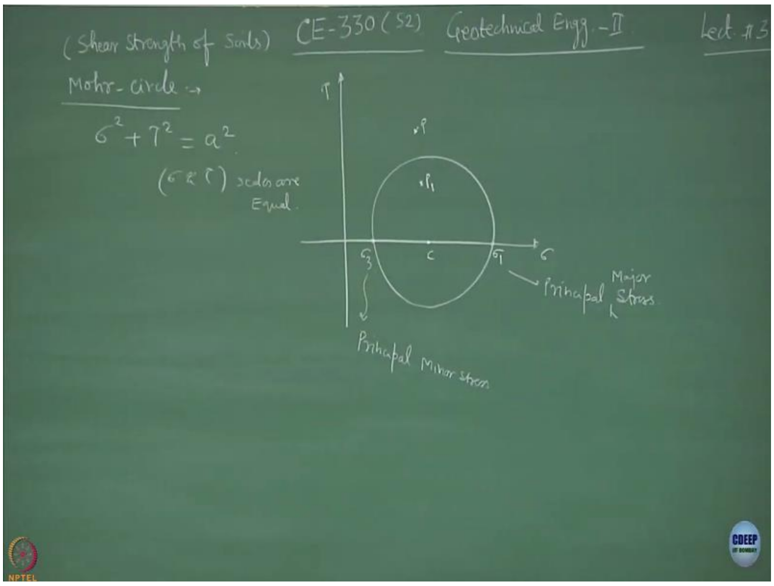

And in the process, I had derived the relationship between sigma square and tau square as a square and this is the equation of the circle which is known as Mohr circle. Now, the interpretation of the results is like this that if I put the σ and τ scales as equal. So, σ scale and τ scale are equal. This is very important to remember. The whole analysis depends upon the fact that σ and τ scales have to be same. So, this is a circle which you get and as we discussed in the previous lecture, this is the center of the circle.

Now, this is what is known as σ_1 and this is σ_3. We call this σ_1 as principal stress, principal major stress and σ_3 is this point, we call it as principal minor stress. So, in short, this Mohr circle defines the state of the material at which the system remains in equilibrium. I hope you can realize that point P, the state of stress at a point P is not going to be possible.

It is a virtual situation because before the point P is achieved, the material has already failed. Similarly, if I say that the point P1 which is lying within the Mohr circle that place also the material is not going to be in a failure state. So, in short, the Mohr circle defines the state of stress acting at a given point which is sitting on a plane along which the failure is about to take place. So, for all practical situations, the point is still in equilibrium.

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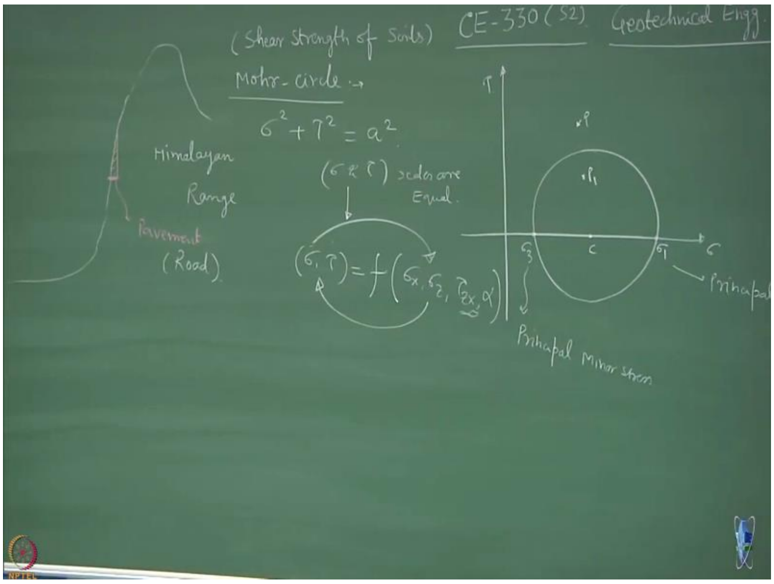

Now, if you remember what we did is we wanted to know σ and τ. The question is why? And I cited a few examples in the previous lecture. A simple situation would be let us say those of you who have gone to hilly terrains, this is a huge let us say Himalayan range. And for the sake of, creating infrastructure, what I will do is I will cut the slope over here. I will create a bench, remove this much of the soil and this is where the road or the pavements are going to sit. So, this becomes a pavement on which the road is going to be sitting.

I hope you realize that the situation is quite critical. It is very easy to draw something like this on the board and then say that I have constructed a road over here. So, whenever you get a chance to go to the hilly terrain, you will realize that most of the infrastructure is being developed on the hills by cutting and levelling the ground or the hillocks. I will take you through a real situation in which I was involved. To give you more idea about how these concepts which are so simple, preliminary can be utilized to analyse these situations.

The larger picture here was that we wanted to know state of stress as a function of σ_x, σ_z, τ_{xz} and α. It could be a reverse problem also. If I know this, I would like to know this. This is also a situation and that is where this Tamaguno, he was talking about τ_{xz} thing comes in handy or the forward situation would be if I know this and I would like to compute σ and τ. Both ways its possible. So, in all these situations, what we have to do is we have to first draw the Mohr circle which represents a state of stress and once this has been done I can analyse the way I want to analyse it.

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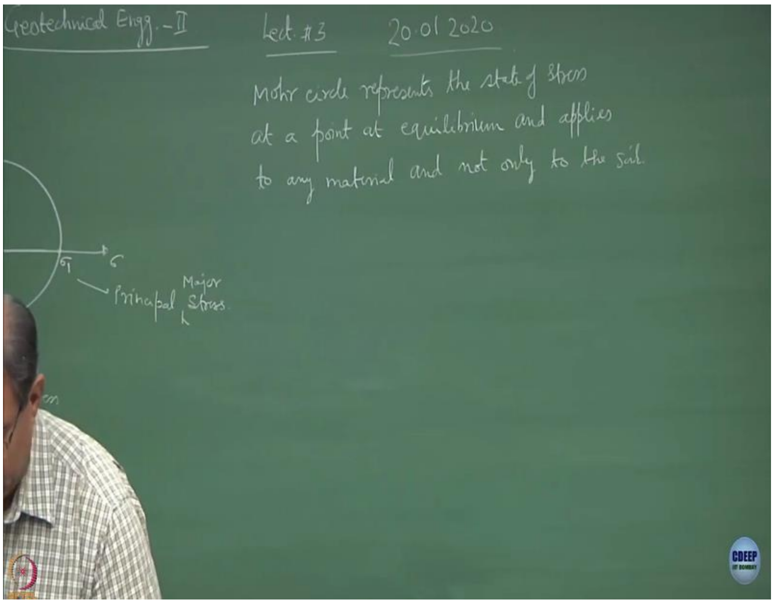

So, there are a few characteristics of the Mohr circle which you should write down and try to understand. The Mohr circle represents the state of stress at the point and this point I hope is understood is lying on a plane and that happens to the plane along which the shear is going to take place or the slippage is going to occur, at a point at equilibrium and applies to any materials.

I am sure when you are doing strength materials, you must have not realized that this concept can be utilized to any material. Now, in this case, we are going to utilize this concept for the geomaterials like rocks and soils. The mechanical engineers would be utilizing this for composites, steel, different types of girders, different types of I-beams, and whatnot or any material. Any material and not only to the soil.

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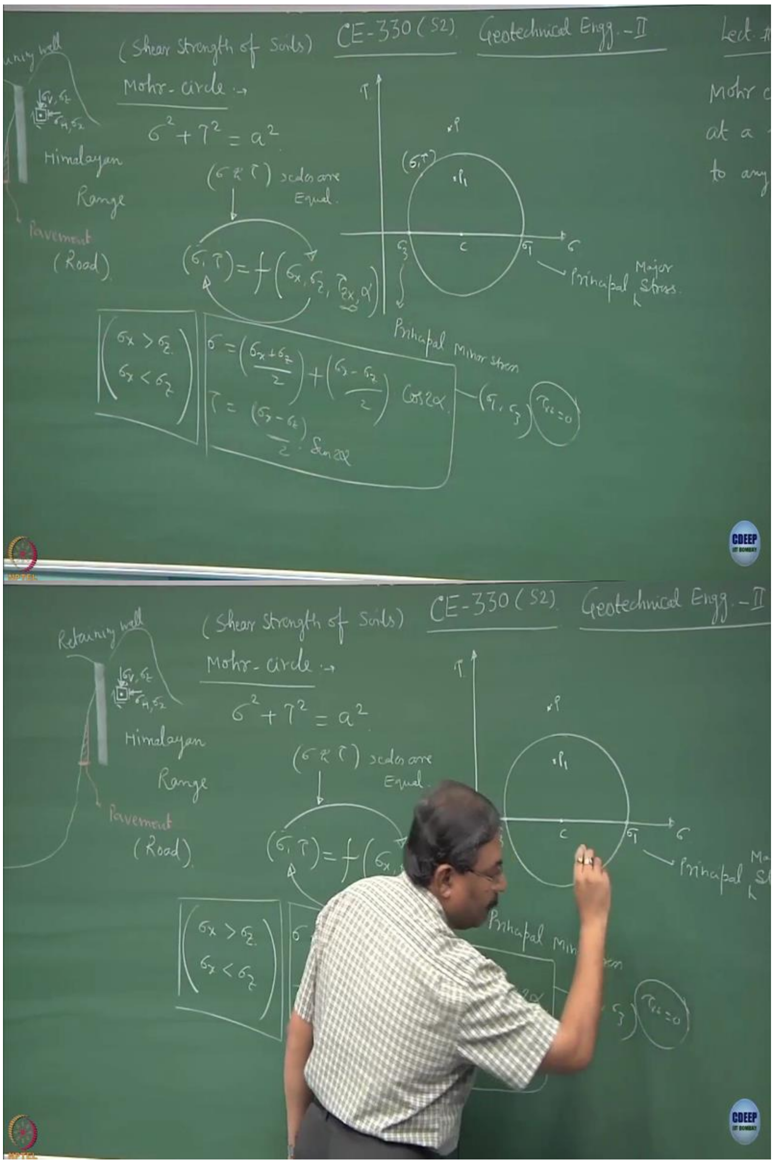

So last time I wrote an expression where we had defined σ and τ as

$$\left[\frac{\sigma_x + \sigma_z}{2}\right] + \left[\frac{\sigma_x - \sigma_z}{2}\right] \times \cos(2\alpha)$$

and shear stress is

$$\left[\frac{\sigma_x - \sigma_z}{2}\right] \times \sin(2\alpha)$$

I can interplay the stresses in such a manner that these expressions can be written now in the form of σ_1, σ_3. So, I can always say that σ_x; please understand this concept this will be very useful for the rest of the course and σ_z, σ_x could be greater than σ_z.

A simple situation would be if I consider a point somewhere here in the soil mass. Now this is σ_v, this is σ_x or σ_h. I normally defined σ_h as σ_x and σ_v as σ_z. These are the state of stress. And of course, to complete this if I am dealing in terms of σ_x and σ_z there will be shear stresses which are going to act. This is what is known as state of stress existing in the material at a given point.

There could also be a situation where σ_x becomes less than σ_z. Can you guess when this type of situation might occur? So, the way I read it is the lateral stresses are more than the vertical stresses in the first case. In the second case, the lateral stresses are less than the vertical stresses. So, these are two situations which you might be analysing in day-to-day civil engineering practices.

Now, the common sense says, if the vertical stress is more than horizontal stress, I still require some confinement over here. This becomes a retention system which we will analyse. So, this becomes a retaining wall. You must have seen when you move around the hilly terrain, what do they do is they try to stabilize the entire slope by putting a retention scheme.

So, this becomes a retaining wall. There could also be a situation where the reverse happens. Lateral stresses are more than the vertical stresses. They are defying the gravity. The lateral stresses are so high that they are defying the gravity. Tectonic motion: two plates coming and hitting each other and formation of mountains. Correct?

So, it is very interesting to start with this simple concept if I can project and if I can solve the state of stress which exists in the geomaterials for different real-life situations. So, this is a typical case of let us say, tectonic motion, formation of a hillock, plates coming and hitting each other. Lateral stresses are more than the vertical stresses. This is a reverse situation.

So, I can still use these equations. What I will have to do is, I will have to just substitute the values of σ_x and σ_z to σ_1 and σ_3. This I can do analytically. And when I am dealing with σ_1 and σ_3, your τ_{xz} vanishes. Why? Because σ_1, σ_3 always acts on the plane where the shear stress is 0. Look at this diagram. σ_1 and σ_3 is acting on the plains where the shear stress component is 0. And that is the reason we call them as principal major and minor stresses. So, these are the planes on which shear will never act or it will be 0.

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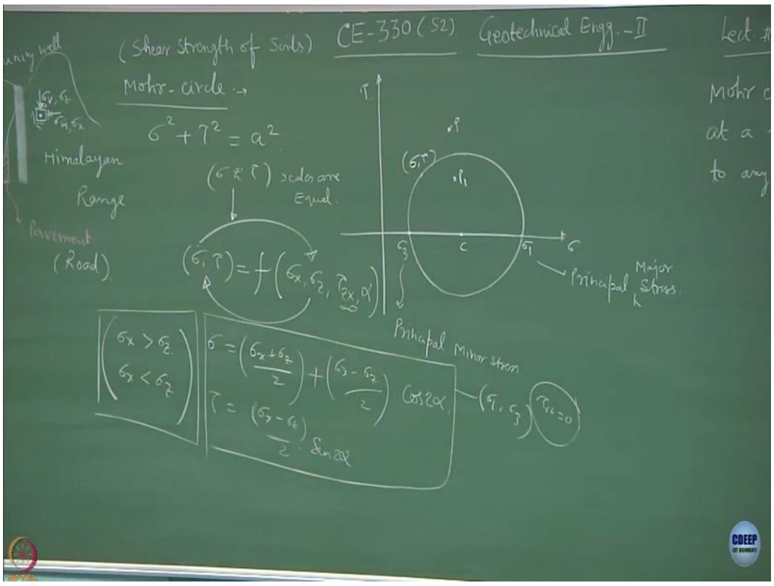

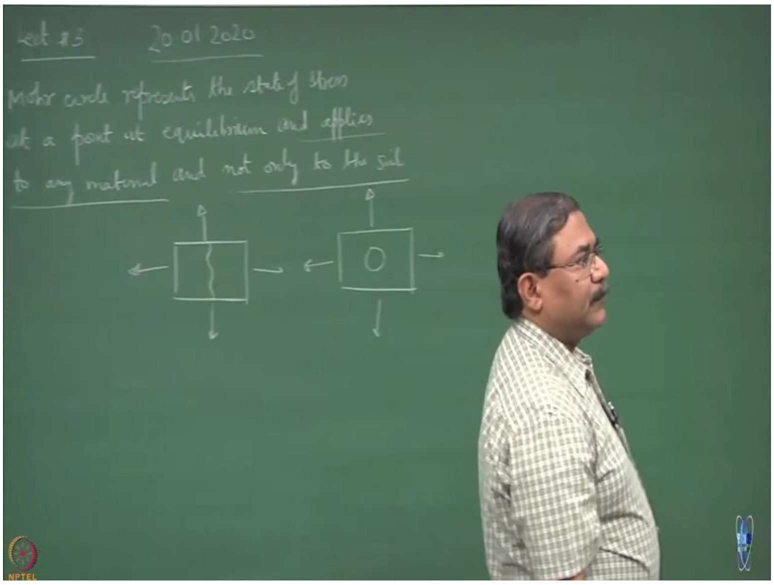

But suppose if I consider a point over here, and if I define this as σ, τ. Now, this is a state of stress which is acting at some plane and we will discuss about this. The moment this becomes critical, the failure takes place. Is this part clear? Now, what I have done is I have written here that it might be applicable to any material and not only to the soil. This could be rocks; this could be steel plates. It could be anything. So, I am sure in your strength of material course you must analyse the situations where suppose if I give you a metal platen, and if I asked you to stretch it or compress it.

Normally metals have to be stretched. So, suppose if I say this is a state of stress and find out how the fracture is going to take place? Is it okay? This is a failure plane or the fracture plane. I could reverse the situation also and I could make it more complicated by saying that I need a situation where there is a hole-punched plate. In your steel structures, you might be using it for gussets, rivets. Is it not? For the gusset plate when you connect with the I-beams or the sections you do punching so that you can put a rivet and you can connect the two.

And suppose if I ask you what is now happened to the material if I stretch it? Very complicated-looking problem but very simple to solve once you have the Mohr circle with you. Remember, this is devoid of the material. We have not put the material properties over here.

Now, the question is, if I want to really make it valid for a material like soil or for any material, what I should be doing? I have to define the shear strength of the material. Did you get this point? And that is the reason when we start talking about the shear strength of soils, we, first of all, try to understand the state of stress and superimpose on this the material properties. Is this fine?

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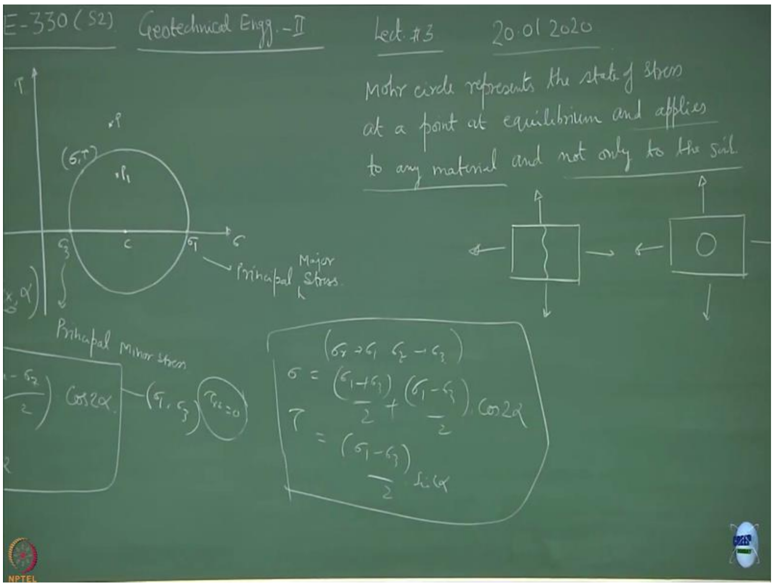

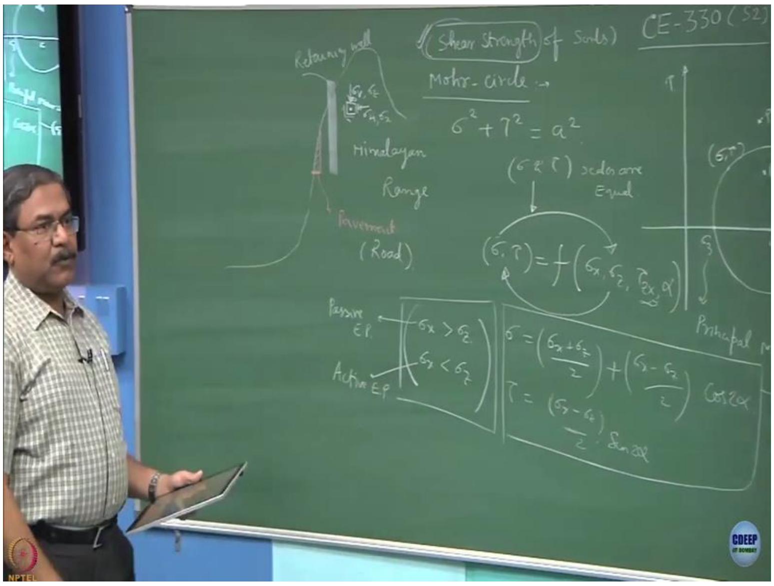

Now, there is another characteristic of; so, I can replace the whole thing and I can say that σ and τ would become just replace σ_x by σ_1 and σ_z by σ_3. Later on, we will study that this type of a situation where the horizontal stresses are more than the vertical stresses become say passive state of the material (passive earth pressure) and as long as the lateral stresses are lesser than the vertical stresses, this becomes the active state of earth pressure state. EP corresponds to earth pressure.

So, if I just interchange these things, I will be getting σ as

$$\left[\frac{\sigma_x + \sigma_z}{2}\right] + \left[\frac{\sigma_x - \sigma_z}{2}\right] \times \cos(2\alpha)$$

and τ will be equal to

$$\left[\frac{\sigma_1 - \sigma_3}{2}\right] \times \sin(2\alpha)$$

So, this becomes the generalized law.

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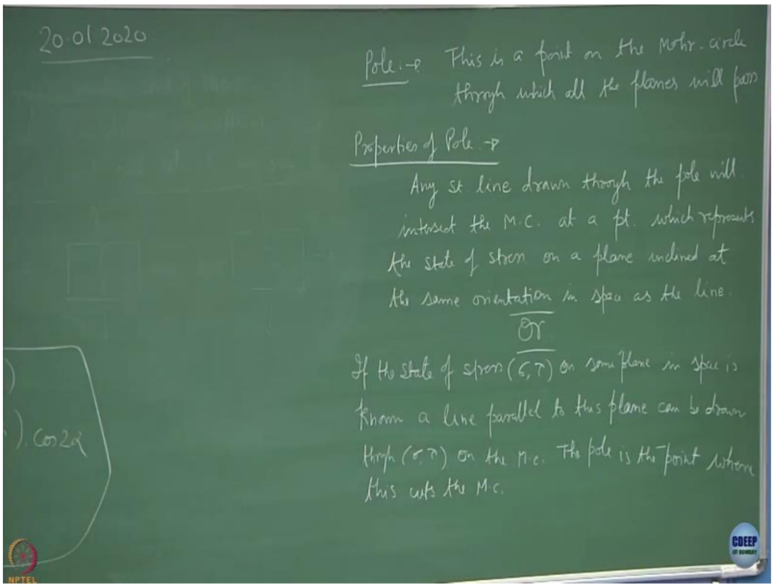

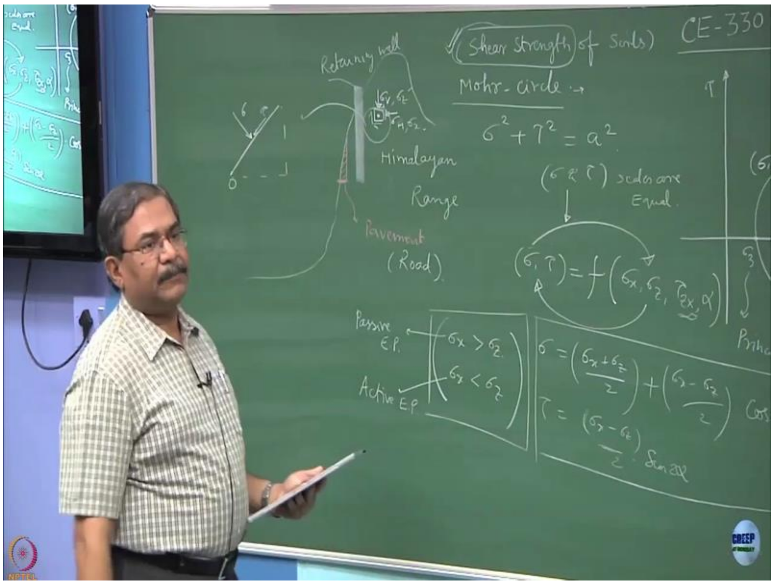

Now, when we deal with the Mohr-Coulomb circles or the Mohr circles to be precise. Not Coulomb. Coulomb I have not brought yet in the picture. We define a point which is known as pole. Hope you must have done it in the strength of materials. Have you done it or not? No? No issues. Do you remember you have not been exposed to this? No issues.

Basically, the characteristic of the pole is that this is a point on the Mohr circle through which all the planes will pass. Fine? This is the fundamental nature of this pole. The reverse is also possible. Alright? Any plane starting from a known state of stress wherever it intersects the Mohr circle that becomes the pole. Fine? Both ways.

Now, we will talk about this a lot. So, what are the properties of the pole? The simplest possible theorem is any straight line drawn through the pole will intersect the Mohr circle at a point which represents the state of stress on a plane inclined at the same orientation in space as the line. Please read this a bit and then we will go further. Go through this statement and try to understand what I have written.

So, pole is a point on the Mohr circle through which all the planes will pass. These are the planes on which the state of stress is known. Remember what I did? I took out an element from here. And then I said we can continue on this itself. We know the state of stress here. τ also might be acting. If these are the principal stresses τ will be 0. We have σ_1, σ_3 and then we are trying to find out what is the state of stress at this point. When I zoomed it, it became like this. This is a point O let us say. This is σ, this is τ and then we did some simple analysis.

So, the property of pole: any straight line drawn through the pole will intersect the Mohr circle at a point which represents the state of stress on a plane inclined at the same orientation the space as the line is. Let us talk about the reverse what we call it as a lemma. Or if the state of stress that is σ and τ on some plane in the space is known, a line parallel to this a line parallel to this plane can be drawn through σ and τ on the Mohr circle. The pole is the point where this cuts the Mohr circle where this cuts the Mohr circle.

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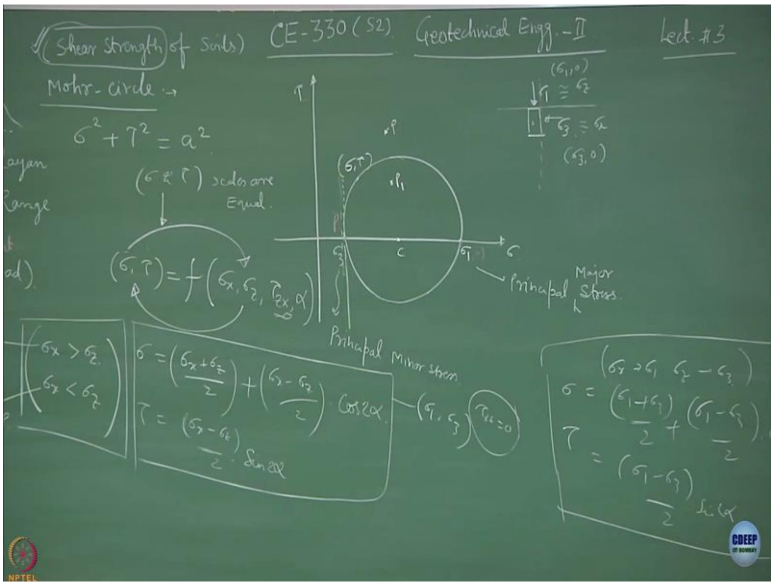

In the simplest possible form at this point if I show the element this is σ_1, this is σ_3. Alright? And we are assuming that σ_1 is equivalent to σ_z and σ_3 is equivalent to σ_x. σ_1 is the plane is the stress which is acting on the horizontal plane. This is the horizontal plane. Correct? σ_3 is acting on a vertical plane. The state of stress at this point is σ, τ that means σ_1,0. By virtue of being a principal stress, shear stress is 0. Clear? At this point the state of stress is σ_3,0.

Where are these points located on the Mohr circle? I have shown them as this point and this point. Where is the state of stress is acting? σ_1? Where it is acting? Horizontal line. So, starting from the state of stress which is σ_1,0 if I draw a line which is horizontal because σ_1,0 is acting on a horizontal plane. Clear?

Now, this going to cut the Mohr circle at σ_3. So, by the first definition, this becomes a pole. Now, reverse the situation. So, σ_3,0 is this point and on which plane is acting? Vertical plane. Clear? So, if you draw a particular plane passing through $\sigma_{3,0}$ how it looks like? This is how it will look like.

Please excuse me for my poor drawing but anyway and this is going to be a tangent. So, by definition, the state of stress at a given point is known, the plane passing through that intersecting the Mohr circle is going to give you the pole. So, pole is a unique thing. And what we have done? We have cross verified. Either you start from σ_1,0 take a horizontal line cutting the Mohr circle at point P or reverse σ_3,0 is known draw a plain, point of tangency. This is the pole.

So, what you have done in short is we have identified the pole and this pole has a lot of interesting peculiar characteristics. Read this. If the state of stress on this plane in the space is known, a line parallel to this plane when is drawn through σ, τ will you know on the Mohr circle, this will also result in the state of stress of the material.

Reverse what we discussed just now, starting from the pole, if I draw a horizontal plane it is going to go and cut over here the Mohr circle. Clear? So, this becomes your σ_1,0. Starting from this pole if I draw a plane which is perpendicular, is going to cut the Mohr circle at this point itself. This becomes σ_3,0. Is this part clear? Please understand this thing clearly because henceforth I will not be discussing this but I will be utilizing this whole thing. And what you will observe is 80 percent of the course is going to be a discussion only on these states of stress.

See last one, what we did is first we started from the known state of stress. The known state of stress is σ_1,0 acting on a horizontal plane. Clear? Where is σ_1,0 acting? At this point. Draw a plane which is parallel to this plane cutting the Mohr circle. Point becomes pole. Fine? State of stress here is σ_3,0 acting on a vertical plane. Draw a vertical plane. It cuts at this point, this becomes the pole.

The reverse process. If I draw a line passing through the pole, wherever it cuts the circle, it gives you a state of stress on that plane. So, starting from pole, if I draw a horizontal plane, it cuts over here, the state of stress on this plane is going to be σ_1,0. Starting from this point, if I draw a vertical plane, it cuts at this point itself and hence it gives σ_3,0, state of stress. Fine?

So, I am sure you must be realizing slowly and slowly what we are doing starting from simple models now we are approaching complexities and trying to answer the real-life situations. What I have to do is only in this model which is devoid of the material properties, I am just going to input the material properties. And what material properties I need? I need shear strength characteristics. That we will discuss later.