Seismic Analysis of Structures
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Lecture – 12
Response Analysis for Specified Ground Motion (Contd.)

In the previous class, we discussed about the time domain analysis of structures for specified motion and in that we described 2 Numerical Integration Schemes:

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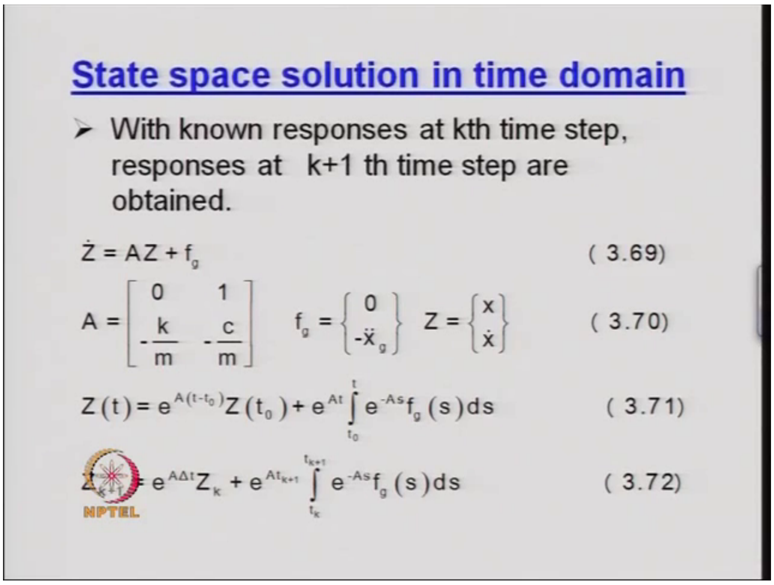

One is the Duhamel Integral. Other is the New Marks beta method. Both the integrations schemes were cast in the recursive form so that if one knows the displacement, velocity, and acceleration at k-th time, then one can find out the displacement velocity and acceleration at k plus one-th time. Provided the force acting at the k plus one-th time is known, in fact in all cases the load is specified or load is known before hand therefore there is no problem.

The specific advantage with the Duhamel integral was that for the functions which are not integrable for that one has to carry the integration in the Duhamel integration by a numerical technique in the recursive formulation that was presented in the last lecture in that it was assumed that between two time station or the within the interval of time delta t, the load varies linearly or the acceleration is assumed to vary linearly that is the ground acceleration. And once we do that then we don't have to perform any numerical integration one can divide the loading into a constant rectangular loading plus a triangular loading for which standard Duhamel integration can be performed in closed form, so using the responses for then we formulated a recursive formulation for the Duhamel integral.

Next let us see the state space solution in time domain in which the state space equation as I described before is written in these form that is Z dot is equal to A into Z plus f s, where f, A is this matrix, f s is the excitation matrix and j is the state vector. Now if we solve this equation in time then the solution of this equation turns out to be this that is Z t can be obtained provided we know the response at time t 0, then this is equivalent to Z t is equal to e into A into t minus t 0 multiplied by Z t 0 that is known plus this integration.

In this integration one can see that it is a e to the power A matrix is involved here also e to the power minus A into s is involved and f g s is the loading function running between t and t 0 or within the interval so this can be now written that in the form of the discrete variable that is Z k plus 1, if that is the quantity that is to be known then it is equal to e into A delta t and then multiplied by Z k and that Z k is known and here this integration is performed from t k to t k plus 1.

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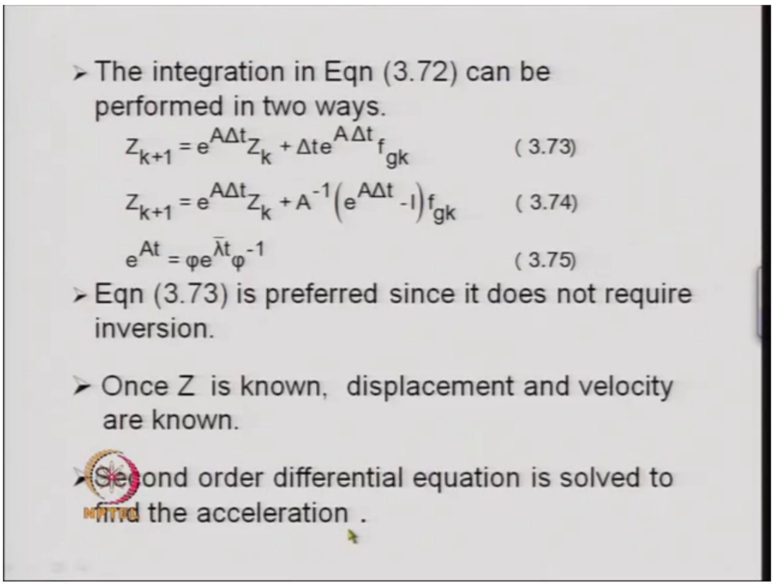

Now, the integration that which we are shown before that is this integration they can be performed in various form; the two forms are given over here so that one can get 2 expressions for Z k plus 1, this is the first term which involving Z k which is known and it is e to the power A delta t and this is the result of the integration which we were discussing and in the previous slide.

So, the other result for the integration could be in this form now out of these 2 forms these form is preferred because this is simpler in the sense that it does not require the inversion of matrix A. Note that the excitation which is to be known is f g k that is the excitation at k at time t, now e to the power A delta t is difficult to be as such because A is the matrix, but e to the power A t can be obtained in this particular fashion that is phi multiplied by e to the power lambda bar t into phi inverse where phi is the Eigen vectors of matrix A and lambda bar is the Eigen values of matrix A and e to the power lambda bar t in fact is a diagonal matrix in which each diagonal is of the form of e to the power of lambda bar I into t, where lambda bar I is a I-th Eigen vector.

So, the size of this matrix is same as the size of matrix A and one can obtain the Eigen values and Eigen vectors for the matrix A which is known and in this particular way one can obtain the value of e to the power A t. So, in the entire problem the numerical computation which is somewhat intensive is the computation of e to the power A t, once we are able to obtain the value of e to the power A t then the rest of the formulation is very simple and one can write down the value of Z k plus one provided one knows the value of Z k.

Once we have obtained the values of Z at a particular time station that is the state of the system s k and s dot k then we go back to the second order differential equation that is the original equation and from there we obtain the value of the acceleration. Next let us come to the Frequency domain analysis.

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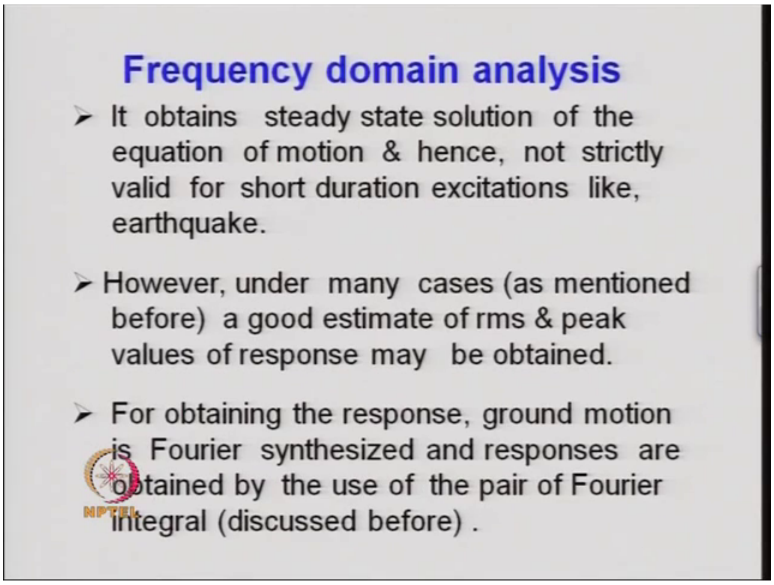

In the Frequency domain analysis, the analysis is performed for the structure for a specified ground motion and in that the input is the frequency contains of the ground motion and how to obtain the frequency ground motion with the help of Fourier series and Fourier integral we have discussed before therefore, you know how to obtain the frequency contents of the ground motion these frequency contents go as an input for the analysis.

Frequency domain analysis, in fact provides a steady state solution for the structure it cannot take care of the transient phenomena that is the transient part of the response that comes before because of the initial state of the system. So therefore for short duration excitations like earthquake or any other impulse kind of loading which is a short duration loading for that the frequency domain solution may not be applicable that is the steady state solution they are does not represent the true response of the structure.

The influence of the initial condition that is the transient part of the solution they are placed an important role because for the short duration excitation the interval of time t, for which the free oscillation takes place they are the free oscillation may not become very small or may not die down. Therefore, the steady state solution using frequency domain analysis may not be applicable for short duration earthquake.

However there are many cases where one can get reasonably good estimate of the root means square value of the response and the peak value of the response using the frequency domain steady state solution and that is a condition where the structure period if it turns out to be much smaller than the predominant frequency of excitation then these frequency domain analysis may reasonably provide a good estimate of the rms and peak responses. So, we will ah generally use the frequency domain analysis for such condition.

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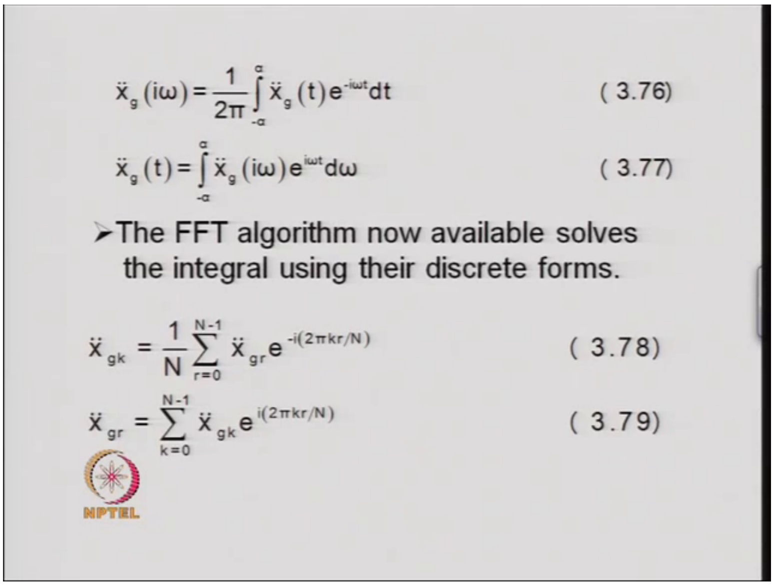

And in most of the practical cases we find that the structures period is smaller compared to the predominant frequency of excitation. Now, if we recall instead of Fourier series analysis for finding out the frequency contents of the ground motion, we introduced FFT algorithm or the Fourier transform algorithm in which the parent equation is the pair of Fourier integral which is given in equation 3.76 and 3.77.

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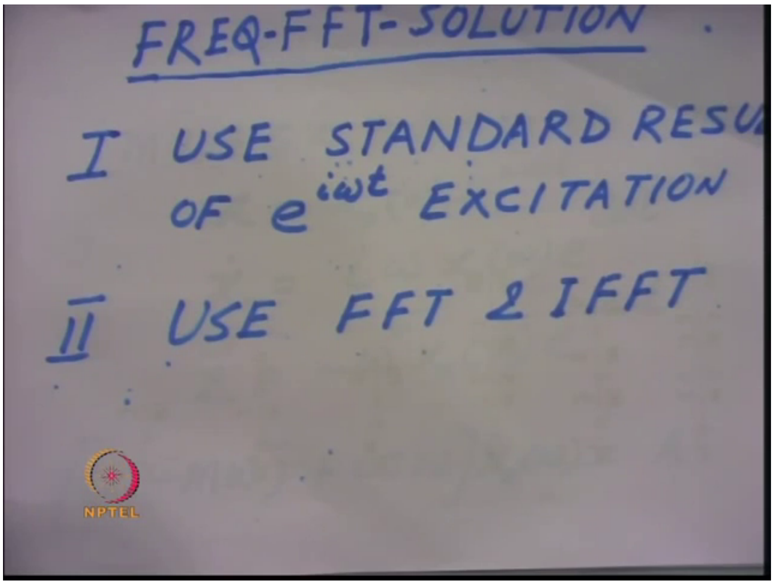

And the discrete part of the Fourier integral is the one which is given in equation 3.78 and 3.79 once we give the X double dot g r that is the ground acceleration sampled at delta t interval and put it into FFT then the FFT gives us a frequency contents of the ground motion in the complex form and if that frequency content in the complex form is given in IFFT then the IFFT gives back the original time step ground acceleration.

So, we use these pair of discrete Fourier transform to obtain the frequency contents of the ground motion and the solution here is then called a frequency domain FFT solution of the problem that means, here we are using frequency FFT solution here we use 2 important component of it; first one is the use standard result of e to the power i omega t excitation for a single degree freedom system and in the second stage we use FFT and IFFT to get the response of the structure in the form of a time history IFFT provides the response in the in the time scale.

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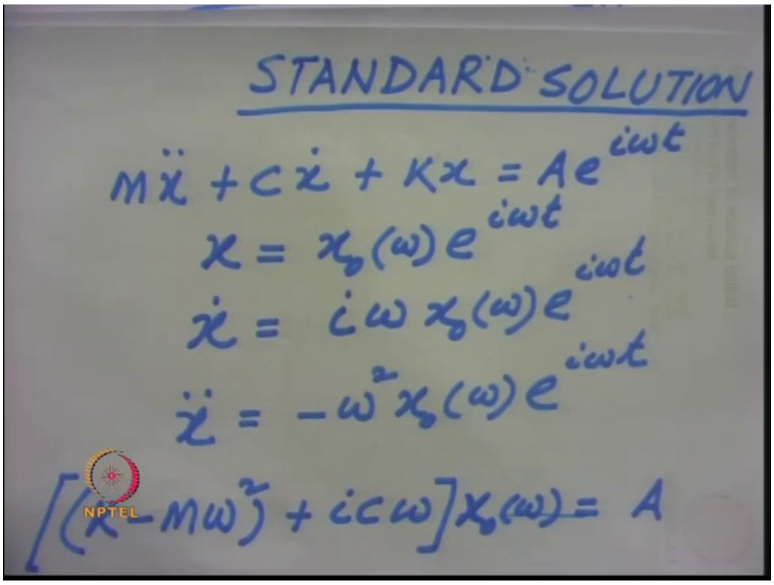

Now, in order to understand the procedure let us look into the standard solution for A to the power e i omega t so this is known as the complex harmonic function A into t to the power i omega t so if the single degree of freedom equation is subjected to a complex harmonic instead of the ordinary harmonic function. Then the steady state part of it can be written as X is equal to X 0 omega into e to the power of i omega t. Since the excitation is of the form e to the power i omega t the response also will be of this form.

If we differentiate it once we get X dot, if we differentiate it twice we get minus omega square X 0 omega e to the power I omega t, substituting the values of X dot and X double dot into this equation we land up in a equation like this k minus m omega square plus i c omega into X 0 omega is equal to a.

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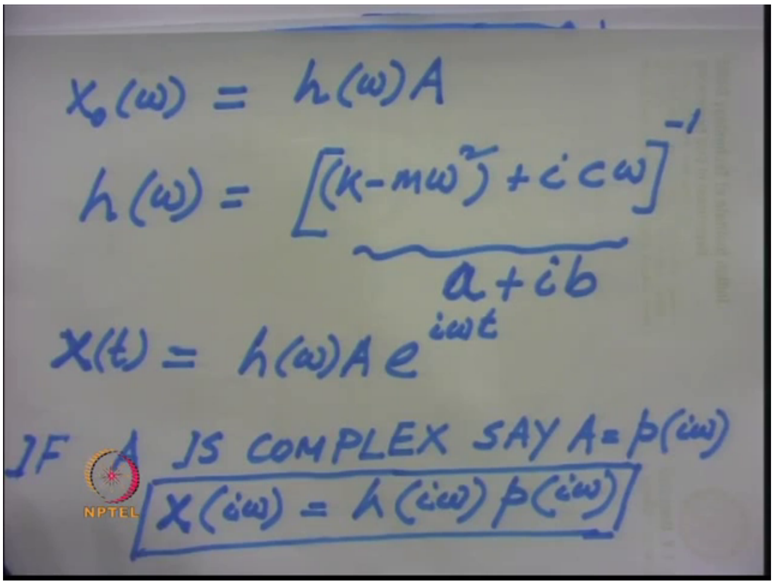

Now once we have this equation from this we can write down the value of X 0 omega as h omega multiplied by A, where h omega will be equal to the inverse of k minus m omega square plus i c omega and in fact if we make this inversion then it will come in this form as a complex number A plus i b the X t that we are wanting to find out that X t can be obtained now by multiplying h omega with A into e to the power i omega t with the help of this we can get the value of X t. So, the technique is that to get the value of response at time t then we first out h omega which is known as the complex frequency response function of the system and once it is known then one can obtain the value of the response by this equation.

If A happens to be a complex quantity say A is equal to p i omega which will be the case for us for ground motion for specified ground motion. Then one can write down X i omega to be is equal to h i omega multiply it by p i omega so with this background now we try to explain the frequency domain analysis of the system or the single degree freedom system for a specified ground motion for that first what we will do?

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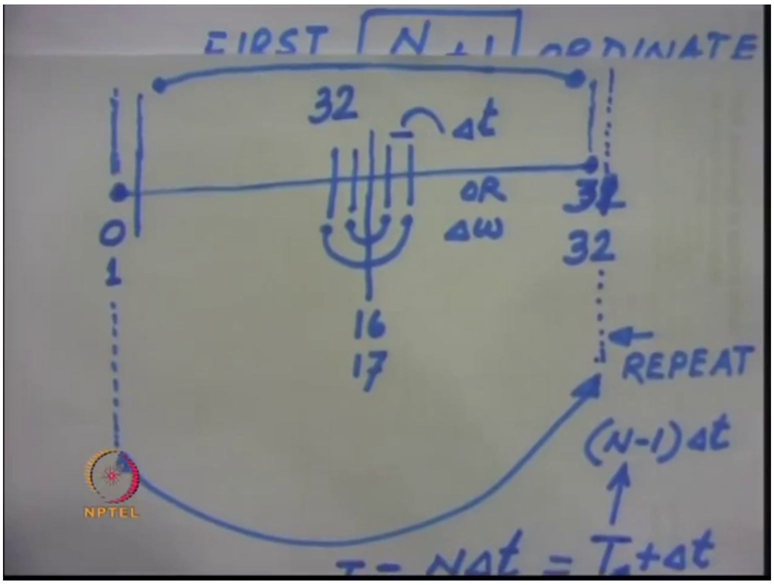

We will use the FFT for finding the frequency contents of the ground motion the I solved a problem in connection with the Fourier series analysis of the ground motion in order to find out the Fourier spectrum. Let me again repeat it little bit for your recapitulation say if we have got 32 ordinates, a sampled ordinates of the ground acceleration, then these 32 ordinates are sampled at a value of delta t and is given to the MATLAB and we click on FFT, the FFT would provide the 32 numbers of the complex quantity.

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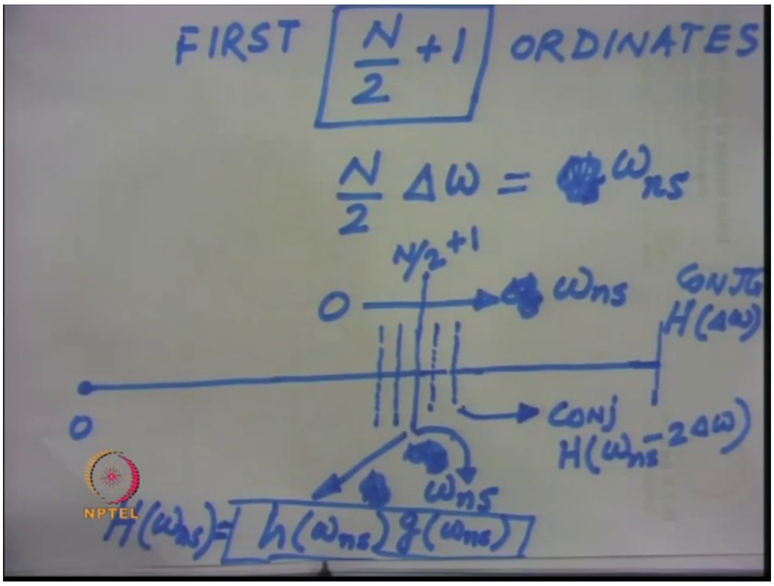

And those out of those 32 complex numbers we choose the first N by 2 plus 1 ordinates that is the17 ordinates we choose so this is your N by 2 plus 1 ordinate over here and the Nyquist frequency or the cut off frequency is equal to omega n s. How to find out the value of omega n s that is N by 2 multiplied by delta omega is equal to the Nyquist frequency where delta omega is equal to 2 pi by t, where t is the period of the excitation so here the period is taken to be is equal to the entire duration of the excitation plus delta t after that the repetition takes place. So, in other words if t is the duration of excitation then t plus delta t becomes the period and that also happens to be is equal to n into delta t.

Now, with this first ordinate first state of ordinates from the FFT, we obtain the value of h omega and the for each value of delta omega or each value of an omega at an interval of delta omega we obtain the value of h omega and then we multiply this with the frequency contents of the ground motion. The frequency content of the ground motion will be equal to the g omega that is again the first 17 ordinates of the frequency contents of the ground motion that we take at an interval of delta omega and multiply these with h omega, the h omega can be obtained ah from the equation that I have shown before that is k minus m omega square plus i c omega and inverse of that, that gives you the value of h omega.

So, once we get these multiplication of the these quantity two quantities then this is given as an input to IFFT, in the IFFT what we do is that after the 17th value, the next value that we give over here or the next ordinate that you consider over here is the complex conjugate of this one.

Similarly this value ordinate will be the complex conjugate of this so these will be the additions that has to be provided from the information that we have got that is the multiplication of these 2 quantities at different frequencies and we take complex conjugate of them and place them in this particular fashion so the last value over here will be equal to a value which we will get just after delta omega interval that means, this ordinate not this ordinate.

The ordinate at 0 in fact is the repetition ordinate from where again the entire things repeats therefore we take up to this. So, in that fashion we can have say if we have 32 sampled values of the ground motion then we will get 32 h omega and capital H omega value this capital H omega value is nothing but the small h omega value multiplied by the frequency contents of the ground motion and this multiplication of them is a quantity which is a complex number.

Now, once we give this 32 complex number that means, on this side we have complex conjugate numbers then the IFFT would give us the response of the single degree of freedom system to the specified ground excitation of X g t, X g t is the ground motion so that is the frequency domain analysis and in that frequency domain analysis you see that we are using the FFT algorithm and also we are using the standard solution which is known for a single degree of freedom system excited by e to the power i omega t. So, the frequency domain analysis can be carried out in a very routine fashion using FFT and it requires very less time compared to the time integration techniques so the cases where we are able to use a frequency domain analysis we go for that because this states the FFT algorithm is available in most of the standard libraries in the computer.

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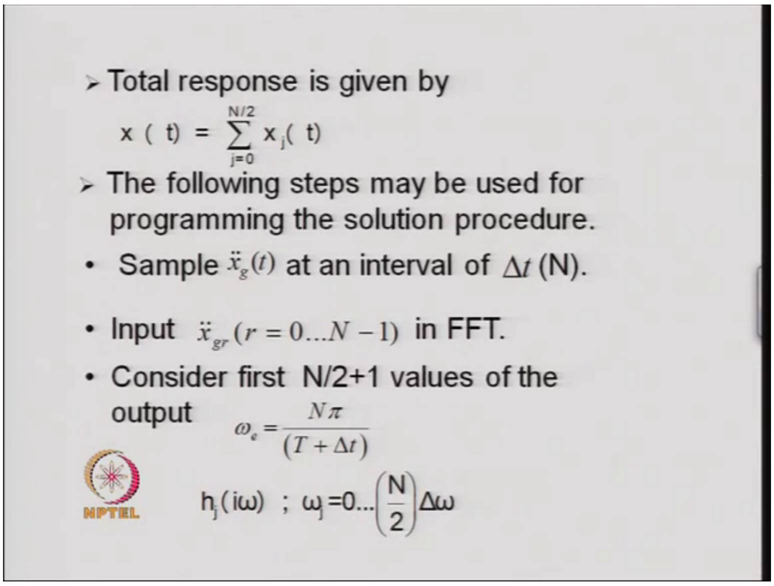

The summary of the frequency domain analysis is the given over here that is sample X double dot g t at an interval of delta t that will give a value a N values of the ordinates, then input these sample values in FFT then consider the first N by 2 plus 1 values of the output and we get the Nyquist frequency and the Nyquist frequency is given by this and we obtain h j i omega which is the complex frequency response function for the single degree of freedom system which is nothing, but is equal to k minus m omega square plus i c omega inverse of that.

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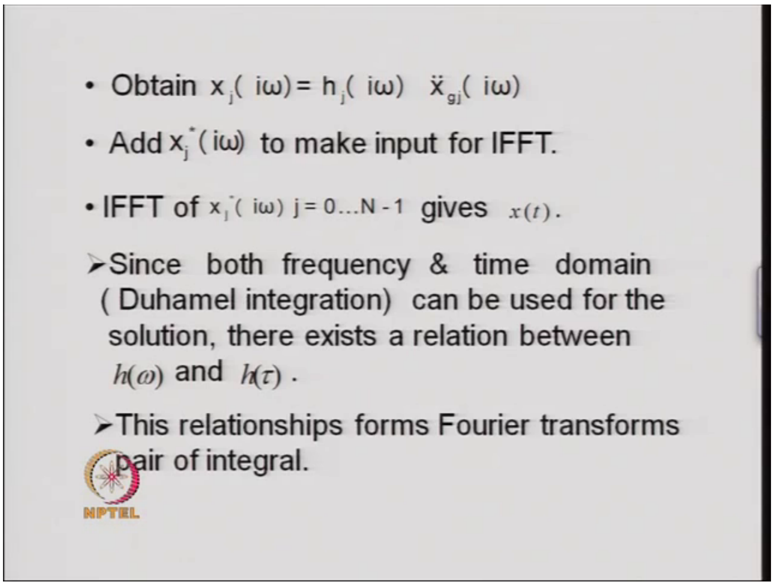

Then obtain this quantity that is X j i omega is equal X j i omega multiplied by X double dot g j i omega X double dot g j i omega is the frequency contents of the ground motion, then add appropriately the complex conjugates as I explained before then put it as an input to IFFT and the IFFT would give the values of the response in time.

Since both frequency and time domain solution can be used for the solution it is expected that there should be a relationship between h omega and h tau. Now h tau specifically arises in the Duhamel integration that is for the unit impulse the integration that we perform for obtaining the Duhamel integration is called h tau and there exists a relationship between the frequency response function and h tau.

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$$h(i\omega) = \int_{-\alpha}^{\alpha} h(t) e^{-i\omega t} dt \qquad (3.83)$$

$$h(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} h(i\omega) e^{i\omega t} dt \qquad (3.84)$$

And this relationship is nothing, but again the Fourier pair of Fourier integral only difference here is that 1 by 2 pi is not associated with the first equation which is usually which is the generate the case for finding the frequency contents of any time history. But here one by 2 pi is associated with the IFFT part of the equation. So, FFT part of the equation does not have 1 by 2 pi whereas the IFFT part contains 1 by 2 pi however, if we interchange this then also the results of the Fourier first Fourier transform do not change.

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Example 3.7: Single bay portal frame in Fig 3.11 is subjected to Elcentro earthquake. Find $x(t)$ with $x(0) = 0;\ \dot{x}(0) = 0;\ \xi = 0.05$

$\Delta t = 0.02s$

$\omega_n = 12.24$ rad/s

$\omega_d = 12.23$ rad/s

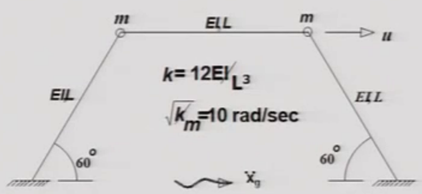

Fig 3.11

Next let us look into a response of a single bay portal frame and in this single bay portal frame we are interested in finding out the response of at the sway response of the frame that is u subjected to a ground acceleration of X double dot g. The two masses are lamped at this two points and the E I and L values are specified for the incline portal frame. The natural frequency of the system is equal to 12.24 radian per second and the damped natural frequency is 12.23, they are very close to each other because the damping is coefficient is very small and the initial condition for them is given as X 0 is equal to 0 and X dot is 0 is equal to 0, that is the initial condition is given and the excitation is sampled at an time interval of 0.02 second.

Now, first let us see how we can obtain the natural frequency for such a system which is a incline portal frame. So, the stiffness first let us take the stiffness of this frame to corresponding to the sway degree of freedom so that will be the k value and they are the combination of these 2 masses will form the effective mass and root over k by m is equal to the value of the frequency natural frequency.

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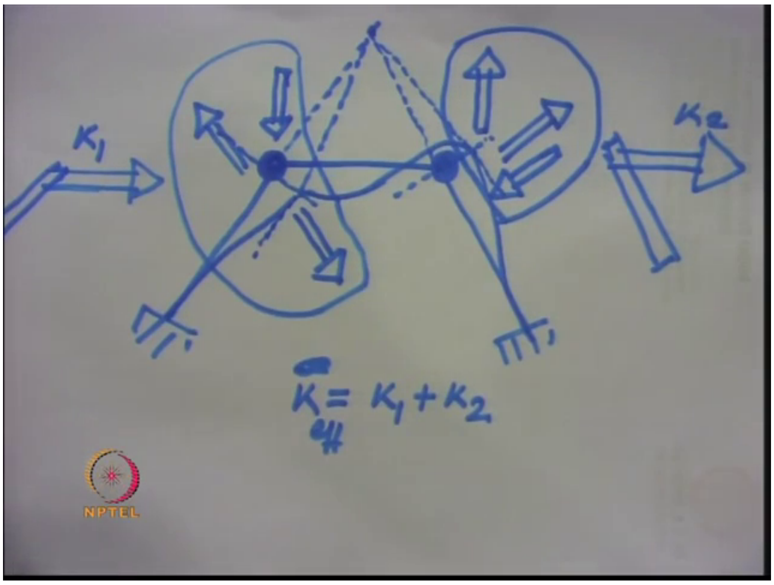

In order to obtain the stiffness corresponding to these degree of freedom let us try to recapitulate how we can find out the stiffness of the system we are interested in obtaining the force required to produce unit horizontal displacement over here.

So, for that what we do we make the instantaneous center of rotation over here and give consider these as a sets square and rotate this about the instantaneous center such that the horizontal component of this movement becomes equal to unity. Now this movement will be perpendicular to this member or perpendicular to this dotted line, this movement also will be perpendicular to this member or perpendicular to this member in that case the length of this and this they remain unchanged length of this and this remain unchanged according to the small displacement theory.

And in this triangle if these 2 lengths remain unchanged after the rotation then the third length that is the joining this point and this point third length also will not change. So, if they don't change then the length of this member remains unchanged length of this members remain unchanged because of this point is moving perpendicular to it so therefore, it remains unchanged. So, all the 3 members after this given in a rotation about the instantaneous center will not undergo any change of in length therefore the condition of inextensibility is maintained.

Now, once we have that then one can find out the forces that is generated at this points due to this rotation so we have first let us consider these beam in these beam the relative displacement in the vertical direction between these 2 points can be worked out from the knowledge of this and this and for that relative displacement will have a force a shear force acting in this direction over here and shear force acting in this direction over here and there will be a rotation also because of this rotation at these in our 2 points there will be again a shear force which will be acting in this direction and in this direction. So, in the beam finally, we will have a shear force acting in this way and acting in this way.

Next let us take these particular inclined lake, in the inclined lake since it is moving perpendicular to the member over here there will be movement produced there will be a shear force in this direction plus there is a rotation say in this direction these direction then this direction we give raise to a shear force in this direction.

In the same way one can find out the shear forces due to the displacement and the rotation at this point for this incline lake, so they are these forces. So, we have got now 3 forces acting at this joint and 3 forces acting at this joint these 3 forces now are resolved one along the this lake that is along this direction and other in the horizontal direction or the direction in which the sway takes place.

Similarly here also these 3 forces can be resolved one along this length and other on the horizontal direction that is the direction of the sway some of these two in fact will be the force which will be required to produce unit displacement and that defines stiffness of this inclined portal frame corresponding to a sway movement of unity and these forces which are coming onto the lake they simply pass to the support and there is no shortening of the member in this direction because it is assumed that the members are inextensible. So, in this particular fashion one can find out the value of the stiffness for the incline electron portal frame.

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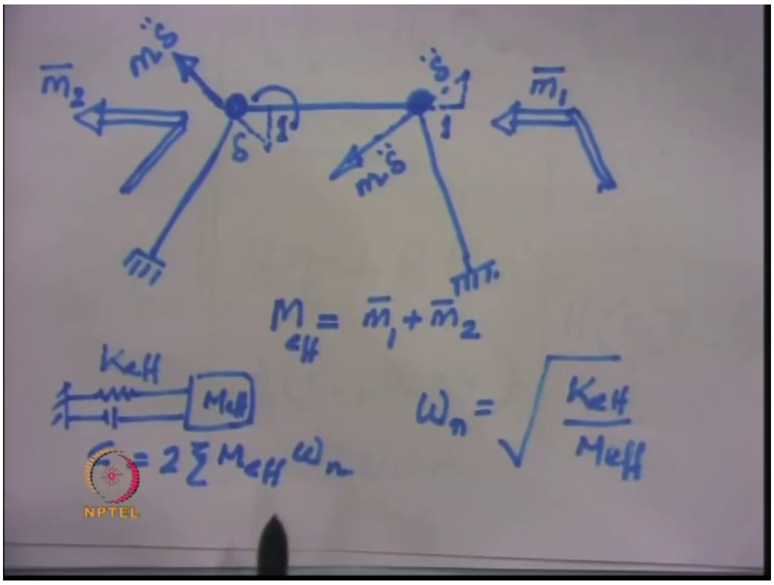

Now, once we get the stiffness for the portal frame then we have to find out the effective m, now effective m mind you is not is equal to m plus m because if we give unit displacement s y unit displacement in this direction then the entire system would move like this, this point will come over here and this point will come over there as I had shown before because the inextensibility condition has to be satisfied and for that the inertia force which will be generated will be in this direction minus m or m multiplied by delta dot where delta double dot is the acceleration caused due to the unit displacement in this direction

Similarly m multiplied by delta double dot will be the acceleration produced in this direction and hence mass multiplied by delta double dot will be the inertia force. So, these are the 2 inertia forces that will be generated over here because of the movement of the structure in the horizontal direction, maintaining the inextensibility condition and these 2 forces are resolved one along this direction and the other along the member similarly here it is resolved in this direction and one along this member.

So, sum of these 2 inertia forces are the mass matrix or the mass equivalent mass of the system. So, the omega n for calculating omega n we make use of square root of k effective divided by m effective, where m effective is this one and the k effective we have found out before. And then one can convert the entire system as a single degree of freedom system with a spring having k effective and a mass which is m effective and the dash pot can have a constant which will be equal to twice psi m effective into omega n; however, psi is the specified value of 0.0 5.

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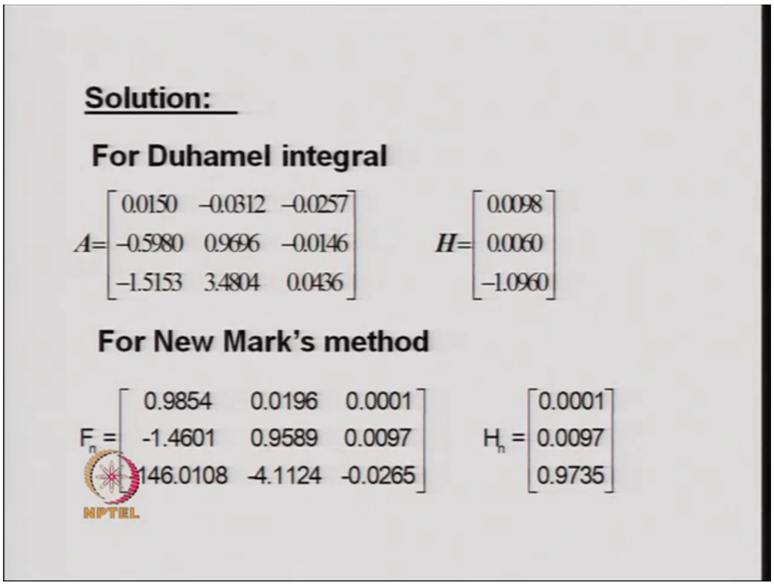

So, we now solve this equation by various methods that I have outlined previously. So, if we use the Duhamel integral then in the Duhamel integral the type of recursive formulation that we used or proved is this is the Duhamel integration.

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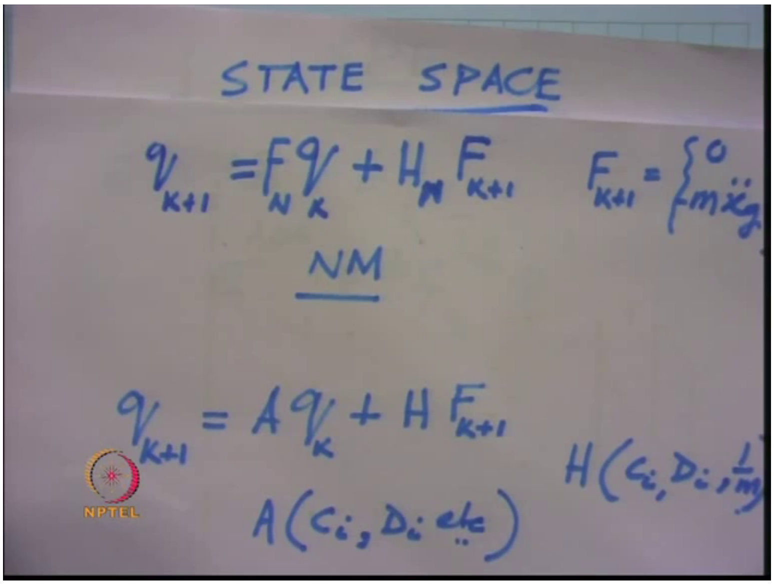

If you for the capitulation we use the q k plus 1 is equal to F n multiplied by q k and H n multiplied by F k plus 1, F k plus 1 is known that is the for the New Mark's beta method and for the Duhamel integral we have A multiplied by q k, so this A matrix is the matrix which is given here is in the solution this is the A matrix, this is the A matrix for the Duhamel integration and the F n matrix is the New Mark's beta method which is given here and H n is this matrix.

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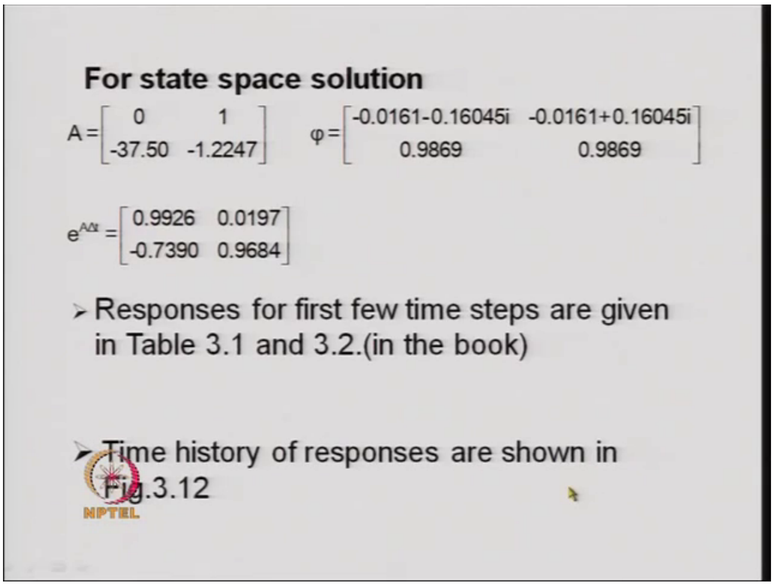

So, with the help of these matrices one can find out the response at different time steps in the state space solution we have seen that we require e to the power A delta t and A is this matrix state space matrix the Eigen vectors for this matrix is given by phi and is obtained like this, this is the Eigen vector and the Eigen values matrix multiply it by delta t that is that we can e to the power A delta t when can be is obtained like this. So, with the help of these quantities one can easily obtain the response of by in the state space solution at different time t.

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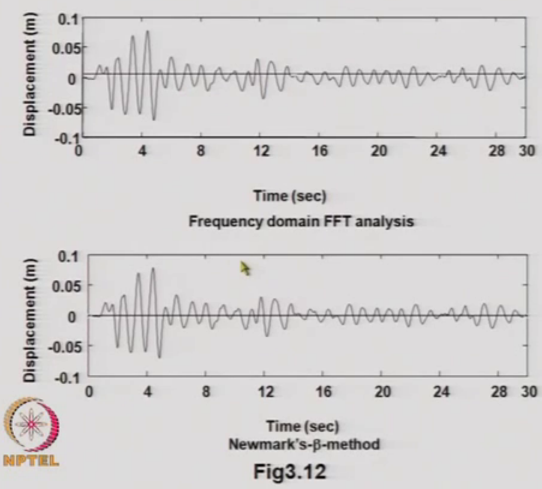

Fig3.12

Now, the time histories of the responses that are obtained is shown over here I think the FFT solution also was carries out that I have not explained over here, but explain before how to the response of the system using frequency domain FFT analysis. And then it was compared with New Marks beta method and we found that for both the cases the results were exactly the same. The reason was that the initial condition that was taken for this problem was that it was at rest that if there was no displacement and no velocity at time t is equal to 0. Therefore, the both the frequency domain solution and the New-Marks beta method solution they provided nearly the same result.

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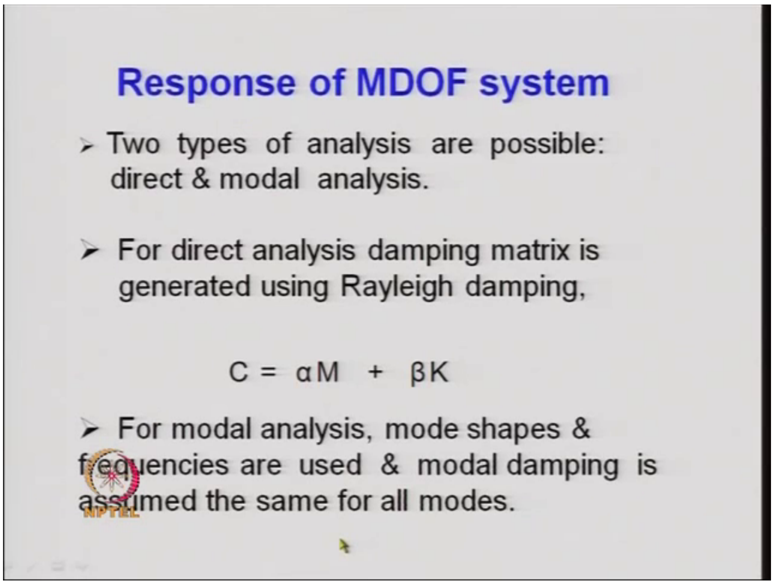

Next we come to the response for the multi degree of freedom system and in the multi degree of freedom system as I told you before we classify the problem into either a single point excitation or a multi point excitation. In the single point excitation we have i influence vector or i influence matrix that it generally consisting of 1010 etcetera. For multi support excitation the i is replaced by r, r is a matrix that is called the influence coefficient matrix that is generally obtained by performing a separate static analysis for the differential support movement at the base and from that how we obtain the value of r that we have explained with the help of a few example also.

Now in response analysis for m (Refer Time: 43:26) system again we can have 2 types of solution: one is a direct solution, other is a modal analysis. So, first we will take up the direct analysis in which the damping matrix must be explicitly known, because the equation of motion becomes M X double dot plus C X dot plus K X is equal to minus m i or m r into X double dot g.

So, the C matrix must be known and C matrix is obtained by assuming it to be a Rayleigh damping that all of you know and the that is equal to alpha times the mass plus beta times K and this alpha and beta values can be obtained from the 2 frequencies of the system it depends which 2 frequencies you take most of the people take the first 2 frequencies but it is not necessarily that one should take first 2 frequencies.

In fact, any 2 frequencies can be taken and which 2 frequencies will describe the damping matrix best that is a matter of debate and research but generally for ah you for the solution of the problems we satisfy our self by considering only the first 2 frequencies.

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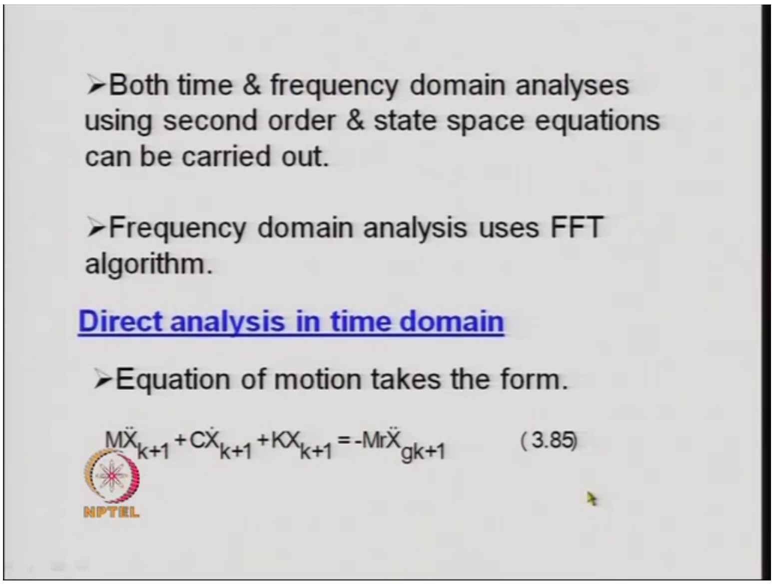

For modal analysis, we require mode shapes and frequencies that we will discuss later. Again in the case of multi degree of freedom system one can have a time domain analysis and a frequency domain analysis. So, first let us discuss about the direct analysis in time domain.

In the direct analysis in time domain we have the; this is the equation that we are trying to now solve here the r is the influence coefficient matrix for multi support excitation if we change r to I then it will become a single support excitation system.

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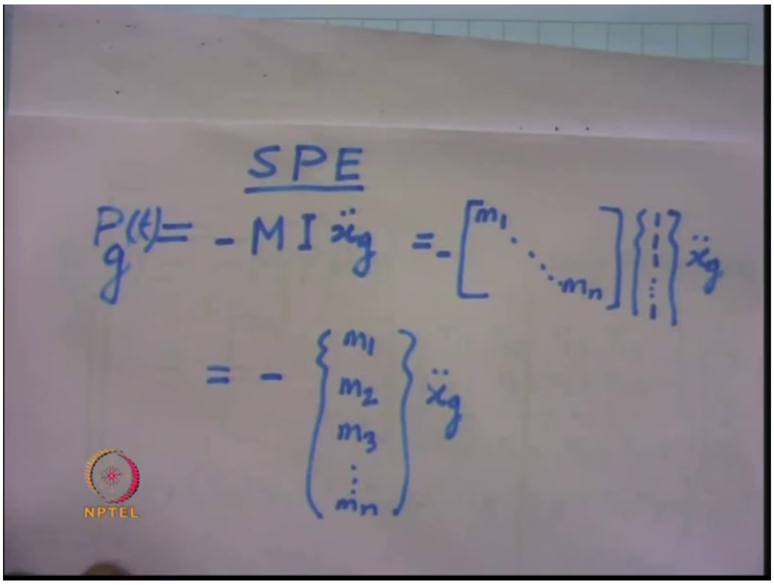

And this is discussed over here say if it is a single point excitation system then minus M I x double dot g that is the right hand side force vector and if we expand it this is a diagonal mass matrix I is a vector for the multi storey buildings 1111 into x double dot g that gives a right hand side load vector as minus m1, m2, m3 up to m n into X double dot g.

So, we have got n number of forces and this n number of forces is such that it is components each component of this force vector is m1 x double dot g, m2 x double dot g, m3 x double dot g so on. And if one has to perform the solution in frequency domain then each one of these forces has to be furious synthesized or FFT algorithm is to be used for each one of these forces.

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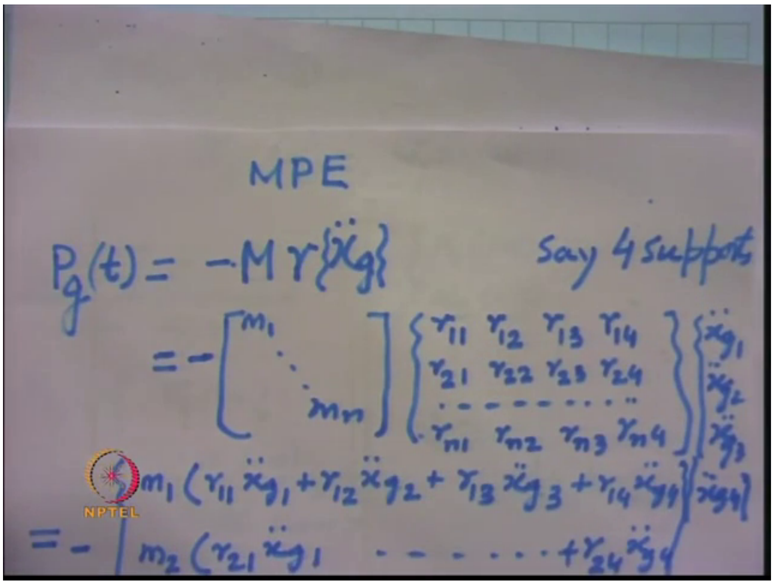

If we consider the multi support excitation then the this is the form of the equation that is minus m into r into x double dot g this is the mass matrix diagonal mass matrix the r matrix say we are considering 4 support excitation then it will be a n multiplied by 4 matrix, n is the number of degrees of freedom and the r matrix would look like this and x double dot g1 and x double dot g 2 and x double g 3 and x double dot g 4 they are the support excitation.

Now once we multiply this vector with these matrix the and then multiply with the mass matrix then we find out a vector of size of n and each element of the vector we will ah have terms like this that is m1 multiplied by r11 x double dot g1, r11 into x double dot g 1 plus r12 into double dot g2, r13 into x double dot g3 so on so that is the first element similarly one can obtain the second element, third element so on.

So, these quantities r11, m11 they are all known therefore, the this particular vector is known. So, one can construct n number of time histories and these n number of time histories would be the excitation vector. Now if we again go for a frequency domain analysis then this n number of time histories have to be furious synthesized or FFT algorithm is to be used for finding out the frequency contents of all this n time histories.

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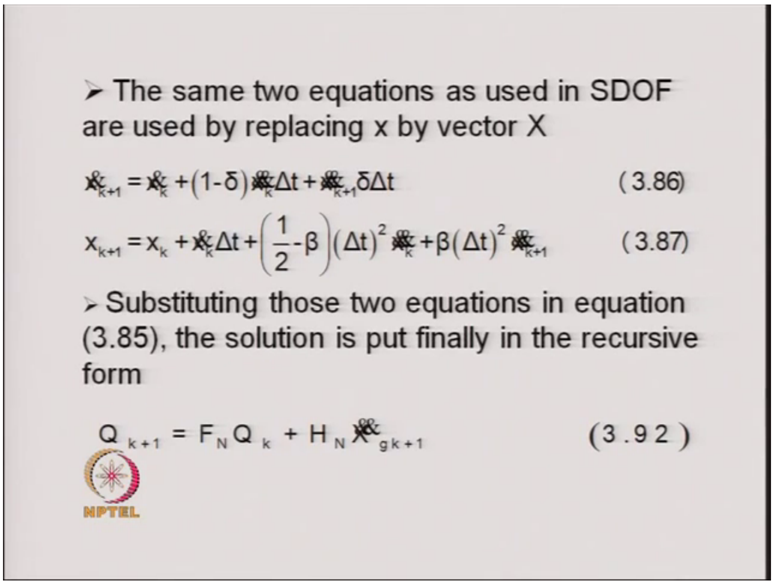

Two cardinal equations that we used for New Marks beta method are given in equation 3.86 and 3.87 the equations are written using linear variation of acceleration within the time interval delta t. Note that the variables of the equations are vectors using these two equations New Marks beta method can be finally cast into the recursive form of equation as we had done for the case of single degree of freedom system and is shown in equation 3.92, the difference between the two cases that is equation 3.92 and the previous equation for single degree of freedom system is that here the F n matrix contents turns which are all matrices.

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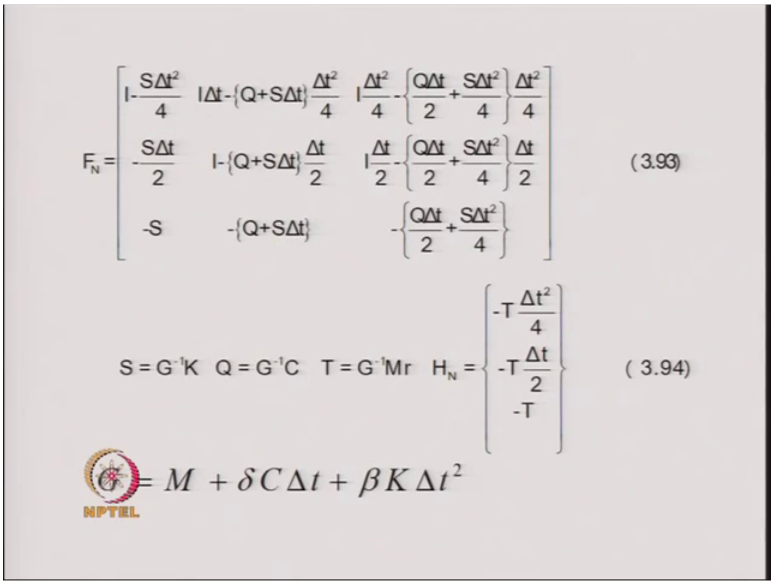

For example S is a matrix then G is a matrix, S is equal to G inverse K, Q is equal g inverse C and you can see that the Q matrix is appearing in F n matrix then there is a t matrix which also appears in F n matrix and t matrix is given by G inverse M into r where r is the influence coefficient matrix H n matrix is given by equation 3.94 that is minus T into delta t square by 4, then minus T into delta t by 2 and minus T. The G in this matrix which requires an inversion to find out S Q and t, this G matrix is simply is equal to mass matrix plus delta into C matrix into delta t plus beta K into delta t square.

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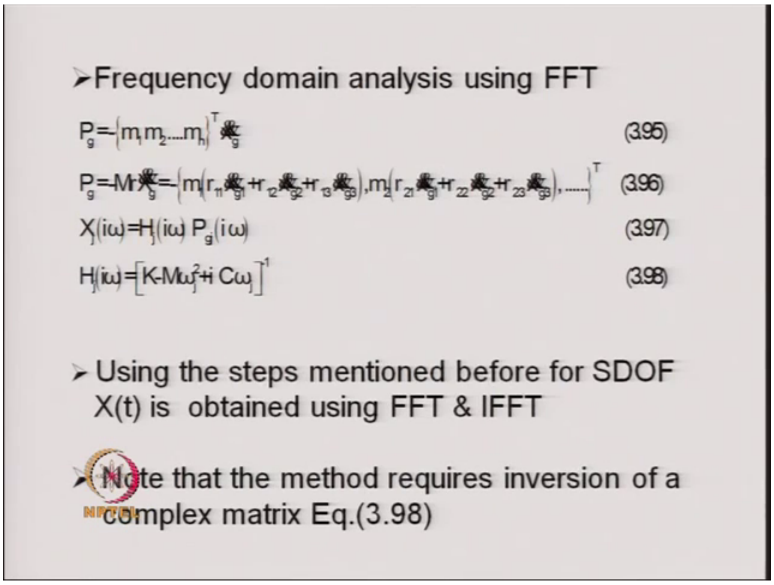

Now, with these matrices defined one can find out the value of the responses at K plus 1 time station given the value of the responses at the k-th time station and the direct time domain analysis using New Marks beta method can thus be obtained.

Now, let us come to the frequency domain analysis using FFT in the frequency domain analysis the right hand load vector for single support and multi support excitations are given in equation 3.95 and 3.96. In equation 3.95 the term which are shown are very straight forward with x double dot g as a single time history where as in equation 3.96 the equation is little involved in the sense that the terms of the load vector turns out to be the mass matrix of a particular floor say first floor multiplied by this sum of r11, x double dot g 1 plus r12 x double dot g 2 and so on where r11, r12 etcetera they are the elements of the r matrix that we have obtained before and the excitations are x double dot g1, x double dot g 2 or that is the excitations at different supports.

Similarly, one can obtain the second element of the vector that is the mass of the second floor multiplied by similarly r11 x double dot g one etcetera equation 3.97 gives the necessary equation to find out the frequency content of the displacement vector X i omega and that is equal to H j i omega multiplied by P g i omega, where p g is a given by either equation 3.95 or equation 3.96 depending upon whether the load vector is defined for a single point excitation or a multi point excitation. Note that H for computing H j i omega one has to obtain a complex inversion of a matrix given by equation 3.98, using the steps as we have discussed before for single degree of freedom system X t then can be obtained using FFT and IFFT.

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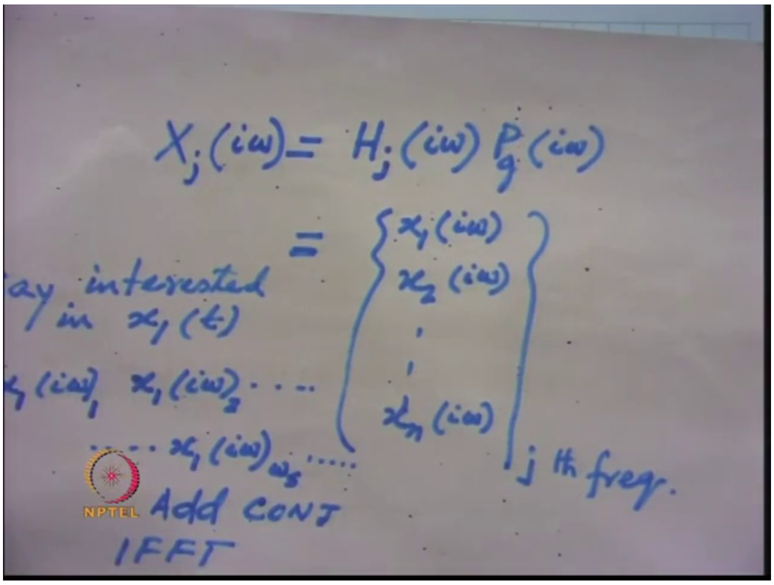

We use this relationship that is X j i omega that is the responses frequency components of the responses is related to the H j matrix and the P g i omega vector H j i omega matrix is in fact the matrix which will be K matrix minus m omega square plus i C omega which you have got for the same kind of expression you have got for the single degree of freedom system only you replace K m and C by k matrix, m matrix and C matrix and inverting this matrix you get the value of H j omega and this is n by n matrix. If the number of degrees of freedom are n P g i omega will be the frequency contents of the ground motion and this frequency contents of the ground motion will be the time histories of the ground motion or the forces that we get on to the right hand side, so those different forces all of them are furious synthesized and we get the value of P g i omega and this again becomes a vector for a specified value of omega this becomes a matrix for a specified value of omega if you multiply these 2 then there is it becomes a vector and for a specified value of this omega one can have a vector of size n.

Then each one of them can be taken out separately and the first N by 2 plus 1 quantities of x can be written down or put in FFT algorithm and then we add on to that the complex conjugates of them as I described before then we have the complete set of x j i omega for the response one and put it into the IFFT, the IFFT gives the response of x1 in time that is a we get a time history.

Similarly, we take x 2 and again add onto it the complex conjugates give it to i 50 and from there we get the time history of the response C 2t so that way one can get the value of the responses for all the n degrees of freedom so that is how one can obtain the response of a multi degree freedom system using FFT and the frequency response function matrix over here.