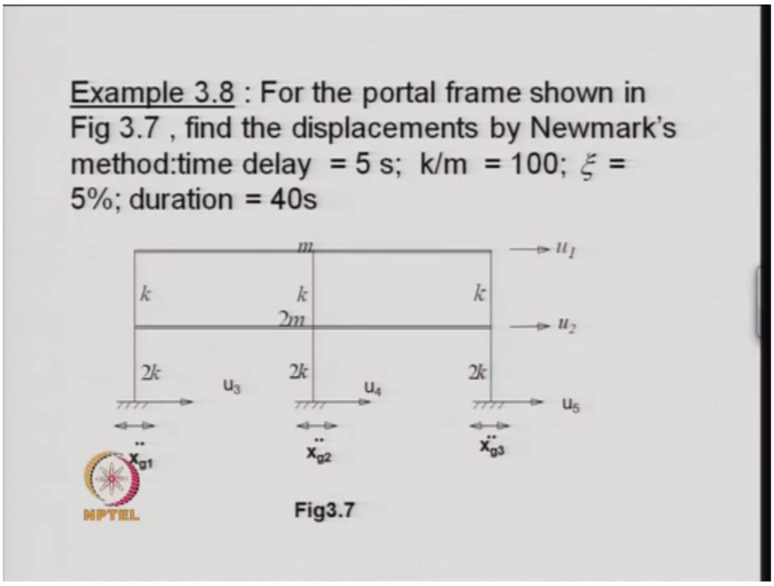
Seismic Analysis of Structures

Prof. T.K. Datta
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture – 13
Response Analysis for Specified Ground Motion (Contd.)

(Refer Slide Time: 00:45)

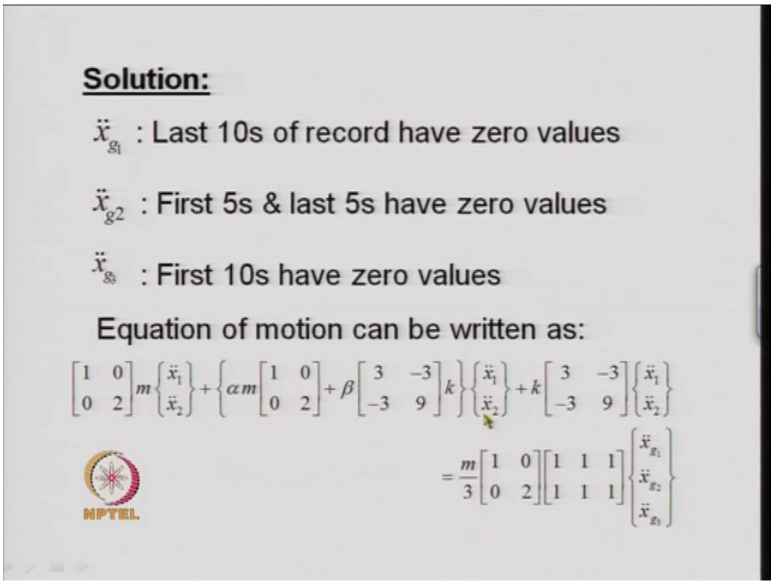


In the last lecture, we discussed about the frequency domain solution of the single and multi-degree of freedom system and in that we discussed about the in obtaining the frequency contents of the ground motion and with the help of that frequency content of the ground motion, we solved the problem in frequency domain obtain the frequency contents of the response and then inverse Fourier transform it to obtain the time history analysis for the multi-degree of freedom system, we require the r matrix whereas, for the single point excitation system, we require i matrix or i vector whatever be the case.

Now, here is an example problem which illustrates; how we can obtain the response of a multi-degree of freedom system using both time domain and frequency domain in a multi-degree of freedom system for example, in this frame we have 3 ground accelerations and these 3 accelerations are different although the same earthquake wave is passing through all the 3 supports, but because of the time delay that has been assumed the excitations at the 3 supports would be different.

So, in order to understand that we take 30 seconds duration of the ground motion and which is travelling when it arrives here at this particular point, then the ground motion at this point would be 0 and next here also it will be 0 when 5 seconds of ground motion has passed through this particular point, then we get the ground motion over here and at that point of time ground motions still at this point is 0 when 10 seconds of ground motion have passed through this point, then the we get the ground motion here as well.

(Refer Slide Time: 03:42)



So, as a result of that the excitation caused by the same travelling wave of ground motion is different at this 3 different supports and we can obtain this by the following considerations from the 30 seconds of record of ground motion that is available to us the last 10 second of record have 0 values for the first support; that means, when all the 30 seconds have passed through this point then then we have the next 10 seconds of the time history of excitation would be 0 the; in this case the first 5 seconds and the last 5 seconds have 0 values and in between we have got the 30 seconds of the ground motion and for the last support, we will have the first ta 10 seconds will have 0 values rest of the 30 seconds will have non zero values as a result of that effective total duration of ground motion is taken as forty seconds and that is how the 3 different records of excitations differ on a scale of measurement of forty seconds.

Now, with these 3 ground motion distinct 3 ground motion, now we write down the equation of motion for this system this has 2 non support degrees of freedom and the support degrees of freedom are 3 and they are shown over here that is the these are the 3 ground accelerations and the r matrix we computed before as one third one; 1 1 1 1 1. So, for this particular problem, we had seen before that the participation of the ground motions in creating responses at the 2 nonsupport degrees of freedom where equal and was equal to one third one third one third. So, this was of course, for this problem; for other problems it may not be like that that is what you have seen before the r; r matrix could be different for different kinds of structures and it is to be obtained from of the matrix condensation relationship.

(Refer Slide Time: 07:14)

$$\omega_1 = 12.25 \ rad/s; \ \omega_2 = 24.5 \ rad/s; \alpha = 0.816; \ \beta = 0.0027; \Delta t = 0.02s$$

$$F_n = \begin{bmatrix} 0.9712 & 0.0272 & 0.0193 & 0.0006 & 0.0001 & 0.0 \\ 0.0132 & 0.9581 & 0.0003 & 0.0192 & 0.0 & 0.0001 \\ -2.8171 & 2.7143 & 0.9281 & 0.0611 & 0.0096 & 0.0003 \\ 1.3572 & -4.1751 & 0.0302 & 0.8973 & 0.0002 & 0.0094 \\ -281.7651 & 271.4872 & -7.1831 & 6.1402 & -0.0432 & 0.0343 \\ 135.7433 & -417.5091 & 3.0701 & -10.2531 & 0.0171 & -0.0602 \end{bmatrix}$$

$$H_n = \begin{bmatrix} -0.0 & -0.0 & -0.0 \\ -0.0 & -0.0 & -0.0 \\ -0.0033 & -0.0033 & -0.0033 \\ -0.0032 & -0.0032 & -0.0032 \\ -0.3301 & -0.3301 & -0.3301 \\ -0.3182 & -0.3182 & -0.3182 \end{bmatrix}; \qquad \dot{x}_0 = [0 \quad 0]^T$$

Now, with this particular equation now we put on the values of a x double dot g 1 x double dot g 2 and x double dot g 3 in time from the different records that we have obtained and these records are used for solving the problem in time domain using Newmarks beta method and when we use the Newmarks beta method using direct integration technique in the recursive form, we require the c matrix and the c matrix is obtained with the help of this 2 coefficients alpha and beta and this is the F n matrix of the Newmarks beta method and this is the H n matrix for the Newmarks beta method the initial condition that was used is equal to 0 velocity and 0 displacement with the help of that we went for the recursive equation. So, solution of the recursive equation to obtain the response of the system if in order to remind you what is the form of the recursive equation let us have a look at the recursive equation.

(Refer Slide Time: 08:17)

in which

$$q_i = \begin{Bmatrix} x_i \\ \dot{x}_i \\ \ddot{x}_i \end{Bmatrix} \qquad H_N = \left\{ \frac{1}{m\alpha} \right\} \begin{Bmatrix} \beta(\Delta t)^2 \\ \delta \Delta t \\ 1 \end{Bmatrix} \qquad (3.67)$$

$$F_N = \frac{1}{\alpha} \begin{bmatrix} \alpha - \omega_n^2 \alpha (\Delta t)^2 & \Delta t - 2\xi\omega_n \beta (\Delta t)^2 - \omega_n^2 \beta (\Delta t)^3 & \frac{1}{2}\alpha(\Delta t)^2 - \beta(\alpha+\gamma)(\Delta t)^2 \\ -\omega_n^2 \delta \Delta t & \alpha - 2\xi\omega_n \delta \Delta t - \omega_n^2 \delta (\Delta t)^2 & \alpha \Delta t - \delta(\alpha+\gamma)\Delta t \\ -\omega_n^2 & -2\xi\omega_n - \omega_n^2 \Delta t & -\gamma \end{bmatrix} \qquad (3.68)$$

$$\alpha = 1 + 2\xi\omega_n \delta \Delta t + \omega_n^2 \beta (\Delta t)^2$$

So, that yeah this was the recursive equation that we use for Newmarks beta method. So, this is F n matrix and this is the H n matrix.

(Refer Slide Time: 08:41)

- Using recursive Eqn. (3.92), relative displacement, velocity and accelerations are obtained.
- Time histories of displacements, (u_1 and u_2) are shown in Fig 3.13

(Refer Slide Time: 08:47)

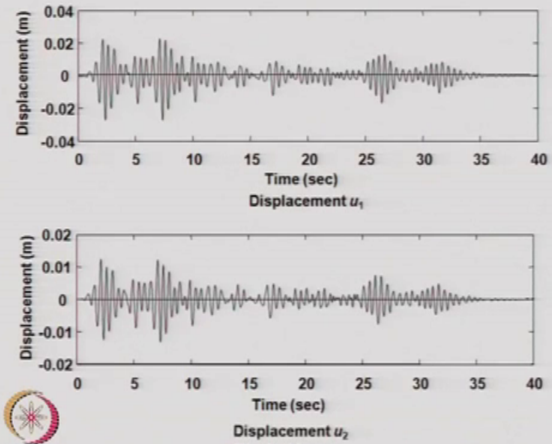


Fig3.13

So, plugging in the different values that we use in the solution of the problem we get the solution and we get relative displacement velocity and acceleration the displacements at u 1 and u 2 that is the 2 nonsupport degrees of freedom the time histories of that are shown over here, it can be seen that although the 30 second is the duration of the earthquake, but the excitation duration was effectively 40 seconds and therefore, the response that we get at u one and u 2 are extends beyond 30 second and it almost goes to 40 second of course, in this part the responses are very very small.

(Refer Slide Time: 09:26)

Example 3.9: For the pitched roof portal frame shown in Fig 3.8, find displacements 4 & 5 for zero & 5s time delay between the supports.

$$\left(\frac{EI}{mL^3}\right)^{1/2} = 2 \quad rad/s; \quad \xi = 5\%$$

Solution:

$\omega_1 = 5.58 rad/s;\ \omega_2 = 18.91 rad/s; \alpha = 0.4311;\ \beta = 0.004;\ \Delta t = 0.02$

Next we solve the pitched roof portal frame and here if you recall the in the pitched roof portal frame we had the degrees of freedom one in the horizontal direction and other at the crown in the vertical direction these were the 2 degrees of freedom and for that we assume the 5 second time delay between the 2 supports by solving the Eigen value problem we get the frequency as omega 1 and omega 2 as these values and from that we computed alpha and beta as that and delta t was taken as 0.02.

(Refer Slide Time: 10:13)

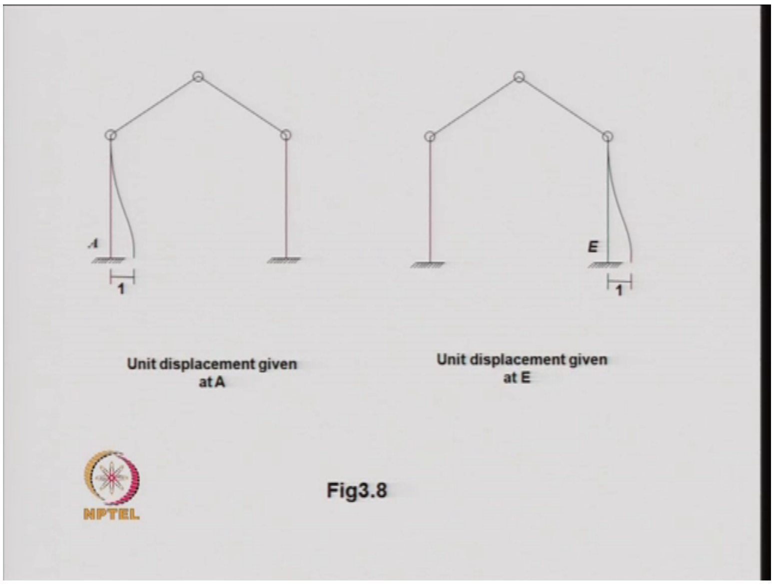
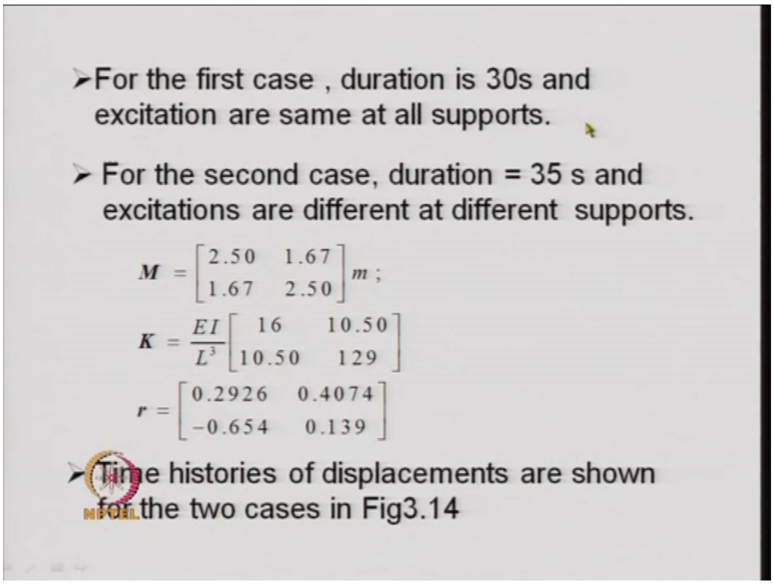


Fig3.8

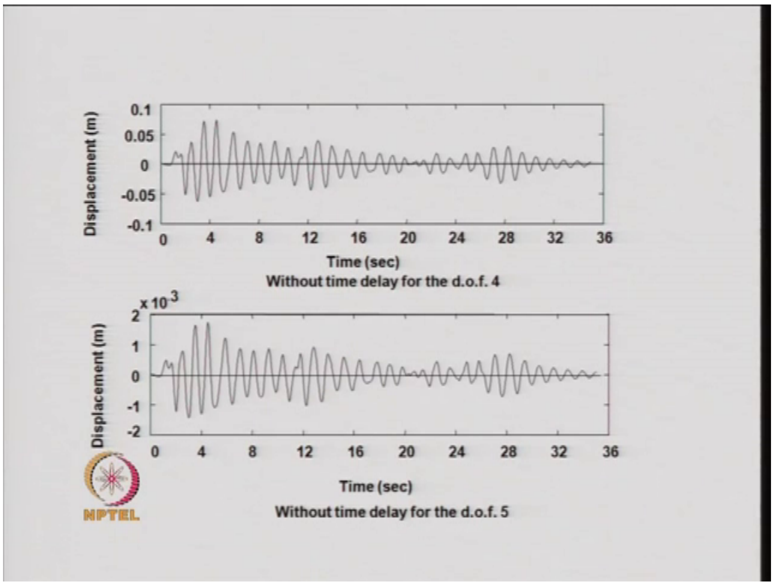
And this is how we obtained the r matrix if you recall that is we give a unit displacement over here and find out what is the forces developed here and here and that gives the stress stiffness matrix and from that one can obtain the value of the displacements caused at this points due to the unit displacement similarly if we give a unit displacement over here one can find out what is the vertical displacement here and what is the horizontal displacement over here and they constitute the r matrix that is what we had shown and for this example when you were working out the r matrix for different structures and the degrees of freedom; let me repeat again is the horizontal degree of freedom here one and this another horizontal vertical degree of freedom over here these are the 4 and 5.

(Refer Slide Time: 11:21)



So, we calculated the mass matrix this calculation must shown; how did we calculate this mass matrix for the pitched roof portal frame this is the stiffness matrix condensed stiffness matrix corresponding to the 2 translation and degrees of freedom and this is the r matrix that we obtain for the 2 degrees of freedom again. So, now, by writing down the equation as m x double dot plus c x dot plus k x is equal to minus m r into x double dot g with the using that.

(Refer Slide Time: 11:56)



We obtain the response of the system, but before that let me tell you that in this case although the duration of ground motion earthquake record is 10 second, but the duration of excitation at the 2 supports are taken as 35 seconds because there is a 5 second of time delay between the 2 support that is what is a assumed therefore, in the first case there will be the first 30 seconds will be non-zero values and the la end 5 seconds will be 0 values for the second support the you know first 5 seconds will have 0 values and the next 30 seconds will have non zero values that is how we construct the 35 seconds of excitation for the 2 supports and we use this 2 excitations and obtain the values.

Now, in this problem the; it was solved for 2 cases in one case we consider a time delay and in another case we obtain that there we assume that there the there is no time delay between the 2 supports. So, this is the result shown for the without time delay case that is in both the supports, we had the same ground motion of 30 seconds and we continued the integration till 35 seconds. So, after 30 second the responses that we see is due to the few vibration of the problem and the u 1 rather the degree of freedom 4 that is the horizontal degree of freedom we see that the ground displacement or the displacement of the response is about 0.075 whereas, the vertical response at degree 5 that is of the order of 2 to the power 10 minus 3 it is expected because since the ground motion is in the horizontal direction therefore, the vertical response of the structure would not be very high and therefore, we get a low value.

(Refer Slide Time: 14:29)

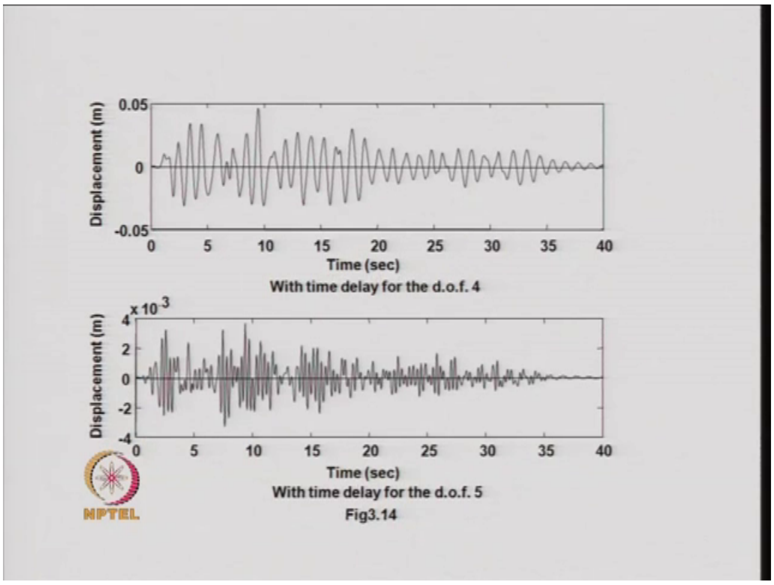
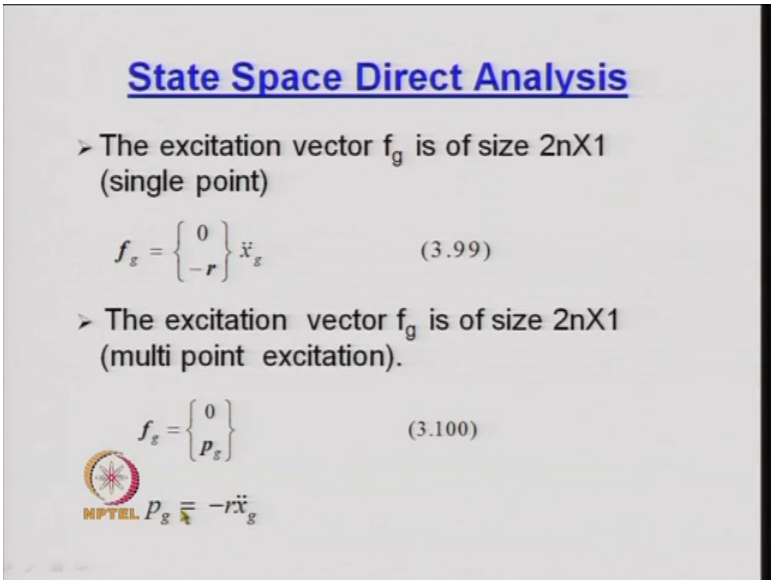


Fig3.14

This is the scenario when we have got the time delay effect considered in the analysis and we see that the there is a difference of course, between this case and the previous case that is in the previous case we had 0.075 almost at the maximum value here the maximum value it goes up to 0.048 or so.

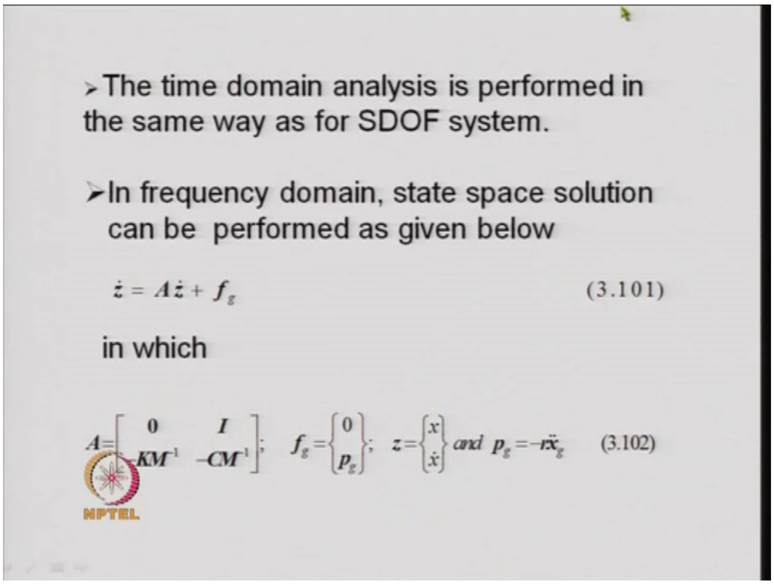
So, we see that there is a with time delay the response is reduced that is expected whenever there is not a perfect correlation of the ground motion between the 2 supports the responses at least the displacement responses generally is decreased and the, but the important thing is that we see that the vertical in the vertical degree of motion the responses has increased that has become nearly equal to 4 into 10 to the power minus 3 wherein the previous case the; it was almost near 2 into 10 to the power minus 3. So, we see that the horizontal in the horizontal direction the response of the structure is decreased due to the time delay effect whereas, in the vertical direction in which the ground motion is not acting. So, in that vertical direction the response of the structure is increased.

(Refer Slide Time: 16:17)



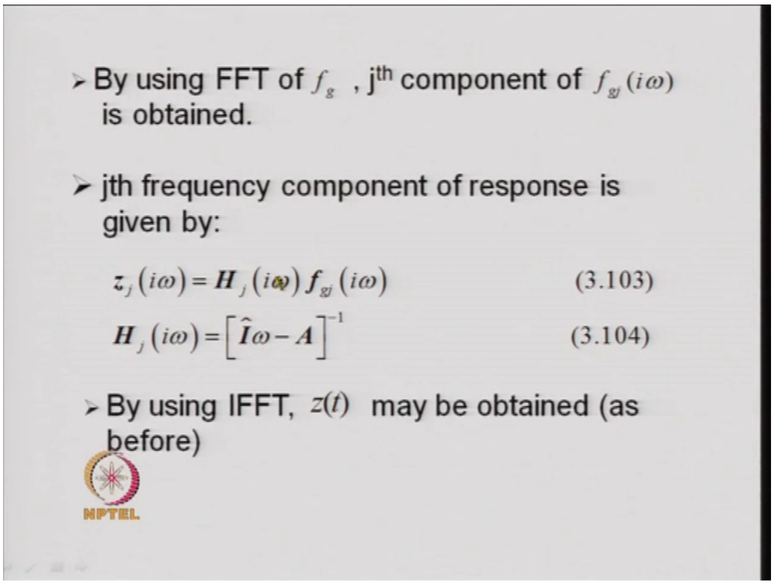
Next let us come to the state space direct analysis and we have written down the equation of the state space before and in that we have an a matrix and f matrix and the form is equal to z dot is equal a z plus f if you recall where a is a matrix z is the state of the system consisting of x and x dot that is a displacement and velocity. Now here this is for the single point excitation system this is a not r, they should be i that is influence coefficient vector and this is for the multi point excitation system this is becomes r; r into x double dot g that becomes the force and the first part; that means, these vector is of size 2 n if it is n degree of freedom then the vector will be of size 2 n. So, first n values will be 0 and the last n values will have minus r minus i into x double dot g over here and here it will be minus r into x double dot g and how we construct r into x double dot g that we have explained in the previous problem.

(Refer Slide Time: 17:42)



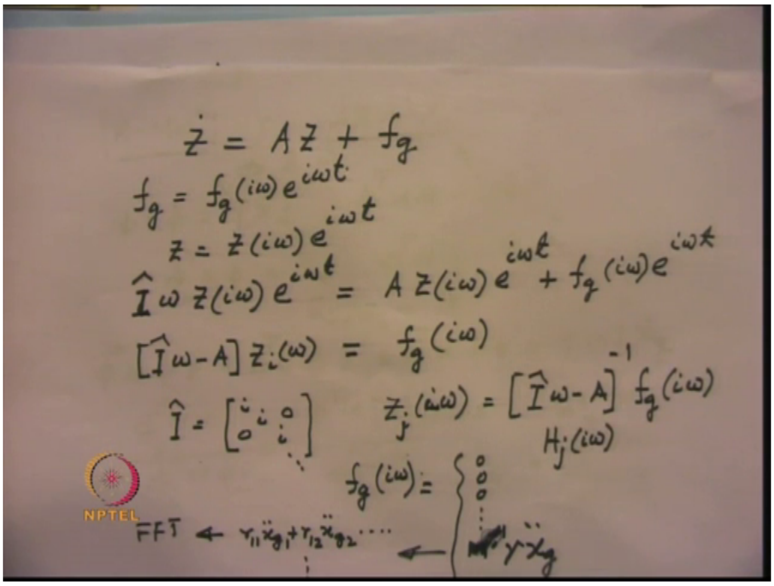
So, we can solve the problem for the system in time domain as well as in frequency domain and in the time domain one can solve the problem in using the Newmarks beta method or Duhamel integration for a single degree of freedom system, but when we solve the same problem in for multi-degree of freedom system generally we are not able to solve the problem using the Duhamel integration. So, there is a problem and therefore, we generally do not try to solve the problem using Duhamel integration and for multi-degree of freedom system when we try to solve it is in the state space form.

(Refer Slide Time: 19:06)



So, in that case we generally use some other integration scheme than the Duhamel integration these problem in the state space can also be solved in frequency domain and in the frequency domain what we do is that we try to find out the H matrix that is the frequency; frequency response function of the system. So, let me first clarify this.

(Refer Slide Time: 19:26)



Say this is the equation z dot is equal to A z plus f g and we say we are able to Fourier synthesize this f g which is described in time mind you f g is a vector and the top values are 0 and the last values have got non zero values corresponding to the time history.

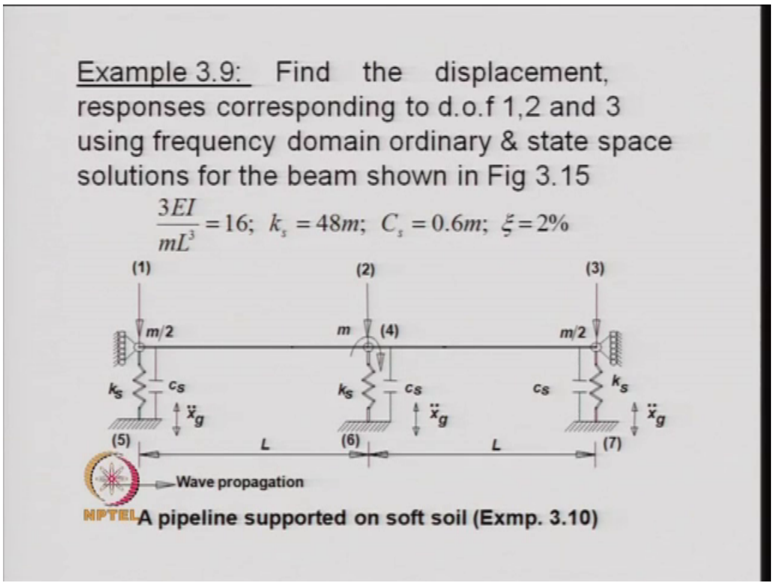
Then this f g t can be Fourier synthesized and can be written in this form f g i omega into e to the power i omega t f g i omega can be easily obtained from using the FFT algorithm that we discussed before now if the excitation is of this form then it is reasonable to assume that the steadied state solution of the system be also of this form z will be equal to z i omega into e to the power i omega t. Now if we substitute this into this equation that is we differentiate this once then the differentiated form will be equal to this capital I cap omega z i omega e to the power i omega t where the i cap is nothing, but a matrix with all diagonals at imaginary val imaginary quantity i i i and the non diagonal components are all 0. So, this is the definition of your i cap with omega multiplied by z i omega into i e to the power i omega t. So, that becomes the z dot and in the case of z we simply substitute this value.

So, we have got a multiplied by z i omega e to the power i omega t and for f g we write down the FFT of f g, then if I take this onto the left hand side, then we get this particular matrix i cap omega minus a into z z i omega that becomes equal to f g i omega because e to the power i omega t cancels from both sides and then one can write down for a particular frequency say a jth frequency if we can write down z j i omega to be is equal to i cap omega minus a inverse of that into f g omega and this is what is called H j i omega capital H j i omega for the jth frequency and f g i omega can be obtained from the FFT that I told before and how we get the individual components or the elements of this vector when we have the r matrix that we had discussed before.

That means, the one element typically one element of this vector would be of the form of r 1 1 1 x double dot g 1 plus r 1 2 into x double dot g 2 and so on that is that will be the one element and in time one can compute this and can get a particular value in time and that is how we can have all the elements values of the elements at every instant of time t that we feed in the FFT. So, we have to now carry out FFT for the n number of the time histories because here will corresponding to all n degrees of freedom will have one excitation, will have one time histories which are inputted into the FFT algorithm to find out the vector of f g i omega.

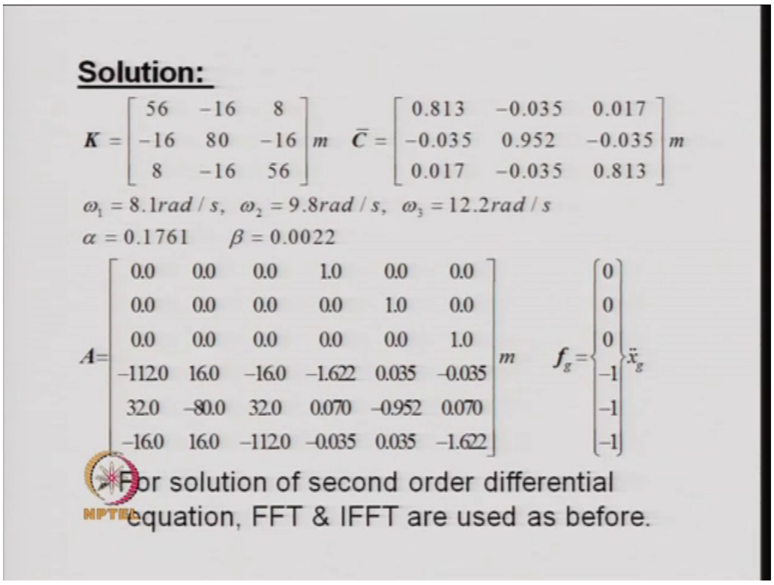
Now, with this background. So, for the jth response quantity in frequency domain we can write down this relationship and h j i omega is defined like this that is what we explained before and then once we get the value of z j i omega then for different values of omega we obtain this z j till the Nyquist frequency or the cut off frequency and then after that we add on the complex conjugate values that is again what we discussed before and that entire values of or the values of the z j omega at a interval of delta omega. So, all the values including this com complex conjugate values that are given an i FFT and the i FFT gives the value of z t and the z t contains not only the displacement, but also a velocity as well.

(Refer Slide Time: 25:03)



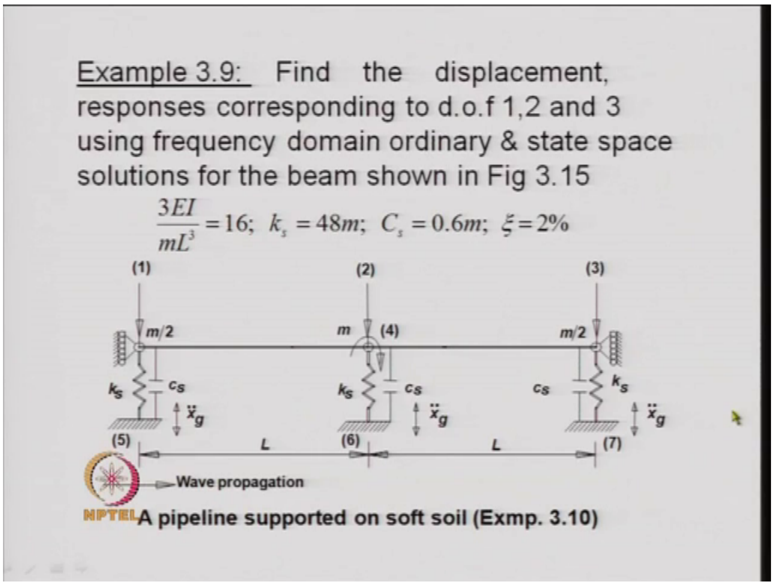
So, that is how one can solve the problem in state space in frequency domain. So, now, we look into another example in which there is a pipeline and in this pipeline is lying on the ground the resistance provided by the ground is represented by this springs over here and the damping provided by the soil that is shown here as a dash board. So, at this 3 supports we provide this equivalent spring and dashboards for the soil and these are the masses which are lamped at these 3 points and we have got 3 degrees of freedom in the vertical direction 1, 2 and 3 and a rotation here. So, what we do first is that we condense this the rotational degree of freedom we condense out and then have only a 3 by 3 matrix corresponding to this vertical 3 degrees of freedom with the these values which are specified over here.

(Refer Slide Time: 26:31)



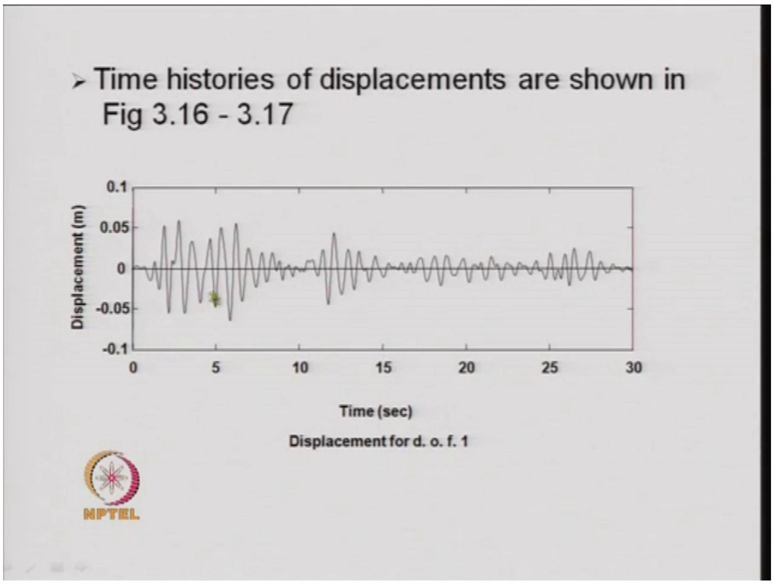
Next we obtained this is the k matrix which is obtained this is the c matrix which is obtained using assuming it to be a classically damped system and in which the values of alpha and beta are this. So, with these values the c matrix is generated then we form the a matrix and these becomes the f g matrix and since we assume.

(Refer Slide Time: 27:08)



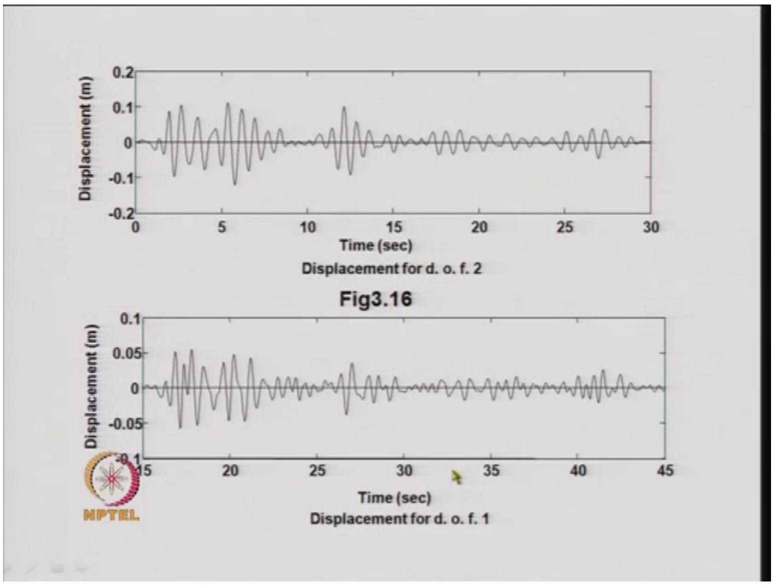
That the ground motions at these 3 supports they are same; that means, there is no time lag between them that is why 1 1 1 over here this is minus 1 1 1 that is the last 3 values are this and the top 3 values of course, will be 0, then we Fourier synthesize this f g and once we Fourier synthesize this f g, then one can obtain the values of the responses of the 3 displacements in frequency domain that it z z j omega and using i FFT, we obtain the response of the 3 degrees of freedom in time domain.

(Refer Slide Time: 28:16)



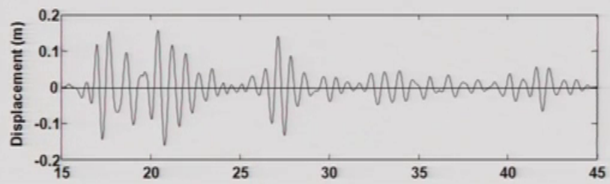
So, this is the time history of the displacements for degree of freedom one that is shown.

(Refer Slide Time: 28:26)



Then we have the time history of displacement at 2 and time history of displacement this is 1 and 2; these 2 are shown and these result is for the solution that we obtained by solving the equation not in the state space, but by solving the problem in the as a second order differential equation there also we can obtain the solution in the frequency domain. So, the objective of this problem was to solve this problem by 2 methods that is using the frequency domain ordinary and state space solution. So, ordinary means that the solution was second order differential equation and the state space solution is the one in which the h j matrix is different than the h j the small h j matrix that we obtained for the second order differential equation.

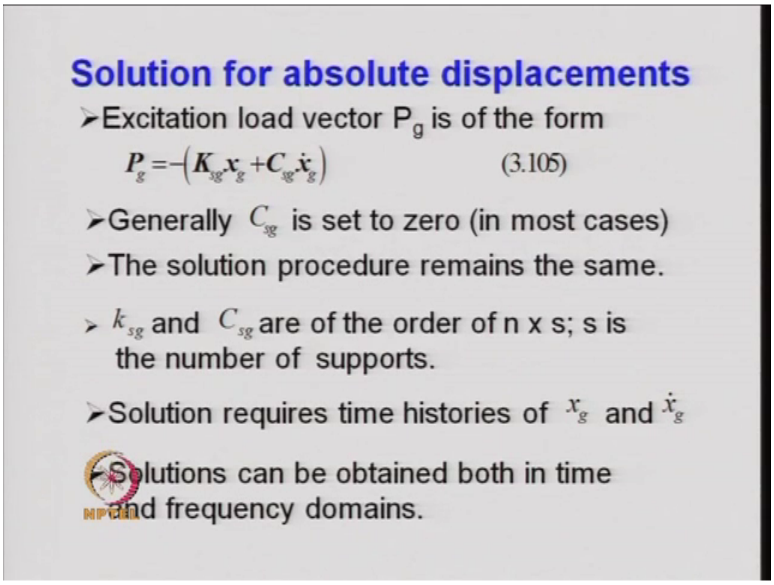
(Refer Slide Time: 29:42)



Displacement for d. o. f. 2
Fig3.17

And the results you can see that the compare very well that is the displacement of degree of freedom one over here has a maximum value of almost 0.05, they are you can see that again the maximum value is almost 0.05. So, the responses obtained by the ordinary solution of the ordinary differential equation that state space equation; they are obtained as same.

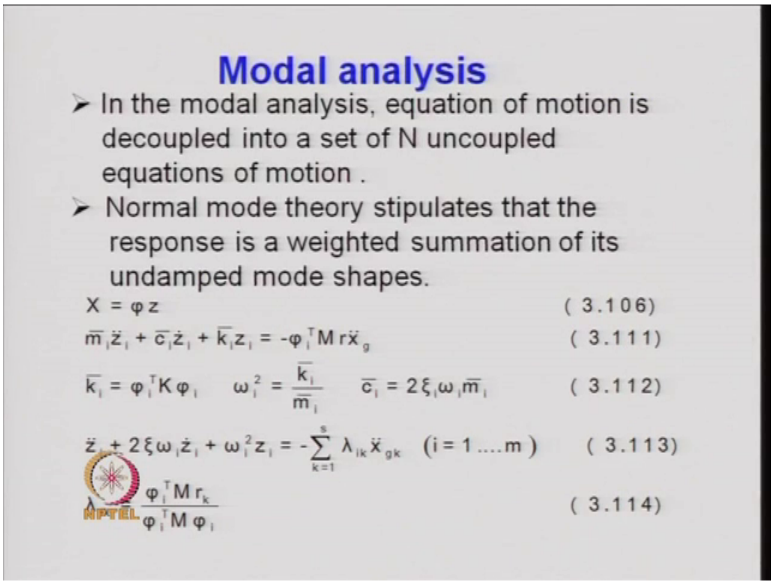
(Refer Slide Time: 30:17)



Next if we are wanting to solve the problem for absolute displacement then we use a different equation and in that differential equation on the left hand side of the equation you have all the terms written in terms of the total displacement that is total displacement total velocity and total acceleration on the right hand side instead of the acceleration we require the knowledge of displacement and the velocity that we discussed before and generally we assume that this C s g that is the coupling term in the damping matrix they are generally set to 0 because that is very small and in most cases we only consider p g to be is equal to minus K s g into x g.

So, by knowing not the ground acceleration, but the ground displacement one can obtain the value of p g vector over here and once we know the p g vector, then we can solve the problem either as a second order differential equation or an in the state space and we can use whatever method we want that is we can solve within a time domain or in the frequency domain.

(Refer Slide Time: 31:55)

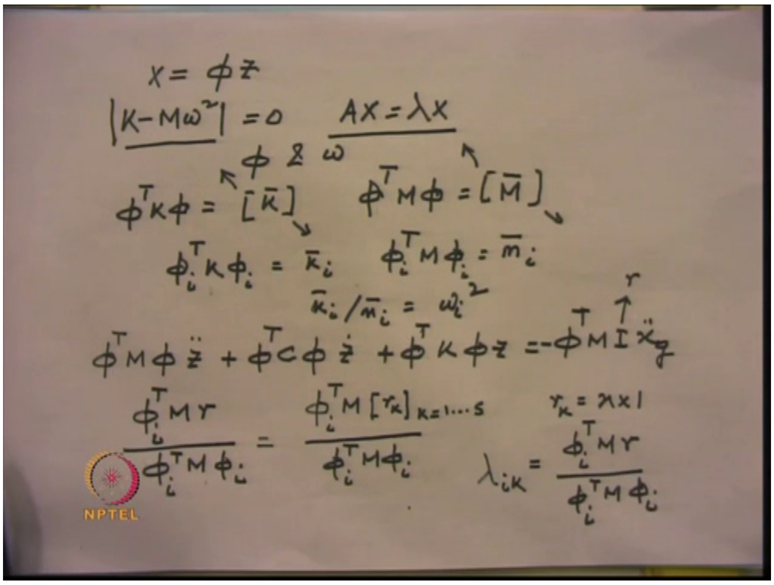


Now, we come to what is known as the modal analysis which is very popular in the dynamics in the beginning of the multi-degree freedom system I mentioned that the solution for the multi-degree freedom system can be obtained for 2 kinds of excitation one is for the single point excitation other for the multi point excitation again for both types of excitations we have a direct solution and the solution obtained by modal analysis when we use the direct solution, then we require the C matrix to be defined and for defining the C matrix, we assume that the damping matrix is a is of classical nature or we say that the it is a classically damped system and when we assume the it to be a classically damped system then c matrix is written and to be is equal to alpha times m plus beta times k where alpha and beta values are obtained for using the 2 frequencies of the system generally the first 2 frequencies are considered to obtain the values of alpha and beta.

So, once we have the c matrix for the system then one can go ahead with the direct solution either solving it in as a second order differential equation or by solving it as a coupled up first order differential equation which is called the state space equation and one can use both time domain solution and the frequency domain solution in the time domain solution for multi-degree of freedom system we generally adopt Newmarks beta method for solving the problem Duhamel integral is somewhat what you call complex if we extend it to the multi-degree of freedom system.

Now, we come to the modal analysis and in this analysis we try to take advantage of the properties of the mode shapes or undamped mode shapes of this structures and i am sure all of you know from your the knowledge of your dynamic analysis how we carry out the modal analysis, but still for recapitulation; let me try to summarize what you have already learnt in your dynamics for the modal analysis we write down the displacement that is the x or the dynamic displacement of a multi-degree of freedom system that as equal to a phi matrix that is the mode shape matrix multiplied by z which are known as the generalized coordinates or sometimes also known as the modal coordinates.

(Refer Slide Time: 35:06)



So, with this defined we go to the main second order differential equation we substitute in the main equation the phi into z in place of x.

So, once we do that then these becomes m into phi multiplied by z double dot the c matrix becomes c into phi into z dot and this one becomes phi k into phi into z. So, in place of x we write phi z here in place x dot we write phi z dot and here in place of x double dot we write down phi z double dot once we do that; then we pre multiply by their transpose of the phi matrix and once we pre multiplied by phi t, then here we get a term as phi t here; we get phi t here; we get phi t and on the right hand side also we multiplied by phi t, now for a single point excitation system.

It will be i for a multipoint excitation system it will be r, then the comes the property of this phi T m phi and phi T c phi and phi T k phi the mode shapes of the undamped system obtained by solving the Eigen value problem that is K minus M; M omega square absolute value of that will be equal to 0 or in standard form it is written as a x is equal to lambda x and we give this a matrix into in any standard program for Eigen values, then the it gives the Eigen values and Eigen vectors of matrix a. So, those Eigen values and Eigen vectors of matrix a, those Eigen values and Eigen vectors are nothing, but the mode shapes and the frequencies of the system.

Now, the property of this Eigen vectors or the mode shapes of that it is orthogonal with respect to k matrix and it is orthogonal with respect to mass matrix orthogonality condition means phi T k phi will lead to a diagonal matrix that is we will this will be having only a diagonal values as non-zero all of diagonal values will become 0. So, this turns out to be a diagonal matrix similarly m matrix becomes a diagonal matrix because this is also orthogonal with respect to the mass matrix. Now if one assumes that the c matrix is proportional to the alpha times m or is proportional to mass and stiffness matrix then one can write as c is equal to alpha into m plus beta into k therefore, phi T c phi also will become diagonal because phi T k phi will be diagonal phi T m phi will diagonal. So, therefore, this phi T c phi also becomes diagonal.

Now, if the left hand side these things becomes diagonal then we can see that the in the left hand side the equation of motion becomes uncoupled; uncoupled in the sense that only you have diagonal terms over here. So, the each one of this equations are independent of each other that is one can write down a equation only in the form of this that is this m bar i into z double dot i c bar i into z dot i k bar i into z i this is the ith equation and this is a single variable z i is a single variable and one can write down such equation which is consisting of n number of equation that is i varies from one to n where n is the degree of freedom for the multi-degree of freedom system.

So, this is the basis of the modal analysis and if the k bar i and m bar i that are taken and we divide k bar i by m bar i then we get frequency natural frequency square right that is the ith natural frequency we get like that and one can write down c bar i that that is to be is equal to twice into twice xi i omega i into m bar i and if we divide all through by m bar i then we land up in a equation in a single degree of freedom equation, in which it is defined with respect to frequency of the mode ith mode and the damping of the ith mode generally we assume the constant damping for all the modes therefore, we do not write down here xi i, but we simply write xi indicating that it does not vary with modes.

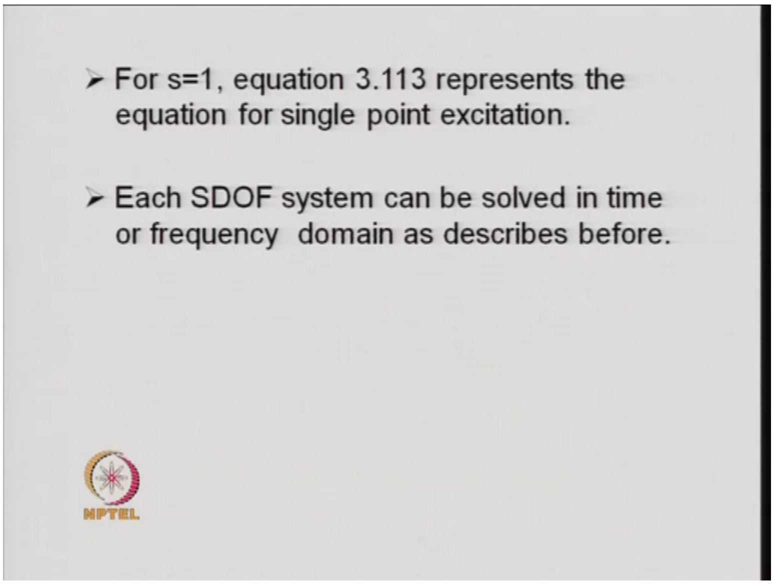
Then on the right hand side the entire thing this is phi i T m r into x double dot g that can be written as a summation of lambda i k multiplied by x double dot g k now the k indicates the supports; that means, in a multi-degree of freedom system if you have got s number of supports then this s number of supports can be considered in the analysis and this and at this n supports we have different ground accelerations that is represented by x double dot g k; k stands for the number of this support. So, this x double dot g k varies from support to support and lambda i k is a quantity which also varies from support to support. So, here we have got lambda i k defined as phi i t multiplied by m into r k where r k is the ith column of the r matrix.

So, divided by phi i T m phi i. So, that is the definition of lambda i k. So, it is support dependent that is it has an index k. So, this summation is done for all the supports over here and we generate a time history of the ground excitation for the structure; that means, for the entire structure for the ith generalized coordinate we will have only one time history of excitation which will be called the generalized excitation for the ith mode and they are the effect of the different types of excitations at different supports will be all summed together to find out the time history of the generalized excitation at the ith mode.

So, now we can solve this equation written in terms of the general coordinate or we call this modal equation we can solve as a single degree of freedom equation either in time domain or in the frequency domain in time domain again we have option we can use Duhamel integration or we can use new mark beta method or we can use any other integrations scheme let me just try to elaborate this lambda i k if the ground acceleration is not varying from support to support; that means, at all supports if you have got the same excitation time history or the ground acceleration then in place of r k it becomes i that is the influence coefficient vector 1 1 1; in that case, it becomes the k it becomes independent of k and we have a unique value of lambda i defined for the ith mode.

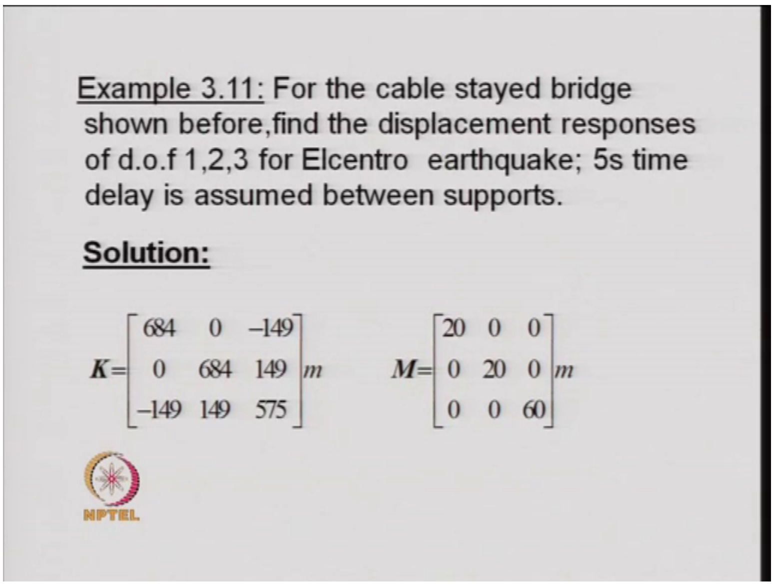
So, that quantity is known as the mode participation factor that has been discussed I am sure in your dynamic analysis class. So, the for the single point excitation you have this mode participation factor as a single quantity, but if it is a multi support excitation system then there is not a single value of the mode participation factor these mode participation factor then becomes support dependent; that means, for each support we have one mode participation factor defined.

(Refer Slide Time: 46:35)



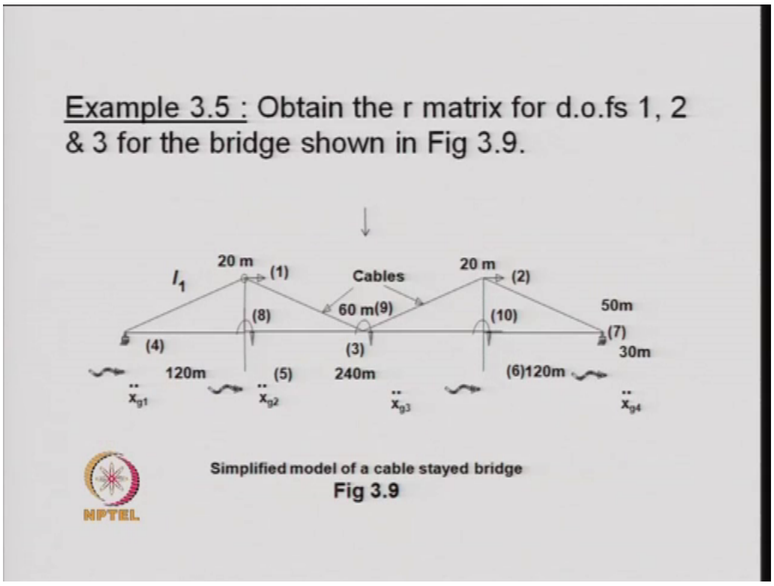
Obviously if s is equal to 1 that is if you have got single support excitation then it these generalized formulation can be made what to call made same as single point excitation system.

(Refer Slide Time: 46:57)



Now, let us look into the example an example solved for this multi support excitation system with the modal analysis. So, here we take the example of the cable state bridge if you recall in that cable state bridge we had the many supports or rather 4 supports were there in the cable stayed bridge; let me show you the diagram for that for your recapitulation yeah.

(Refer Slide Time: 47:48)



This is the cable supported bridge that we had taken up in order to obtain the r matrix and in that you can see that we have got one support second support third support and fourth supports at this 4 supports the ground accelerations were different and the degrees of freedom that we consider were 3 degrees of freedom that is at the 2 top of the towers or the at the at the top of the 2 towers, we have got 2 horizontal degrees of freedom and a vertical degrees of freedom is considered at the center of the depth.

So, these were the 3 degrees of freedom dynamic degrees of freedom that we considered. So, we condensed out all the rotational degrees of freedom then we obtain a equation of or obtain an expression for the stiffness matrix and the mass matrix corresponding to this translational degrees of freedom and then the found the stiffness matrix we again take took out the 3 nonsupport degrees of freedom that is 1, 2, 3 and the rest were the support degrees of freedom and from there, we constructed the r matrix for the system and the r matrix for the system was obtained like this this was the r; r matrix corresponding to all the 4 support degrees of freedom and the r matrix; obviously, was 3 by 4.

(Refer Slide Time: 49:17)

- Using them the above matrices the r matrix is obtained as

$$r = -\begin{bmatrix} -0.781 & -0.003 & 0.002 & -0.218 \\ -0.218 & 0.002 & -0.003 & -0.781 \\ -0.147 & -0.0009 & 0.0009 & 0.147 \end{bmatrix}$$

Equation of Motion in state space

$$\dot{Z} = AZ + f \qquad (3.20)$$

in which

$$Z = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}; \quad f = \begin{Bmatrix} 0 \\ -r\ddot{x}_g \end{Bmatrix}; \quad A = \begin{bmatrix} 0 & I \\ -K_{ss}M_{ss}^{-1} & -C_{ss}M_{ss}^{-1} \end{bmatrix} \qquad (3.21)$$

Because the 3 at the non-support degrees of freedom. So, with these r matrix in position we try to solve this problem and the solution for the problem yeah the solution for the problem were obtained with the K matrix that is the for corresponding to non support degrees of freedom that was the K matrix that was obtained and the mass matrix was a diagonal mass matrix because we lamped the masses.

(Refer Slide Time: 50:10)

$$r = -\begin{bmatrix} -0.781 & -0.003 & 0.002 & -0.218 \\ -0.218 & 0.002 & -0.003 & -0.781 \\ -0.147 & -0.0009 & 0.0009 & 0.147 \end{bmatrix}$$

$\omega_1 = 2.86\,\text{rad/s} \quad \omega_2 = 5.85\,\text{rad/s} \quad \omega_3 = 5.97\,\text{rad/s}$

$\varphi_1^T = [-0.036 \quad 0.036 \quad -0.125]$

$\varphi_2^T = [0.158 \quad 0.158 \quad 0]$

$\varphi_3^T = [-0.154 \quad 0.154 \quad 0.030]$

- First modal equation

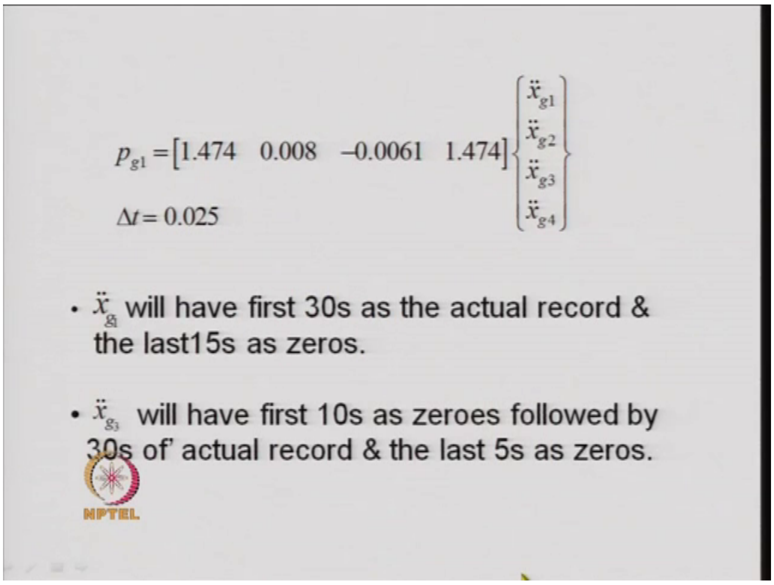
$$\ddot{z}_1 + 2\eta\omega_1\dot{z}_1 + \omega_1^2 z_1 = -\frac{\varphi_1^T M r}{\varphi_1^T M \varphi_1}\begin{Bmatrix} \ddot{x}_{g1} \\ \ddot{x}_{g2} \\ \ddot{x}_{g3} \\ \ddot{x}_{g4} \end{Bmatrix}$$

At the 3 degrees of freedom and then this was the r matrix that i had shown you before and then omega one omega 2 omega 3; 3 frequencies are obtained the phi 1 T phi 2 T, the phi 3 T that is the 3 mode shapes well like this and then we have the first modal equation that we can write in this particular form.

Now, the you can look at the nature of the mode shape it is interesting to see the nature of the mode shapes the nature of the mode shapes is the first mode basically is such that you have these are the first 2 degrees of freedom that is at the top of the pylons and you can see the in the top if the pylons if one is moving in this direction the other moves in the this direction that is you have a situation like this and with the; at the center we have a downward upward displacement.

So, the phi 2 basically were such that both the pylons are moving in the same direction and there was no what to call displacement at the center and this basically is again a mode shape in which the both pylons moved in the opposite direction and the deck center of the deck moved down rather than up. So, these were 3 mode shapes that were obtained. So, using these 3 mode shapes. So, we write down the 3 modal equation and for this 3 modal equation this r phi i phi 1; T m m and this one will be equal to r 1 not r; r 1 that multiplied by g that would give the rho g 1.

(Refer Slide Time: 52:24)



So, this is the value of the generalized ground acceleration or generalized excitation of the first generalized equation and we take delta t defined as 0.02 phi, these thing can be in integrated, but in order to obtain the values of x double dot g 1 x double dot g 2 and x double dot g 3 and g 4 for that we take the ground acceleration of 30 second which is travelling and there is a 5 second of time delay between the supports as a result of that you will have for the first support the 30 seconds of the record first 30 seconds will be non-zero and rest 15 second will be 0 values. Similarly for the second support you will have first 5 seconds as 0s, then you will have got 30 second of the record and a last 10 second will be 0. So, that way we can construct the different excitations at different supports from the 30 second of the actual earthquake that is travelling and with a 5 second time delay between each support.