Seismic Analysis of Structures
Prof. T.K. Datta
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture - 14
Response Analysis for Specified Ground Motion (Contd.)

In the previous class we were discussing about an example problem.

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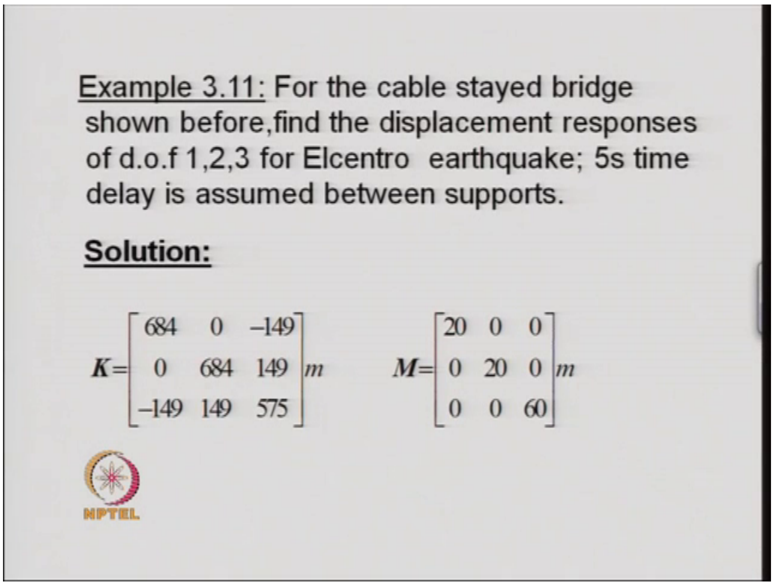

In which we are wanting to find out the displacement response of the structure using model analysis technique. The problem was that of a cable stayed bridge. In which these cable stayed bridge we had solved before for finding out the value of the r matrix. And in that the degrees of freedom those were considered as the degrees of freedom which are dynamic or dynamic degrees of freedom that we identify.

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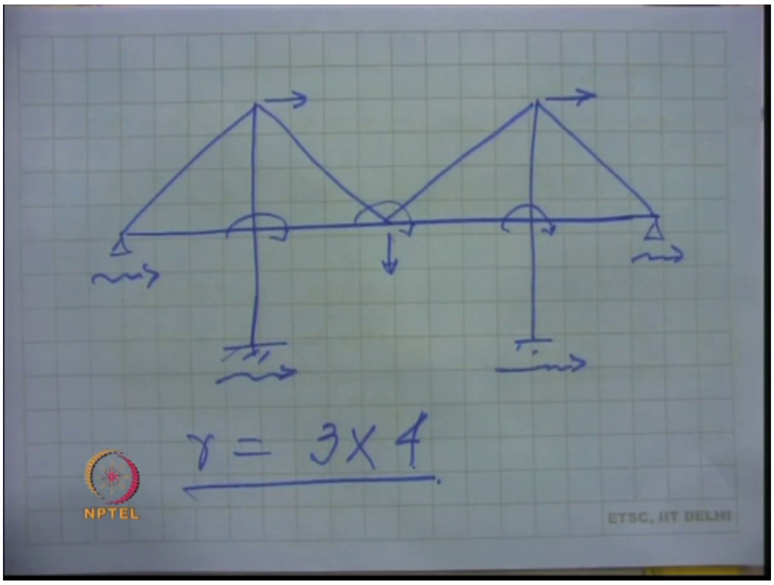

Are the these two displacements at the top of the tower and a vertical displacement at the middle of the deck. There were 4 support degrees of freedom and 3 rotations. So, we condensed out the rotational degrees of freedom first and they then formed a 4 plus 3, 7 by 7 matrix. Out of that again we took out the three non support degrees of freedom that is partitioned at stiffness matrix. So, that the three non support degrees of freedom are identified. And from there we obtained a r matrix that is the influence coefficient matrix we described before.

Now, using that influence coefficient matrix, now we are wanting to solve this problem using the model analysis technique. And one can see here that the matrix k for the 3 degrees of freedom are the least matrix that is 3 by 3 matrix.

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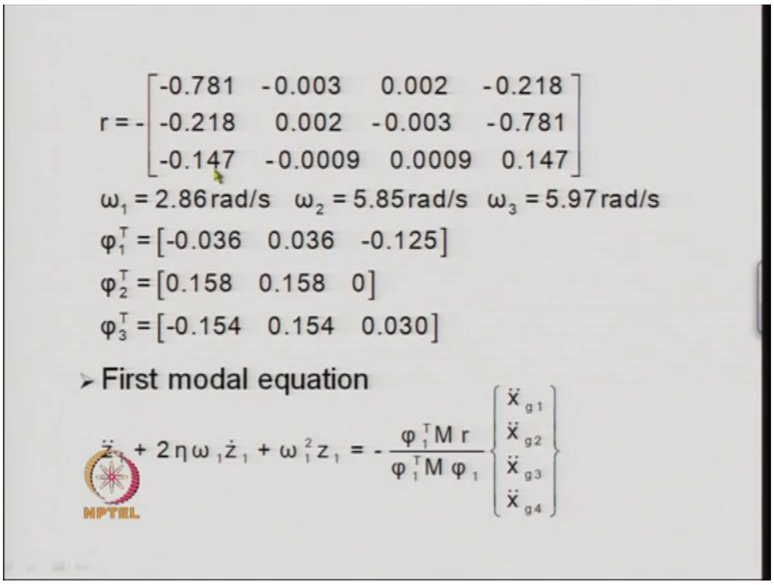

And the mass matrix again is a 3 by 3 diagonal mass matrix. The r matrix that you had already obtained for this cable stayed bridge that is shown over here 3 by 4 r matrix, the 3 frequencies that we obtained for the structure was omega 1, omega 2 and omega 3. Corresponding to each frequency we had a mode shape that is phi 1 T, phi 2 T and phi 3 T shown over here. You can see that the first mode shape is such that the 2 pylons are having displacement in the opposite direction that is 1 in the negative with the negative sign other with the positive sign, but by the same amount and the deck is going up rather than the rather than going down.

The second mode is a mode in which both the pylons move in the same direction and there was no displacement at the centre of the day, third mode is such that in which the 2 pylons again move in the opposite in the opposite direction and the deck instead of going up goes down. So, these are the 3 mode shapes for the structure. So, using that mode shape 1 can obtain the 3 general model equations the first model equation is written in this form and here we have phi 1 T and this is phi 1 t m phi 1 that is we take the first mode shape.

And here these r basically would be the first column that is the first column of this r matrix. So, with this we obtain the phi it a M r and we get these as the quantity that we obtain.

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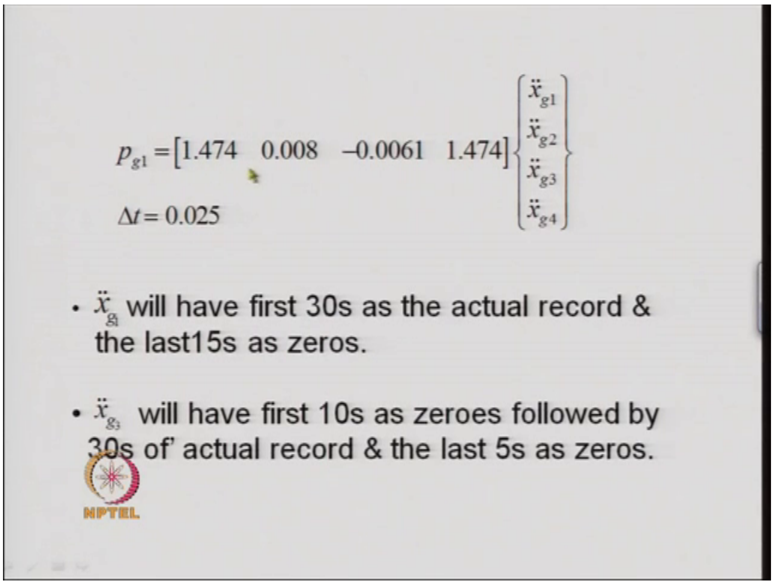

Once we multiply m into r when you multiply that then we get this quantity and with these quantity we found the P g 1 matrix that is informing the P g 1 matrix we multiply 1.474 into x double dot g 1 plus 0.008 into x double dot g 2 and so on and them together to get a the load at a particular time t for the P g 1 load.

So, this P g 1 is a time dependent load which is called the generalized load and at every instant of time t one can obtain a value for this using this equation, delta t is taken as 0.025. So, far as the accelerations at the 4 supports are concerned we construct this time history of acceleration at the different supports from the 30 seconds of actual earthquake record since we have 4 supports.

So, we have got 3 spaces in between and if we assume that 5 second is the phase time lag between the 2 supports then the total record of the excitation would be 30 plus 15 second that is 45 second, x double dot g 1 will have first 30 seconds as the actual earth quake record and rest of the 15 seconds will have 0 values. Similarly x double dot g 2 we will have first n seconds as 0 values then there will be 30 second of the actual earth quake record and the last 5 second will be again 0 values that way we can obtain the time histories of excitation for all the supports and then we perform the analysis using the model analysis technique.

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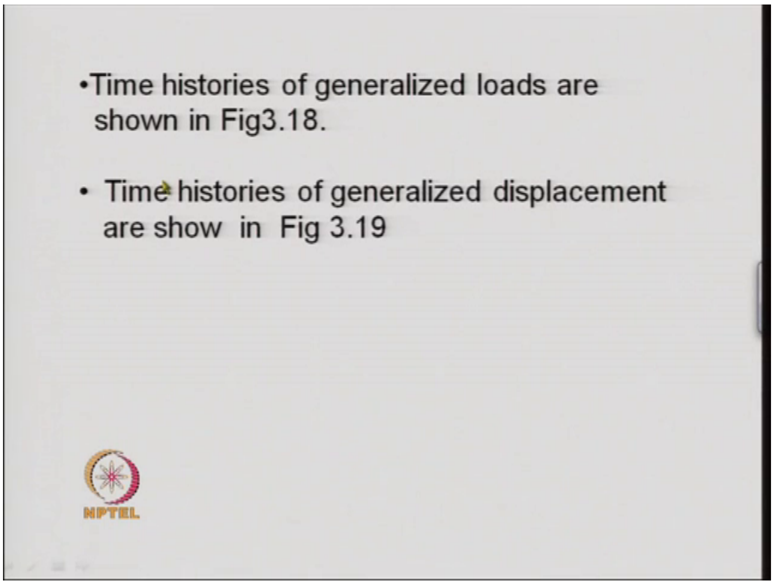

The results are shown over here.

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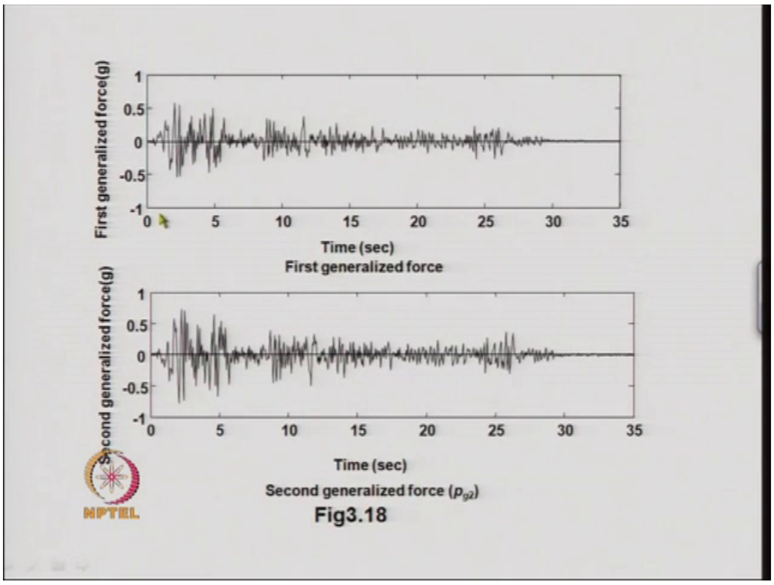

This is the first generalized force the time history of that is plotted over here, this is the second generalized force it is plotted again over here and 1 we have taken a plot up to 35 second. In fact, you can see that after 30 second these values of the generalized load are very small.

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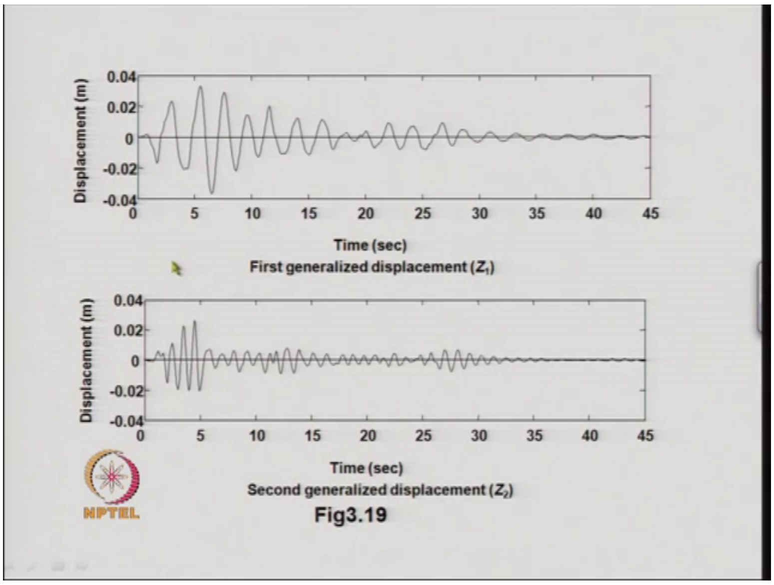

Fig3.19

We have the first generalized displacement time history like this and we can see that after n 30 second the displacement decreases and continues up to 45 second that is the total length of time of excitation at each support. Therefore, we have the responses after 45 seconds this is the second generalized displacement that is again shown over here.

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Table 3.4 Comparison of generalized displacement

Solution	Z_1		Z_2		Z_3	
	rms (m)	peak (m)	rms (m)	peak (m)	rms (m)	peak (m)
Time history	0.0091	0.0369	0.0048	0.0261	0.0044	0.0249
Frequency domain	0.009	0.0368	0.0049	0.0265	0.0044	0.0250

We solve the problem both using the time history analysis and the frequency domain analysis using model analysis technique. So, the results were compared for the generalized displacement z 1, z 2, z 3. So, we took all the 3 modes as a result of that the model analysis and the direct analysis would not show up any difference. If you had taken less number of modes for solving the problem there would have been a difference between direct analysis and the model analysis, because in the direct analysis we take the contributions from all modes. Whereas, in the model analysis the number of modes that you consider in the analysis depends how we choose the number of modes that is considered to be important.

Therefore, many a time we do not take contribution from all the modes and we may be taking the first contributions of first few modes, in this particular example we have taken all the 3 modes and the solutions obtained from the time history and frequency domain analysis they were compared over here and we can see that both in terms of rms value and the peak value the results are matching quite well. The reasons for this is that while performing the time history analysis we assume 0 displacement and 0 velocity in the beginning, that is the initial condition was 0 and 0.

Therefore, the transient part of the solution was not significant at all and that the time history and frequency domain analysis they messed very well in terms of rms and peak values.

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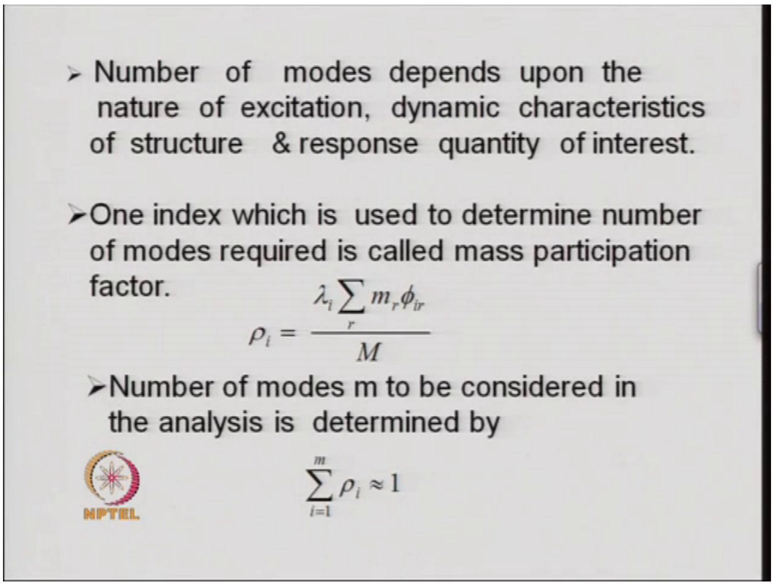

The crucial point in the model analysis is the number of modes that is to be taken into analysis for example, if we have a structure having 100 degrees of freedom then we will have 100 mode shapes and 100 frequencies and one of the objective of the model analysis was to obtain a problem which would be uncoupled problem that is a coupled differential equation need not be solved. So, we should we shall solve a single degree of freedom equation and we wish to also solve as less number of equation of motion as possible, yet at the same time we should get a good result that is the result which is almost correct.

So, with this end in view the number of modes that is considered for the analysis that depends upon the nature of excitation, dynamic characteristics of the stress structure and the response quantity of interest. Looking at these 3 quantities we decide how many number of modes we should consider in the analysis lesser the number of modes will require less computational time, one index which is used to determine the number of modes required for getting fairly accurate result is called the mass participation factor and the mass participation factor is what tau using this expression. Here lambda i this is equal to the mode participation factor m r is the mass attached to the r-th degree of freedom and phi ir is the mode shaped coefficient in the i-th mode for the r-th degree of freedom.

So, if we sum this over the all degrees of freedom then we get the mass participation factor for the i-th mode and these summations at the top that we obtain that is of course, multiplied by the total mass of the system. So, this shows the ratio between the total mass of the system and the mass that that is considered in the analysis through the number of modes that we are considering in the analysis.

So, that is why it is called a mass participation factor that is in the i-th mass, i-th mode what is the percentage of the mass that is considered in the analysis, the number of modes in to be considered in the analysis is determined by this equation. That means, for each mode we find out the mass participation factor sum them together over the number of modes and when we get it to be more or less equal to 1 then we decide about what should be the number of modes that we should take into consideration. Generally, it is found that if we take say first say first or first 10 modes in a structure we are able to take into consideration about 95 of the mass of the system.

So, in that case we satisfy our self by considering 5 to 10 modes of a very large structure. And therefore, for a large structure we benefit by reducing the calculation because we have to then solve 5 or 10 single degree of freedom equation to get the final answer.

Now, this is so far as the displacement response is concerned if the bending moment shear force or any other quantity is required then not necessarily that this particular concept would work. Because the for example, for bending moment the participations of the higher mode is very important, because the higher the number of mode the more curvature we get into the mode shades and since the curvature influences the bending moment therefore, we should not miss out higher modes.

So, there this general guideline is applicable for finding out a good displacement response.

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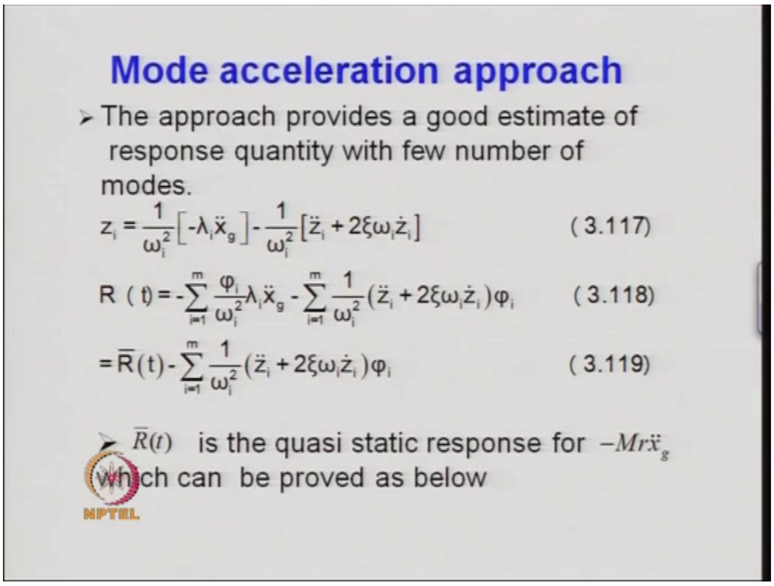

Now let us say come to a another method called mode acceleration approach the it was evolved because of the fact that we wish to obtain a very good response of any type that is whether bending moment shear force or displacement with less number of modes that is our intention.

Therefore, the classical mode summation approach that is the classical model analysis technique is improvised to obtain a method which is called a mode acceleration method or mode acceleration approach, it is found that these mode acceleration approaches provides a good estimate of the response quantity with few number of modes and the reason for this is shown over here. If we write down the equation of motion for the i-th mode then the equation would look like this.

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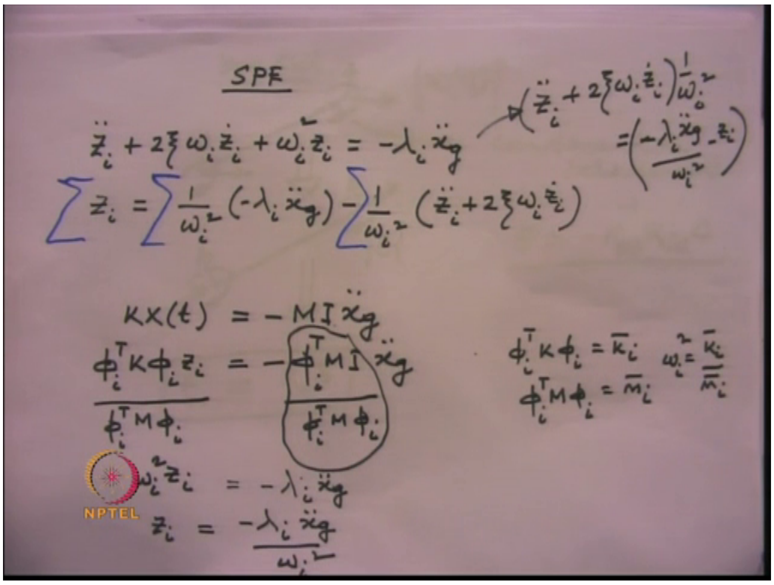

Where a lambda i is the mode participation factor in the i-th mode Z i is the generalized displacement in the i-th mode, this equation can be now rewritten in this form that is Z i is equal to 1 by omega i square into minus lambda ix double dot g that is we divide this by omega i square then this entire thing is taken on to this side therefore, it becomes negative and we have 1 by omega i square within bracket this enter quantity

Now, we can have now the summation; that means, the Z i the summation of Z i for all the modes over here these summation now can be written like this that is summation of these Z i multiplied by phi i that gives the true response of the quantities provided we take the contribution for all the modes. Now, here what we do is that we do not take the contribution from all modes, but we take contribution for say for the first m modes. So, the summation is done for m modes only. Therefore, this submission is also made for m modes and this summation is also done for the m mode.

Now, let us look at the equation a quasi static equation in which we write down k multiplied by xt that is we are performing in quasi static analysis that is equal to minus m I x double dot g that is the for a single point excitation system that is the force earthquake force that is acting at every instant of time t. So, for this basically we solve this equation we will get a quasi static response of xt not the dynamic response. Now, this equation can be now converted into the model equation, model quasi static equation by multiplying by pre multiplying k by phi it and substituting for x t as phi into z phi i and Z i that is the contribution coming from the first mode.

Then on this side also if we multiply it by phi i and divide this by phi i T and phi i by this that is for the i-th mode we find out the generalized mass and on the right hand side also we divide it by phi i T m phi i. Now, we know that from the model equation or the model analysis that phi i T k phi is equal to k bar l this will be a single quantity and phi i T m phi i is the generalized mass at the i-th mode and if we divide k by k bar i by m bar i then can we get the natural frequency square for that particular mode.

So, these enter quantity over here will turn out to be omega i square Z i and this quantity is known as the mode participation factor for the i-th mode and this actually all of you know from your dynamic analysis. So, we can substitute this and write minus lambda i into x double dot g. So, from this we can get the value of Z i is equal to minus lambda i x double dot g divided by omega i square. So, this is the, if I perform a quasi static analysis for this system then the first mode shape will have a contribution to these quasi static response which can be described by this that is minus lambda i x double dot g omega i square.

Now, if we look into this equation. So, this equation is nothing, but a summation of these quantity that is the first generalized displacements contribution to the quasi static response in the i-th mode and that is summed over for all the modes or n number of modes that is what we said before. Now, if we wish to get a correct value for this quasi static response then instead of summing it up for the m number of modes we should sum it up for all the modes. Now, if we sum it up for all the modes then the response that we get that will be equal to x t and these x t can be obtained from a quasi static analysis of the structure.

So, what we say is that if we wish to find out the response of the system by adding up the contribution from different modes then we can split this response into 2 parts, one is a quasi static part which can be obtained by solving this quadratic equation another is a dynamic part, Now, in this quasi static part it is possible to consider the contribution of all the modes that is all m number of modes because if we instead of solving this, if we solve this quasi static equation then we can straight away get the value of x t and these x t is a quasi static response of the system and 1 can replace these entire summation of course, it should be multiplied by phi i also mode shape coefficient. So, we can replace this entire thing by the quasi static response.

So, that is what is shown over here that is the in the mode acceleration approach we write down the response to be is equal to the phi i that is the mode shape coefficient in the i-th mode lambda x double dot g divided by omega i square. So, this constitute what is known as the contribution to the quasi static response and this is the contribution for to the dynamic response. So, if I sum it up from 1 to m is equal to m that is all modes then it comes to be equal to r bar t that is the quasi static response of the system by solving the equation kxt is equal to minus m mi x double dot g. So, that can be easily obtained from a static analysis and then we consider the number of modes which are less than the total number of modes and we can choose may be first type or 10 modes of a structure over here.

Since in this calculation we have considered 1 part of the response, in calculating 1 part of the response we have taken into consideration contribution of all modes therefore, this response is better than the usual mode analysis technique because in the usual mode analysis technique we consider only the first few modes for obtaining the solution. But here we have divided the response into 2 components. In the first component we consider the contribution from all the modes and that we are calling as r bar t and then here we are considering only first few modes for obtaining this dynamic response. And in most of the cases we see that this part this quasi static parts predominates the response quantity of interest.

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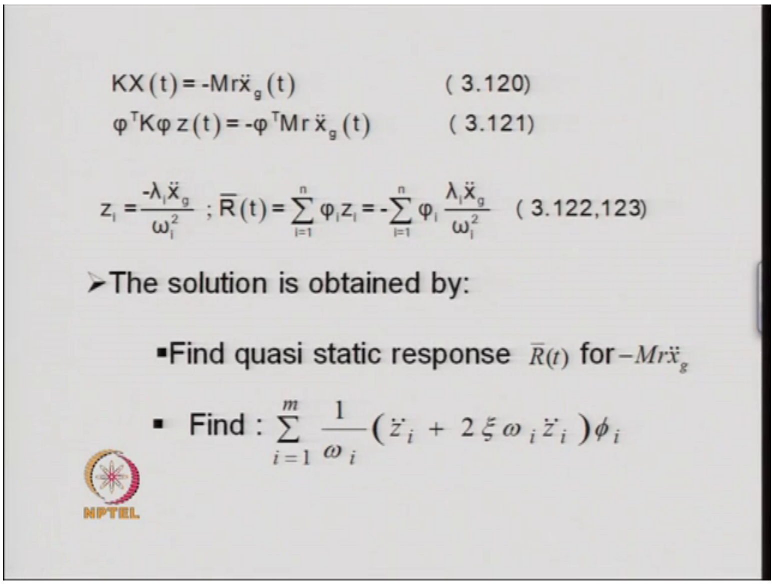

So, therefore, we obtain the response by this mode acceleration technique and we can get a better result by considering only first few modes of the structure. So, the solution is obtained like this first we find out the quasi static response of the system that is we solve the problem statically for a load of minus this will not be r will be i we are because we are talking of a single point excitation system.

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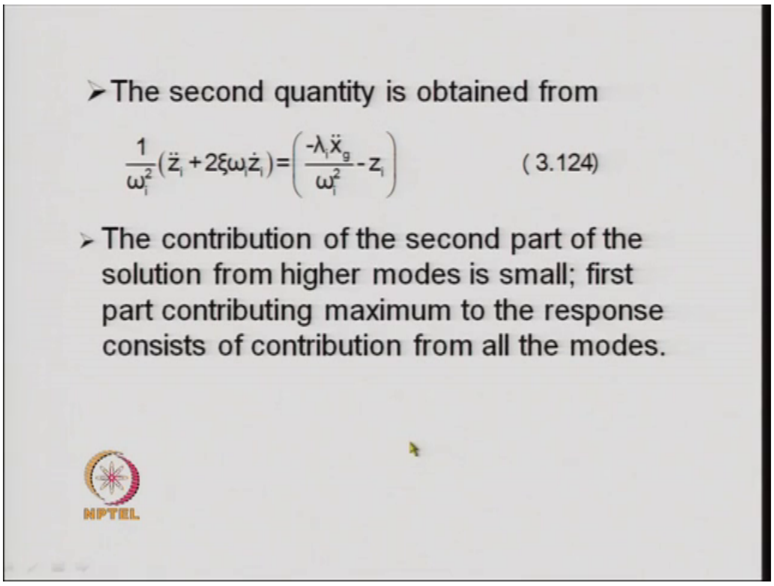

So, minus m ix double dot g and then find this quantity from the model equation and the we can see that this quantity is nothing, but is equal to minus lambda x double dot g divided by omega square minus Z i and Z i we have already obtained minus lambda ix double dot g omega i square that is already known to us. So, therefore, this quantity gives these z double dot i plus twice theta omega i z dot i divided by omega i square this quantity is automatically known.

So, therefore, this quantity; that means, the acceleration and the velocity that the generalized acceleration and generalized velocity need not be computed separately we can get the information about these 2 summations divided by omega i square by knowing Z i only because lambda is known as double dot g are known. And therefore, we can see that by knowing only z Z i we can obtain the summation of these 2 quantities. So, for that makes basically the entire calculation shorter and the simpler here the main important thing is that we perform is quasi static analysis along with the dynamic analysis.

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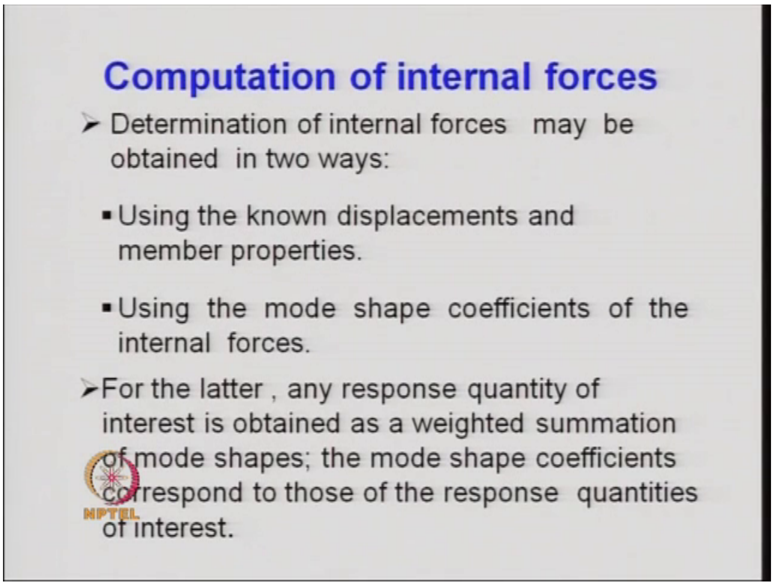

Now, let us look at the entire thing that we have said before you in finding out the displacements, in finding out the displacement responses now what we do is that either we can use mode acceleration approach or the classical model analysis. In classical model analysis the number of modes that is to be considered is obtained to form the mass participation factor, but that guideline is valid for the displacement response. If one has to calculate the internal forces or the bending moment shear forces etcetera then one has to take into consideration higher number of modes.

If we use the mode acceleration approach then for any response quantity of interest we can obtain a better estimate of the response quantities by using a less by using less number of modes in that we perform 2 analysis. One a quasi static analysis in which we consider the contribution from all modes and the other is a dynamic analysis in which we need not calculate z the generalized acceleration and generalized velocities by knowing the generalized displacement itself we can obtain the contribution that is coming from the dynamic analysis and that contribution may be considered only for first few modes and we can sum them together.

So, that mode acceleration technique is found to provide better results for finding out any kind of response digit the bending moment shear force dripped or whatever up with a response quantity that you want. Now, let us look into how we can find out the bending moment and shear forces in a particular structure because the dynamic analysis that you have talked about. So, far provides the displacement and from that displacement we have to finally, calculate the bending moment and shear force and each of the members that is called the internal forces.

Now, one can obtain these internal forces by 2 approaches, first approach is that once we know the displacements at every degree of freedom of the structure and since we know the member properties or in other words the stiffness matrix for individual members then from that 1 can obtain the bending moment and shear forces. So, let us look at that how do we do this.

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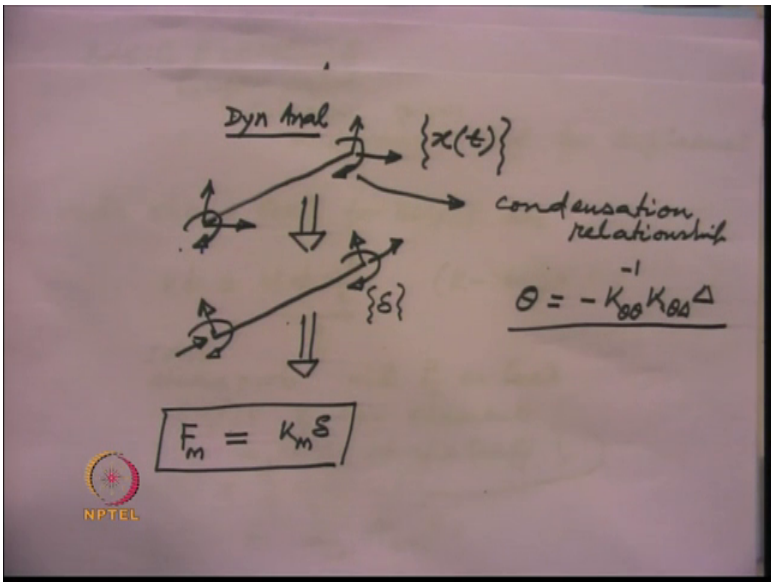

So, if we consider a particular mode over here and a mode over here at these 2 modes the dynamic analysis provides us say this displacement and these displacements. So, because the dynamic analysis generally condensed out all the rotational degrees of freedom and the translational degrees of freedom are considered as dynamic degrees of freedom. So, the response analysis provides us xt that is the displacement. Now, for finding out the bending moment and shear force in this member we need also the rotations that are taking place at the 2 modes.

And in order to obtain that we use the condensation relationship that we had obtained before for finding the condensed stiffness matrix corresponding to dynamic degrees of freedom. And if you recall the rotational degree of freedom is written as minus k theta inverse into k theta delta into delta where delta is the dynamic degrees of freedom or the translational degrees of freedom and theta basically are the rotational degrees of freedom that you wish to condense out.

So, we substitute in the other set or in the other set of equation this theta and by that substituting that we get a condensed stiffness matrix which corresponds to only delta degrees of freedom, where and k theta, theta is the partitioned matrix corresponding to theta degrees of freedom only and k theta delta is the coupling matrix between the rotation and the displacement degrees of freedom. So, with the help of this equation we can find out the value of theta at each node, because we know delta that has been obtained from the dynamic analysis. So, we now know the rotations also at each degree of freedom, once we know the rotation then if I take any member then at the 2 ends of the members we have the displacements and rotations, but all of them are in the global coordinate.

Now, from the global coordinate we convert these to local coordinate system for that we use the transformation matrix, using the transformation matrix one can obtain these displacements and rotations in the local coordinates and once we get this displacement. And rotations in the local coordinate which is called as delta vector then the member and forces internal forces it be written as km into delta where km is the member stiffness matrix in the local coordinate and delta are the degrees of the displacements at the 2 ends of the member in the local coordinate

So, this is the first approach by which one can obtain the internal forces in the structure after we have performed the dynamic analysis.

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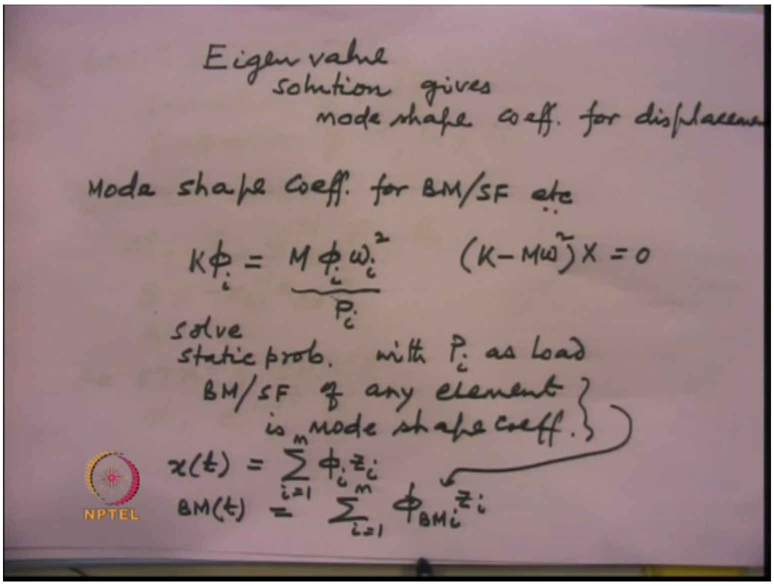

The second approach is that we use the mode shape coefficient for the response quantity of interest. Now, what we mean by the mode shape coefficient for a response quantity of interest would be clear from this the eigenvalue solution that we have performed that provides us the mode shape coefficient for displacement. So, therefore, a phi that we use you know on model analysis that phi correspond to the mode shape coefficient for displacement. Now, this if you are wanting to find out the mode shape coefficient for bending moment or shear force or for any other response quantity of interest then we can obtain it in this particular fashion. If I take the i-th mode the i-th mode can be represented by this equation k into phi i is equal to m phi i omega i square that follows from the eigenvalue problem that is that we are solving here this particular equation can be written kx is equal to m omega square x, now for the i-th mode the x becomes phi i. So, for n-th mode this is valid.

So, now if you look into this equation then this is simply a static equation, in that the k into phi i that is the left hand side of the static equilibrium equation and if I consider the entire thing as pi a load then this is the right hand side load. So, we have a static equation matrix equation which we can write as k phi is equal to pi and pi is known because mode shape is known mode shape or displacement is known, omega i is known and we multiply it by m matrix. So, we get the value of pi. So, pi is known now we can view the entire thing like this in the first mode this equation represents that as if the entire structure is subjected to a static load of pi and if I analyse it then the displacements that we get will be equal to phi i.

Once we know the value of the displacements phi i then from there one can find out the bending moment and shear force for any element and we will proceed as before that is first from the displacements we will obtain the rotations and once we know that rotations then one can get the values of bending moment and shear force at the ends of any element or any member. Therefore, the these bending moment and shear force that I will get for a particular mode by solving this equation that is solving this static problem then that bending moment and shear force is called the mode shape coefficient for bending moment or shear force or any response quantity of interest.

So, we can sum up like this that usual eigenvalue solution provides mode shape coefficient for displacement, if we wish to find out the mode shape coefficient for any other response quantity of interest then we can solve a static problem in which we apply a static load in each mode equal to m into phi into omega i square as a static load. And for that we find out the bending moment shear force or any response content of interest and that quantity will be called as the mode shape coefficient for that response quantity of interest. So, what we do is that then we use the usual mode summation technique xt is equal to phi i into Z i I is equal to 1 2 m whatever you wish to consider number of modes I will sum it over there that is the displacement.

Similarly, if I want to find out bending moment at any particular section then we sum it up for all the modes this product that is phi bending moment I that is for the i-th mode the bending moment that you have obtained that is the bending moment coefficient for that mode multiplied by Z i. So, one can obtain the bending moment shear force or any internal forces by this technique as well, it depends upon the individual which technique they prefer, but in the beginning of the program if one can compute after finding out the mode shapes for the structure if 1 can find out the mode shape coefficients also for different response quantities and store it then this method turns out to be a better method for finding out the different quantities of interest.

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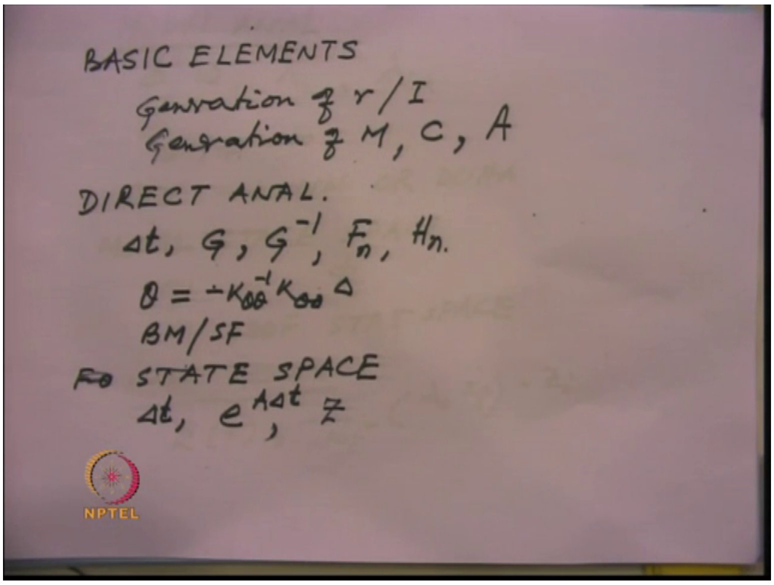

Next we come to the state space analysis.

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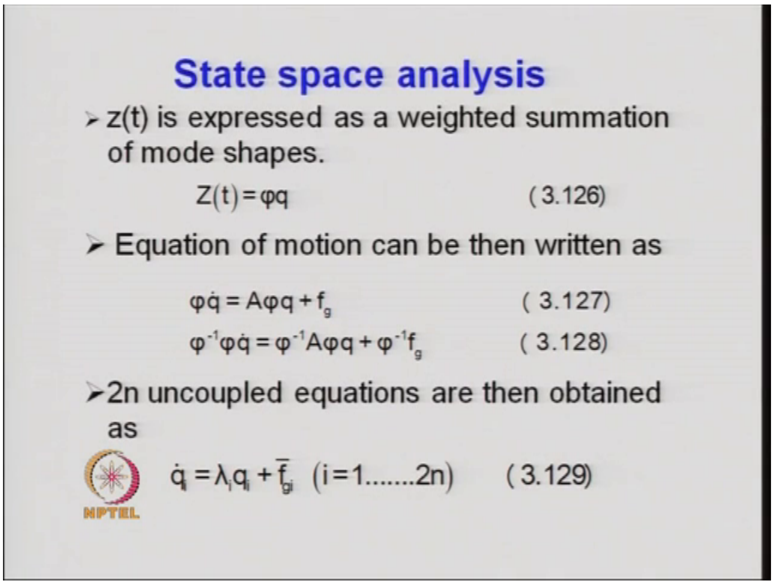

If we wish to obtain again the model analysis or use the model analysis technique for the state space equation then we follow in a in the same fashion that is we use z t we write down the displacement z t is equal to phi into q where phi is the mode shape matrix and q is the generalized displacement and then substitute into the state space equation the state space equation is of the form of z dot is equal to a z plus f g we have seen before. So, in place of z dot to write down phi into q dot, it comes from this equation and in place of z we substitute phi into q and then we add onto it the f g that is the load vector. Then this is multiplied, pre multiplied by phi inverse on this side. So, on the side also if we multiply it by phi inverse and this load vector is also multiplied by phi inverse.

So, since phi inverse phi will provide us a diagonal unit matrix therefore, that diagonal unit matrix multiplied by q dot that becomes a diagonal matrix consisting of only q dot i and phi inverse a phi again becomes the decoupled because that is the property of the eigenvalue. The eigenvalues that is the eigenvalue vectors right that the rather than it is a property of the eigenvectors and the property of the eigenvector is that it is orthogonal to the a matrix. So, therefore, this product that which is product of these 3 matrices again becomes a diagonal matrix and once it becomes a diagonal matrix then there we get 2 n uncoupled equations where n is the degree of freedom and each equation is a first order differential equation and in this one first order differential equation lambda i that becomes the what is known as the eigenvalues of the matrix a. So, we can write down such equations which will be 2 any number.

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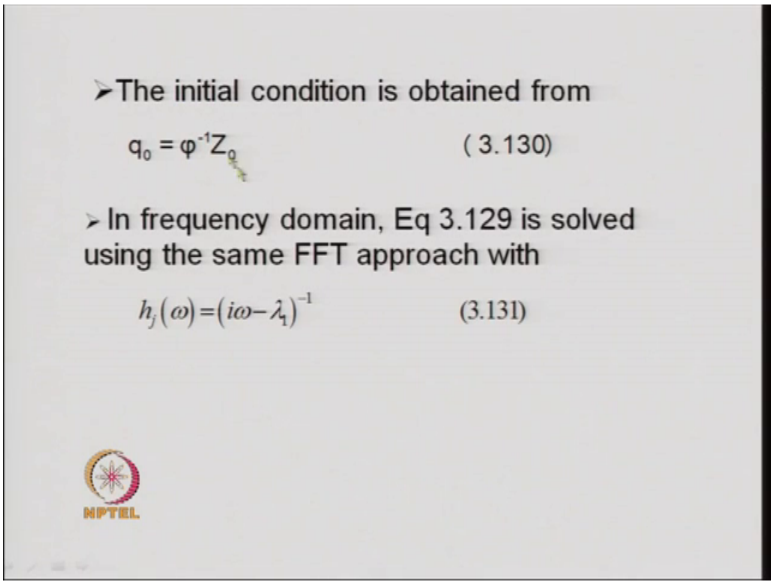

Next for solving this problem in time domain each one of these uncoupled equation will required initial condition. So, this initial condition is obtained from that, that is q 0 if you wish to find out; that means, this situation is in terms of the generalized coordinate whereas, the z is the actual coordinate system or the structural coordinate system. So, since we are solving the equation in the model coordinate system or the generalized coordinate system then we have to provide the proper initial condition in terms of the generalized coordinate. So, that we obtain by this inverting; that means, finding out q over here q is equal to phi inverse z.

So, that is what we write in this equation. So, q 0 is equal to phi inverse z 0 and from there we get the initial condition that is to be used for solving each 1 of the model equation in the generalized coordinates. In fact, at that the in the same way we should find out the initial condition for all the model equation that it that we have the that you have shown for the second order differential equation that we have solved before.

So, both in the case of a state space analysis and the usual second of all those couple second order differential equation we need to find out the initial condition and corresponding to the generalized coordinate and in for actually coordinate we know the initial conditions. So, the relationship that exist between the initial condition of between the generalized coordinate and the actual coordinate system that relationship is shown over here so we required in that case the phi inverse. If we wish to solve this problem; that means, in the frequency domain for the state space analysis then we take up again this equation and it is as a single degree or a single state space equation and this single state space equation for that h j omega is written as I omega minus lambda i to the power inverse of it.

So, that also has been discussed before while solving a state specification for this single degree of freedom system and in the frequency domain as before we used the affective algorithm that is we first find out the frequency contents of the response by multiplying hj omega which the frequency contents of the load generalized load that is. This is the generalized load phi inverse f g that will be generalized inverse that will be Fourier synthesized after it is Fourier synthesized we will multiply with hj omega and then we add on to that the complex conjugate and the entire numbers would be given as input to I FFT and the I FFT would provide us the responses in time.

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Example 3.12: For the frame shown in Fig3.20, find the base shear for the right column; k/m = 100; ξ=5%

Solution:

$$K = \begin{bmatrix} 2 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 6 & -4 \\ 0 & 0 & -4 & 8 \end{bmatrix} k \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} m$$

$\omega_1 = 5.06\,\text{rad/s}$; $\omega_2 = 12.57\,\text{rad/s}$

$\omega_3 = 18.65\,\text{rad/s}$; $\omega_4 = 23.85\,\text{rad/s}$

So, an example is solved.

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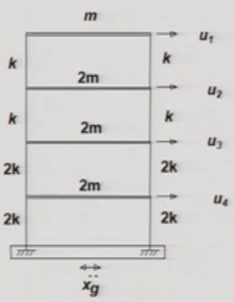

Fig3.20

$$\phi_1^T = [-1.0 \quad -0.871 \quad -0.520 \quad -0.278]$$
$$\phi_2^T = [-1.0 \quad -0.210 \quad 0.911 \quad 0.752]$$
$$\phi_3^T = [-1.0 \quad 0.738 \quad -0.090 \quad -0.347]$$
$$\phi_4^T = [1.0 \quad -0.843 \quad 0.268 \quad -0.145]$$

In which we had a problem of this type is simple frame and it is subjected to the same ground acceleration at the two supports and for that this is the k matrix, this is the m matrix and these are the 4 frequencies of the system and these are the 4 modes corresponding to the 4 frequencies.

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$$C = \alpha M + \beta K = m\begin{bmatrix} 1.500 & -1.140 & 0.0 & 0.0 \\ -1.140 & 3.001 & -1.140 & 0.0 \\ 0.0 & -1.140 & 4.141 & -2.280 \\ 0.0 & 0.0 & -2.280 & 5.281 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ -2.0 & 1.0 & 0.0 & 0.0 & -1.454 & 0.567 & 0.0 & 0.0 \\ 2.0 & -2.0 & 1.0 & 0.0 & 1.134 & -1.495 & 0.567 & 0.0 \\ 0.0 & 1.0 & -3.0 & 2.0 & 0.0 & 0.567 & -2.062 & 1.134 \\ 0.0 & 0.0 & 2.0 & -4.0 & 0.0 & 0.0 & 1.134 & -2.630 \end{bmatrix}$$

And we obtained a c matrix for that finding out alpha and beta from the first 2 frequencies and once we get the c matrix then we can obtain the state space matrix a for the entire thing and go for a state space analysis.

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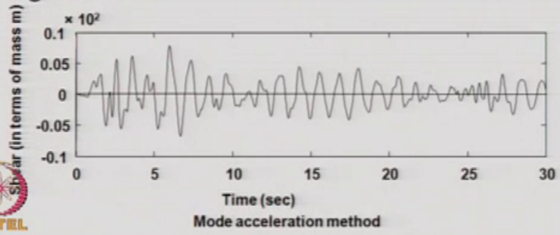
$\lambda_1 = -1.793 + 1.571i$; $\lambda_2 = -1.793 - 1.571i$;

$\lambda_3 = -1.157 + 1.461i$; $\lambda_4 = -1.157 - 1.461i$

- Time histories of the base shear obtained by mode simulation, mode acceleration and modal state space analyses are shown in Fig 3.21.



The states the space that is a the eigenvalues of the a that turns out to be a complex number and these are the 4 eigenvalues of the problem and using these 4 what you call the eigenvalues we decoupled the state space equation into a single degree of freedom state space equation. So, in this case we had the since there are 4 degrees of freedom. So, we will have 4 single degree of freedom state space equation and we can also solve this problem using model, mode acceleration approach or mode summation approach in time domain as well as in frequency domain.

So, the all the results are shown here, the base shear is the quantity of interest over here rather than the displacement therefore, you can see that here we are wanting to find out the response quantity as base shear therefore, it is possible to solve the problem in 2 different ways. Either by finding out the base shear directly from the displacements that we get a different stories and finding out the forces at each storey level and sum them together to find out the base shear or we can straight away find out the mode shape coefficient for the base shear and each mode and multiply it by Z i and sum it over the number of modes that you consider.

Since, here we are considering only it is only a problem of the 4 degree of freedom. So, we have considered all the 4 degrees of freedom or all the 4 modes in finding out the responses. So, this result shows that by mode acceleration approach the responses that we have obtained for the base shear.

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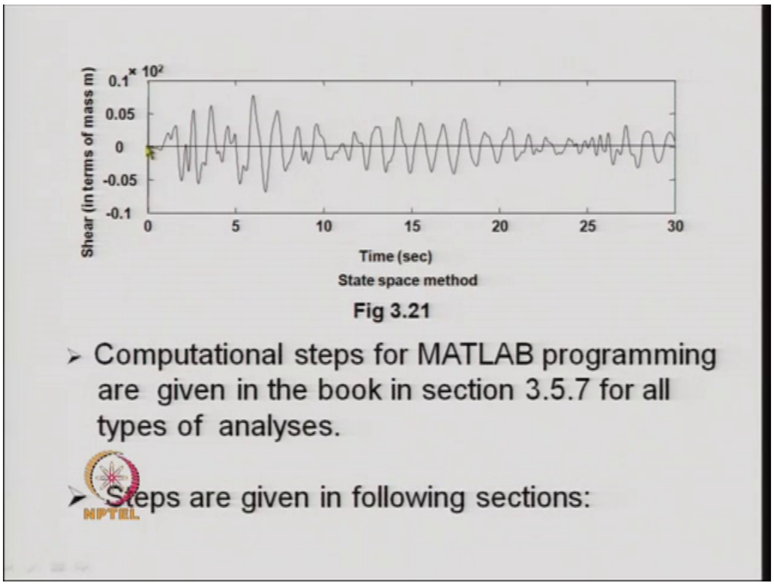

This shows same response that is the base shear using that state space analysis and both these analysis are obtained this one we are we have we have used the mode acceleration method that is we have used the solution in the time domain here we obtained by the state space analysis here we have used the frequency domain analysis. That is your Fourier transform the load and use the response or obtain the responses using FFT.

Now, at the end of this chapter that is chapter on the response analysis for a for a specified ground motion the response analysis for a specified ground motion the at the end of this chapter in the book, I have outlined the computational steps that are required for developing a program in the mat lab by using all the methods that we have discussed before and the steps.

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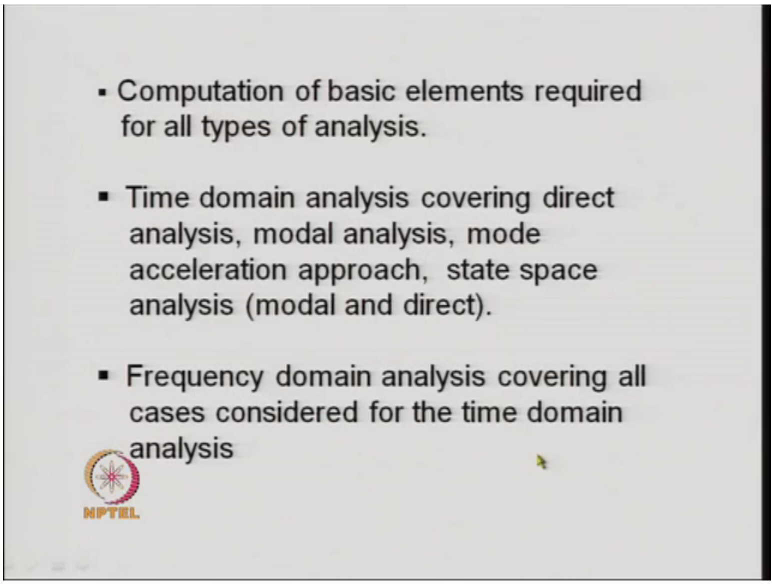

That are given what to call divided into 3 segments.

The first segment consists of computation of basic elements required for all types of analysis for example, you have to obtain the overall stiffness matrix, then condensed stiffness matrix, then from the condensed stiffness matrix again the non support degrees of freedom and support degrees of freedom matrices they are isolated then coupling matrices are considered. So, all these come. And then one can obtain the quasi static analysis of the system for the ground motion, the mode shapes frequencies and for each mode shape one can also find out the mode shapes for the bending mode shapes shear forces etcetera. All these kinds of computations can consist are contained in the first part of the program and the results can be stored.

In the second part of the program the time domain analysis, direct model and mode acceleration approach and state space approach all of them are outlined. And one can obtain the solution of any problem or any structure in time domain. And the third part or the last part is a frequency domain analysis covering again all the cases that you have considered in the time domain analysis; that is the direct analysis, model analysis, state space analysis and so on.