Seismic Analysis of Structures
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Lecture – 10
Response Analysis for Specified Ground Motion (Contd.)

In the previous lecture, we discussed about the different forms in which the equation of motion of a single degree freedom system subjected to a support excitation can be written.

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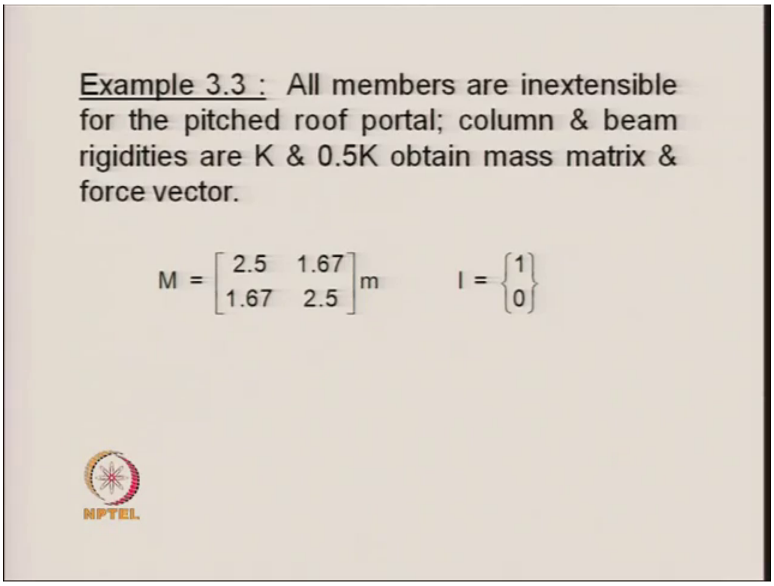

They are the first equation is written in terms of the relative displacements of the mass with respect to the support. And it is a second order differential equation, or with the relative motions as unknown. On the right-hand side of the equation we have the mass times the ground acceleration. The second form of equation, was in terms of the total displacement of this structure in which the second order differential equation at the total motions as the unknowns. On the right-hand side, we had the displacement; that is the stiffness multiplied by the ground displacement, plus the damping coefficient multiplied by the ground velocity. Then these equations can also be written in in state space form; that is in a form in which second order differential equation is written in terms of a state of coupled first order differential equation. The state of the system is defined by a vector consisting of a displacement and velocity. And both the state space equations can be written one in terms of the relative displacement, other in terms of the total displacement.

After that we discussed about the equation of motion for a multi degree phenomenon system, in which we classified the multi degree of freedom system on the 2 classes. First one was single support excitation, and the second one was multi support excitation. For the multis single support excitation the base of this structure moves as a rigid body, as a result of that the displacements which are induced at the different floor levels of the frame; they consist of the relative displacement of this structure with respect to the base caused by the inertia forces. And the second part is the quasi static displacement produced due to the movement of the support as a rigid body. And this rigid body movement therefore, produces the same quasi static displacement at all supports. The on the right-hand side we have an influence coefficient vector or influence coefficient matrix; which can be written from inspection in most of the cases. And these vector or the matrix consist of either 1 1 1 1 1 or 1 0 1 0 1 0 like that. And in most of the cases or most of the problems, these influence coefficient vector can be easily written without performing any calculation.

Then we explained the equation of motion of a 3-dimensional frame as symmetric frame with a rigid slap on the top of it. The idea was to illustrate that in writing down the equation of motion, the mass matrix need not be always a diagonal matrix as it is thought in the case of point lamp point mass lamping. It was shown that depending upon the degree of freedom that is chosen the mass matrix could be a diagonal mass matrix mass, matrix could be also fully populated mass matrix or a coupled mass matrix. We continue with that to show that for also other kinds of structures, these kind of scenario may happen that is the point lamp mass matrix may be also a coupled mass matrix in place of a diagonal mass matrix. For that we take the example of a pitched roof portal frame and for that pitched roof portal frame the mass matrix is shown over here, the mass matrix here is a coupled mass matrix; that is of diagonal terms are not 0. They are having some value and the influence coefficient vector corresponding to that is 1 and 0.

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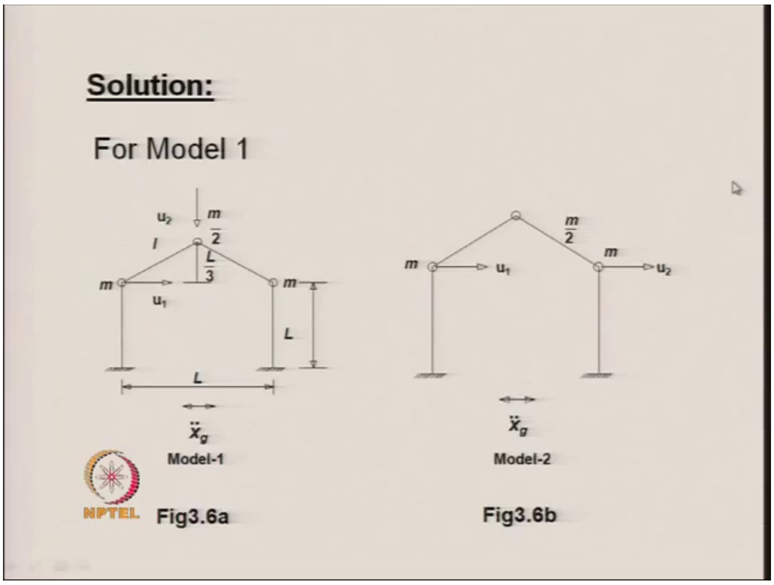

The problem for which this mass matrix exists is shown over here. This is the pitched roof portal frame, this is the pitched roof portal frame which is shown here. And one can model it in 2 ways. The first model is this one in which the degrees of freedom, kinematic degrees of freedom are the rotations at this point rotation at this point rotation at this point. And if we assume the inextensibility of the members then 2 translational degrees of freedom will be existing. And one can choose it in different ways for model one we choose the displacement degree of freedom 1 over here, and the other at the top; that is the vertical motion of the crown.

In the second model the 2 translational degrees of freedom are chosen. One here as a horizontal degree of freedom, and the other here as a horizontal degree of freedom. And for both one can write down the mass matrices, and the these matrices as will see will be a coupled mass matrix. For deriving the mass matrix for this particular 2 cases. Let us consider this figure.

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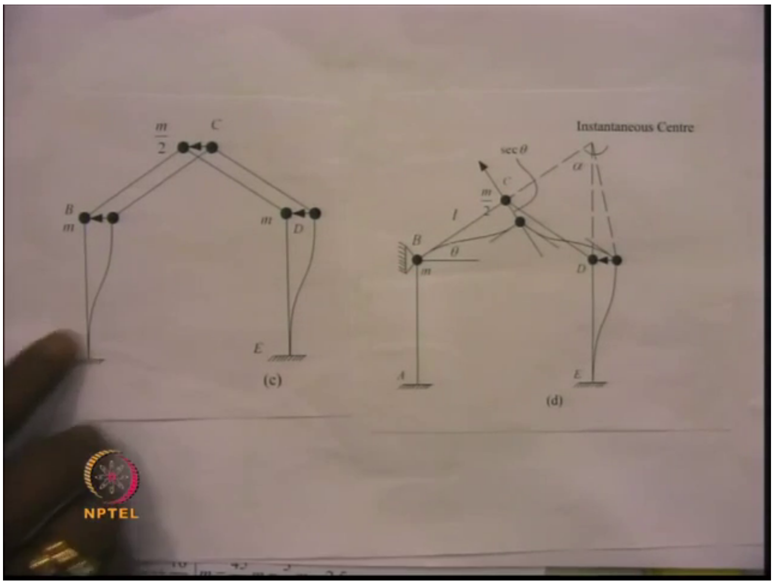

In this figure we have a the frame and A in this frame; what we do is that we give a unit displacement at this point. And locking the vertical movement of this particular point. As a result of that these moves these point also moves, and these moves also horizontally without any vertical displacement. The displacement is given such that a unit acceleration is induced, and that is acceleration will be in the opposite direction to the direction of motion. So, the unit acceleration is induced over here. Unit acceleration is induced over here, and unit acceleration is induced over here because of the these displacement. The mass times the unit acceleration; that is m into unit acceleration will be the force inertia force that will be in induced due to the unit acceleration provided over here. Here the force also will be equal to inertia force will be equal to mass time the unit acceleration. And at this point also will have an a inertia force which will be m by 2 multiplied by the unit acceleration acting in the horizontal direction.

Since we do not have any degree of freedom declared at this point, the mass time unit acceleration these force is decomposed. One in this direction, other in the direction along this member. And this force is finally transferred to this point. At this point now, we have got 2 forces. One force along this, the other force in the horizontal direction. We take the component of these 2 forces, one in the vertical direction that is the direction of the chosen degree of freedom, translation and degree of freedom, and the other along this member. So, the component of this 2 forces which are acting along this member they are finally, transferred to this point. And at this point again we resolve those 2 forces. One in the horizontal direction, other in the vertical direction. The forces which are dissolved in the vertical direction they pass through the column through this support.

Here also the force which was existing in the vertical direction, because of the dissolution of this force, there is also pass to the support. The forces which are the existing over here that is 3 forces. One force is this 1 plus the 2 forces which are the components of the forces coming from this point, and at dissolved in these direction, these 3 forces are added together. And these 3 forces will be equal to by definition m 1 1. That is the inertia force produced at this point due to the unit acceleration over here. Similarly, the 2 vertical forces or the component of the vertical forces in this directions that we are obtained. These 2 forces together, we give raise to m 2 A; that is the forces generated at this point due to the unit acceleration produced over here. So, these 2 forces were found to be is equal to 2.5 and 1.67. So, they are shown over here. They are the first column. And 2 point is the force which is equal to m 1 1 1. And 1.67 is the force which is equal to the m 2 1.

Now, if you are wanting to find out m 2 2, then what we do? We give a unit vertical displacement to this particular point to the crown, and lock this as a result of this these kind of deformation takes place. The deformations takes place in such a fashion so that the lens of the member do not under go any change. And that can be conceived by with the help of an instantaneous center of rotation. Now if we extend this line and extend this line, then they join over here. And these point is the instantaneous center of rotation. And these as a sets square, these triangle as a set square if we give a rotation at this particular point.

Then these point will move perpendicular to the member, for displacements theory these point will move perpendicular to this member according to the small displacement theory, and they do not undergo any change in length because they are moving perpendicular to the members. And if the 2 sides they undergo any change in length then the third side also will not undergo any change. Therefore, these length which was there initially the same length remains over here. So, the condition of in inextensibility is maintained. Similarly, since this member is moving perpendicular to this member there is no change in length and also since this member is moving perpendicular to it then there is no change in length of this member. So, thus by giving these kind of movement, we can ensure that the condition of in s inextensibility is satisfied.

Now, the this point not only moves vertically down, but also moves horizontally or in other words these point moves perpendicular to this. Now one can find out that for a given displacement on this side so that there is a unit acceleration produced in the vertical direction, will have a component of that acceleration in this in client direction, mass times or m by 2 multiplied by that acceleration will be the force induced over here. Similarly, mass times the acceleration which will be produced over here, because of the unit acceleration give over here, will be the force which will be acting in the horizontal direction at this point. Since there is no degree of freedom which is defined at this point, these horizontal force will be decomposed one indices of vertical direction, which will straight away go to the support. The other force will be dissolved in this direction this will be transferred to this particular point. And then this particular force which is acting along this member plus this force. These 2 forces will be resolved in the vertical direction, other along the direction of this member. The component of these 2 forces in the vertical direction, by definition will be equal to m 2 2 that is the inertia force developed at this particular point in the vertical direction due to the unit acceleration given to the crown in the vertical direction. And that component of the 2 forces which will be acting along this member is finally transferred to this point. The horizontal component of those 2 forces one will be added together, will give the force which will be called 2 m 1 2. So, that is how one can find out the forces. And the second column of these matrix; that is the matrix which is shown over here, that is 1.67 and 2.5. They are the forces that are second column of the mass matrix denoting m 2 2 and m 1 2 2.

So, in the similar fashion, one can obtain the mass matrix for the second model; that is model 2, and here we have the degrees of freedom which are defined like this; that is degree of freedom one degree of freedom over here, other degree of freedom over here. And for that one can straight away go this particular figure, say that in the in the case when we are considering the these degree of freedom, then we give a unit or a displacement here, such that unit acceleration is induced over here. And at this point we lock it. And because of the unit displace or the displacement which is given over here, these point will move down. And we have forces which will be induced over here, and this force will be resolved will be equal to the force which is induced at this particular point because of the unit acceleration induced here.

The force which is generated here, due to the movement of this point along this particular line, these force will be resolved in 2 particular or in in particular direction; that is the force which will be will be acting at this will be resolved one in this direction, other in this direction. And the force which will be resolved in these particular direction, will be ultimately transferred to this particular point. And this component of this force in the horizontal direction will be is equal to m 2 1, and the other component of force which will be in the direction of the column that will simply pass through the column and go to the support.

The component of these force on this particular member along this member will finally, be transferred through this particular point. And this force will be then dissolved one in the vertical direction, other in the horizontal direction. The component of the force which is in the vertical direction will simply pass through this support, and the horizontal component of the force will be added to the previously calculated value of m 1 1, and the addition of these 2 values will be the final value of m 1 1. So, that way one can calculate m 1 1 and m 2 1 for the model 2 over here which is shown in this particular figure.

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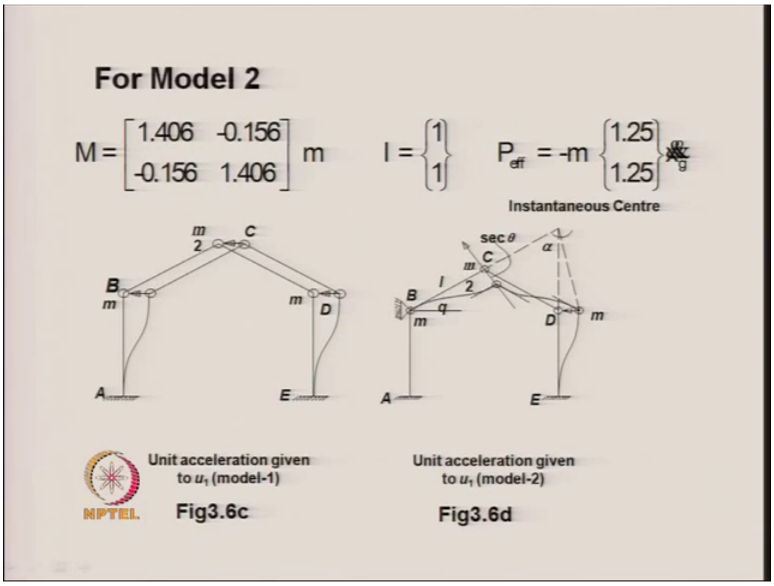

And if we look at the ma mass matrix the first column; that is 1.406 and minus 1.15 size they are the forces inertia forces, corresponding to the unit acceleration provided degree of freedom 1, which is equal to 1.406 that is m 1 1; that is minus 1.156 which is equal to m 2 1.

Similarly, one can find out the second column of the mass matrix by considering the unit displacement at the second degree of freedom, locking the first one and dissolving the forces in the same fashion as we have discussed before. The I matrix or the I vector will be 1 1 in this case. Because of the horizontal ground motion for the masses both at the 2 ends; that is at d and d or the masses will be moving horizontally therefore, the I vector consist of 1 1. In the previous case for model one it was 1 and 0; that is for the mass at b, the mass moves in the horizontal direction, but the m by 2 mass which is acting at c, that mass cannot undergo any vertical movement therefore, it I was is equal to 1 and 0. If we multiply the I vector with the mass matrix, then the effective if we multiply the I vector with the mass matrix then the effective earthquake force for the 2 degree of freedom for the second model.

They would become 1.25 and 1.25. And if the ground motions at the 2 supports are the same, then d and d, at b and d the same replacement will take place, and as a result of that effectively will have only 1 degree of freedom, and for that the total mass will be equal to 2.5 as we can see from the left-hand side of the figure. And the effective load vector minus n is equal to 1.25, 1.25 when this 2 components are added together then also we can see; that the total mass for the case of the same excitation at the 2 support that turns out to be 2.5 n. So, that is how one can calculate the mass matrices. For the different types of the degrees of freedom that can be chosen for a pitched roof portal frame. And there could be structures where these mass matrix even for the log mass case could be of this type; that is, they could be fully populated, and the this kind of mass matrix arises for or different kinds of structures with the degree of freedom. Chosen that is for a particular structure if we choose the degrees of freedom in a different way the it could be that mass matrix can become diagonal. If we choose the degrees of freedom some other way then the mass matrix could be fully populated, as we have seen for the case of a 3-dimensional frame with a rigid slab at the top.

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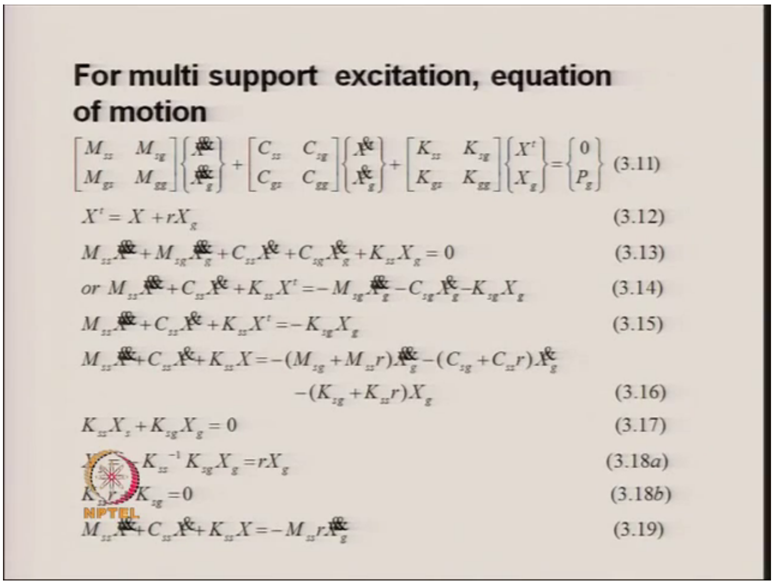

Now let us come to the equation of motion that we write for multi support excitation. And before we come to that, let me also explain to you how we can derive the mass matrix that we have solved for the pitched roof portal frame using the virtual work method.

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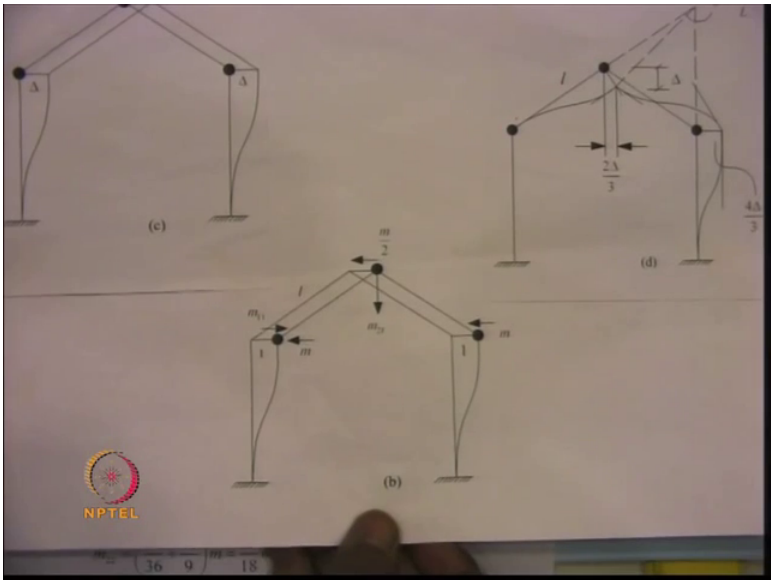

And for that we take this figure again, and in this figure if we consider the first case; that is the case where we have a translational degree of freedom defined here, and the vertical degree of freedom defined at the crown, for that we give a displacement to this particular degree of freedom locking this one, and the displacement is given such that the unit acceleration is produced at this particular point. And the unit acceleration all would be also produced at this point, and also unit acceleration will be produced at this point. As a result of that inertia forces that will be developed here will be equal to m into 1, here it will be m into 1, and at the crown or the inertia force which will be developed is equal to m by 2.

Now let us say this is m 1 1 and this is m 2 1. So, in order to obtain m 1 1, what we do is give a virtual displacement of delta over here; such that no movement takes place. So, then there is a virtual displacement of delta, which is at this point. Virtual displacement of delta which is at this point, and again there will be virtual displacement delta. So, we write down now the virtual work equation.

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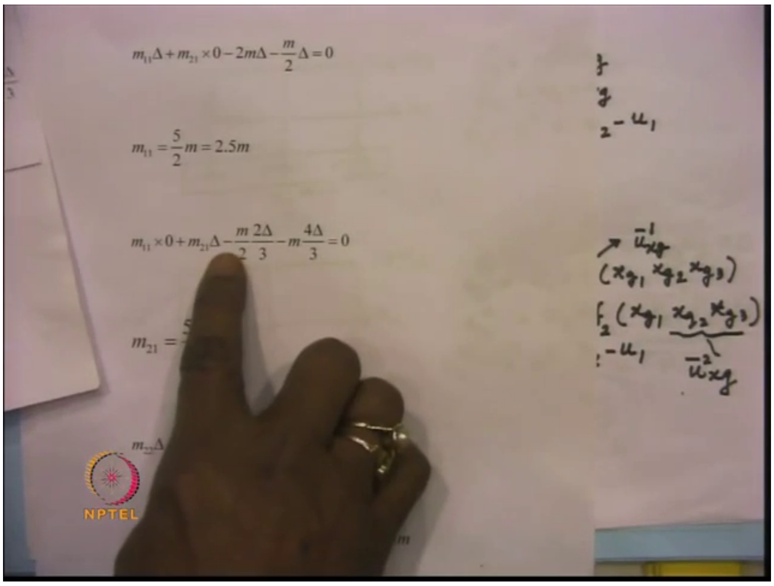

The virtual work equation will be m 1 1 into delta that is the force on the left-hand side which is acting over here m 1 1 into delta, multiplied by m 2 1 into 0. M 2 1 is this vertical force, and that is no vertical displacement therefore, m 2 1 multiplied by 0. And then the forces inertia forces, they will be acting and since the inertia forces are acting in the opposite direction, then it becomes minus m into delta. M into delta for this mass, m into delta for this mass. So, this because minus 2 m into delta. And here the inertia force is acting in this direction m by 2 and the delta in this direction, again we write down minus m by 2 into delta. And that becomes equal to 0, and from there one can derive m 1

1 to be is equal to 2.5 m which we got also in the previous case. In order to obtain the m 2 1, what is done is that we give a unit displacement to this degree of freedom in the vertical direction, making sure that there is no the making sure that there is no extension of the member for maintaining inextensibility condition. As a result of that these moves by delta. These moves by delta by 3, and these horizontal displacement becomes equal to 2 delta by 3. So, these are all these can be these are all arising because of the geometry of the move movement of this particular pitched roof portal frame, considering an instantaneous rotation over here.

Now, once we get that then one can write down m 1 1 into 0, because the here there is no displacement. So, m 1 1 will be multiplied by 0 plus m 2 1 into delta that is m 2 1 is acting in this direction and we have given the virtual displacement of delta over here. So, m 2 1 into delta, then minus m by 2 into 2 delta by 3 the inertia force which is acting over here is equal is equal minus m by 2, and the displacement that takes place over here is 2 delta by 3. So, this is my minus m by 2 into 2 delta by 3. And then this mass this force undergoes a displacement under 4 delta by 3. So, we multiply minus m into 4 delta by 3. So, that gives m 2 1 is equal to 5 by 3 or is equal to 1.67. Now which we got in the previous calculation also. In this particular in this way one can also find out in 2 2 and m 1 1. So, using the method of virtual work, one can also obtain the mass matrix of a particular system if it is not a diagonal mass matrix.

Now, before we start the multi support excitation, let me physically explain the difference between the single support excitation and the multi support excitation, and also explicitly mention the difference between the 2 in so far as the force static displacements that are induced at the non-support degrees of freedom, because of the single support excitation and the multi support excitation.

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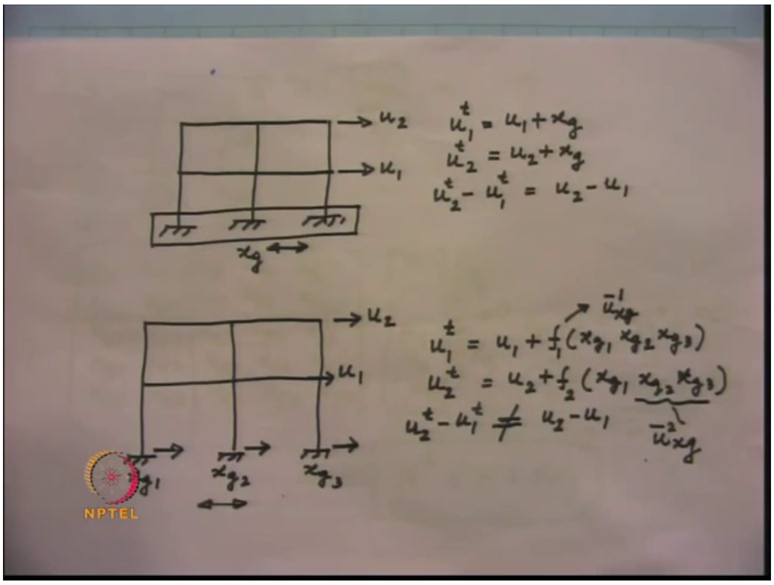

And for that let us take these 2 figures. In the first figure the all the supports are undergoing the same displacement as a result of that the quasi static displacement at this point and in this point, will be same as a x g. The relative displacement of these 2 points with respect to the support will be caused by the inertia force time varying inertia force, and therefore, at every instant of time t will have a relative displacement u 1 at this point and u 2 at this point. So, the total displacement at this point and this point will be is equal to u 1 t, which will be equal to u 1; that is the relative displacement caused due to the inertia force with respect to the base plus x g; that is the quasi static displacement which will be added to that. This will be mid you this is also a function of time t this is at every instant of time t.

Similarly, u 2 t will be equal to u 2 plus x g, because the same ground displacement will be passed on this support or this non-support degree of freedom and this non-support degree of freedom. As a result of that the if we take the difference between the total displacements of these 2 points, then the difference between the 2 total displacements will be same as the difference between the 2 relative displacements. And in working out the bending move inertia for etcetera which generally require the relative displacement of one point with respect to the other. And if the total the relative displacement considering the total displacements and considering the relative displacements they remain the same.

Then there is no need for writing down the equation of the motion in terms of the total displacement we can write down the equation of motion in terms of the relative displacements. And that is why for single support excitation, we have written down the equation of motion in the relative displacement coordinate. With the right-hand side, a having the mass time acceleration, and this is pre-of course, pre-multiplied by the influence coefficient vector.

Now, when we come the multi support excitation, then the we see that the ground displacements; that are there are different at different supports. Now this is equivalent to a differential support movement problem; that means, at a any instant of time t x g 1 x g 2 and x g 3 will be different leading to a differential support movement problem, and these differential support movement will give rise to certain stresses within the members, and also some displacement at this level and at this level.

Now this can be obtained by purely a static analysis; which can be performed at time at n instant of time t, in order to find out the displacement that takes place at this point and at this point because of the differential displacement of the supports; obviously, that to those 2 displacements will be of function of x g 1 x g 2 and x g 3. And let us say for the first floor that displacement which is caused due to the differential displacement of these 3 points is written as u bar 1 x g.

Similarly, one can find out at point 2 at a displacement produced due to this differential support movement, and that will be a function of again another function of x g 1 x g 2 and x g 3, and let us say that is equal to u bar 2 x g. Now since u bar 1 x g and u bar 2 x g they are different. Therefore, the total displacement when will be obtaining for u 1 and u 2 for that relative displacement u 1 which is caused due to the inertia action with respect to this support; that u 1 when we add to u bar 1 x g, then u 1 t that is total displacement at this point is obtained. Similarly, for this point we get the total displacement at as u 2 plus u bar 2 x g, that becomes the total displacements. Now when we deduct these 2 displacement or total displacement between these 2 points in order to find out the displacement of one point with respect to the other, you find out the stresses or bending stresses, then you find that that does not become equal to u 2 minus u 1. Because u bar 1 x g and u bar 2 x g will not cancel with each other.

As a result of that u 2 minus u 2 t minus u 1 t will not be equal to u 1 and therefore, in order to get the bending stresses in this particular member, we have to know the values of the total displacement at this floor level and at this floor level. And because of that reason on the equation of motion should be written in terms of the total displacement.

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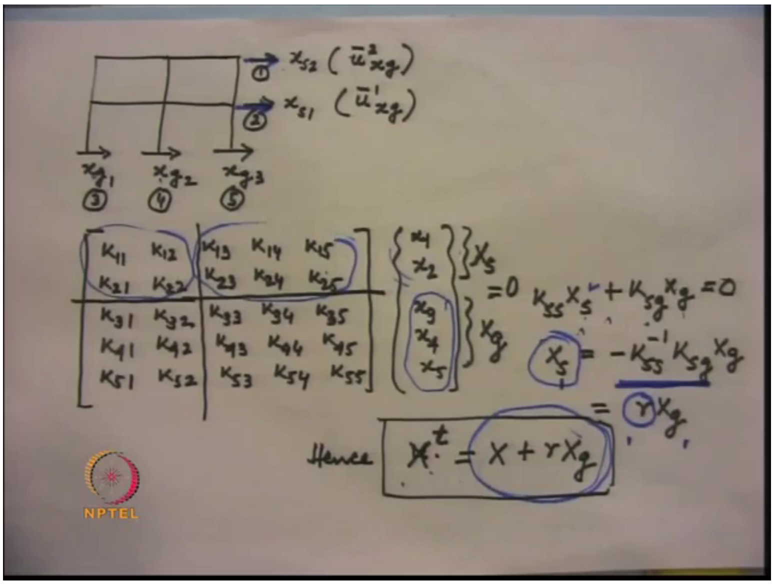

Now, let us see how we can find out the displacement or the quasi static displacements that are obtained at this point and this point due to the differential movement support movement at this point this point this point. Say at any instant of time t, x g 1 x g 2 and x g 3 are the 3 displacements. And because of 3 displacement the displacement which is caused over here is equal to u bar 1 x g and here it is u bar 2 x g. And that we are calling as s x one and s x 2; that is the non-support degrees of freedom, that is the structural displacement one and structural displacement 2. Now considering all the degrees of freedom that is degree of freedom, this degree of freedom, this degree of freedom, this also another degree of freedom, this is another degree of freedom, this is another degree of freedom. We have been all 5 degrees of freedom.

We can write down a stiffness matrix in terms of only the displacement degrees of freedom. And in that if the rotational of degrees of freedom are presented then we condense them out. And finally, write down the stiffness matrix corresponding to these 5 degrees of freedom. And once we write these 5 degrees of these stiffness matrix with respect to this 5 degrees of freedom, then we partition them into nonsupport degrees of freedom, and the support degrees of freedom. Nonsupport degrees of freedom are called say x s, and support degrees of freedom are called say x g; that is these degrees of freedom. And if we consider this partition matrix then one can write down K ss into x s; where K ss is this matrix, and K sg is this matrix multiplied by x g; that should be equal to 0. And from there one can write down x s; that is the at the non-support degrees of freedom the displacement produced due to x g, which is known to us will be equal to minus K ss inverse K sg into x g that is the x s; that is the displacement these 2 displacement can be obtained from the known displacement at this 3-points x g. We can write down a x s in general to be is equal to r into x g, where r is a matrix, this matrix consisting of minus K ss inversing to k s g.

So, for a given frame one can find out this r matrix very easily. And then one can write down the quasi static displacement to be is equal to r times r matrix times the x g; that is the displacement which is which are taking place at the supports. And in when we are talking about the total displacement x t, then this x t can be written as x plus r x g. Where x is the displacement at the non-support degrees of freedom produced due to the inertia effect with respect to the base that is the relative displacement, relative dynamic displacement plus the quasi static displacement which r x g is the quasi static component of the displacement these 2 displacement together get the value we get the x g. And we write down the equation of motion in terms of x g.

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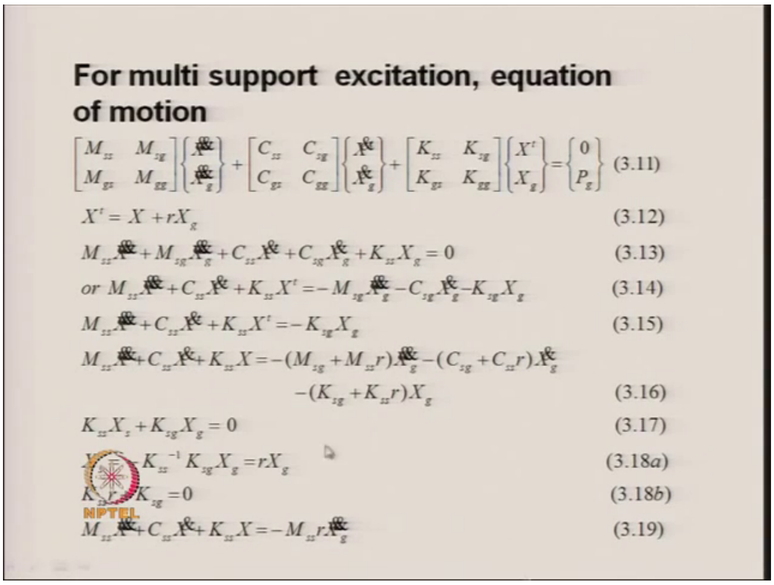

And that is what is shown over here, say in this particular problem the first equation the matrices M ss are the masses corresponding to the non-support degrees of freedom. The M gg; that is the masses with the respect for the support or at the support. And s g and g s they are the coupling matrices. And these will be X double dot t and this is X double dot g X double dot g are the ground accelerations at the supports. Whereas, X double dot t is the total displacement at the non-support degrees of freedom. Similarly, C matrix and K matrix can be defined. And on the right-hand side, since there is no force external force acting we write it to be 0, and here at the supports if we consider solid structure interaction problem, then there may be some forces which may exist at the level of support therefore, that force will be equal to p g g. Now this force for the case of no solid structure interaction will be equal to 0.

Now, we substitute here for x t, x t is equal to x plus r x g that what we are pro proved, before and once we substitute this; then these gives a equation and in this equation, we have got M ss into X double dot t plus M sg into X double dot g plus C ss into X double dot t plus C sg into X double dot g plus K ss into X g, plus there is one term missing over here. This will be equal to K s g multiplied by X g. So, this term somehow the is missed out over here. It should be present. Now if I keep all the total displacement terms on the left-hand side that is as associated with the non-support degrees of freedom, then the left-hand side of the equation become M ss X double dot t plus C ss x dot t plus K ss x t is equal to minus n s g into X double dot g minus C sg into x dot g and minus K sg into X g.

Now what we do for the multi support excitation problem, the coupling mass matrix M sg and C sg they are generally ignored. In fact, in most of the cases M gg that is that turns out to be 0 for no solid structure interaction problem, and M sg or this coupling masses are also 0. Therefore, one can assume M sg is to be equal to 0, and C sg also to be equal to 0 not contributing much to the equation of motion. And if we ignore that then we get equation 3.15 in which on the left-hand side we have all the quantities which are the total displacement total velocity and total acceleration or in terms of the total motion, the equation is written. On the right-hand side, we have minus K sg into x g. Mind you K sg cannot be ignored K sg is the coupling stiffness between the non-support degrees of freedom and the support degrees of freedom which can be easily calculated. Thus, if we wish to solve 3-point equation 3.15, then we need x g or the ground displacement to be specified. And then one can straight away obtain the displacement as total displacement, and from there one can work out the any other quantities of interest of any other response.

Now, if you are wanting to write down the same equation in terms of the relative displacements. Then we rearrange this equation 3.15 by substituting for x t with the help of equation 3.12, and once we do that then on the left-hand side we keep all the quantities in terms of the relative motion; that is x x dot and X double dot and on the right-hand side we bring in hm M ss into r which will be coming from this relationship, and C ss into r and K ss into r. And we see that on the right-hand side we have these all these 3 terms which are associated with the ground displacement, ground velocity and ground acceleration. Now since the definition of r was equal to minus K ss inverse K sg into x g, from that one can easily show that cases into x s into plus K sg into x g that is equal to 0. Therefore, in equation 3.16 these terms becomes equal to 0.

Next what we do is that we assume that C sg which was assumed to be 0 in the previous case, and the component of C ss into r, that basically also do not contribute much to the response. As a result of that we ignore this particular quantity. Now for many problems; however, one may not set it to be 0, one may keep one may sets C sg to be 0, but one may keep C ss into r. In that case we should have also the ground velocity to be specified; however, it has been found that for lightly damped system the C sg plus r into C ss they turn out to be almost equal to 0, or they are effect on the solution is very negligible.

Therefore, the entire thing is assumed as 0. And so far as the mass term is concerned, the M sg is assumed as 0 as before. Therefore, we have the equation 3.19 where we write down the equation of motion in terms the relative motion of this structure that is relative displacement and relative velocity and relative acceleration. And on the right hand side you have got minus M ss r into m s double dot g. The motivation behind writing the equation of motion in terms of relative displacement, and representing it in the form of equation 3.19 is to bring a similarity of the equation of motion that exist between a single point excitation system, and the multi point support excitation system. Or in other words we try to represent the multi support excitation system equation of motion, in the same way as we defined for the single point excitation system.

Here the difference is only that for the multi support excitation system the M ss or the mass matrix is multiplied by a not influence coefficient matrix I, but a r matrix which is also a influenced coefficient matrix, but these coefficient matrix are is to be computationally obtained by solving a static problem. Or in other words by solving the frame problem by a using a matrix method; in which we will find out the values of the non-support degrees of freedom in terms of the degrees of freedom which are existing at the supports. And that gives the r matrix, and how we have obtained r matrix that I have shown before. So, thus a the mass multiplied by the r matrix into X double dot g vector that gives the p effective or the effective r quake force.

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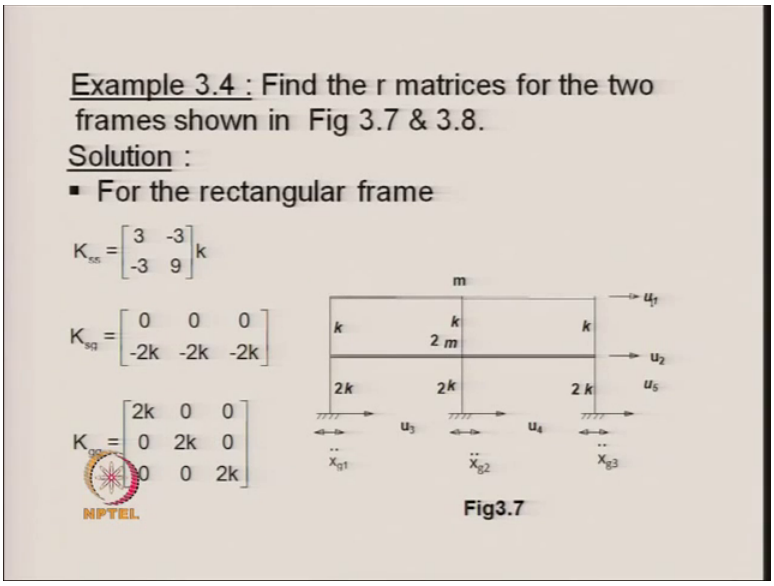

Now, these concept is explained with the help of these problem, equation 3.14. In this problem we are wanting to find out the r matrix for the frame in the figure 3.7. Here this is a shear frame, and for this shear frame we have the degrees of freedom. 2 degrees of freedom at the non-support degrees of freedom. And at the 3 supports we have got 3 degrees of freedom. Therefore, we can easily write down the stiffness matrix, and partition them so that K ss is the stiffness matrix corresponding to non-support degrees of freedom. U 1 and u 2 and K sg is the coupling matrix.

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$$r = -K_{ss}^{-1}K_{sg} = -\frac{1}{k}\begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ -2k & -2k & -2k \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- For the inclined leg portal frame

$$K_{rr} = \frac{EI}{3.6L}\begin{bmatrix} 38.4 & 12 & 0 \\ 12 & 48 & 12 \\ 0 & 12 & 38.4 \end{bmatrix} \qquad K_{ru} = \frac{EI}{L^2}\begin{bmatrix} -6 & -20 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ -6 & 12 & 6 & 6 \end{bmatrix}$$

$$\bar{K}_{uu} = \frac{EI}{L^3}\begin{bmatrix} 24 & 16 & -12 & -12 \\ 16 & 181 & 0 & -16 \\ -12 & 0 & 12 & 0 \\ -12 & -16 & 0 & 12 \end{bmatrix} - \frac{EI}{L^3}\begin{bmatrix} 8 & 5.53 & -8 & -4 \\ 5.53 & 52 & -5.33 & 6.33 \\ -8 & -5.33 & 8 & 4 \\ -4 & 6.33 & 4 & 3.69 \end{bmatrix}$$

$$\bar{K}_{ss} = \frac{EI}{L^3}\begin{bmatrix} 19.56 & 10.51 \\ 10.51 & 129 \end{bmatrix} \qquad \bar{K}_{ssg} = \frac{EI}{L^3}\begin{bmatrix} -4 & -8 \\ -5.33 & -22.3 \end{bmatrix}$$

$$r = -\bar{K}_{ss}^{-1}\bar{K}_{ssg} = \begin{bmatrix} 0.0661 & -0.0054 \\ -0.0054 & 0.0082 \end{bmatrix}\begin{bmatrix} -4 & -8 \\ 5.33 & -22.31 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2926 & 0.4074 \\ -0.654 & 0.1389 \end{bmatrix}$$

So, using the coupling matrix K sg and K ss, one can find out the r matrix, and the r matrix is computed like this, and we see that r matrix for this problem is equal to 1 third 1 1 1; that is for all the this particular problem the influence of all the 3 degrees of freedom are the same on the non-support degrees of freedom. For the inclined leg portal frame, that we have discussed before for which we have obtained mass matrix. That also was solved over here. And here again we obtained the values of the K rr K ss, and then we condensed out the rotational degrees of freedom, and after that we obtain the 2 plus 2 of 4 translational degrees of freedom with res with respect to that we obtain the stiffness matrix; that is the condensed stiffness matrix then this was partitioned to the non-support degrees of freedom and support degrees of freedom. And this is the stiffness matrix corresponding to the non-support degrees of freedom, and this is the coupling matrix between the support degrees of freedom and non-support degrees of freedom. And then we obtain the r matrix in the same fashion as before.

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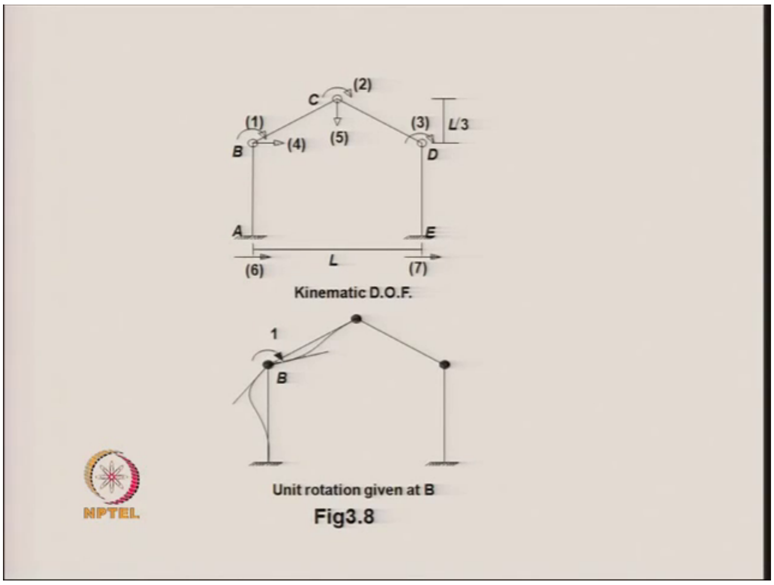

So, one can obtain the different for different degrees of freedom the r matrix and the corresponding m matrix.

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