Seismic Analysis of Structures
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Lecture – 21
Response Spectrum Method of Analysis

In the last lecture, we discussed how response spectrum method of analysis is developed. And then we discussed the accuracy of the response spectrum method.

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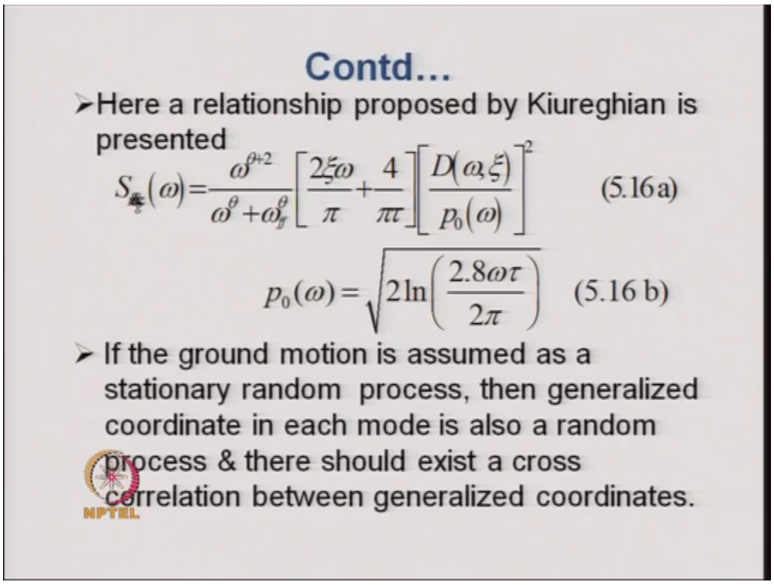

In the sense that there is an approximation involved in the combination of the maximum modal responses, and because of this approximation it is called an approximate method of analysis; however, it was found that there for most of the cases it fairly predicts the mean peak value of the response.

We developed the response spectrum method of analysis for single support excitation. That is the I is a vector of 1 1 1 or 1 0 1 0 1 0, whatever be the case. And then briefly we mentioned that the 3 different rules of the combination of the maximum responses or peak responses, they are how they are approximate and how this secure schedule and SRSS rule have been developed. Let me recapitulate it again. That is, we assume that the modal responses; that is the response in each mode of vibration is a quantity which is random, and it is uncorrelated to the other mode of response. Or in other words there is no correlation that exists between the 2 modal responses. And with the with this assumption the SRSS rule is developed.

However, if we consider that the there exists a relationship between the first mode response and the second mode of response and so on, then we say that they are correlated. And we must have a correlation function defined between these any 2 a modal responses, and improve the SRSS rule by convert it into ACQC rule which is called a complete quadratic combination rule, in which we bring in the rho i j, that is a correlation between 2 modal responses. And in that connection, we say that the derivation of CQC rule and SRSS rule, they are best obtained with the help of the random vibration analysis technique, assuming that the responses in each mode of vibration, that is the z 1 z 2 z 3 they are the random variables. And there should exist a correlation between these random variables. And that is how CQC rule was developed and CQC rule since it considers these correlation between z 1 z 2 etcetera, they are more exact compared to SRSS.

Then we also mentioned that since the power spectral density function of ground motion, and the response spectrum of ground motion both of them indicate the frequency composition of the ground motion as such, or the frequency composition of the future ground motion that is going that is expected. Therefore, they are should exist a relationship between the 2. And that relationship was obtained by Kiureghian using the equations in which the power spectral density function of ground acceleration was related to the displacement response spectrum, and p 0 omega which peak factor which varies with omega, that is involved. And these peak factor we have discussed before when we are discussing about the response of the structures to random ground motion in the PVR in the previous sets of lecture.

Now, if the ground motion is assumed as a stationary random process, then generalize coordinate a in each mode is also a random process. And there should exist a cross correlation between the generalized coordinates as I mentioned before. So, with this background in view the response spectrum method of analysis can be extended for multi support excitation, and today we are going to discuss about that.

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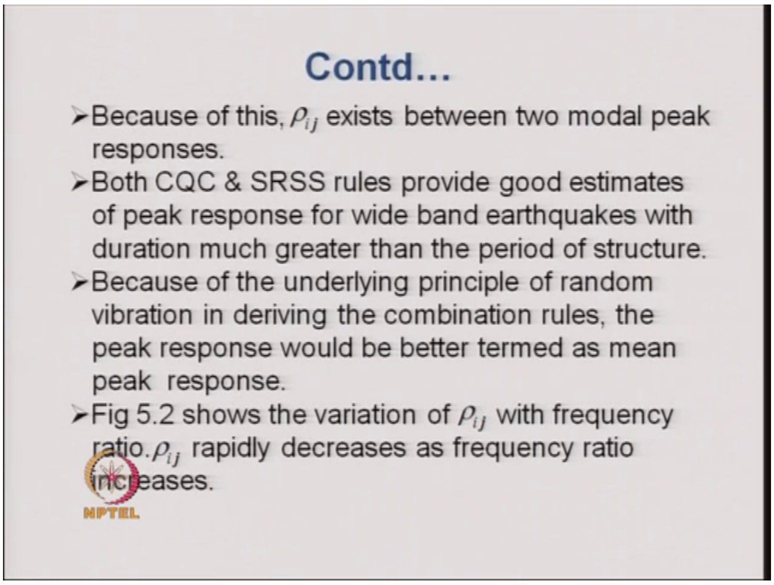

Because there exist a cross correlation between 2 modal peak responses, and we if we consider these rho i j then the CQC rule provides a better estimate than the SRSS rule, that of combination that we mentioned also. So, let us seek what basically rho i j looks like; that is the correlation between the 2 modal responses rho i j, that is plotted with respect to omega i by omega j, that is the frequency between the 2 modes and if we look at that plot.

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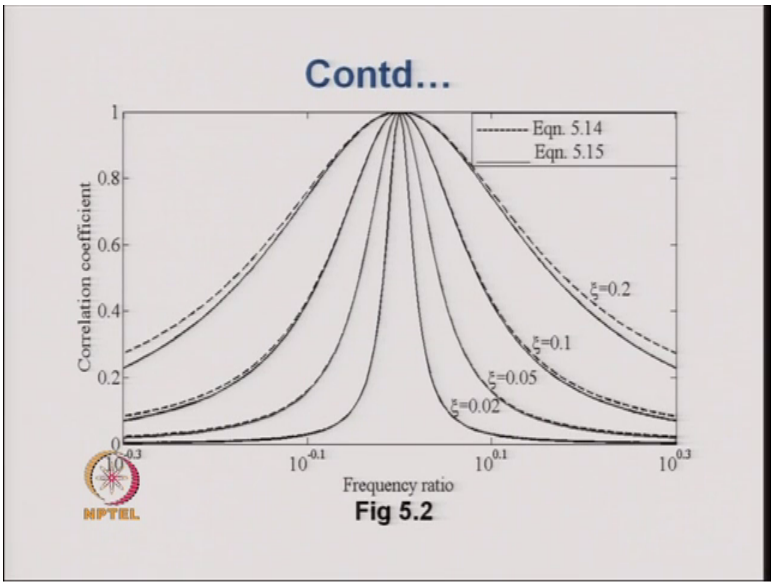

Then this plot looks like this, and the corresponding equation 5.14 and 5.15, that we discussed in the last lecture.

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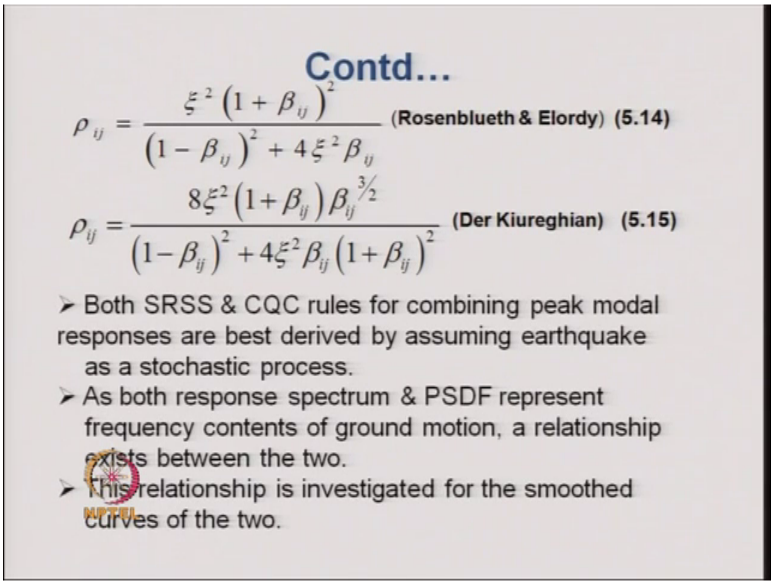

The these 2 equations that is rho i j is a function of beta i j, and beta i j is the ratio between omega i and omega j. And xi is the specified damping ratio.

So, 2 such empirical equations for rho i j were developed one by rosenblueth, and other by Der Kiureghian. And if we plot them against beta i j; that is the frequency ratio, then the plot looks like this. And one can see that on the right-hand side, the left-hand side is a symmetric half of the right-hand side. So, if we look at the right-hand side that is the positive frequency ratios. We can see that as the frequency ratio increases that is omega y by omega j that increases, then the rho i j basically sharply falls down. And beyond a certain value of the ratio the correlation coefficient is almost equal to 0.

So, that justifies the use of SRSS rule; that is SRSS rule is applicable when the 2 frequencies are well separated. And once the 2 frequencies are well separated, then the frequency ratio is also high. And as a result of that the correlation between 2 modal responses can be ignored, and they can be considered as independent random variables. So, using this figure and 2 equations, that is the empirical equation on rho i j that was developed. We take care of the correlation between the 2 modal responses. And using CQC rule, one can obtain and the combination between the 2 more responses.

Now, let us look at some example.

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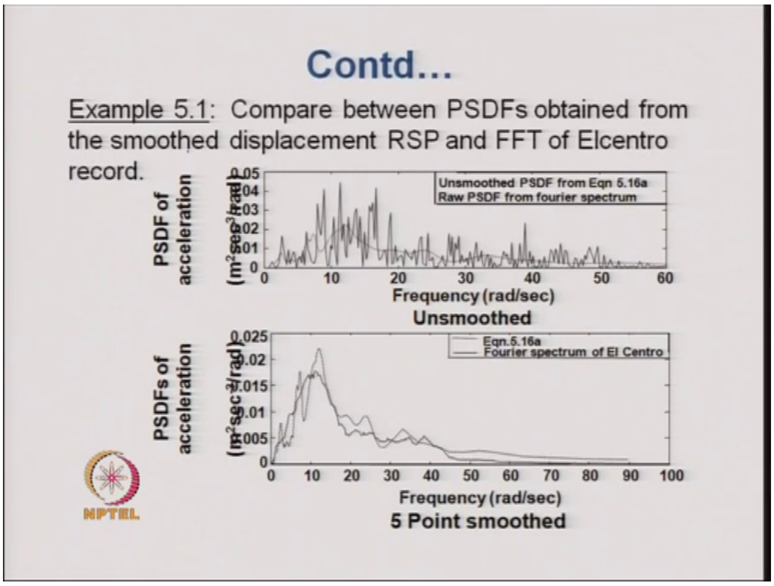

And in this example, the what has been done is that, the power spectral density function have been obtained from the smooth displacement RSP and FFT of elcentro record. That is the one we have used the elecentro record and from the elcentro record we obtain a response spectrum, and smoothened it. Then also for the same time history we obtain the power spectral density function using effective. And it was smoothened, and then this 2 were compared in this figure. You can see the unsmooth comparison; that means, unsmooth power spectral density function obtained by these 2 approaches are compared in the first figure. And then when they were smooth with 5-point smoothing, then you can see that the both PSDF compare quite well. That is the equation that is used for obtaining the power spectral density function of ground acceleration from the displacement response spectrum for a given damping that is quite valid. So, and that is seen from this example.

Now, let us come to the application of the response spectrum method of analysis for 2D frames.

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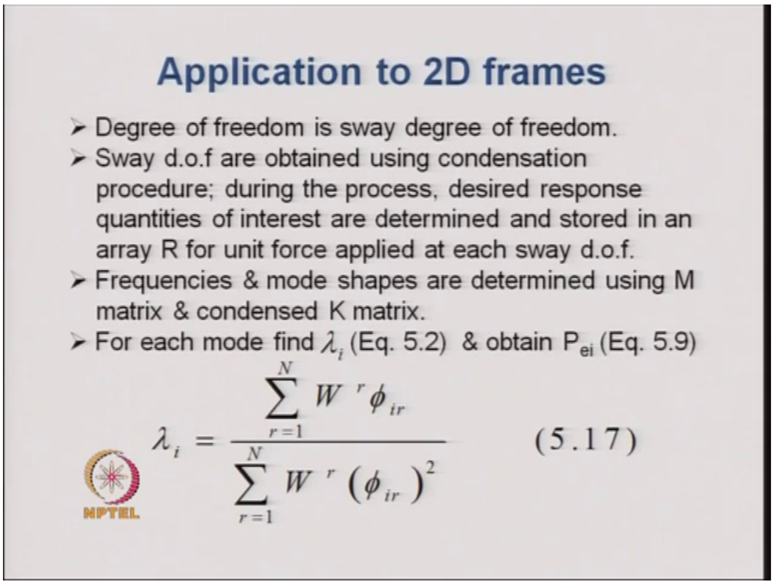

2D frame means, it is a building frame multi bay multi story building frame. And for that we proceed in this particular fashion. The first the dynamic degrees of freedom are considered, and generally the dynamic degrees of freedom for 2D frames they are considered as the sway degrees of freedom. These sway degrees of freedom are obtained by the condensation procedure. And this condensation procedure is obtained slightly in a different way than the condensation procedure that we have discussed before.

Here what we do is that, we write down the entire stiffness matrix of the frame, with the rotations included. And then apply unit load in the sway deduction one by one; that is, first we provide a unit load at the top of the frame, and put 0 corresponding to other sway degrees of freedom, and perform an analysis of this structure. And for that we find out the sway responses at all degrees of freedom and put them into a vertical column.

Similarly, any other response quantity of interest that can be also obtained for this case n can be stored in the computer in a matrix or a vector R as the case may be if we consider more than 2 3 responses, then it can be stored in the form of a matrix. If we are interested only say bending moment at a particular cross section then this will be a vector. So, and that vector or matrix is called r and these R matrix or the vector R and the matrix of the structure displacements, that is the that that what we obtained that is called the flexibility matrix of the structure, corresponding to the sway degrees of freedom. So, this flexibility matrix of the structure can be inverted to obtained the condense stiffness matrix for the sway degrees of freedom of the frame. Sometimes there are algorithms which require only the mass matrix and the flexibility matrix of the structure rather than the stiffness matrix with the help of that also the Eigen values can be obtained.

So, either we invert the flexibility matrix, or we directly use the flexibility matrix to obtain the mode shapes and frequencies of the structure. And by the time we do that we have also in an array R the response quantities of interest that are stored for unit forces applied at different sway degrees of freedom. Now for each mode of vibration one can calculate lambda i using the equation that we describe before. And this instead of the mass m into i we replace it by a summation convention. That is w r into phi i r. And sum it up for all the floors. And w is the weight of the floor rather than the mass and since the mass appears both in the numerator and the denominator. Therefore, it does not matter whether we reconsider mass or debate. So, this standard expression that is used in most of the practices in finding out lambda i they use this particular formula.

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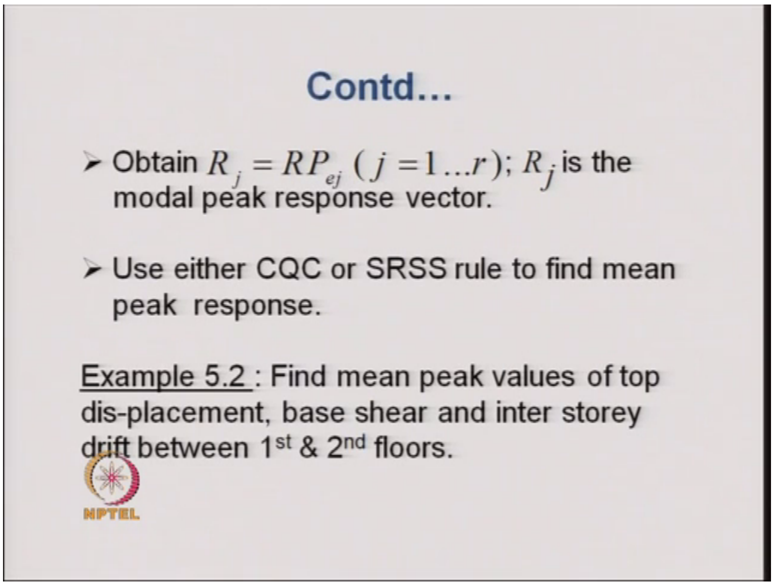

Once we obtain the lambda i, then we obtain the p i, that is the equivalent lateral load that exist on the structure for each mode of vibration from j is equal to 1 to r r is the number of modes that we are considering. And P e j how it is calculated that we have discussed before for single point excitation. And any response quantity of interest say R j say bending moment shear force drift or any other response quantity interest. That can be obtained by multiplying the equivalent lateral load, with the coefficient matrix for the response that is obtained before and stored, when we are obtaining the flexibility matrix of the structure by applying unit load at different sway degrees of freedom. After that one can combine the modal peak responses, either using CQC or SRSS rule and that provides us a mean peak response that is desired.

So, a problem is this now solved in order to obtain the mean peak values of top displacement, base shear and inter storey drift between first. And second floor for a 4 storey 2D frame which was solved before for the ground motion, which is deterministic. That is the for the same elcentro ground motion, the time history of ground motion was applied to this particular frame 4 storey frame. And the results the peak values of the results are known to us. And the same frame is now analyzed using the response spectrum method of analysis here the response spectrum which is used is the response spectrum of the elcentro earthquake.

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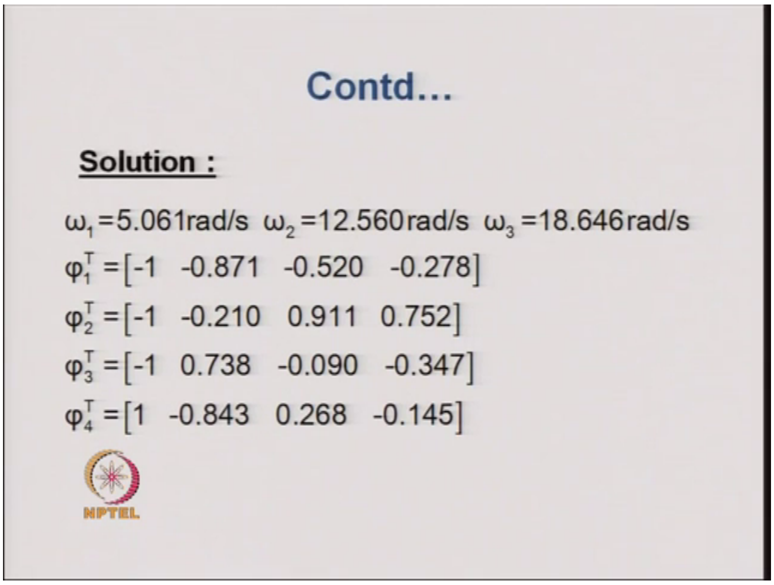

The digitized values of these response spectrum are available in many books including the books including the book, which is the reference book for this slides. The frequency is omega one omega 2 and omega 3 the 3 frequencies of that structure are given over here. And then the corresponding mode shapes are shown. We have given all the 4 modes.

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Table 5.1

Approaches	Disp (m)		Base shear in terms of mass (m)		Drift (m)	
	2 modes	all modes	2 modes	all modes	2 modes	all modes
SRSS	0.9171	0.917	1006.558	1006.658	0.221	0.221
CQC	0.9121	0.905	991.172	991.564	0.214	0.214
ABSSUM	0.9621	0.971	1134.546	1152.872	0.228	0.223
Time history	0.8921	0.893	980.098	983.332	0.197	0.198

We compare the responses or rather the mean peak values of the responses obtained by different methods in table 5.1. And you can see that when he considered 2 modes only, then SRSS rule give 0.9171 as the response. Then the CQC rule that gives 0.9121 quite close. The abssum gives 0.9621 which is expected to be greater than the CQC or SRSS rule and the time history response that provides 0.8921 that we obtained in the in chapter 3 when you are discussing the response of the same frame for elcentro ground motion using time integration technique.

And one can see that the time history response analysis, and the response spectrum method of analysis they give quite close results. When we consider all modes, then we can see that there is not much change in the response. And that is all the foremost are considered. And the result between the time history analysis and the response spectrum method of analysis using CQC and SRSS rule they compare very well. So, for as base shear is concerned, it is expressed in terms of mass, the when you consider the 2 modes only then you can see the comparison, the time history results give 980 as the maximum base shear whereas, the CQC gives 991 and the SRSS give higher value 1006.

So, CQC rule and the time history rule are quite close to each other. When we consider all modes, then the results compare very well between CQC and a time history analysis. Similarly, when you look at the drift between the first floor, and the second floor we see that the CQC rule and the time history analysis they are quite close to each other. Thus, we can see that in the response spectrum method of analysis, if one uses CQC rule for cases were the frequencies are not too well separated. Then the results compare quite well with those obtained from the time history analysis.

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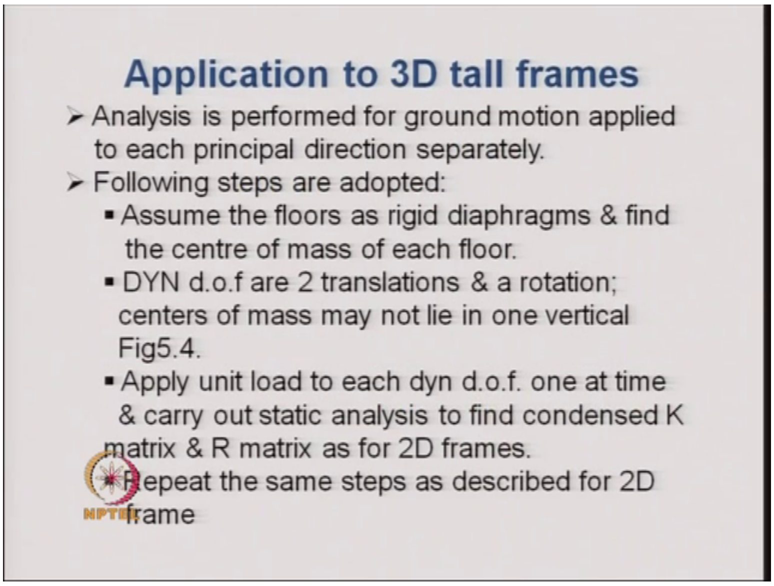

Now, let us look at the application of the same response spectrum method of analysis for 3 dimensional tall frames. In fact, in the code of practice the response spectrum method of analysis and their formulas are to be used they are given specifically for 2D frames. For 3D tall frames the code does not provide the kind of guidelines that will be following over here. That is, the code does not provide the responses using the response spectrum method of analysis, the way it should be performed. It considers the torsional effect of the 3D tall building slightly in a different way; however, here will be following the response spectrum method of analysis for 3D tall frames, which is just an extension of that what is done for 2D tall frames.

Now, the for the 3D tall frames we find out the principal direction of the 3D tall frame, and for each principal direction, we apply the ground motion separately and find out the response. And following steps are adopted. Firstly, what we assume that the floors are rigid like a rigid diaphragms. And for each floor we find out the center of mass. Then dynamic degrees of freedom at each floor that is 2 translation and a rotation about vertical axis which is consider. And in these dynamic degrees of freedom are shown in this figure.

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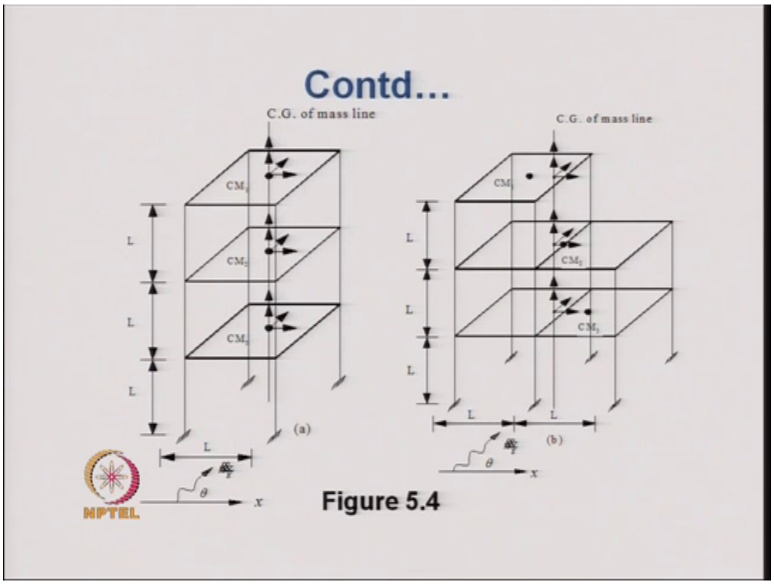

Left hand side figure is a symmetric building in which the center of mass for all the floors are lying on the same vertical line. In the second figure we have offsets, and the center of mass for each floor they are not coinciding in the sense that they do not fall in the same vertical line. And therefore, we construct a C G of the mass line. And the intersection of the C G of the mass line with the floors that is where the degrees of freedom are consider and 2 or 3 degrees of freedom are consider at each floor at that intersection point 2 translation, and the rotation about a vertical axis.

Now, once we have done that then we apply unit load to each dynamic degrees of freedom one at a time, and carry out the same kind of static analysis that you had performed for the 2D frame. To find out the condense stiffness matrix and then array of R or a R matrix of the responses of desired quantities. And then we go ahead in the same fashion as we have done for the case of 2D frame. We have now different mode shapes and frequencies, and for each mode will get a mass mode participation factor. And once we use the mode participation factor for each mode, then one can obtain the general single degree freedom equation written in terms of the generalized coordinates and because of that coupling of the mode shapes. That is in certain cases we have a translation in 2 directions as well as a torsion. So, that kind of mode shapes are known as the couple mode shapes.

So, for the couple mode shapes we have a torsional moment created at the intersection point of the vertical line, or the mass vertical line with the floor. And apart from the 2 forces that can act in the 2-translation direction. So, for asymmetric building even if we are applying a single component earthquake, in a particular direction which may consider coincide with x direction or y direction. We can expect that there could be a force in the x direction force in the y direction, and a torsion about a vertical axis for a particular mode where which is a couple mode.

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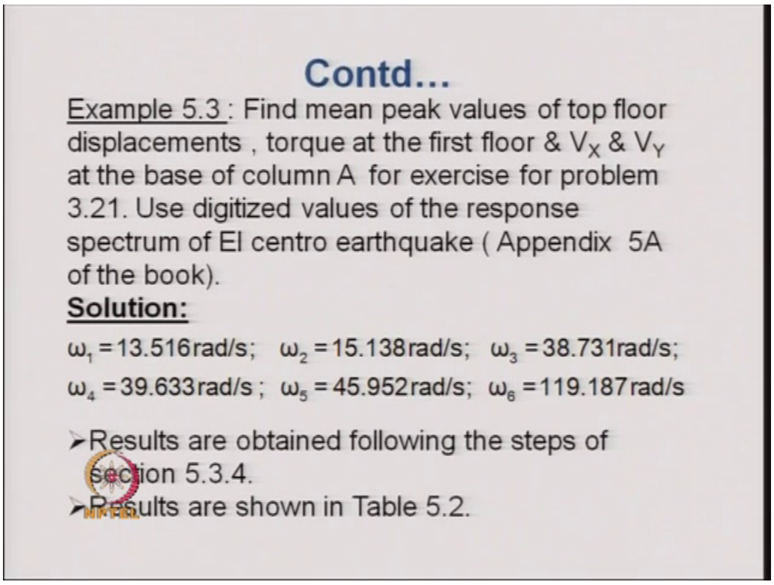

So, the equivalent static load that is P e i that we had seen before for the 2D frame. In that P e i now we expect that there could be the torsional moments apart from the translational force in x and y direction for a 3D frame. And thus, for an asymmetric building we expect both a torsional motion about the vertical axis and 2 translational displacement in x and y directions. An example of this type is solved for this particular problem in which we have taken a 2 storey frame.

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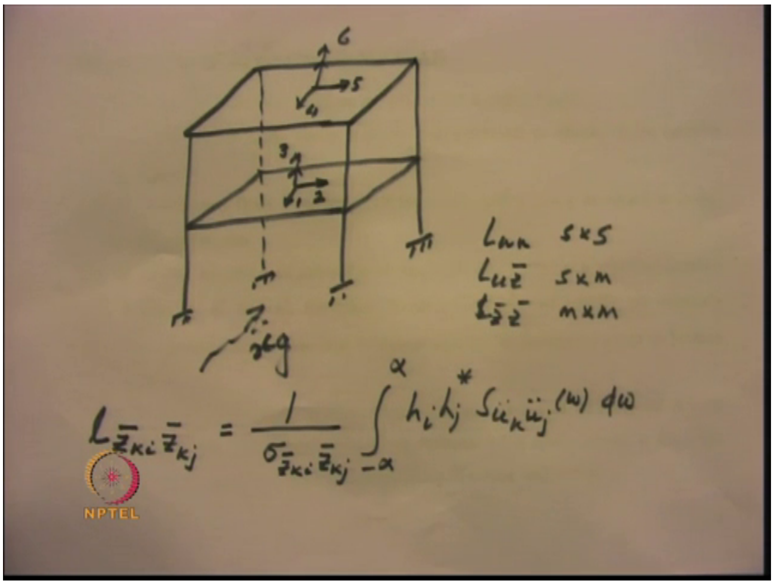

And in these 2 storey frame the degrees of freedom are considered as this 1 2 3 and 4 5 6 these are degrees of freedom. And corresponding to these 6 degrees of freedom, we obtain the mode shapes and frequencies of the structure that is we had 6 mode shapes and 6 frequencies. And the response quantities that we require, that is the torque at the first-floor level and v x and v y at the base of column a that is to be found out.

So, for these response quantities we obtain a an array of matrix R; that is the R matrix that we talked about, when we were obtaining the flexibility matrix for the structure by applying unit load one at a time at each degree of freedom. The results of the analysis are shown in this table. That is, when we consider the SRSS rule, CQC rule and time history analysis for the same frame. We find out that the displacement that is there for direction 1 and 2 these displacements are 0.0431 and 0.1325. They are obtained by SRSS rule, and CQC rule respectively. And the time history analysis provide a response maximum value of the response as 0.1206.

In the for the second degree of freedom, that is at these in these degree of freedom the earthquake excitation is acting for in this degree along the direction of these degree of freedom.

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TABLE 5.2

Approaches	Displacement (m) (1)	Displacement (m) (2)	Torque (rad) (3)	V_x(N)	V_y(N)
SRSS	0.1431	0.0034	0.0020	214547	44081
CQC	0.1325	0.0031	0.0019	207332	43376
Time history	0.1216	0.0023	0.0016	198977	41205

- Results obtained by CQC are closer to those of time history analysis.

Whereas, the second direction is perpendicular to that and one can see, that the displacements in that directions are quite small compared to the directions in which the excitation or the ground motion is applied. And the value of the displacement or maximum value of the displacement obtained from the time history analysis is 0.023 whereas; we obtain from using CQC rule as 0.0031.

The torque, that is obtained using CQC rule and the time history analysis about the vertical axis and they compare quite well. So far as the shear forces are concerned in the x and y direction, we can see that the CQC rule and time history analysis they gives fairly or they compare valuable.

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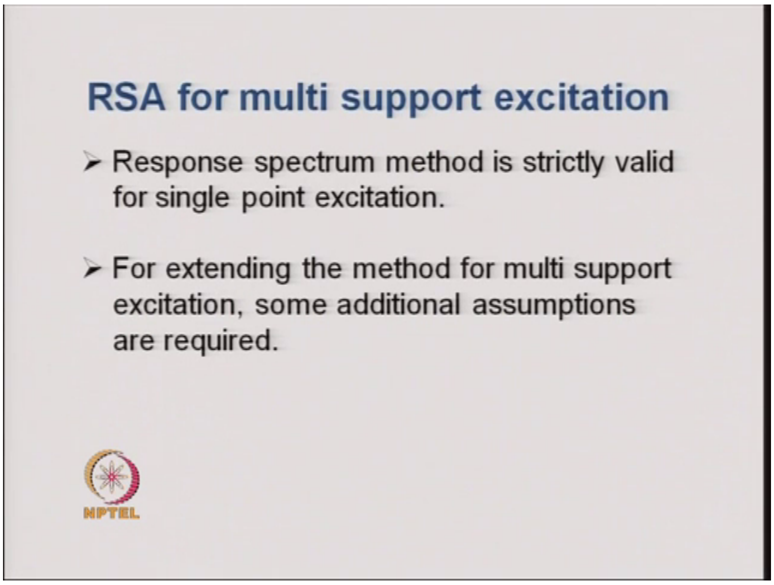

Next let us now try to develop or extend the method of response spectrum analysis to multi support excitation case. And as I told you before that the response spectrum method of analysis is not strictly valid for the multi support excitation. System it is strictly valid for single point excitation for which it was derived; however, if we wish to extend it for the multi support excitation case, then some additional assumptions are made.

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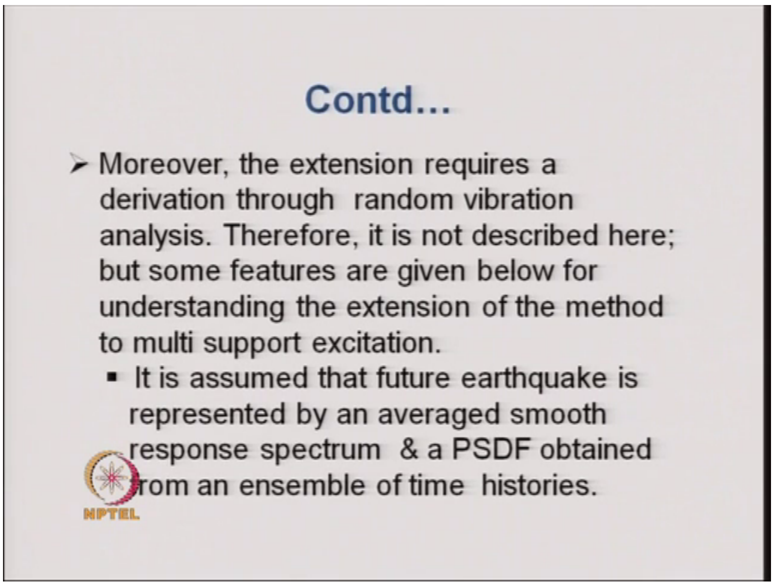

Then the derivation of the response spectrum method of analysis for multi support excitation requires a random vibration analysis. That is, a part of it uses the random vibration analysis to arrive at the required equations that can be used for finding out the mean peak response in the spirit of response spectrum method of analysis.

Since it requires some random vibration analysis therefore, the entire derivation is not described here; however, some features of that derivation are mentioned in order that the extension of the method for multi support excitation can be understood. In the first place it is assumed that future earthquake is represented by an averaged smooth response spectrum, and a power spectral density function both of them are obtained from in ensemble of time histories.

So, we assume that their exist an ensemble of time histories or the past time histories of earthquake. And using that ensemble of time histories we have established first that the that is a stationary random process, and then we obtain a power spectral density function, for that ensemble of time histories that is we have obtained the unique mean square value for that ensemble, and the distribution of that mean square value gives us the power spectral density function what we describe before.

Many a time one can use the assumption of a good city in order to obtain the power spectral density function easily with the help of a single time history using Fourier transformation method. The response spectrum of the earthquake can be obtained from the single time history record. The weight is done and we have described that the procedure we have describe when we are discussing about the seismic input to the structure. And both response spectrum and Ps power spectral density function thus obtain are smoothened.

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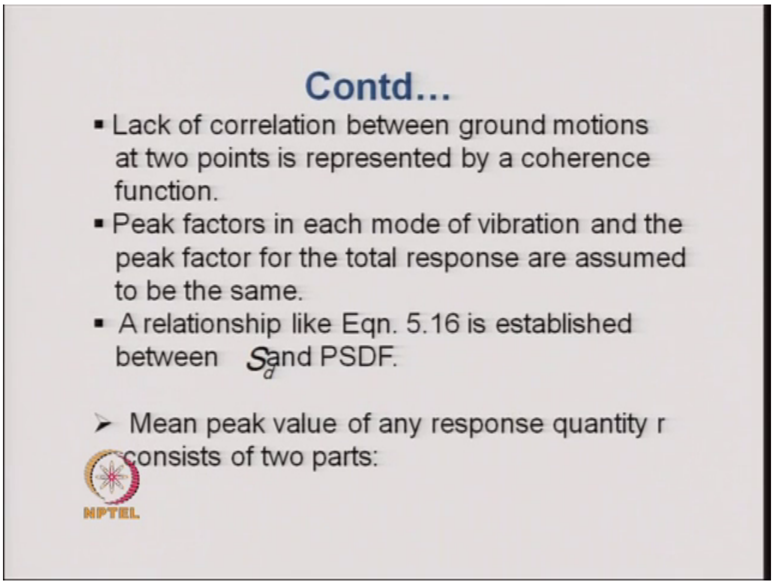

Then the most important point that comes in the extension of the response spectrum method of analysis from single point excitation to the multi point excitation is the consideration of the lack of correlation that exist between ground motions at 2 points. And that we have seen that can be obtained either by knowing the time lag between the 2 supports, or by using a coherence function. And for the case of random vibration analysis we have seen that we have a number of empirical equations which are available to represent the coherence function to represent the lack of correlation between the ground motions at any 2 points. And we can use any one of those coherence function. Or the coherence function that is appropriate for that particular site or the area.

Next assumption that we make is the about the peak factor. We have seen that them peak response can be obtained by multiplying a peak factor with the standard deviation of response, or Rms value of the response for the 0-mean process. And that peak factor can be obtained with the help of the first 3 moments of the power spectral density function. And we also had shown an equation which provides a value of the peak factor provided we know the duration, and the moments of the power spectral density function. Using that equation, one can find out the peak factor.

Now, here we make an assumption that the peak factor in each mode of vibration. And the peak factor for the total response that is when we combine all the modes of vibration, and obtain the total response. Then the peak factors they are assumed to be the same. In fact, these peak factors in each mode of vibration may vary. And the peak factor for the total response could be different than the peak factors that we obtain for each mode of vibration.

However, for the development of the method, a crucial assumption is made that these peak factors are all same. Then a relationship like equation 5.16 is established between the power spectral density function of the ground motion and the displacement response spectrum. And one such equation on empirical equation was shown before in equation 5.16, then the mean peak value of any response quantity of interest r that is consisting of 2 parts.

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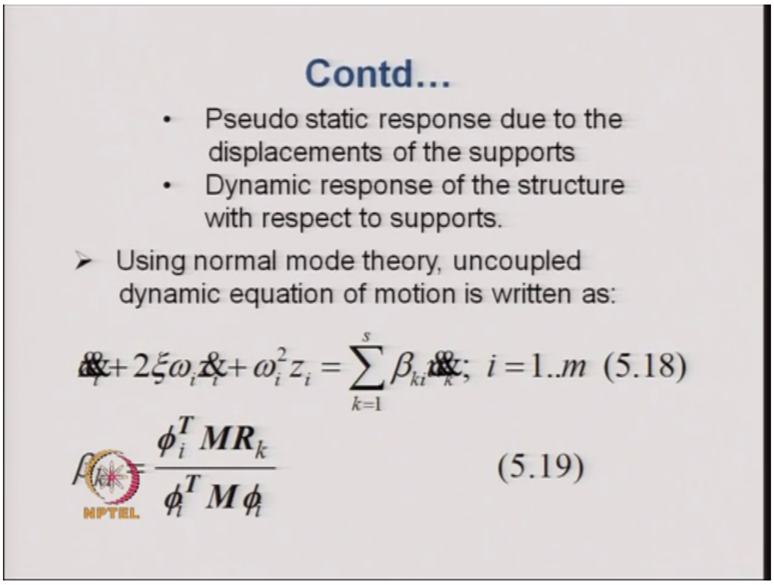

The first part is called a pseudo static response due to displacement of the responses. And the second part is the dynamic response of the structure with respect to supports.

Now, that they have been discussed before. If we are wanting to find out the absolute displacements at the different nonsupport degrees of freedom, then we have to add 2 responses. One response or one displacement will be the dynamic displacement of those nonsupport degrees of freedom with respect to the base. And then to that will add the displacement that is caused due to the ground displacement at different supports. And that would create a differential displacements between different support resulting in some displacements at different sway degrees of freedom.

So, these are called the pseudo static response. And this pseudo static response is added to the dynamic response in order to get the total absolute value of the response either for displacement, or for any other response quantity of interest. So, in the usual derivation of the response spectrum method of analysis, the second part is taken into consideration. That is a dynamic response of the structure with respect to support that is considered. Because for single support excitation the response of the structure at nonsupport degrees of freedom.

They basically are the dynamic response, plus the uniform ground displacement that takes place at different nonsupport degrees of freedom. Because of single point excitation they are added together, and we can get the absolute value of the displacements at different degrees of freedom; however, these absolute values of the displacement at different degrees of freedom. Do not carry much meaning, because if you are wanting to find out the bending moment or shear force at any particular cross section. Then the moment we try to find out the relative displacement between the 2 ends of the member. Then the effect of the ground displacement at different supports, they cancel because they are the same at different degrees of freedom because of the single component excitation.

So, the second part of the response is good enough for single point excitation; however, when we consider the multi support excitation then we have to take into account these part also. And because of the presence of this part we require some kind of additional information or additional assumption, and also, we have to go into a random vibration analysis in order to take into account it is effect, into the response spectrum method of analysis.

So, first what we do is that we find out using a normal mode theory, uncoupled dynamic equation of motion for each degree of freedom. And here instead of a single mode participation factor lambda i we have now beta k i represents the mode participation factor. And these beta k i these mode participation factor, for a particular mode has different components, that is for different supports we have one mode participation factor in a particular mode. And that beta k i is given by this equation, and you know how it has been derived.

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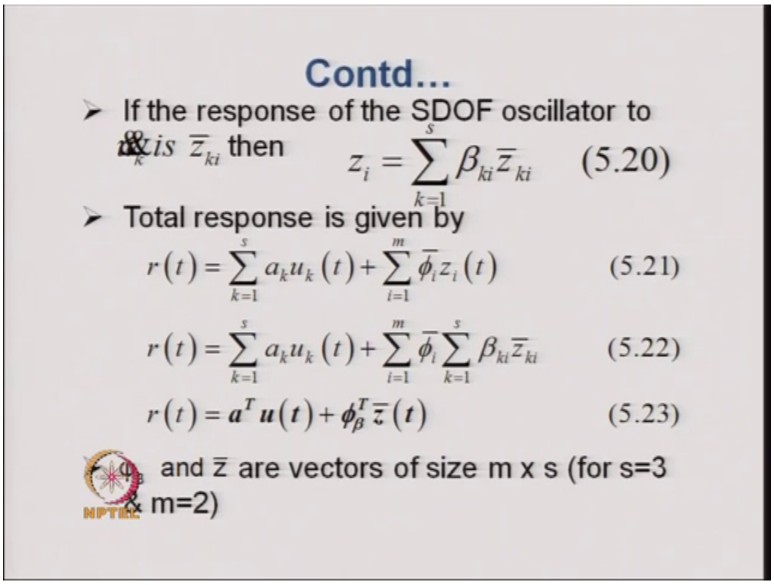

Next, we consider a single degree freedom oscillator subjected to a ground acceleration u double dot k, that is the this is the u double dot k. That is the ground excitation at the kth support, if we consider that, then the response for the single degree of freedom oscillator to that ground or that that support excitation is a z bar k i. Then the total generalized displacement in the ith mode can be written by this summation that is beta k i multiplied by z bar k i and summation is taken over all the supports. Now once we are able to obtain the value of z i, then the total response which will be obtaining will be equal to 2 components of the response that we discuss before. This is the pseudo static part, and this is the dynamic part which is coming from the dynamic analysis, or modal dynamic analysis that we have performed over here.

Now, phi bar i is the mode shape coefficient for the response quantity of interest. It could be displacement. It could be bending moment. It could be shear force, but these phi bar i has been obtained before. And how we obtain the mode shape coefficient for different response quantities of interest that we have describe before while discussing the modal analysis technique.

This a k is the coefficient matrix. That is if I give a unit displacement at the kth support. Then and keeping all other supports locked or fixed. Then what is the displacement that is obtained at different nonsupport degrees of freedom. And that can be arranged in the form of a matrix. And the elements of the matrix can be taken, and those using those elements of the matrix one can find out a response quantity over all the pseudo static response quantity over here using this summation. Now this a k could be a coefficient for displacement, could be a coefficient for bending moment could be a coefficient for shear force, what about be the response quantity of interest. So, this is the pseudo static part, and this is the dynamic part. Then what we do is that we substitute in place of z i this equation as a result of this, this r t any response quantity of interest can be written in this particular form.

Now, this equation can be written in the matrix form in which a T is the transpose of the matrix the coefficient matrix for pseudo static response. U t is the vector of the ground displacement at different supports, and phi beta t is a transpose of a matrix called phi beta will shortly explain it what it is. And z bar t is the response of the generalized or the generalized displacement that we obtain for each mode of vibration. The phi beta and z bar are vectors of size m into s. That is m is the number of modes that we are considering, and s is a number of supports.

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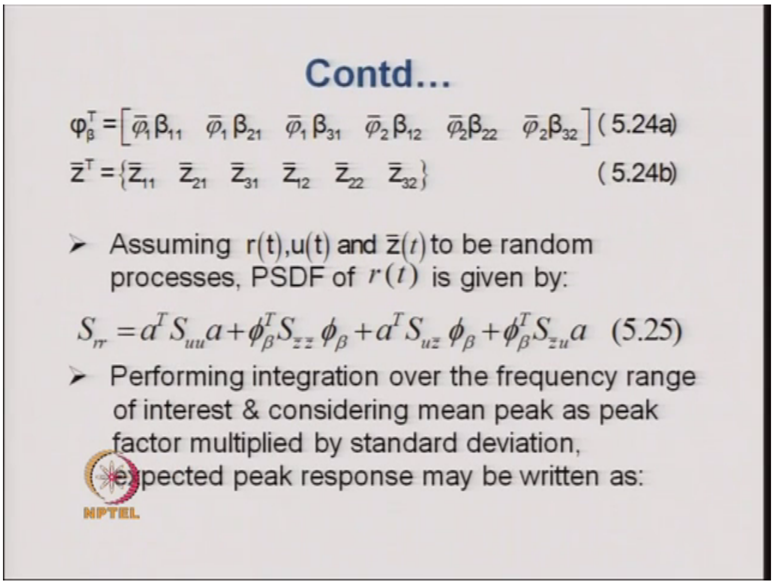

For 3 support and 2 mode one further elaborate this phi beta vector and the z bar vector. So, you can see that phi beta t is equal to phi bar 1 beta 1 1 phi bar 1 beta 2 1 phi bar 1 beta 3 1.

Then phi bar 2 starts phi bar 2 beta 1 2 phi bar 2 beta 2 2, phi bar 2 beta 3 2 and so on if we have got some other or mode. Then it starts with phi bar 3 beta one 3 like that. The z bar t is z bar 1 1 z bar 2 1 z bar 3 1. Since we are considering here the number of supports as 3, then the modes as 3 then z bar 1 1 z bar 2 1 and z bar 3 1 z bar 1 2 z bar 2 2 and z bar 3 2. So, these are the z bar values.

Now, we assume that r t u t and z bar t to be random processes. Since the excitation is random, and represented by an ensemble of record. Therefore r t the response, the u t the displacements at different supports, and z bar t the generalized responses, they are all random processes. Then if r is represented by this equation, that is equation 5.23, then the power spectral density function of r which is called S r r will be can be written as a T S uu a plus phi T beta is z or z bar phi beta, plus a T S u z bar phi beta plus phi T beta S z bar u a.

So, that follows the derivation for the power spectral density function of 2 quantities or 2 processes, or other waited processes, which we had seen before, if you call. If y is a response quantity and is a summation of a into x and plus b into z, then the power spectral density function of S y or power spectral density matrix of S y y if y is a vector can be written by a rule, which we have discussed before. And using that rule we have derived this equation for S r r. Now in order to obtain the mean peak value, we have to in obtain the Rms value or the standard deviation of responses. And that will be discussing in the next class.

Thank you.