Seismic Analysis of Structures
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Lecture – 17
Frequency Domain Spectral Analysis (Contd.)

In the previous lecture, we discussed about SISO that is single input single output system, in which a one excitation is on the single degree freedom system and there is one output, and for that we obtain the necessary relationships.

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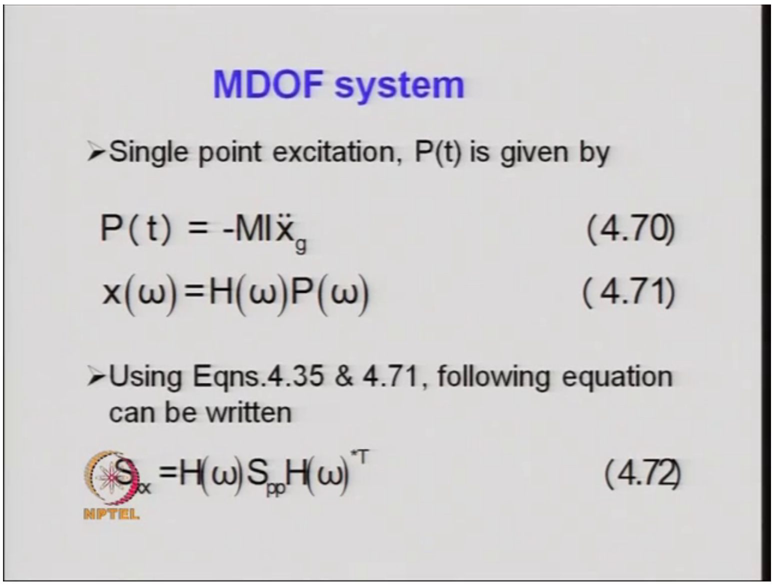

And in that also we discussed several things like, what is the relationship that exist between the power spectral density function of velocity with the power spectral density function of displacement, then power spectral density of acceleration, with power spectral density function of displacement and so on.

Now, before we start the multi degree of freedom system, in which we will have a number of inputs to the system, and there will be a number of outputs to the from the system. And the inputs would be represented in terms of a vector, and outputs will be also a vector. And as we discuss last time, when we have a number of random processes in a particular vector, then the process is described with the help of the power spectral density function matrix, rather than a single power spectral density function.

Let us try to recapitulate some of those fundamental things that we discussed in yesterday's lecture so that we can effectively understand the MDOF system subjected to a random excitation vector.

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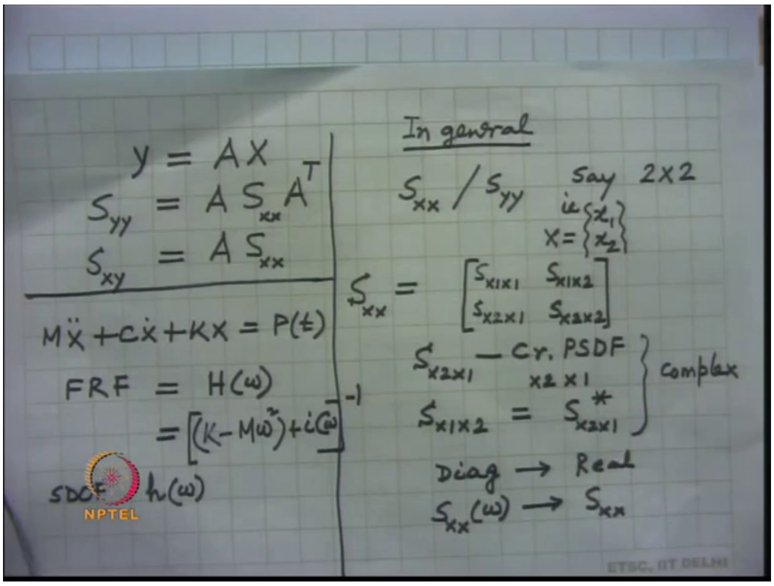

First thing that we discussed yesterday was Y is equal to A into X; where Y is a vector, X is a vector and it is connected through a matrix A. In that case S yy will be a power spectral density function matrix, and S xx is a power spectral density function matrix, S yy would be related to S xx with the help of this equation. S xy; that is the cross power spectral density function between the output and input; that is S ty is will be equal to A into S xx.

Now, if we consider a multi degree freedom system, then the we can write down the equation of motion, and A p t is the excitation vector. Then one can obtain the relationship between the frequency contents of the response vector to the frequency content of the excitation with the help of a FRF; that is frequency response function matrix H omega. And this H omega is equal to k minus M omega square plus i c omega inverse of that, that is what we discussed yesterday. Therefore, X omega can be written as, X omega equal to H omega multiplied by P omega. Where P is omega is the frequency contents of the force vector p t. In case of single degree of freedom system, this H omega matrix is it in a small h omega; that is a single quantity which is a called also the FRF that is frequency response function of a single degree freedom system.

Now, S xx and S yy these matrices, say for example, if it is a vector of size 2, then this will be the nature of the power spectral density function matrix, the diagonal terms are the power spectral density functions of x 1 and x 2, and the off-diagonal terms would represent the cross power spectral density function between x 2 and x 1 and this will be between x 1 and x 2. If we have a known number of elements in the vector then; obviously, will have x 1 x 1 x 2 x 1 x 3 x 1 so on. This will be the cross power spectral density function between different responses. And this relationship also was shown, that is x 1 x 2, cross power spectral density function between x 1 and x 2 is equal to the complex conjugate of the cross power spectral density function between x 2 and x 1.

If the in case of the matrix; that means, if it is not a single power spectral density function, if this is a matrix then the relationship is S x 1 x 2 will be equal to S x 2, x 1 complex conjugate and transpose of that. So, that is the difference between the single quantity as the cross power spectral density function, and the S x 1 S x 2 or the cross power spectral density function and between x 1 and x 2. In that case x 1 and x 2 will be the 2 vectors.

Now, the diagonal terms of the power spectral density function matrix are real, and they are frequency dependent, and generally we do not write omega in front of it, but it is understood that for each frequency we have a matrix like this. Therefore, as we go on varying the frequency, then we have for each frequency one matrix like this. Now with the help of this we would we would now go in to the multi degree of freedom system.

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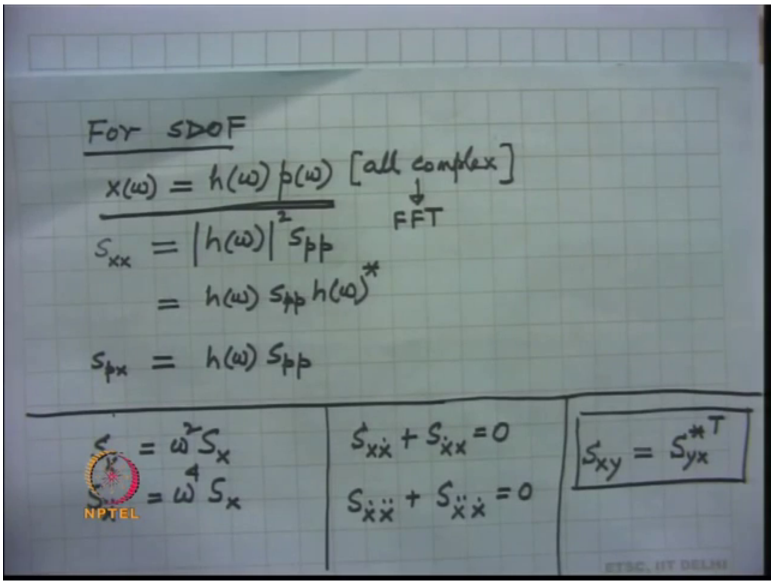

And in the previous lecture also we had shown that X omega is equal to h omega into p omega; that they are all complex numbers p omega is obtained from fast Fourier transform, and S XX the power spectral density function of the response is equal to absolute value square of H omega into the power spectral density function of p.

Now this can also be written as H omega into S p p into H omega star. Because this multiplied by this becomes equal to the absolute value square. And the cross power spectral density between the response or between the force and the response is equal to H omega S p p. Apart from that we also have proved in the previous lecture that the power spectral density function of velocity is equal to omega times the power spectral density function of displacement. And power spectral density function of acceleration is equal to omega to the power 4 into the power spectral density function of displacement.

Now, in case when the this X dot and X they are vectors, then they are matrixes and the power spectral density matrix of velocity is related to the power spectral density function of displacement again through this relationship, only thing that this changes to matrices. Also, we had proved that S XX dot plus S X dot X is equal to 0. That is the cross power spectral density function between the velocity and displacement. That that if it comes into the solution or analysis, then the we ignore that; that means, we set that V equal to 0 because of this reason.

Similarly, if the velocity and acceleration; for that if we have a cross power spectral density function or cross power spectral density matrix. Then we ignore that because of this reason that this plus this that becomes equal to 0. And also, we have seen that S XY is equal to S Y X star T; that is the complex conjugate of this, and transpose of that. So, with this few things that we describe in the previous lecture we come into the response of a multi degree freedom system excited by a force vector p T.

Now, for single point excitation, we know that the force vector p T is equal to minus M into I into X double dot g. Where I is a influence coefficient vector consisting of 1 1 1 or 1 0 1 0 1 0, and so on that you have seen in the case of the deterministic analysis. X omega we can write down to be equal to H omega matrix multiplied by p omega; that is what we have just discussed. Where H omega is the frequency response function matrix for the system, multi degree of freedom system. And p omega is the frequency content vector of the excitation p t, and X omega is the frequency contents of the response.

Now, if this relationship holds good. Then from the just discussion that we had, from that we can write down the power spectral density function of S XX; that is a power spectral density function matrix of X is equal to H omega matrix multiplied by the S p p matrix that is the power spectral density function of the excitation into H omega star T. In the case of the single degree freedom system, if you recall we had we could write it as small H omega, multiplied by S p p multiplied by small H omega star and H omega star small H omega star. And this H omega if you multiply that becomes the absolute value square of H omega square.

If H omega is a matrix, then we have to make it a transpose of the complex conjugate of the matrix H omega. So, then only it will become the absolute value square. So, this is a standard relationship that we use for the case of a multi degree of freedom system. What you have to simply do is that you have to find out the FRF; that is the frequency response function matrix for the system which is equal to k minus M omega square plus i c omega, and inverse of this matrix if you take at every frequency then you get H omega at every frequency. And S p p at every frequency is known. So, you have to carry out this matrix multiplication, and in the and from that you will get the value of the power spectral density function matrix at each value of omega.

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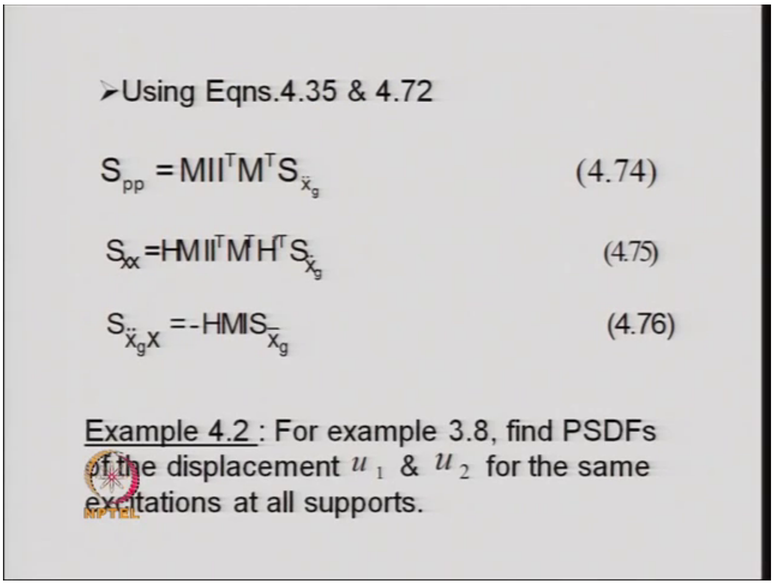

Now, in case of the single point excitation system, S p p that turns out to be M I into I T M T into X S X double dot g. And this follows from this relationship; that is p T is equal to minus M I X double dot g. Therefore, if we wish write down the value of S p p then it will be M I into S X double dot g multiplied by the I T M T; that is a S X A T that what we have discussed just now. So, using that one can write down the S p p, that is it will be M I into I T M T in in place of writing I T M T on this side. We have taken it on this side, because S X double dot g is a single quantity in the case of the single point excitation.

So, therefore, whether we give I T M T on this side of S X double dot g or in this or in this side, it does not matter. Now with the help of this relationship, that we obtain for S p p the S XX that there is power spectral density function matrix of the response can be written in this particular form; that is H M I, then I T M T H star T into X double dot g or X double dot g is the power spectral density function of the ground motion. So, it is a single quantity at each frequency and the cross power spectral density function between the ground acceleration. And the displacement that will be equal to minus H M I X double dot g and that follows from the equation that again we just examine that is S X, S X output input correlation is equal to A into S X. So, in place of S a we write down here now H M I. So, that is that becomes the pre-multiplier for the S X double dot g.

Now, using these relationships a problem is solved.

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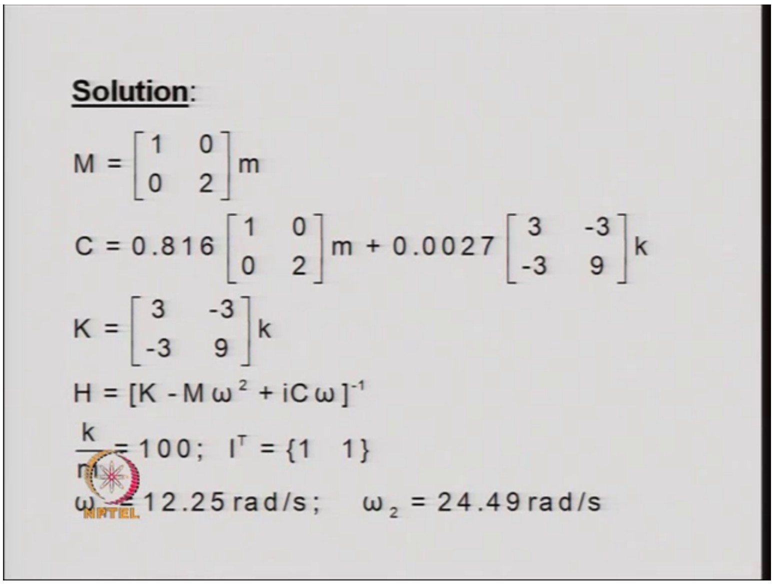

And these problem was the problem in which we had 2 responses at the or nonsupport degrees of responses, the problem is a I think the problem is this was the problem.

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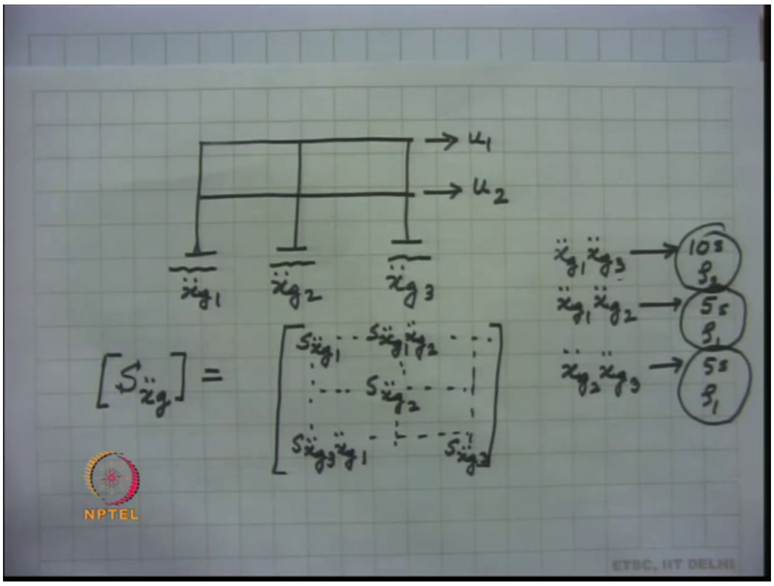

And that is we had u 1 and u 2 as the displacements, and we had 3 ground excitations. And in this case, all the 3 ground excitations are assumed to be the same. And therefore, we had a r matrix; that is in place of r matrix we had only a I matrix.

So, this is the I matrix you can see here.

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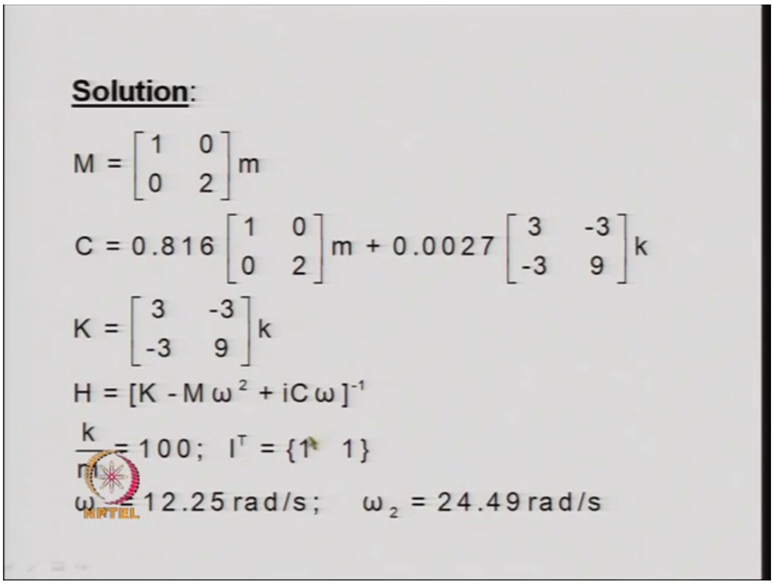

The I matrix is equal to I 1 1; that this is because of the fact that the ground excitations at the 3 supports are assumed to be the same. There are not different therefore, the I influence coefficient vector will be equal to 1 and 1. So, we know the mass matrix we can construct the c matrix from the relationship that is alpha M plus beta k and alpha and beta can be obtained from the 2 frequencies that we have obtained. So, with the help of that the alpha value have become 0.816 and the beta value is 0.0027. This is the K matrix and from there we can obtain for each frequency the H matrix; that is by inverting this matrix. And obtain the value of the power spectral density function matrix of the 2 displacements using the previous equation.

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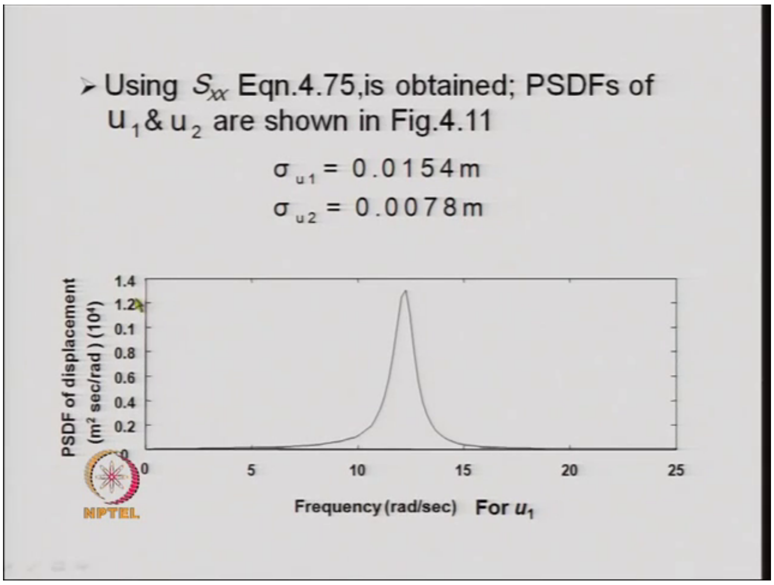

And the diagonal terms of the power spectral density function matrix, that is the PSDFs of the 2 displacements. So, that is what is shown here in the plot. You can see that it is the power spectral density function ordinates for displacement u 1. And this is a frequency and that is how the PSDF of the displacement e 1 varies with frequency and one can see that this is a peaking nearly at about 12.5. And the first frequency of the system is 12.25.

So, therefore, we see that there is a the peak of the power spectral density function of displacement u 1 is around that frequency. This is the power spectral density function of the displacement u 2. These area under the curve of this power spectral density function else of u 1 and u 2 they provide the variance, and the square root of variance gives the standard deviation of the displacement. And in case of the 0 mean possess the standard deviation is equal to the root mean square value. So, the root mean square value of u 1 is 0.0154, and the root mean square value of the displacement n 2 or u 2 is 0.0078.

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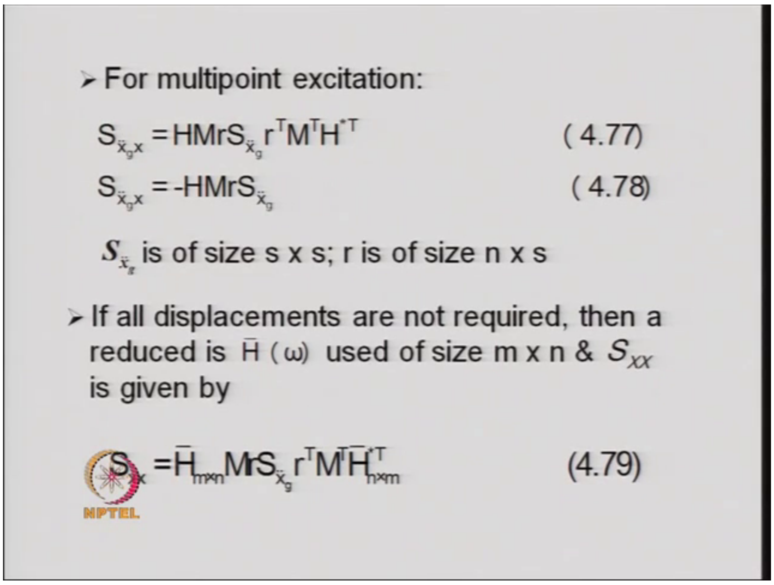

Now, when we consider the multi point excitation system, then the S this will the this is S XX, this is not S X double dot g it is a wrongly written over here. It will be S XX that is the power spectral density function of the matrix of the response is equal to now H M. In place of r now we have in place of I we have got now r. So, H M r X double dot g is a matrix now is not single quantity. Because the size of the matrix would depend on the number of the point of excitations and it is multiplied on the right-hand side by r T M T and H star T.

So, that is the general relationship between the displacement and the excitation. Displacement is a vector the force is a vector and therefore, the this is a matrix of displacement. And this is a matrix of the or power spectral density function matrix of excitation. And r is the influence coefficient matrix that we had discussed when we are solving the problem for the multi degree of freedom system subjected to the deterministic ground motions which cause different excitations have different supports because of the time lag. S X double dot g X that is the cross power spectral density between the excitation and the displacement is given by this expression again. And here S X double dot g again is a matrix. The size of this S X double dot g matrix is S into S if S is the number of support, and r is a influence coefficient matrix that is obtained from a static analysis of the system that we have seen before is of the size of n into S; that is if there are n degrees of freedom. And if there are S number of support excitation, then the displacements that will be caused at the n nonsupport degrees of freedom; that will be represented by a matrix n into s.

Many a time we are not interested in all the response quantities say a only a limited number of response quantities may be involved.

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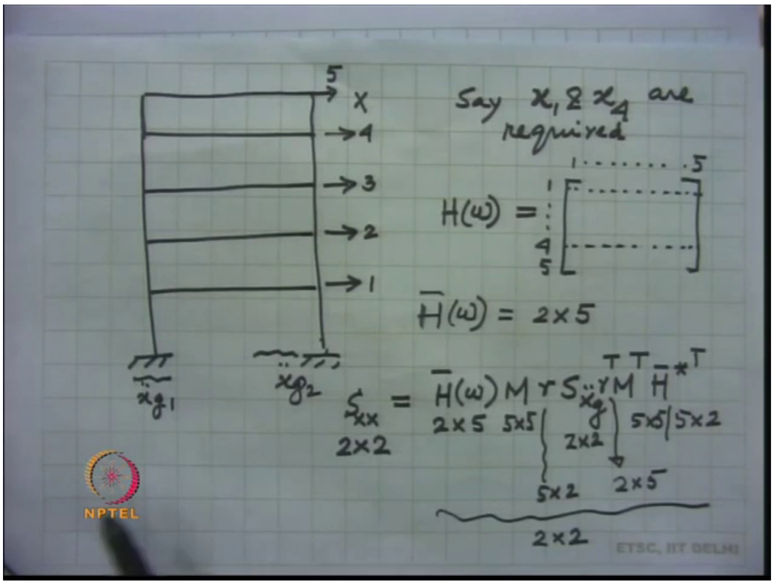

For example, in this problem we consider that this is a frame subjected to 2 ground excitations, different ground excitations arising because of the time lag. And there are 5 degrees of freedom in the system that is nonsupport degrees of freedom. Then H omega matrix will be equal to a 5 by 5 matrix. Now say we are interested in only the response X 1 and X 4. In that case the we can have a reduced frequency response function matrix which would be called H bar omega equal to 2 by 5; that is, we can select the first row, and the 4th row and make a modified H omega matrix called H bar omega matrix and it is size will be equal to 2 into 5.

Now, S XX now will be a 2 by 2 matrix, because we are only interested in the responses X 1 and X 4. So, the diagonal terms of this matrix would give you the power spectral density function of 1 and 4. And the off-diagonal terms will give the cross power spectral density function between 1 and 4 and 4 and 1. So, that is about the S XX matrix will represent. And it has the same equation as that of 4.77 that you had just discussed; that is H M r S X double dot g then r T M T H star T. Only in place of H star and H we write now H bar and H bar star. Where H bar is this modified frequency response function matrix. And if we put the sizes of different matrices, here we can see that these finally, leads to this multiplication finally, leads to a 2 by 2 matrix which will be the size of the matrix S XX.

So, in many problems what we do is that, we may not be interested in all the displacements or all the response quantities. And for that we take a reduced frequency response function matrix and use the same equation that we have stated over here with the help of equation 4.77

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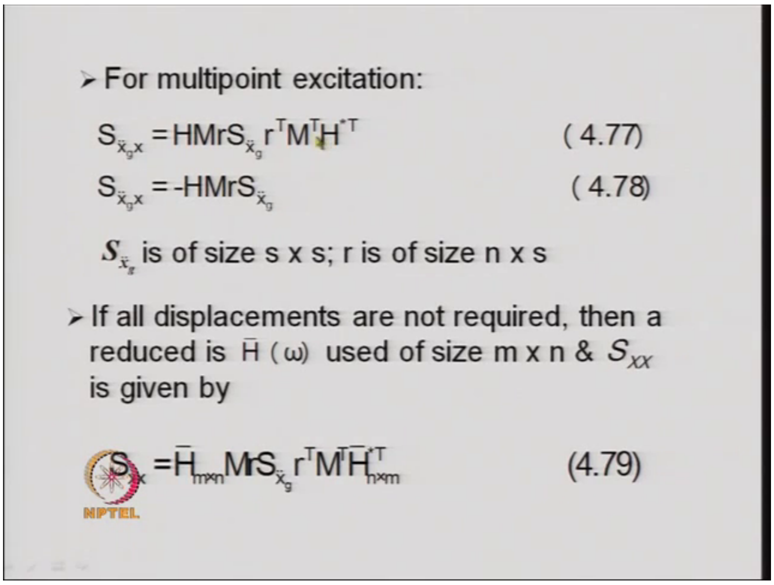

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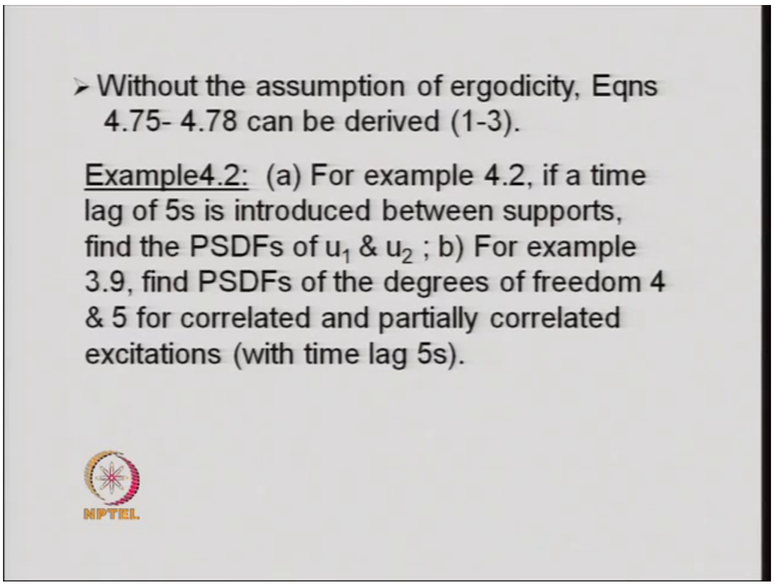

Now the equations that I had shown before or in the previous slides, they have been obtained or derived with the assumption of ergodicity and that we did purposeful because it is easy to understand these relationship by assuming the stationary processed ergodic. Because in that case will be dealing with only one-time history and with the help of the Fourier transform of that we can construct all that relationship.

However all the relationship that we have derive in the previous slide, they can be also derived without the assumption of ergodicity; that is by assuming the process to be a stationary random process, and in that case what we will do is that we will start with the autocorrelation and cross correlation functions of the process, or of the 2 processes. And then take a Fourier transform of them. And using that we would come to all the equations that you have derived before and we have seen before that there exist a relationship between the autocorrelation function, and the power spectral density function through a Fourier transform pair.

Now, let us solve a problem of multi degree or multi point excitation system; that is example 4.2 of if a time lag of 5 second is introduced between the supports, find the PSDF of u 1 and u 2; that is the problem, this problem, in this problem we can now assume that the X double g 1 X double g 2 and X double g 3 are different.

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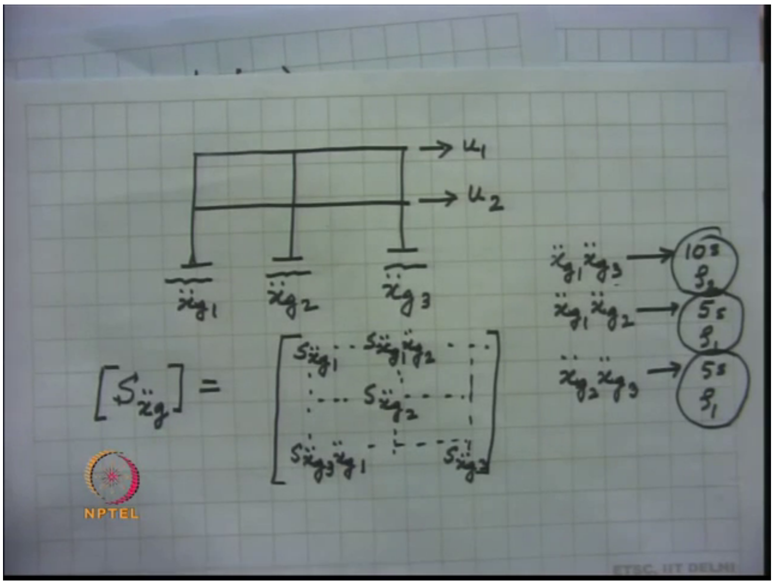

And there is a time lag of the 5 second between this excitations. So, the time lag between X double dot g 1 and X double dot g 3 is 10 second. X double dot g 1 and X double dot g 2 is 5 second, and X double g 2 and X double dot g 3 is 5 second.

So, these are the time lags. Then first we have to construct S XX double dot g matrix that will be a 3 by 3 matrix of the ground motion. And the diagonals of this would be the power spectral density function of the excitations at this point at this point and at this point. And the cross power spectral density functions would represent the cross power spectral density function on between this excitation and this excitation and so on. So, this is the S X dot S X double dot g matrix A 3 by 3 matrix. So, if we have construct that matrix.

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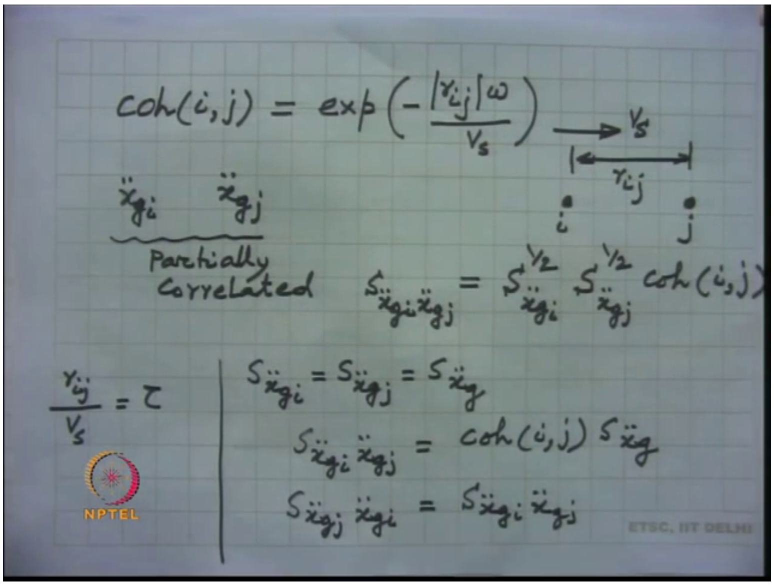

Then let us recapitulate what we discussed in the seismic input part of our study. In the seismic input if you recall, we discussed about a coherence function, and one of the form one of the empirical equation of the coherence function; which is a very simple one is this that is exponential of minus absolute value of r i j into omega divided by V S where r i j is a distance between the point i and j; that is at this 2 points we have excitations, and the distance between this is r i j.

The V S is the seismic wave velocity. So, if the same earth quake train or ground motion or time history of ground motion, that is travelling with a velocity of V S. In that case one can replace r i j by V S by the phase or the time lag; that is r i j divided by V S is equal to the specified time lag. That is the time that will be required to travel from i to j if the velocity is V S. Now this has been specified that is between the supports the time lag is 5 second. So, in this we can replace this quantity r i j by V S by 5.

Now, once we have done that then coherence function between the first support and the third support will be equal to exponential of minus this quantity will be 10 second into omega. Similarly, between the support 1 and 2 that is this support between 1 and 2 you will have 5 second; that means, r i j by V S will be replaced by 5 second. And X X double dot g 2 X g T will be again by 5 second. So, one can construct a matrix easily, because the cross power spectral density functions between the supports are now known. The this cross power spectral density function is given by this equation.

If we recall the seismic input that we discussed, and this is the power spectral density function at support i and this is at support j. And it will be multiplied by a coherence function c o h i comma j. And since this is known now one can obtain the different cross power spectral density function very easily. In the case of a single earthquake ground motion, train of a ground motion moving with a velocity, the power spectral density function at different supports the remains the same; that is equal to S X double dot g that which is specified. Only the change or the cross power spectral density function arises because of the coherence function. So, we write down the cross power spectral density a function between the 2 excitations i and j, as coherence I comma j into S X double dot g.

So, in this particular fashion, one can write down all the cross power spectral density function terms. Note that S X double dot g and X double dot g i that is the cross power spectral density function between j and i is equal to cross power spectral density function between i and j. This happens in this case because S X double dot g is a real quantity, there is no complex quantity involved over here. And since they are real quantity the star that we had used in defining the relationship between the cross power spectral density function between point 1 and 2, and point 2 and one that star is not required and therefore, they become the same.

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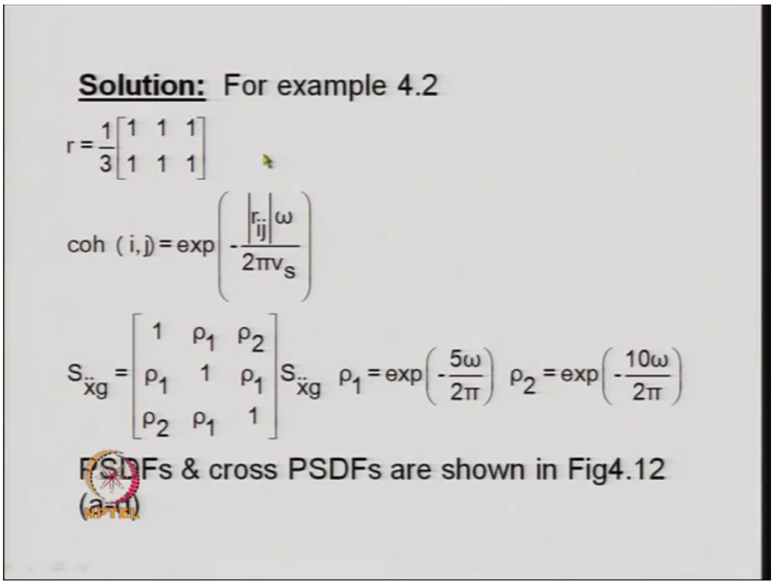

Now, with the help of that we obtained the power spectral density function matrix of excitation; that is given by this matrix. The r matrix we had computed before for this problem the r matrix was 1 by 3 into 1 1 1. And this coherence function we have used sorry, in the in the previous discussion I have simply written is as V S it will be 2 pi by V S. And the values of row 1 and row 2 they are 5 omega by 2 pi and 10 omega by 2 pi.

So, with that values of row 1 and row 2, one can obtain the cross power spectral density function matrix of excitation. So, this is the cross power spectral density function between the support 1 and 2, and this is between support 1 and 3. And the we have taken S X double dot g a single quantity common out of them, because a single power spectral density function is defined for the ground motion. And the excitations at different points vary because of the phase lag. So, these becomes a power spectral density function matrix, and this is the r matrix. So, therefore, with the help of this and the H matrix one can obtain the power spectral density function matrix between of u 1 and u 2 this 2 displacements.

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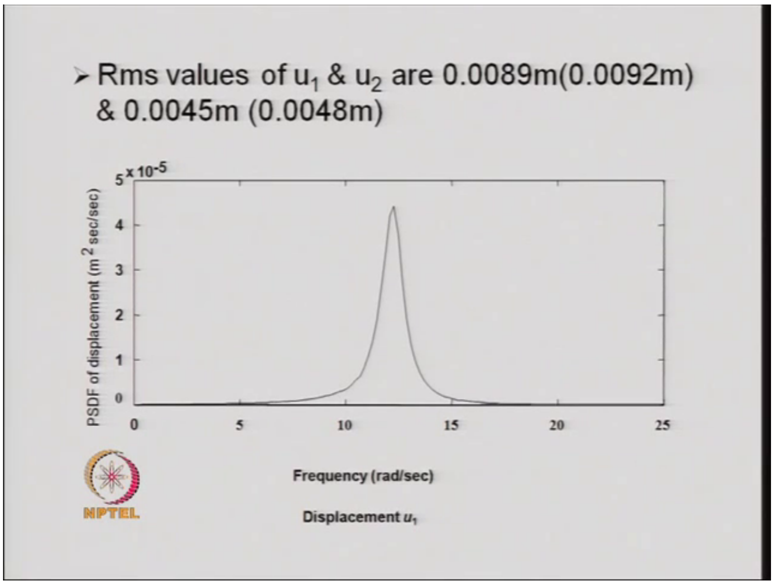

And the diagonal terms of that matrix gives you the PSDFs of u 1 and u 2. So, they are plotted here. And the plot again shows that the maximum value occurs around the first fundamental frequency of the structure, and the values of the or the Rms values of the responses becomes equal to 0.0089 and 0.0045 for u 1 and u 2. So, this is the PSDF of u 2.

The time history analysis that we obtain for the elcentro earthquake; note that the power spectral density function input that you have given here is the power spectral density function of the elcentro earthquake, which is tabulated at the end of the book as an appendix one can use those digitized values. So, we have used those digitize values for obtaining this the power spectral density function ordinates at different frequencies. So, when we obtained we took the same elcentro earthquake, as a time history and we did a time history analysis for the monthly support excitation system. We take time lag of 5 second between supports; we obtain the result as 0.092.

So, that was the value of the Rms value of the response that we obtain from the time history. And the special analysis gives the result at point 0.0089. So, you see that the results are very close to each other. That verifies or validates the accuracy of the result.

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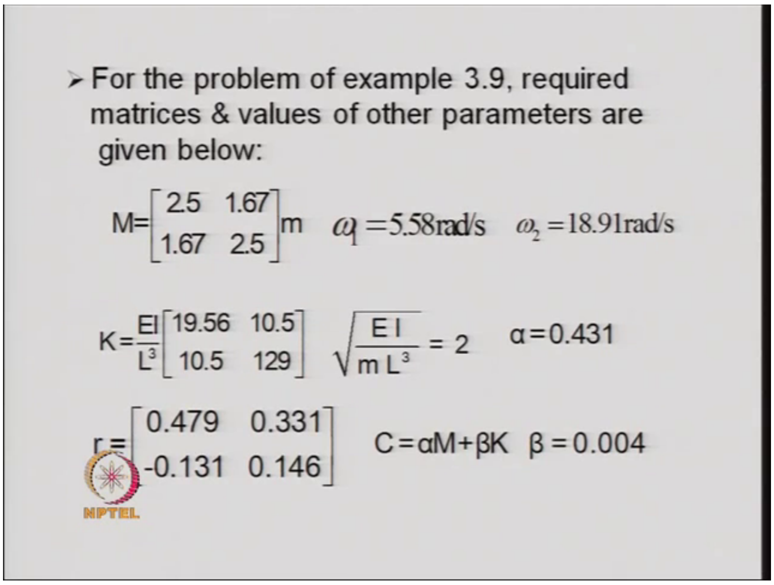

We solved another problem, which was a problem of the pitch reportal frame; that is, we had a pitch reportal frame if you recall.

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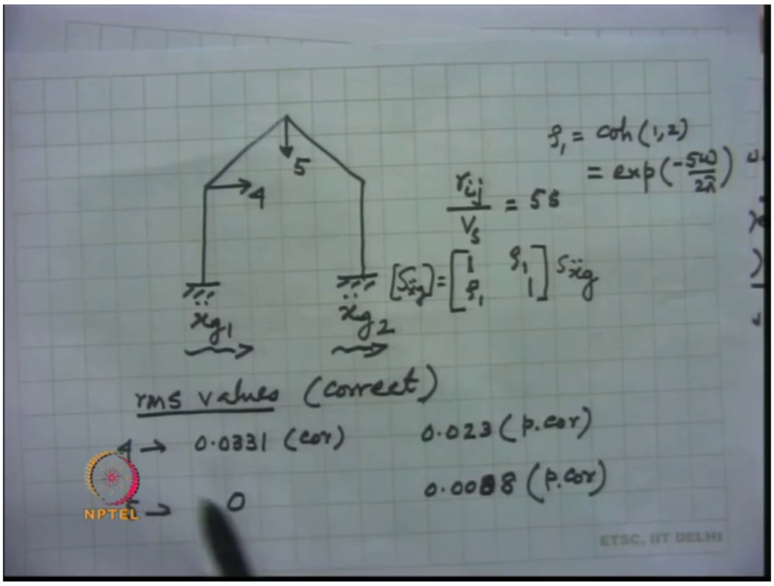

And the there is a degree of freedom here, and a vertical degree of freedom 5. And this is a having a multi support excitation. And these multi support excitation here they are 2 multi support excitations. The time lag between these 2 points is given as 5 second; that is r i j by V S that becomes equal to 5 second.

So, one can have a correlation coherence function with row 1. Row 1 defined as exponential of minus 5 omega by 2 pi. So, with the help of that we can construct the 2 by 2 power spectral density function of excitation. S X double dot g again is the power spectral density function of the elcentro earthquake that you have taken. And with the properties of the pitch reportal frame that is a mass matrix A defined as this. And the K matrix defined as this. We obtain the 2 natural frequencies as 5.58 and 18.91. The alpha and beta that was calculated with the help of the this 2 frequencies where 0.431 and 0.004. And this was the c matrix that you constructed and the r that we determined before; that is the influence coefficient matrix.

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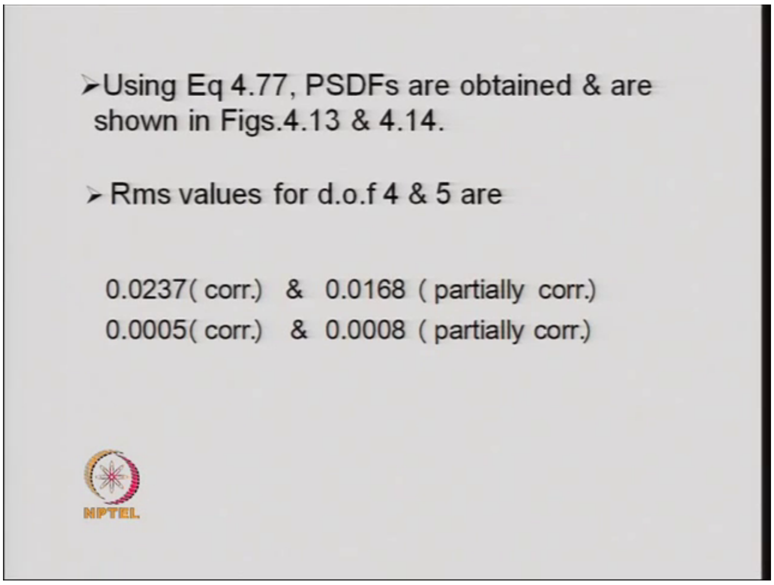

So, with these matrices defined, we again used the same equation for finding out the power spectral density function of displacement 4 and 5, and using that multiplication of M r and the r M and H omega starts on, we obtained the power spectral density function matrix of the displacement that was a 2 by 2. And the results are compared for the 2 cases; that is in one case we assume that it is perfectly correlated, and in other case we assume that there is a phase lag of 5 second.

The results which are given over here is not correct the correct results are shown over here in this paper; that is the Rms value that was worked out for these displacement and these displacement was 0.0331 when it was fully correlated; that is there was no time lag between these 2 points. And when they were partially correlated; that is there was a time lag between the 2 the Rms value of the response was 0.023.

So, one can see that when there is a for fully correlated ground motion; that is for the single support excitation. The response is more compared to when it is partially correlated. For the displacement 5, the when it was perfectly correlated the value was 0 that is expected, because the these 2 supports would be moving as a single unit. That is, it is a it becomes the single point excitation system. And in that case, we expect that the there will be not be a vertical motion over here, the entire thing would be vibrating in this particular fashion. And the for when it is partially correlated, so there is a little bit of displacement that occurs at the crown.

So, this shown the importance of the correlation between the excitations at the 2 supports, if we assume it to be perfectly correlated that is we assume it to be a single point excitation system.

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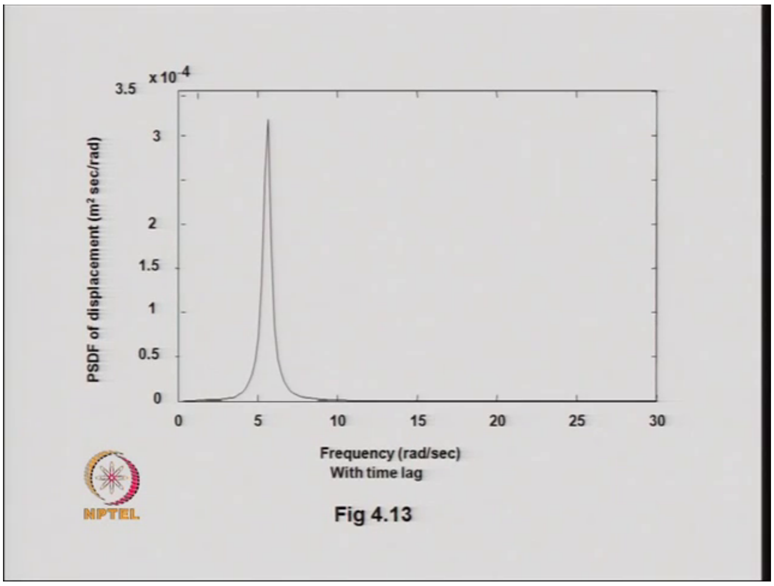

Fig 4.13

Then in some cases we can get some erroneous result for certain degrees of freedom. It is better to consider the partial correlation between the excitation produced by the single earthquake traveling at a certain speed; however, this kind of thing needs to be done only for cases where the 2 supports are spatially separated by a large distance, for small distances these particular partial correlation effect a partial correlation is almost negligible. Though the power spectral density function matrices are or the they are the power spectral density function matrices are shown over here, and one can see that the frequencies at which it is speaking is nearly equal to frequency of the over the first frequency of the system.

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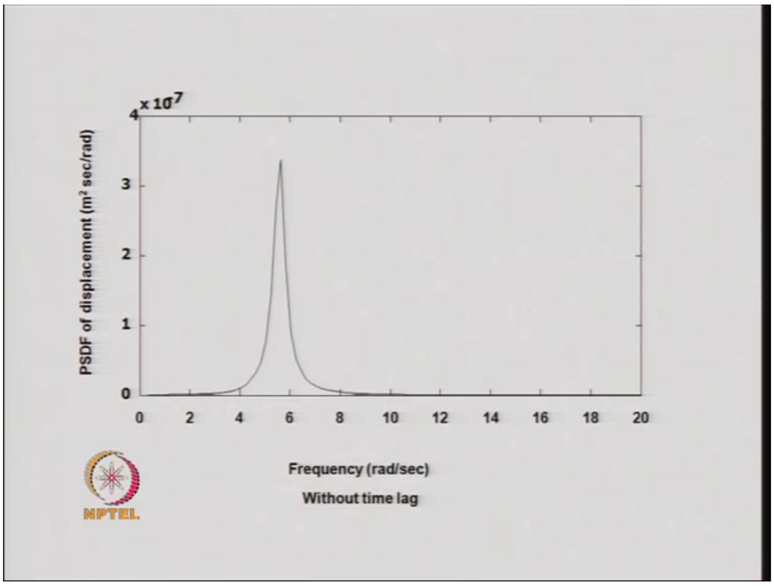

Now, here these the power spectral density functions, they differ a little bit for the case of with time lag and without time lag, and that is shown over here this is with time lag and for without time lag there was only one peak. Whereas, with time lag there are 2 peaks in the power spectral density function.

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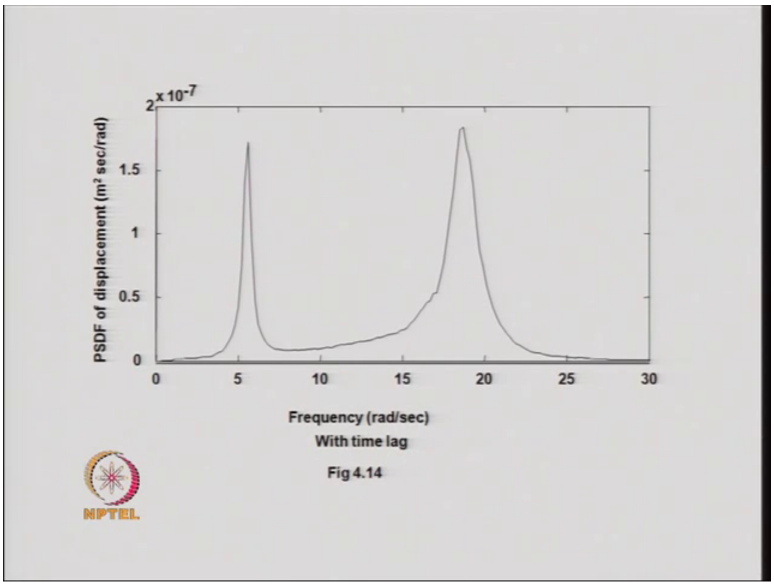

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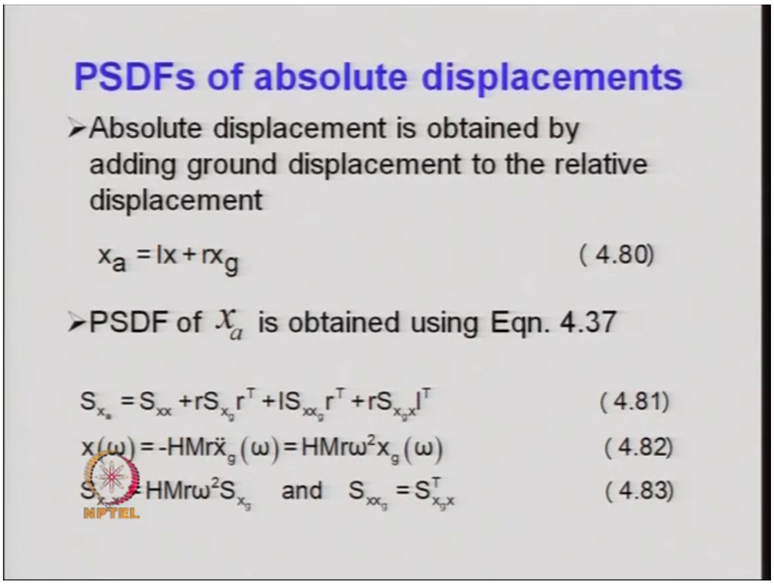

Next let us come to the absolute displacement; that is how to obtain the power spectral density function of absolute displacements of the responses, whenever we have a multi support excitation system, we have seen that there are 2 displacements that one need to consider; that is a relative displacement of the nonsupport degrees of freedom with respect to the support, that is called the relative displacement.

The other one is the absolute displacement in which the total displacements at different nonsupport degrees of freedom are considered. The it is necessary because if we wish to find out the member enforces, then we require the relative displacement of one n with respect to the other in terms of the total displacement for a multi support excitation system. For single support excitation the since the same displacement takes place due to the ground motion at all supports. Therefore, it is only the relative displacement that is good enough to obtain the member enforces from the displacement.

Now, the absolute displacement can be written with the help of this equation; that is, I into X plus r into X g. So, if we look at this expression over here.

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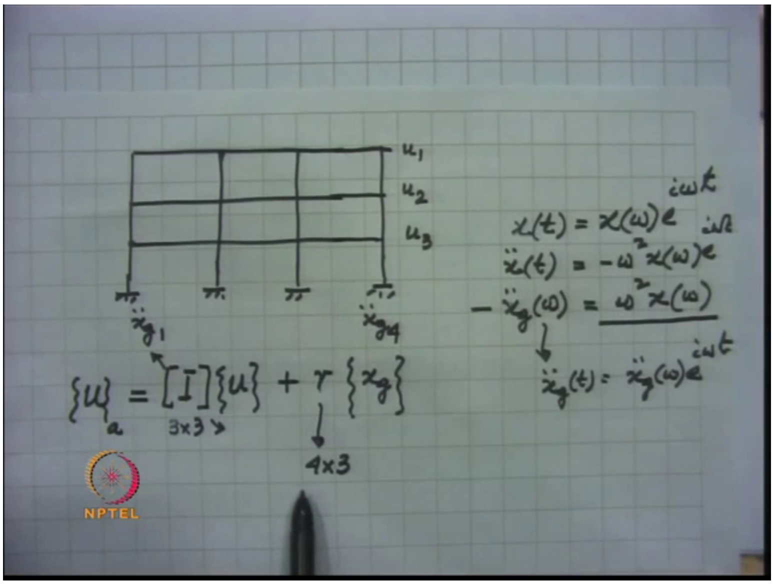

The I is a matrix of identity matrix that is diagonal terms of 1 1 1 off diagonal terms are 0; that multiplied by the 3 displacements u. That gives the relative displacements of the system. And the ground displacements are X X g 1 to X g 4 therefore, ground displacements and therefore, we have a r matrix which would be 4 by 3 r matrix. And when it will be multiplied by this X g vector, then we will get the influence of these displacements coming on to this nonsupport degrees of freedom.

So, the influence of this at these 3 degrees of freedom plus the displacements that is caused at these 3 points, because of the vibration of the structure with respect to the support that is given by this. Addition of this 2 becomes the absolute displacement or the total displacement. Now once we have these equations then using that equation one can write down the power spectral density function matrix of absolute displacement with the help of this equation. This becomes S XX itself because this is an identity matrix. And therefore, the I and I T of that if I multiply with S XX it remains S XX itself. This becomes r into S X double dot g into r T. Then S XX double dot g that becomes or X g that becomes is equal to I S X g r T. And S X g X that becomes into r into S X g X I T.

Now, what is not known to us? This is known to us. S XX is known to us from the dynamic analysis of the system. S X g that is also known, because we know the power spectral density function matrix of the ground acceleration and from that power spectral density function of ground acceleration one can find out the power spectral density function of the displacement. This relationship we had shown before; that is S X double dot g is equal to omega to the power 4 into S X g. So, there exist a relationship between the power spectral density function of ground acceleration and the ground displacement.

So, using that relationship one can find out S X g if S X double dot g is specified. Now here this is the term cross power spectral density function between the response that is the displacement, and the ground displacement that is to be obtained. And once this is known these S X a matrix is completely known. Now one can write down the relationship between the X g and X from all the relationship that exist between these that is S X omega is equal to minus H M r X double dot g omega.

So, this is a basic equation that we write. If we recall for the single degree of freedom system we had written X omega is equal to H omega multiplied by p omega. Now here the p omega is equal to M into r and H is the H matrix. So, X omega vector is related to X double dot g vector that is the ground acceleration vector, with the help of these relationship. And once we have this relationship then X double dot g omega, further can be written as omega square multiplied by X g omega. That follows from the relationship basic relationship that exist between the ground displacement, and ground excitation if they are assumed to be a harmonic excitation. And for harmonic excitation whether we represent it in the complex form, or in the real form we know that the displacement and the acceleration are related through omega square.

So, that is why this X double dot g omega is replaced by omega square into X g omega. And once we have this relationship; that means, this is equal to this X omega then from this I can easily get the cross power spectral density function between X g and X and X and X g. So, by here it would be of course, there will be a since it is a matrix there will be a complex conjugate of transpose. So, that star is missing over here. So, with the help of this relationships that is once we know S X g X and S X g, then one can obtain the value of the power spectral density function matrix of the absolute displacement. So, we stop at this. We will solve a problem in the next class.