Seismic Analysis of Structures
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Lecture – 11
Response Analysis for Specified Ground Motion (Contd.)

In the previous lecture, we were discussing about the multi degree of freedom system with multi support excitations.

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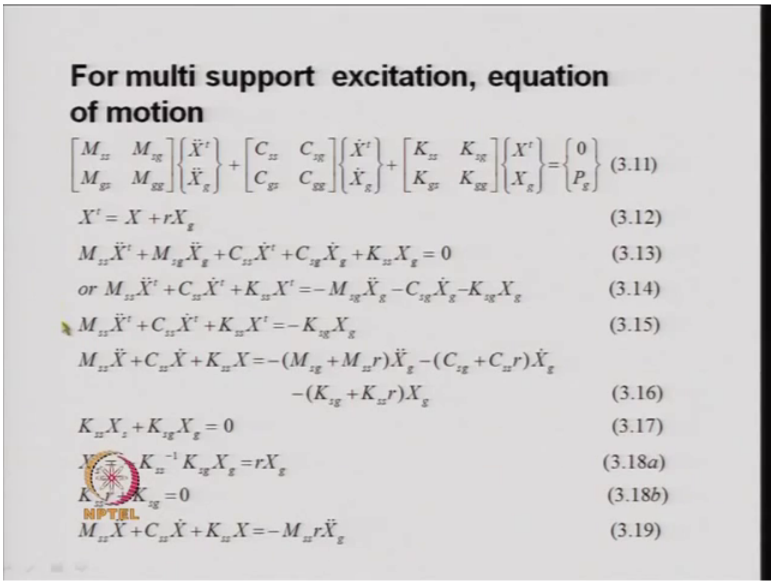

We were looking into the equation of motion the equation of motion can be written in 2 different form; one is in terms of the total displacement that is this equation in which all the quantities; on the left hand side, they are in terms of the total displacement total acceleration and total velocity and on the right hand side. We have minus K s g into X g ma K s g is the coupling matrix between the support degrees of freedom and ground degrees of freedom that is the non that is the K g is the non support coupling between the non support degrees of freedom and a support degrees of freedom that is the degrees of freedom at the ground and X g is the ground displacement vector.

The form was in terms of the relative displacement that is this equation in which all the quantities over here are relative quantity that is relative displacement relative velocity and relative acceleration and in this equation we had the r a matrix called the coefficient matrix and X double dot g happens to be the ground acceleration vector. So, the difference between these 2 equations are that one is written in terms of total displacement other is written in terms of the relative displacement on the right hand side for this equation, we require ground displacement to be support to be specified at this support whereas, here at this support we must know the ground acceleration and then we can solve the problem.

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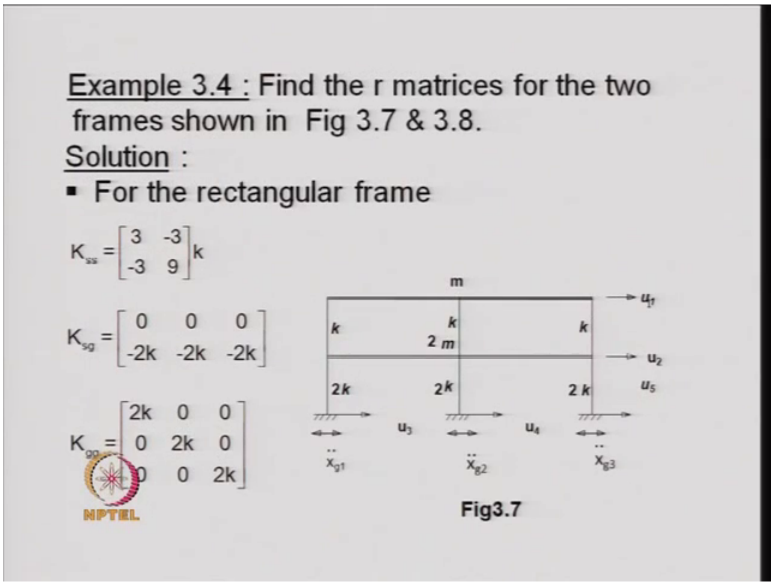

Next we wanted to explain how the r matrix is generated and in the previous slide that was given the r matrix is generated with the help of this equation that is minus K s s inverse into K s g multiplied by X g or the r matrix is equal to minus K s s inverse into K s g; K s s inverse is the partitioned stiffness matrix corresponding to the non support degrees of freedom and K s g is the coupling matrix in the non support degrees of freedom and the ground degrees of freedom. So, once you are able to get these 2 matrices, then one can construct the r matrix. So, we had shown an example for this in the previous lecture, this is a frame in which we had 3 ground excitations at 3 different supports and these were the 2 degrees of freedom which are the non support degrees of freedom. So, the partition matrix for that or K s s matrix was this; the K s g was this and then with the help of the equation and for r.

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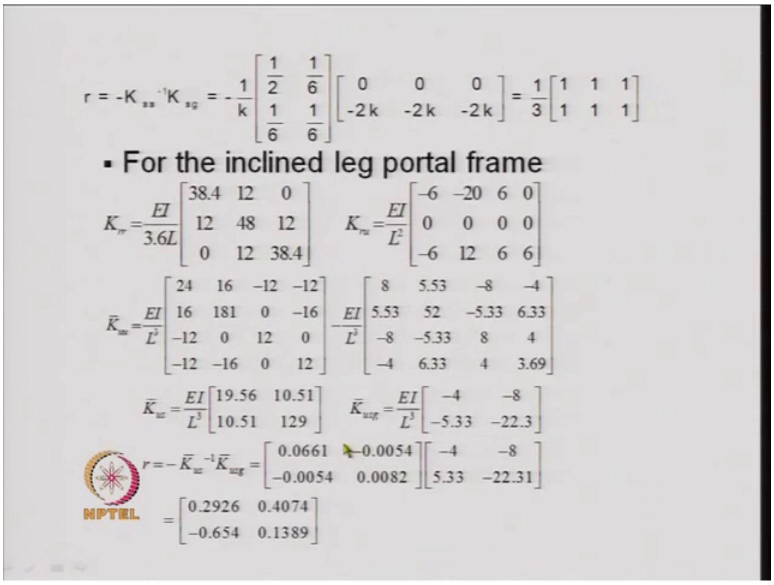

We calculated this quantity that is K s s inverse K s g and that turn out to be one third into 1 1 1 and 1 1 1. So, the 3 ground displacements share equal equally or share equally in producing the non support responses at the non support degrees of freedom.

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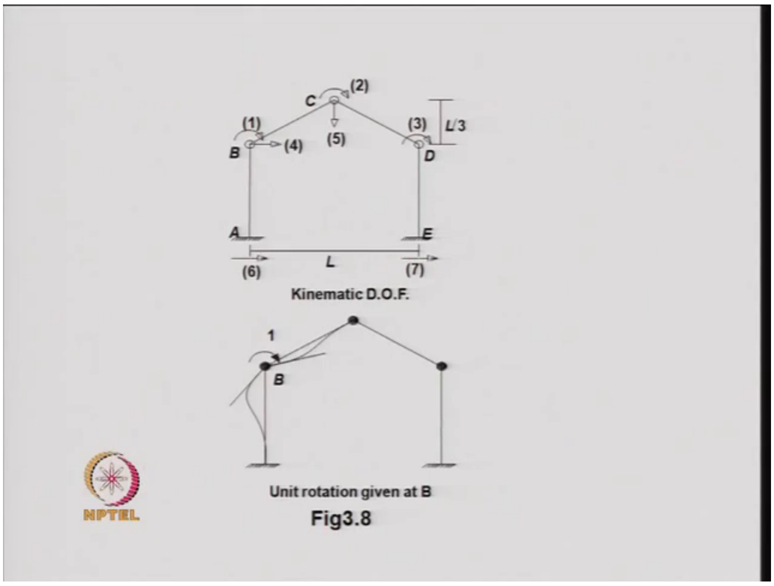

Next we take another example to illustrate the same r matrix; that means, how we can construct the r matrix for a pitched roof portal frame in this pitched roof portal frame, we have the 1, 2, 3; these 3 degrees of freedom are the rotational degrees of freedom and the 4 and 5; they are the translational degrees of freedom in addition to that we have 2 support degrees of freedom that is 6 and 7. So, first what we do we write down the stiffness matrix for a entire thing that is the; for all the 7 degrees of freedom, we write down the stiffness matrix the way, we write down the stiffness matrix for a static analysis and then we partition them.

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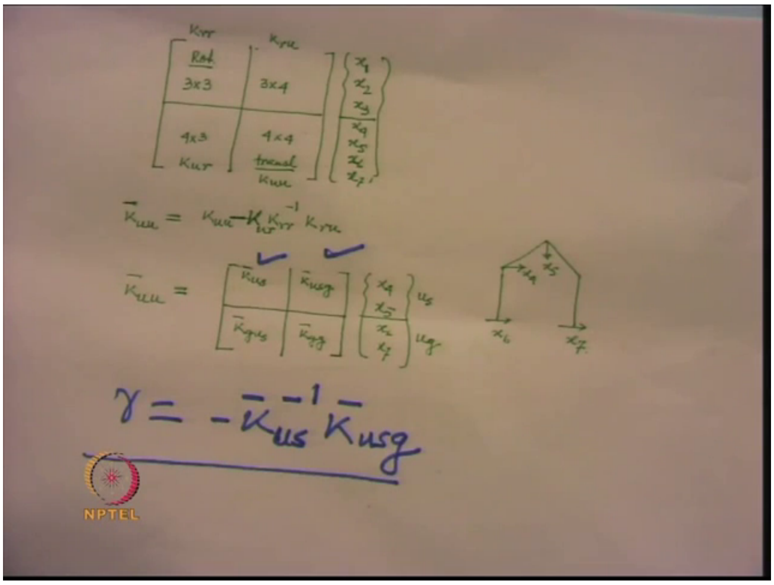

For example x 1, x 2, x 3, they are the rotations and these rotations are taken on the top. So, this is the 3 by 3 matrix corresponding to the rotational degrees of freedom and this is the matrix 3 by 4 that we are calling as ka K r u, this is the coupling between the rotational degree of freedom and the translational degree of freedom and this is these 4 by 4 is the translational degrees of freedom and the 4 by 4 square matrix correspond to that.

So, from this matrix we can obtain a condensed stiffness matrix corresponding to these 4 degrees of freedom that we are calling as K bar u u and this K bar u u will be equal to k u u that is this matrix minus K u g u r; K r r inverse multiplied by K r u. So, that is a standard condensation procedure that I think all of you know and once you do that we get the condensed stiffness matrix corresponding to the translational degrees of freedom. So, these matrix now is a the K bar u u matrix is now of size 4 by 4 and in that the degrees of freedom involved are x 4, x 5, x 6 and x 7; again we partition them over here. So, that these are the; this is the matrix corresponding to the non support degrees of freedom x 4 and x 5 and this is the matrix corresponding to the support degrees of freedom and they are the coupling matrices.

Now, with this we get the value of r using again the previous formulation that is r is equal to the minus K s s or r is equal to minus k u s inverse into K bar u s g and these 2 matrices are given over here. So, with the help of that we get the value of r. So, this quantities are shown here in this slide the this portion that is your K r r matrix this is K r u matrix and then this was the K bar u u matrix and from there we isolated K u s matrix and K u s g matrix and after that we obtain the value of r by simply multiplying K s K u s inverse with K u s g and this is the 2 by 2 r matrix that is generated.

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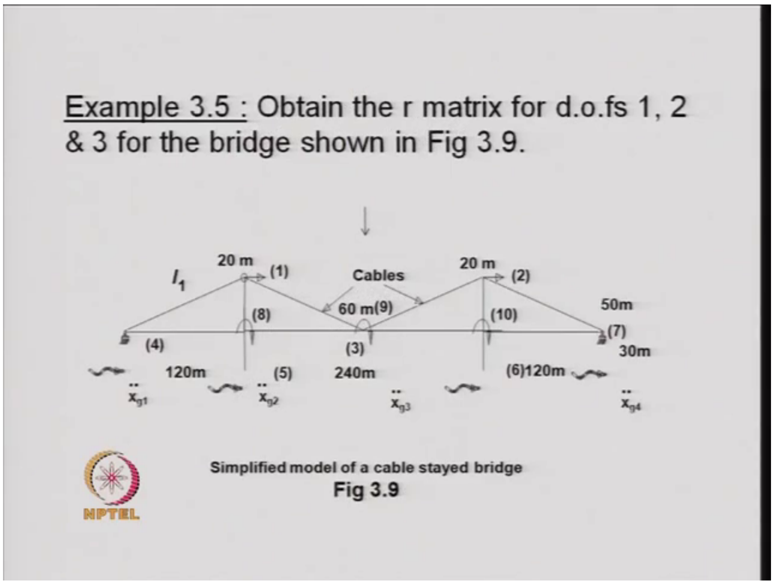

Next, we take another problem it is a model of a cable stayed bridge in that we have degrees of freedom as this at the top; we have a degree of freedom, here we have a degree of freedom to and at the center of the deck we consider another degree of freedom which is in the vertical direction. So, will try to find out the response of the structure for these 3 degrees of freedom and the entire equation of motion is written for these non support degrees of freedom. So, in addition to this degrees of freedom, there are rotations at this points and we have translations at this support point this support point and this support point and this support point. So, we have in all 4 plus 3; 7 translational degrees of freedom out of them 4 degrees of freedom are at the supports 3 degrees of freedom are the non support degrees of freedom that is the translational degrees of freedom as one 2 and 3 and the rotations are 8, 9, 10, they are the rotations. So, what we do is the first, we write down the entire stiffness matrix that is the stiffness matrix you know for the entire system.

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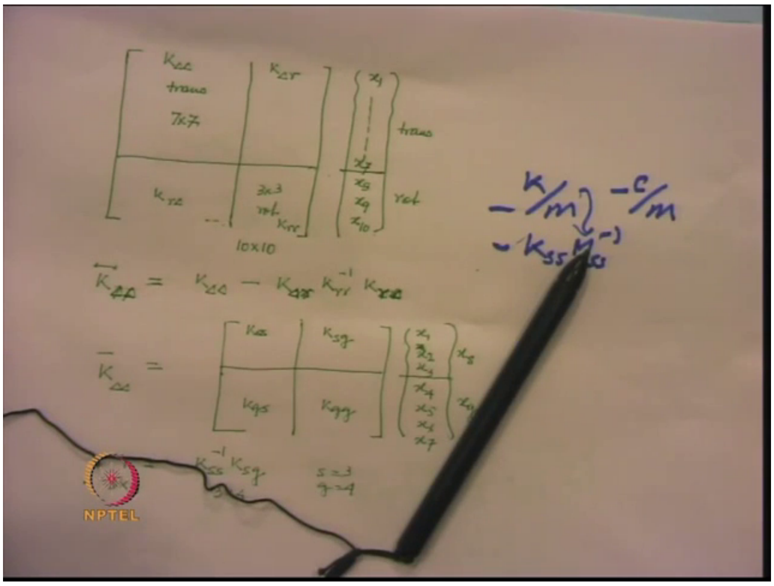

Which is a 10 by 10 matrix out of that the first 7 or 7 by 7 stiffness matrix corresponding to the 7 translational degrees of freedom they are grouped at the top and the 3 rotational degrees of freedom are grouped at the lower part of this vector, then we do this matrix con condensation using this relationship and condense the entire stiffness matrix to the translational degrees of freedom x 1 to x 7.

Once we get the condense stiffness matrix then this condense stiffness matrix is further partitioned into the non support degrees of freedom that is the 3 degrees of freedom which were acting at the 2 at the top of the pylons. And one at the center of the deck these are the 3 translation and degrees of freedom and other 4 degrees of freedom are at the 4 supports of the cable support; once we partition them then using this matrix and this matrix we can obtain the value of the r matrix.

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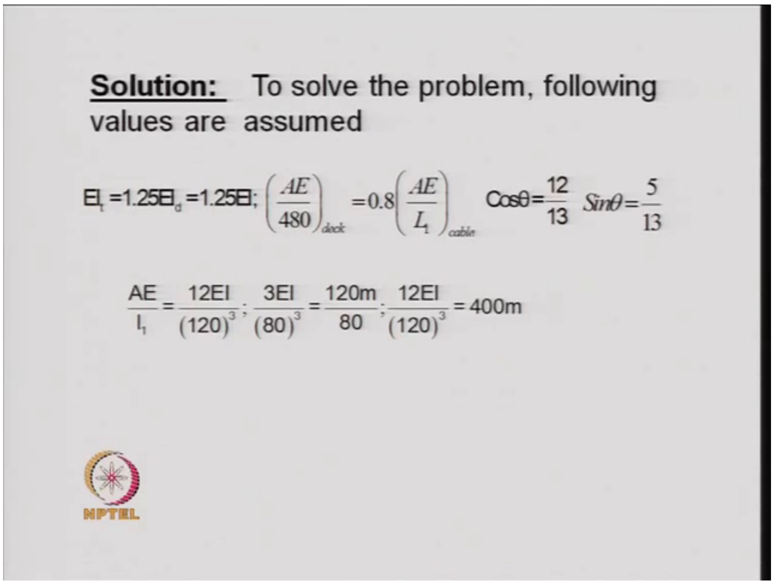

So, in these way; we constructed the r matrix for this problem and the details of this is given here.

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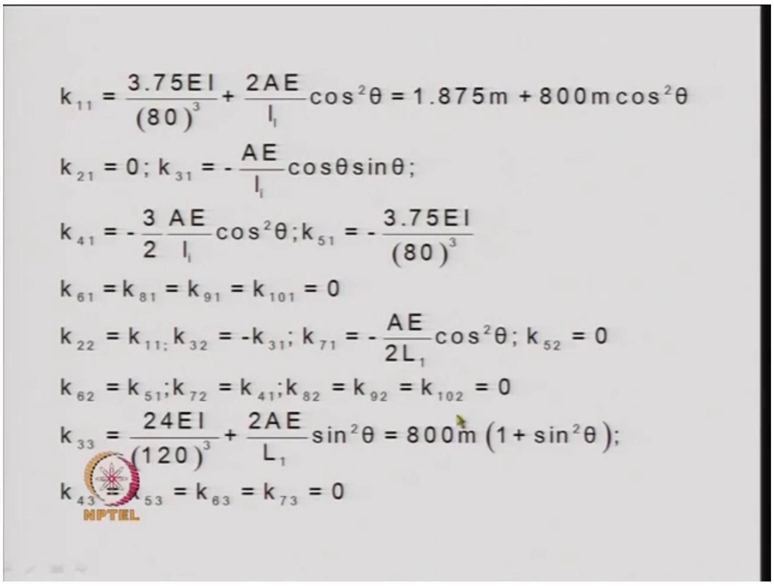

That means element by element we generated the different quantities and then assembled them.

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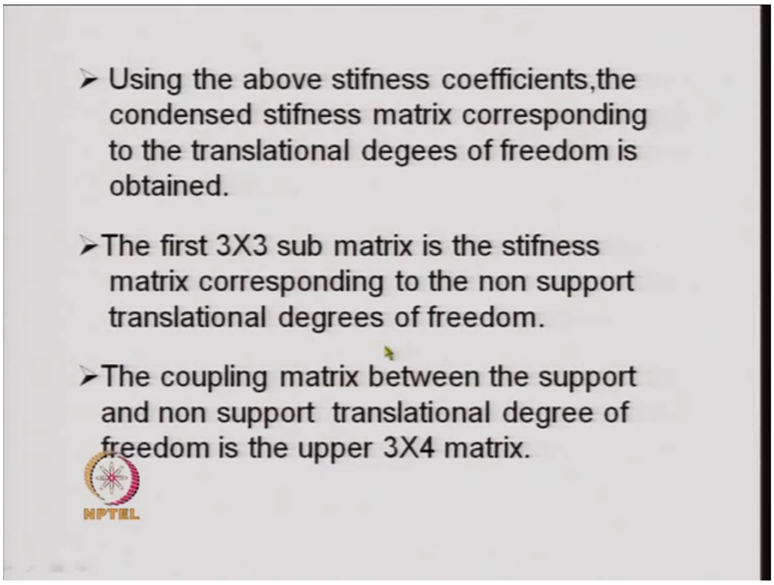

To this 10 by 10 stiffness matrix and after that we condensed them to the translational degrees of freedom and from that we had taken out the 3 by 3 sub matrix corresponding to non support degrees of freedom.

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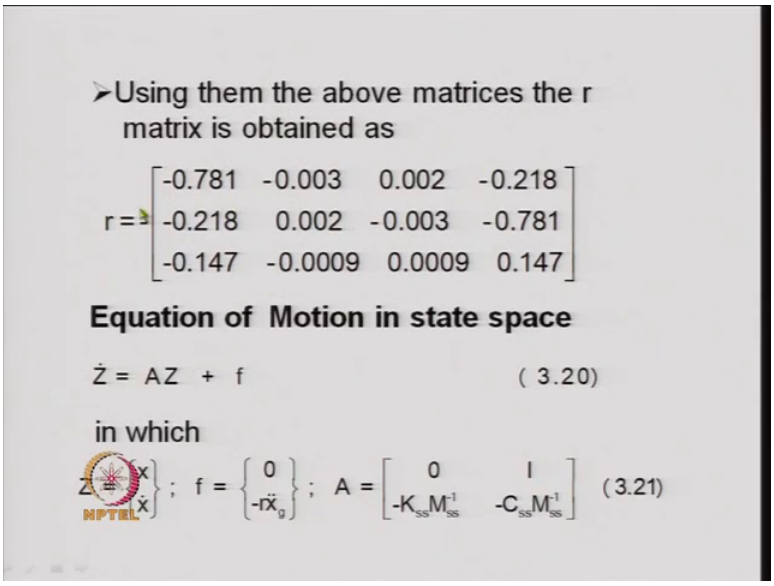

And obtain the r matrix and the resulting r matrix was this. So, here again for the 4 different ground motions we have this 3 by 4 matrix and with the help of this 3 by 4 matrix, we solve the problem that is the problem in which on the right hand side, we have got minus M r x double dot g while x double dot g are the 4 ground acceleration defined at the 4 support points.

Next we see how we can convert these equation of motion that we are written for the multi degree freedom system with the r coefficient or the r coefficient matrix on the right side, in one case in which we are writing down the equation of motion in terms of the relative displacement and in other case we had the equation of motion in terms of the total displacement and on the right side we had the K s g matrix and we require instead of ground acceleration ground displacements. So, those 2 second order differential equation or multi degree of freedom differential equation they can be written in the state space form as before only difference here will be that in the previous case, we had for single degree freedom system we had got minus K by M and minus C by M, these were the terms which were written now in place of that now we have got minus K s s into M s s inverse. So, this term becomes this.

Similarly, the C by M term becomes the; you know C s s into M s s inverse. So, and this one becomes one instead of one it becomes.

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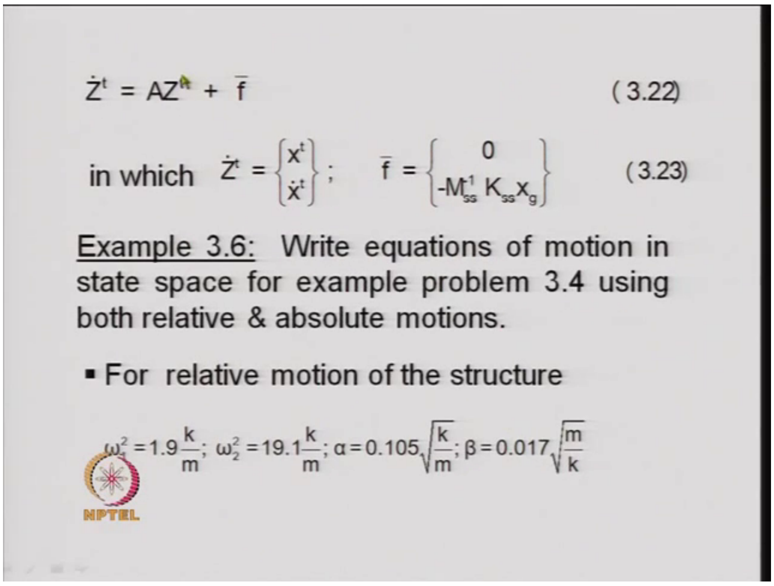

If we wish to write down the second order differential equation in terms of the total displacement as a set of first order differential equation or in terms of the state space formulation then the a matrix remain the same only thing that changes is the f bar 1 and here we see that f bar becomes equal to minus M s s inverse K s g this will be now K s s this will be K s g into X g. So, this will be the only change therefore, the for multi degree freedom system with multi support excitation we can write down the equation in 4 different forms 2 equations can be written as a second order differential equation one in terms of total displacement and other in terms of the relative displacements and correspondingly we can have 2 state space formulation for this 2 equations.

Now, we try to write down the state space form of the equation motion for the problem 3 point 4 that is this problem yes for this problem we wish to write down the entire equation of motion in that state space form. So, here we have seen that if we were wanting to write down the equation of motion in terms of your state space form we have to generate a matrix called a matrix and that a matrix contains C s s or C by M. So, C matrix must be known. So, that is the first thing that we do for the entire structure now we obtain the C matrix. So, for obtaining the C matrix these approach is already known to all of you that one can obtain the C matrix of a particular structure provided we know the first few frequencies of the system. So, here it was a 2 degree freedom problem in the sense that there are 2 non support degrees of freedom.

So, we had 2 natural frequencies. So, once we know; these 2 natural frequencies then with the help of that one can compute the values of alpha and beta.

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$$C_{ss} = \alpha \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} m + \beta \begin{bmatrix} 3 & -3 \\ -3 & 9 \end{bmatrix} k = \begin{bmatrix} 0.156 & -0.051 \\ -0.051 & 0.205 \end{bmatrix} \sqrt{km}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3\rho^2 & 1.5\rho^2 & -0.156\rho & 0.026\rho \\ 3\rho^2 & -4.5\rho^2 & 0.051\rho & -0.182\rho \end{bmatrix}$$

$$\rho = \sqrt{\frac{k}{m}}; \quad f = \begin{Bmatrix} 0 \\ 0 \\ -0.33\left(\ddot{x}_{g_1} + \ddot{x}_{g_2} + \ddot{x}_{g_3}\right) \\ -0.33\left(\ddot{x}_{g_1} + \ddot{x}_{g_2} + \ddot{x}_{g_3}\right) \end{Bmatrix}$$

Which are required to obtain the C matrix considering the C matrix to be mass and stiffness proportional that is we write down C s s to be is equal to alpha times mass matrix plus beta times the K matrix and use using this we can obtain this damping matrix for the entire system and once we have these damping matrix written for the system, then we can use the same equation that is this formulation that is K s s we know; we know M s s inverse and also we know now the C s s matrix and M s s inverse is also known.

So, therefore, using this we can write down the entire a matrix and this a matrix turns out to be like this here it is it will not be 1 8 2 it will be 1 0 2 you can make this correction where rho is equal to root of k by m and on the right hand side the force vector if you look at the expression for the force vector this force vector is equal to minus r into double dot g. So, r for this problem we obtained as the; this was the r that is one third 1 1 1 1. So, using this value of r one can obtain the value of the f that has minus 0 point 3 3 into x double dot g 1 plus x double dot g 2 plus x double dot g 3 that is what we observed before that all the 3 ground acceleration equally influenced the non support degrees of freedom. So, that is how the terms of the load vector over here that is generated.

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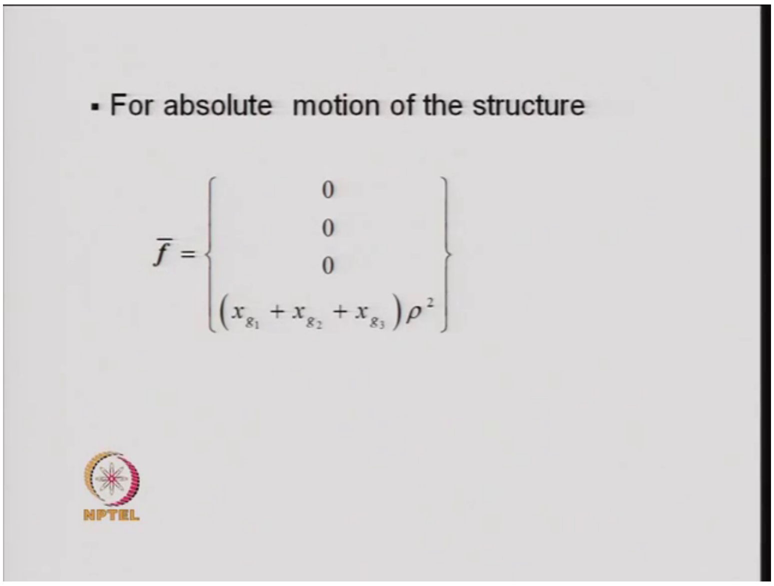

Now, if we wish to write down the equation of motion in terms of the total displacement and again in the state space form, then matrix remain un remains unchanged as I said before. So, a matrix remains this and only thing that changes is the right hand side load vector and the right hand side load vector turns out to be X g 1 plus X g 2 plus X g 3 into rho square and this is nothing, but minus K s g into the M s s inverse. So, K s g into M s s inverse turns out to be this one and here we can see that degrees of freedom that we require are not the degrees of freedom the quantities that are this is required for defining the force is the 3 displacement at the 3 supports.

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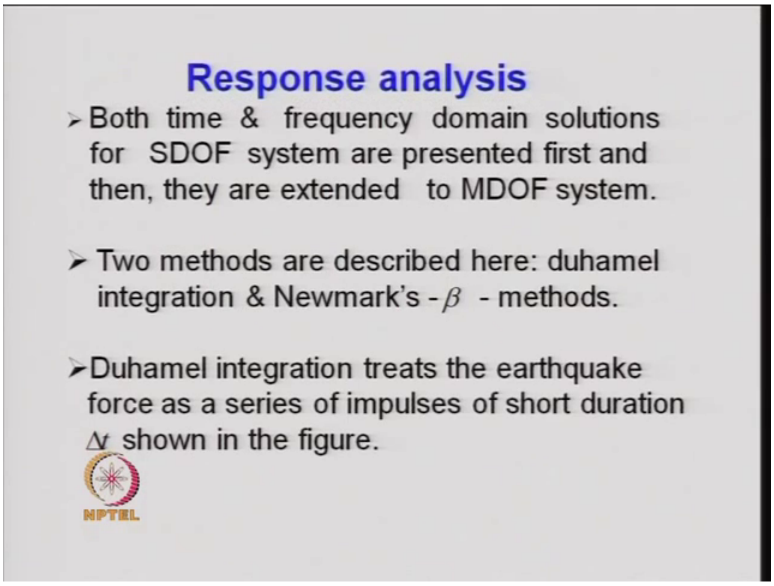

Now, once we are able to write down the equation of motion in different forms that I described that is in the state space form and as ordinary differential equation in terms of relative displacement and in terms of the total displacements then come to how we have solve these equations. So, that is what we are calling as the response analysis now the response analysis can be carried both in time and frequency domain and first we will take up the single degree of freedom system and that will be adopting for solving the single degree of freedom system that will be extended to multi degree of freedom system later.

Now in time domain analysis there are many methods which are available out of that we will take up only 2 methods that is a Duhamel integration and a Newmarks beta method I think in your dynamics course all of you have already done these 2 time estimate methods which are very popular in earthquake engineering therefore, your acquainted with the different important things associated with this 2 integration strategies. however, we will recast these 2 formulation that is the Duhamel integration formulation and a Newmarks beta method formulation in the form of a recursive equations rather than the usual way that you have solved in your dynamics course.

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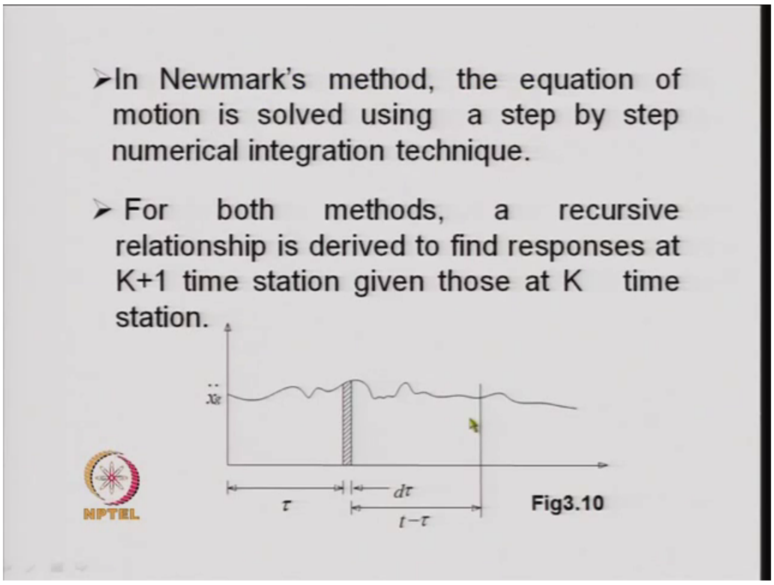

Let us take the Duhamel integral first as you know that a Duhamel integral we consider the load to be a series of impulses that is the load to be consisting of a series of impulses like this and these series of impulses for that if we consider any at any time t, we see what is the response that we obtain and then sum up the effect for or some of the responses for all the impulses to get the final response at this particular point. So, what we do that first we consider a impulse at a time tau from the from the origin that is 0 time and a an impulse of f multiplied by D tau is applied to the system or the single degree of system at time tau then the elapse time is t minus tau that is when the impulse is produced over here then we see its effect at this time that is after a time of t minus tau that is what we call as the elapse time elapse time of t minus tau.

Now, as we all of we know that the problem of producing an impulse to a single degree of system is equivalent to producing a velocity to the system or imparting a velocity to the system. So, we can conceive the problem as a damped free oscillation with the initial condition of displacement and the initial condition of velocity. So, in the free vibration equation if we provide these initial conditions that is the displacement at from here we start counting a time. So, at 0 time if we consider what is the displacement and what is the velocity then we can substitute this displacement and velocity into the equation that provides you the response at any instant of time t for a damped free oscillating single degree of freedom system and that is given over here.

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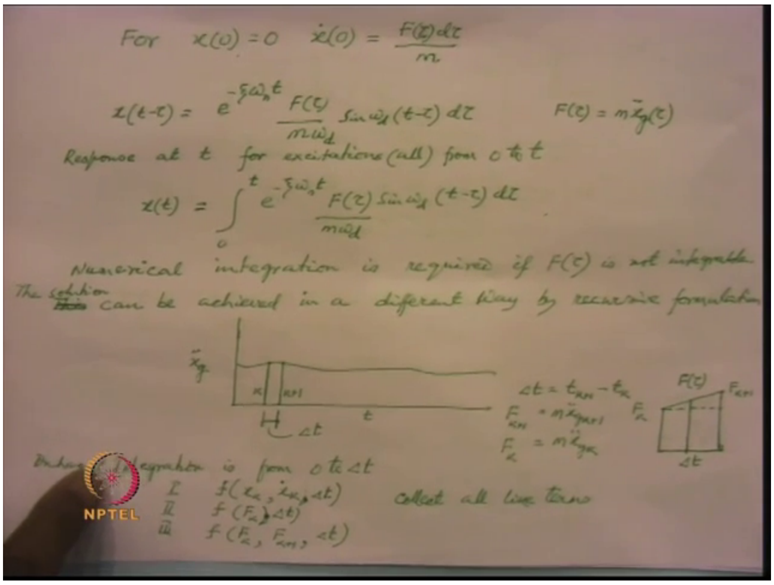

This is a standard this is a standard equation that is the x t minus tau x t minus tau is the response at time t and this is given as e to the power minus tau omega n t and this is the velocity part that if m into x double dot g tau this is the force impulse force that is provided at 0 time.

So, that is F tau that divided by m that becomes the initial velocity and omega d is the damped frequency. So, and sin omega d d t minus tau D tau comes because F tau D tau that is the total impulse. So, that is how D tau is coming into picture over here and these equation is valid for 0 displacement at the initial stage and the velocity is equal to F tau D tau divided by m. So, for this velocity we get this is a standard equation for damped free oscillation now these x t minus tau as we obtain then one can obtain the response x t for all the impulses that acts from this point to a time t. So, integrate from 0 to t these entire expression over here and this is known as the Duhamel integration and using this Duhamel integration one can find out the value of response at any instant of time t if this F tau is an integrable function, then there is no problem, one can obtain the response of this x t in a closed form analytically; however, if F tau is not a integrable function, then one has to perform a numerical integration for this to get the value of x t.

So, instead of doing that numerical integration one can recast the entire formulation in slightly different fashion; let us consider these as the ground acceleration or the support acceleration for a single degree of freedom system and for that we have a interval of time delta t. So, that we can define a time step k and next time step k plus 1 difference between them is delta t at the these 2 steps the values of F k plus 1 and F k; they are known that is the ground acceleration at these 2 points are known. So, we can multiply them by m to get the forces at these 2 points.

Now, if we look at the response at k plus 1 then the response at k plus 1 can be obtained provided we know the response at k that is the k th time station and in this kind of formulation in which we obtain the response at a particular time step with the help of the responses at the previous time steps this formulation is called the time marching formulation. So, in a time marching formulation what we do is start with the 0 time and at 0 time the displacement velocity and acceleration they are known and then we obtain the response at the next time step that is at after interval of delta t and then we proceed in this particular fashion.

So, this is a time marching scheme, but this time marching scheme to be used in the case of a Duhamel integration requires some kind of consideration that is what is shown over here in this slide that.

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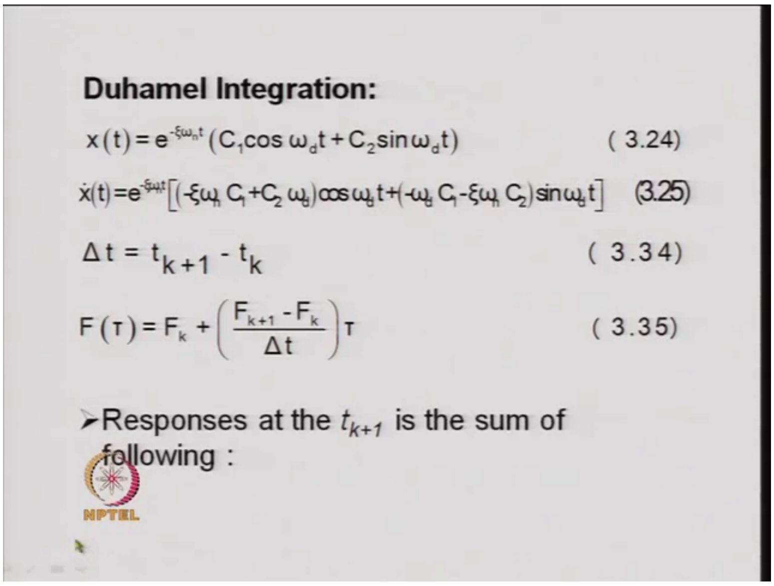

What we do is that at k that is at time t k we know the forcing function F k we also know the forcing function at k plus 1 time then what we consider that the as if the force between F k and F k plus 1 is varying linearly; that means, it is varying in this particular fashion from this point to this point and once we assume that there that is linearly varying, then at any time tau we can obtain the value of this F tau in terms of a k plus 1 and F k. So, this is what we do here in this formulation and we write down the F tau in this particular way and that is it is a sum of a constant force F k and then it is a linear term which is or the triangular part of the equation. So, using this equation one can define F tau at any time tau taken or counted from t k.

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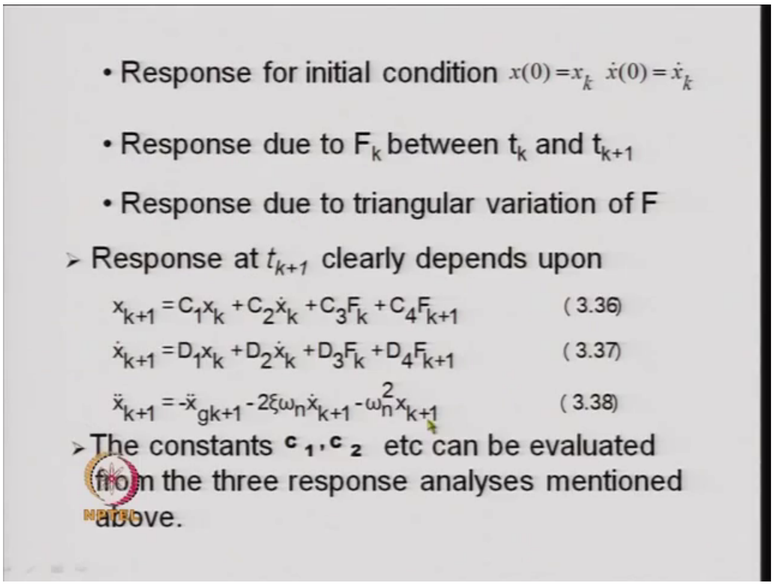

Now, if we look at the responses at time t k plus 1, then we see that the response at time t k plus 1 is consists is consisting of 3 responses that is the initial condition that exists at k or the time step k for that with that initial condition the single degrees of freedom system vibrates as a damp free oscillator and because this vibration there will be some response which will be produced at t k plus 1 that is the next time step. So, this is the first part of the response second part of the response is due to F k that is constantly acting over the duration of time delta t and the third portion is the triangular variation of the load between k and k plus 1.

So, the responses for these 3 would provide the final response at t k plus one. So, t k plus 1 clearly now depend upon these quantities that is x k x dot k. So, they are the initial velocity and displacement at k th time step and F k and F k plus one. So, what we can write down is that x k plus 1 we can write to be is equal to some constant multiplied by C k x k some constant multiplied by a x dot k some constant multiplied by F k and some constant multiplied by F k plus 1.

Similarly, one can write down x double dot k plus 1 in terms of this equation where D 1, D 2, D 3, D 4 are the constants that is to be obtained and then one can obtain the value of x double dot k 1 that is the acceleration at k plus 1 time and that is from the parent single degree of freedom equation that is if we know the displacement and velocity of the system one can find out the acceleration. So, using these 3 equation one can get at any time step k plus 1 all the 3 quantities that is the displacement velocity and acceleration in terms of the displacement velocity at the previous time step that is k and the forces which are acting at k and k plus 1.

They are eventually known therefore, the formulation is centered around finding out this constant C 1, C 2, C 3, C 4 and D 1, D 2, D 3, D 4. Now in this we can see that the first part of the solution that is for the initial condition this will be a function of x k x dot k and delta t second part of the solution that is for a constant F k that is a rectangular F forcing function what is the response at this point. So, this will be a function of F k and delta t and for the triangular part or what third part of the solution requires knowledge of F k F k plus 1 and delta t.

Now, this comes out the first solution comes out form the damped free oscillation in that simply we substitute the initial condition as x k and x dot k and then find out what will be the response at k plus 1 time steps this one is a problem of a Duhamel integral for a rectangular type of pulse and for that the analytical solution is available and all of you have done that similarly for this one the response can be obtained for a triangular kind of pulse and this also can be obtained using the Duhamel integration because it can be obtained analytically. So, for all the 3 the analytical solution is available then once we get all the 3 solutions then these 3 solutions are summed up together and the like terms that is if the terms containing x k terms containing x dot k and terms containing F k and F k plus they are all collected together and the multipliers that will be associated with these variable they are they would form the values of C 1, C 2, C 3, C 4, etcetera.

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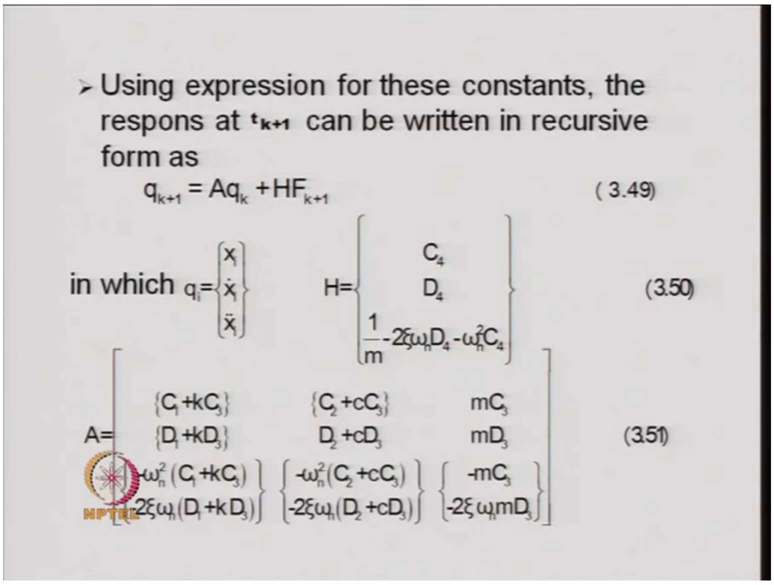

And that is what is done over here and once you are able to find out these quantities that is C 1, D 1, C 2, D 2, C 3 t D C, etcetera, you can write down the equation in this recursive form where u k plus 1 is equal a matrix a into q k plus h into F k plus 1 where q basically represents for all the 3 quantities in a vector form and a is a matrix consisting of these constants and the h is again a matrix consisting of some of the elements some of some of the constants of those equations.

So, the C 1, C 2, C 3, etcetera can be computed the way I told you and they are for your information given over here.

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$$C_1 = e^{-\xi\omega_n\Delta t}\left[\cos\omega_d\Delta t + \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right] \quad (3.39a)$$

$$C_2 = e^{-\xi\omega_n\Delta t}\left[\left(\frac{1}{\omega_d}\right)\sin\omega_d\Delta t\right] \quad (3.39b)$$

$$C_3 = \frac{1}{k}\left\{\left(\frac{2\xi}{\omega_n\Delta t}\right) + e^{-\xi\omega_n\Delta t}\left[-\left(1+\frac{2\xi}{\omega_n\Delta t}\right)\cos\omega_d\Delta t + \left(\frac{1-2\xi^2}{\omega_d\Delta t} - \frac{\xi}{\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right]\right\} \quad (3.40)$$

$$C_4 = \frac{1}{k}\left\{1-\left(\frac{2\xi}{\omega_n\Delta t}\right) + e^{-\xi\omega_n\Delta t}\left[\left(\frac{2\xi}{\omega_n\Delta t}\right)\cos\omega_d\Delta t + \left(\frac{2\xi^2-1}{\omega_d\Delta t}\right)\sin\omega_d\Delta t\right]\right\} \quad (3.41)$$

In these equations and you can see that these C 1, C 2, C 3, etcetera depend a delta t that is a time step and all other quantities are known that is omega d del omega d psi omega and etcetera.

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$$D_1 = e^{-\xi\omega_n\Delta t}\left[-\left(\frac{\omega_n}{\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right] \quad (3.42)$$

$$D_2 = e^{-\xi\omega_n\Delta t}\left[\cos\omega_d\Delta t - \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right] \quad (3.43)$$

$$D_3 = \frac{1}{k}\left\{-\left(\frac{1}{\Delta t}\right) + e^{-\xi\omega_n\Delta t}\left[\left(\frac{1}{\Delta t}\right)\cos\omega_d\Delta t + \left(\frac{\omega_n}{\sqrt{1-\xi^2}} + \frac{\xi}{\Delta t\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right]\right\} \quad (3.44)$$

$$D_4 = \frac{1}{k\Delta t}\left\{1 - e^{-\xi\omega_n\Delta t}\left[\cos\omega_d\Delta t + \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\sin\omega_d\Delta t\right]\right\} \quad (3.45)$$

All of them unknown therefore, one can compute easily the values of C 1, C 2, C 3, C 4, D 1, D 2, D 3, D 4, etcetera.

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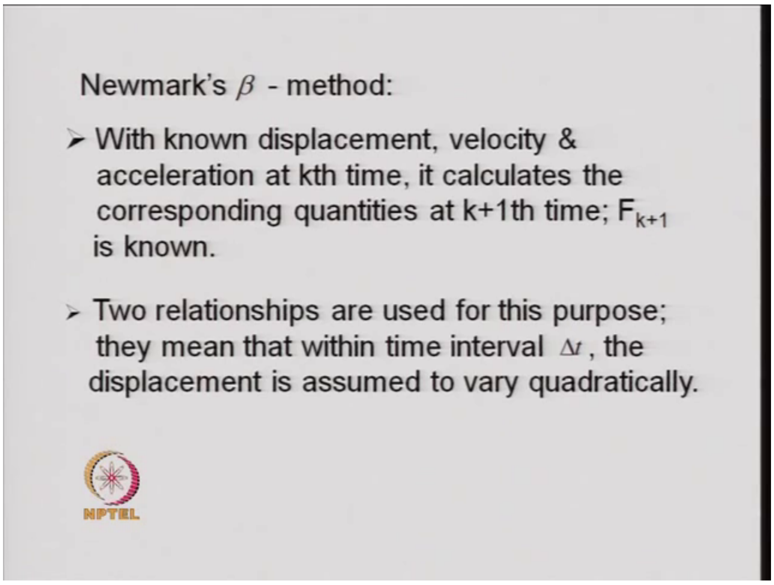

So, that is the recursive form of Duhamel integration and it requires the solution of the problem for certain known cases that is a known cases are that damp free oscillation that is a known case the solution for a rectangular pulse that also can be obtained using Duhamel integral analytically and the response for a triangular pulse that also can be obtained using an analytical solution. So, therefore, with these with these 3 known solutions one can obtain the constant C 1, C 2, C 3, C 4 of the a matrix and one can write down the entire Duhamel integration in this recursive format the advantage of this solution is that if f tau or the force if it is not a integrable one then in any way one has to go for a numerical integration. So, instead of doing that numerical integration one is using the known solution for a triangular pulse rectangular plus pulse and the damp free oscillation. So, using these 3 solutions you are able to tackle the problem.

Next is the Newmark's beta method in the Newmark's beta method we again solve the problem numerically using a time marching scheme.

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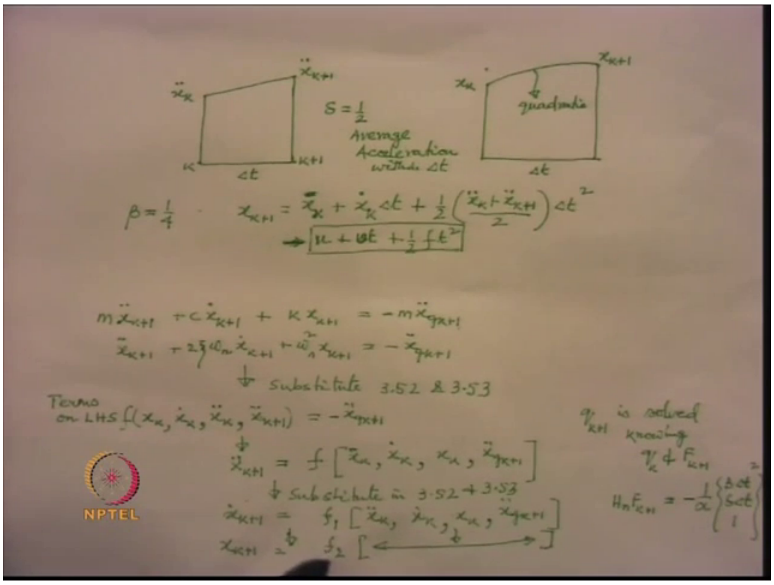

That is a; you try to find out the response at k plus 1 at time steps and with the help of the response at time k which will be taken as known. So, we start with 0 time t where all the quantities displacement velocity and accelerations are specified and we solve for the next time steps using those values and in those particular fashions, we march ahead. Now the 2 e equations that are used in the Newmark's beta method are the once that is shown.

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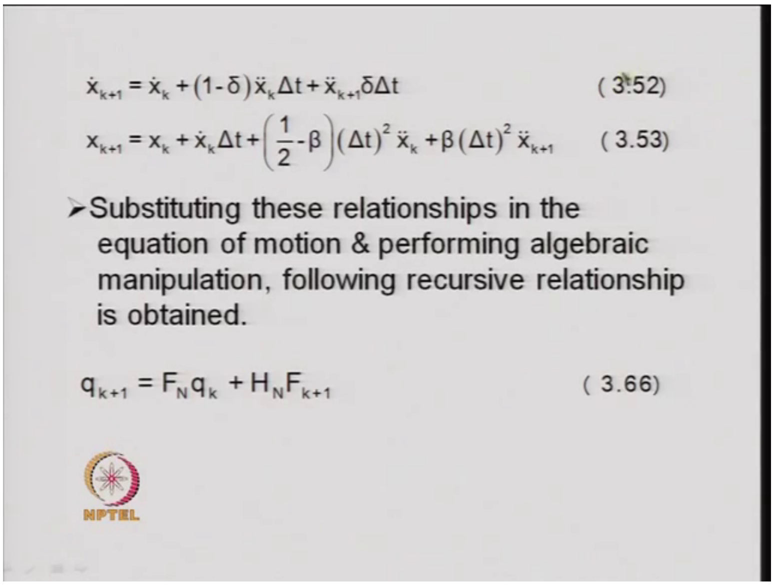

In this equation 3.52 and 3.53 if we look at these 2 equations are already known to you for example, if I ask you to find out the velocity at a particular time given a velocity at some other time t.

Then you can find out that velocity is equal to the previous velocity plus the acceleration multiplied by time t that gives you the velocity x dot k plus 1 only thing; what has been done over here is that the instead of a constant acceleration that we assumed over a time interval here the velocity that is varying or sorry acceleration that is varying from k to k plus 1. So, that varying velocity is considered with the help of this particular formulation that is here we assume that as if the acceleration is varying linearly. Now if we assume that the acceleration is varying linearly, then it is equivalent to assuming that the displacement is varying quadratically because if you integrate twice the acceleration then you get the displacement. So, if there is a linear variation of the acceleration between the 2 points, then if the acceleration is integrated this linear variation will become a; sorry cubic variation sorry; not quadratic cubic variation.

Now, with this assumption that is the acceleration is varying linearly then velocity is varying quadratically and displacement is varying in the cubic form with these assumption we write down this particular 2 equation that is given a value of x dot k plus for a given for a given x dot k, we can find out x dot x dot k plus 1 using this particular linear variation of acceleration then we can write down from that the displacement at k plus 1 at time step given the displacement that time step k and velocity at time step k.

So, this is again a very familiar equation with all of you say if we wish to find out the displacement at a time at some time then at the previous time whatever is the velocity that is say you plus v into t plus half F t square this what we have all done in your physics class. So, it; in fact, is a modification of that equation here this is the first 2 terms remain the same here what we have done we have manipulated the acceleration that is we are not assuming at constant acceleration and acting between the 2 points, but a acceleration which is varying linearly and therefore, we take into account the acceleration at both the points and assumed to vary very linearly between the 2 points.

If we provide the value of delta as half then this particular term becomes an average acceleration between the 2 point that is you can see that it turns out to be x double dot k plus x double dot x k plus 1 divided by 2 similarly if we consider beta to be is equal to one fourth, then these turns out to be half of the average acceleration into delta t square. So, we see that for a value of delta is equal to half and beta is equal to one fourth these 2 formulations or these 2 equations provide us the displacement and velocity at k plus 1 s station using assumption that the acceleration is varying linearly and we assume that the va the acceleration to be an average acceleration over the time interval. Now with these assumption, we can go ahead in this particular sequence that is we write down the equation of motion in the usual form that is m x double dot k plus 1 C x dot k plus 1 into k x x k plus 1 is equal to minus m x double dot g k plus one. So, this is the equation of motion at k th time step, we can rewrite it in this fashion by dividing the entire equation by m.

So, this is the equation which is written in terms of the frequency and damping then what we do is substitute the 2 equations 3.52 and 3.53 in place of x k plus 1 and x dot k plus one. So, x k plus 1 and x dot k plus 1 they are now written using equation 3.52 and 3.53 that we have described before and once we substitute them into the equation then we can see that the equation that will be there on the left hand side would be in terms of x k x dot k x double k and the acceleration x double dot k plus 1.

So, the x double dot k plus 1, these acceleration is not known to you or known to us therefore, from these equation one can find out x double dot k plus 1 and this will be a function of the known quantities that is the acceleration displacement and velocity at the previous time step and the acceleration ground acceleration at k plus 1 at time station and once we get the value of x double dot k plus 1 then these can be substituted into the equation 3.52 and 3.53 that I have shown that is a 2 cardinal equation; in that equation if you substitute for x double dot k plus 1, then you get the value of x dot k plus 1 and x k plus 1. So, all the 3 quantities that is x k plus 1 x dot k plus 1 and an x double dot k plus 1 can be now written in terms of the known quantities of x k x dot k and x double dot k and of course, the load. So, this can be written in a compact form like this that is q k plus 1 is equal to a matrix F n into q k multi plus H n into F k plus 1.

So, F k plus 1 is load that is acting at the k plus oneth station and q k is completely known that is the displacement velocity and acceleration at kth time station and with the help of that we can obtain the value of the response at k plus 1 times station again this is in the recursive form.

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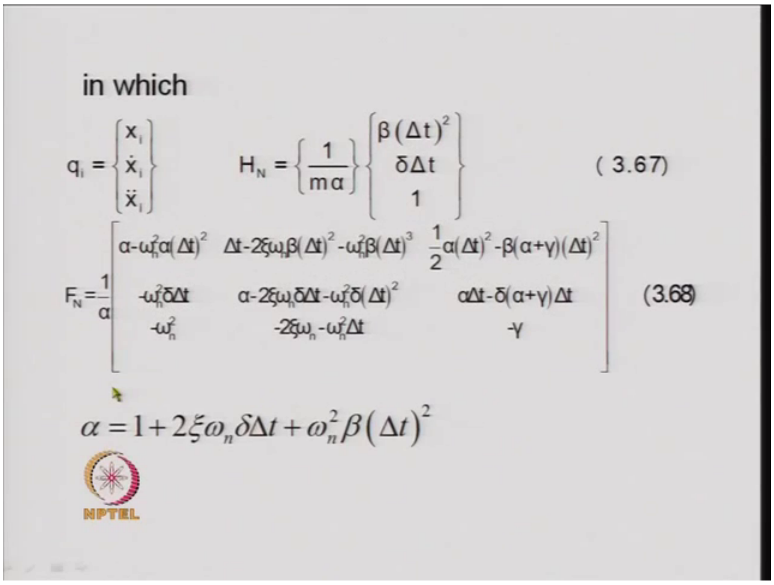

And these recursive equation provides you the responses all the 3 responses at a particular time station with the help of the responses at the previous time station and the F n matrix and the H n matrix are given over here the alpha turns out to be the these quantity which can be compensate.