Seismic Analysis of Structures
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Lecture - 23
Response Spectrum Method of Analysis (Contd.)

In the last lecture, we discussed about the Response Spectrum Method of Analysis for Multi Support Excitation. In that we derived the Response Spectrum Method of Analysis for Multi Support Excitation System. The method is developed through random vibration analysis.

However, the last portion of the derivation that is the results of the derivation laid to a very interesting equation. We use using that equation one can obtain the response spectrum analysis of multi support excitation. That is the response spectrum method of analysis can be performed for multi support excitation using the displacement response spectrum of earthquake and a compatible power spectral density function of ground acceleration. This is achieved through an equation that relates the displacement response spectrum of earthquake and a power spectral density function of ground acceleration.

Apart from that a coherence function is needed, for accounting the partial correlation of the excitation, that takes place at the between the two supports.

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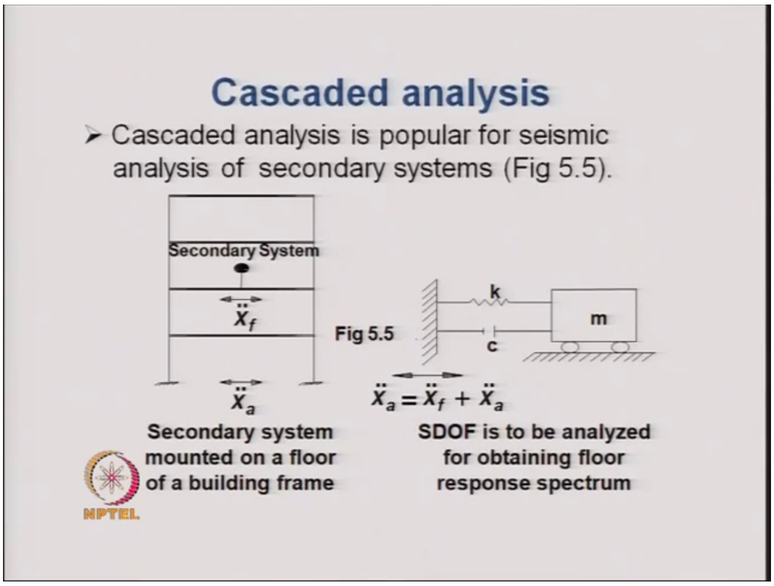

We also illustrated the method with the help of two example problems. Today's lecture we will concentrate on the cascaded analysis; the use of response spectrum method of analysis for the non-classically done system. Cascaded analysis is a very popular for seismic analysis of secondary system. In fact, the cascaded analysis was specifically designed for the earthquake analysis of secondary systems, which are there in many structures specially, for industrial structures, we have many secondary systems.

Similarly, in nuclear power plant structures, we have many secondary systems or many of the structures in the nuclear power plant structures are idealized as secondary system. Then we have structures like buildings on the top of which we have a small tower. Then, the tower is considered as a secondary system. So, there are many such examples of the secondary systems which are used in civil engineering structures.

Now for these secondary systems, if we wish to find out the earthquake response then, one of the popular ways of doing it is by using response spectrum at the base of the applied at the base of the secondary systems. The motivations for developing the cascaded analysis are twofold number one, if we wish to consider the total structure that is the primary and secondary structure together. Then, the number of degrees of freedom maybe very large and therefore, the analysis may entail lot of computational time. Secondly, the cascaded analysis is performed because the cases why do we have secondary systems along with the primary system then, the damping matrix becomes non-classically damped. For example, if we have a piping system connected to the to a main structure or piping systems supported on the floor of a structure made of concrete then, we have two kinds of damping arising.

Similarly, we can have structures in which there is a tower on the top of a building and in that case the damping of the tower and damping of the building would be different, resulting in a non-classically damp situation. So, in order to avoid these non classical damping, one way is to isolate the two system, that is the secondary system and the primary system and analyze them separately. The output of the primary system becomes the input for the secondary system. For example, if there is a piping system mounted on the floor of a particular building then, the response spectrum for the floor become the input for the secondary system or the piping system at the base.

In this kind of analysis where you separate the two systems and analyze them separately; we ignore the interaction effect between the secondary system and the primary system and in most of the cases it is found that if we ignore this interaction effect then, we do not get much error in the analysis or the response of the secondary system. For this purpose, the floor response spectrum becomes an important thing in the cascaded analysis the figure 5.5 shows the a secondary system mounted on the floor of a building there it is excited, the building is excited by a ground acceleration. The absolute acceleration is equal to the, at the floor is equal to the acceleration of the floor with respect to the base and it is ground acceleration is added to it. It will not be a, it will be g X double dot g is a ground acceleration. So, X double dot a is the absolute acceleration of the floor. X double dot g is the ground acceleration.

So, what is done is that first we remove the secondary system from the primary system, only retain the mass of the secondary system to the primary system, analyze it for the given ground acceleration and the results of the analysis provide the time history of the acceleration at the floor level.

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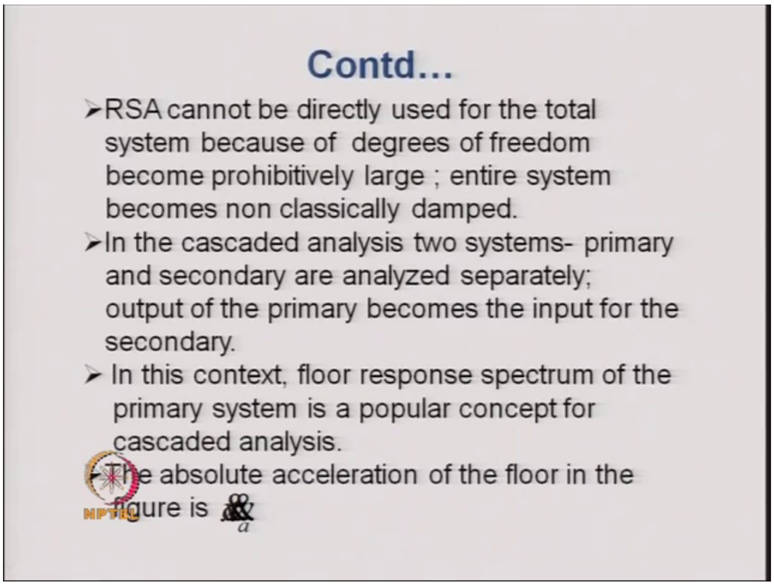

Then to this time history of acceleration we add the ground time history of acceleration to obtain the absolute time history or time history of the absolute acceleration. This time history of absolute acceleration then becomes input for the piping system which is mounted on the floor. We assume that the piping systems are attached to the floor in a way that one can assume ends or the supports of the piping system to be fixed. Thus, for a fixed end support there is a acceleration at support that acceleration is the absolute acceleration of the floor. So, by knowing the time history of the absolute acceleration of the floor one can find out the response spectrum of the floor for the given absolute value of the acceleration time history.

So first, the floor response spectrum is obtained from that time history and once we get the absolute or the response spectrum for the absolute acceleration of the time history then, that response spectrum is inputted at the base of the piping system and we carry out the usual response spectrum method of analysis that we have discussed. In this case we can consider the bases to be a single support excitation case, that is we can assume that there is no time lag between the or face lag between different supports and all the supports are uniformly excited by the time history of the absolute acceleration. If two ends of the supports or sum of the supports are say connected to a another floor or the supports are connected at different floors then, the supports are excited by different excitations.

In that case one can use the response spectrum method of analysis for multi support excitation that we have discussed before. Therefore, in the cascade analysis one can carry out a response spectrum method of analysis by decupling the secondary system from the primary system, finding out the floor response spectrums in case more than one number of floors are utilized for supporting the secondary system at each support of the secondary system, if the excitations are different then also one can perform the response spectrum method of analysis.

So, this has become a very popular technique for finding out the earthquake response of secondary system in structures and specially for nuclear structures. The secondary systems are considered in a different way in the sense that the secondary systems are not clubbed along with the primary system and the total a system is analyzed for the earthquake excitation because in that case the number of degrees of freedom that is involved becomes too many.

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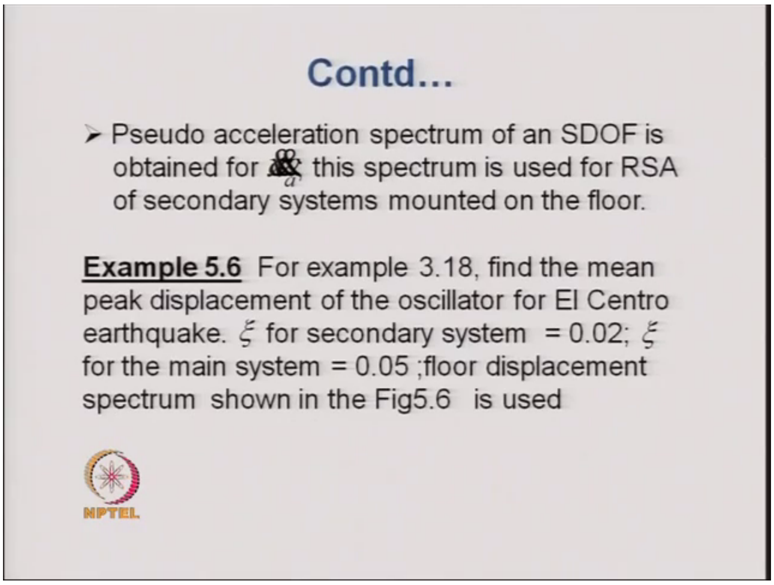

An example problem is solved here to illustrate the use of the response spectrum method of analysis for secondary system.

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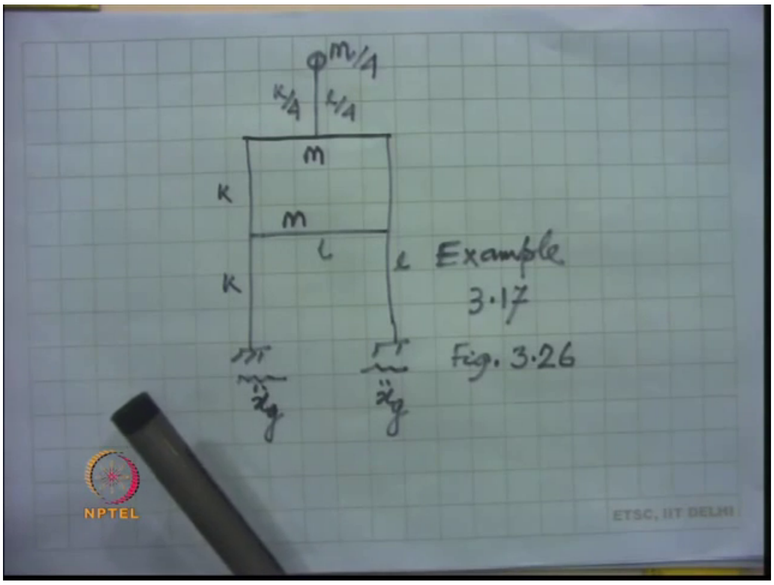

This is a building frame. In this building frame we have a secondary system, the secondary system has a mass m by 4 and it has a length l by 4, the stiffness is k by 4. Whereas, the main building it has a stiffness of k and this is the mass of the two floor. Damping for the secondary system is 2 percent, damping for the primary system is 5 percent. This is an exercise problem given in chapter 3. That is the response analysis for the deterministic ground motion, in that this problem is given as an exercise problem to solve.

Now, for this system, for the specified ground acceleration here, the specified ground acceleration is taken as the El Centro ground excitation. We perform a time history analysis. We can perform the time history analysis either in time domain or in the frequency domain and find out the time history of the absolute acceleration for the top floor level and for that first, we find out the time history of the relative acceleration of the top floor that is a relative acceleration with respect to the base and to that we add the time history of the ground acceleration. So, that provides a time history of the absolute acceleration of the top floor.

Then, for that time history of absolute acceleration we obtain a corresponding response spectrum, response spectrum of acceleration. How to obtain the response spectrum of acceleration? That we had discussed before, that is we take a single degree of freedom system, the base or the support of the single degree of freedom system is subjected to the time history of the absolute acceleration that we have obtained.

Then, for different frequencies for the single degree freedom system that is by wearing K and l, we can have a different frequencies and corresponding to different time periods for that we can obtain the displacement response spectrum first. Then, from displacement response spectrum we derive the pseudo velocity spectrum and from that we can derive the pseudo acceleration spectrum or the acceleration spectrum that we generally use.

These response spectrums are obtained now, for the damping that is the required damping and in this case this damping is 2 percent because the secondary system has a damping of 2 percent. So, once we have the response spectrum of a acceleration for 2 percent damping then, for this secondary system a excitation record acts at the base of the support whose response spectrum now is known.

And therefore, the time period for the secondary system is can be worked out with the help of the stiffness and the mass and once we know the frequency of the secondary system from that we can get the time period corresponding time period. And for that time period one can obtain the acceleration spectrum ordinate and using that acceleration; acceleration spectrum ordinate, we can obtain the force that would be acting for this system.

Since it is a single degree freedom system we need to only consider one mode or there will be only one force static force that will be acting on to the secondary system and we can find out the bending moment shear force and the top displacement of the secondary system. This problem is made easy by providing only one degree of freedom system.

However, one can have a structure which is not a single degree freedom system like this, In fact, there could be a piping system in which there could be two three four supports and in that case at all the supports we can assume that the same time history of absolute acceleration would be acting for which the response spectrum we have obtained, then, carry out the usual response spectrum method of analysis for the piping system

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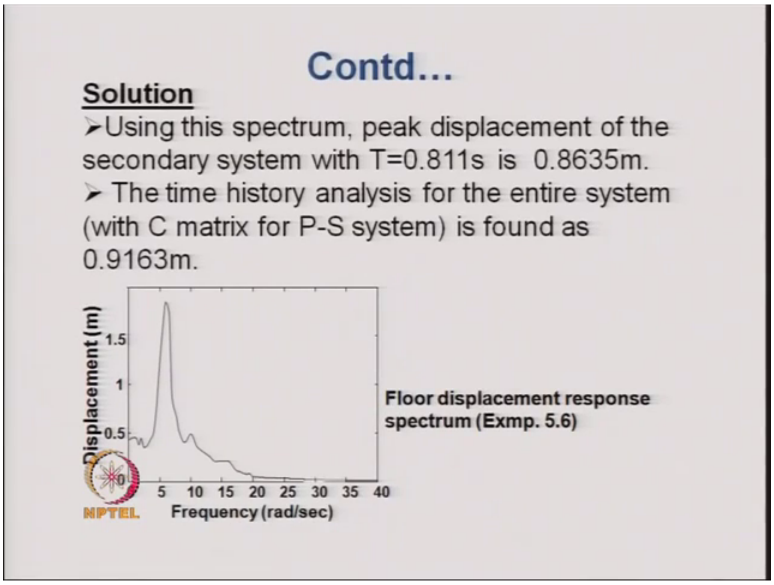

The result for this problem is shown here. The time period of the secondary system was obtained as 0.811 second. The displacement floor displacement response spectrum is given over here. We can multiply the floor displacement response spectrum with the omega square in order to get the pseudo acceleration response spectrum. Once we get the pseudo acceleration spectrum then, corresponding to this time period 0.811 second one can obtain the value of the spectral acceleration and multiply that spectral acceleration with the mass that would give the force acting on the top of the cantilever resulting in a lateral displacement at the top and the corresponding base shear.

The result that we obtained from the cascade analysis was 0.8635 meter as the deflection of the top of the cantilever whereas a proper time history analysis for the entire structure considering the secondary system or the secondary system was included into the entire structures. So, that it becomes a composite primary secondary system, we call it as P-S system; for that, we construct a C matrix for the primary system. The C matrix is constructed using the assumption of Rayleigh damping that is, is a proportional damping with 5 percent, a critical damping for the main structure. The C matrix can be obtained as C is equal to alpha times m plus beta times k. That is how, one can obtain the C matrix for the main structure and then, add on to that the damping for the secondary system. Coupling between the terms of the damping matrix between the primary and secondary system is said to zero.

So, with that C matrix and considering the total k matrix of the system that is, in obtaining the k matrix of the system, we include the degree of freedom at the top of the cantilever. So, we with that k matrix and with that C matrix and the corresponding m matrix one can perform a time history analysis for the entire system subjected to the ground acceleration at the base. The result of that provided a maximum displacement of the cantilever as point 9163.

So, we can see that the actual time history analysis for the P-S system and the cascaded analysis provided nearly the same response and in this the response analysis becomes easier. In the sense that, we are solving two problems, each problem is solved using response spectrum method of analysis. But, the degree of freedom for each problem becomes less compared to the complete P-S system.

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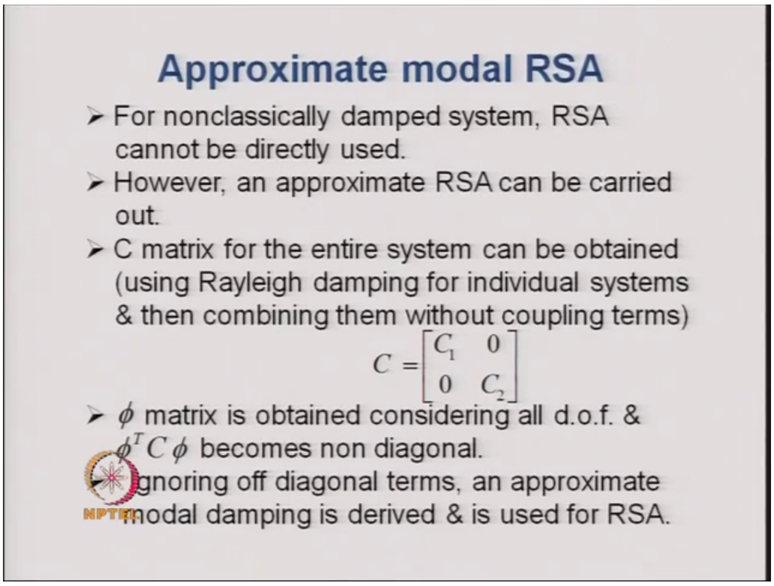

Next we consider the application of the response spectrum method of analysis for non-classically damp system.

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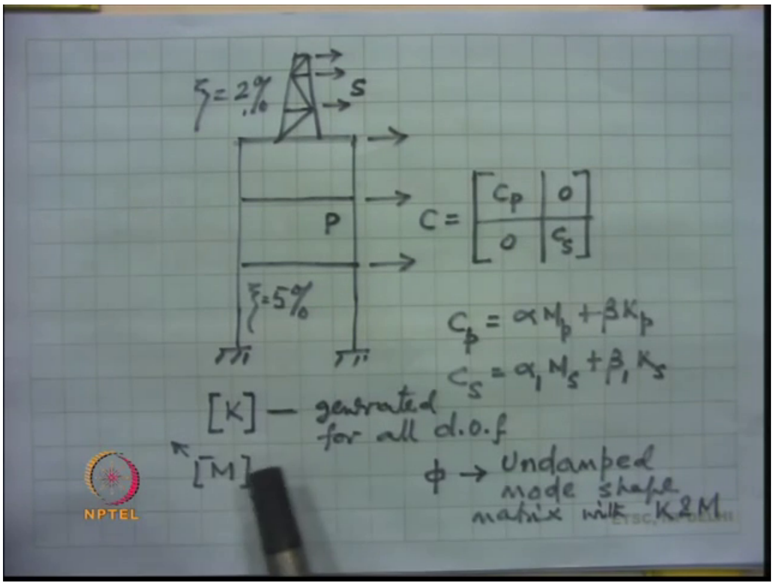

The non-classically damp systems are once in which we have a system like this, in which there is a main system and in that main system say we have a tower. Then, these tower may be made of steel this building is made of concrete both of them have different damping. So, the construction of the C matrix for the composite system that is to be obtained first. So, in this in this case the c matrix is obtained by first obtaining the damping matrix for the primary structure, separately obtaining the damping matrix for the secondary system. So, here say the main structure has 3 degree of freedom. So, we obtain the damping matrix corresponding to this 3 degrees of freedom by assuming it to be a mass and stiffness proportional.

So, we first find out the frequencies of the primary system and in that we completely de couple this secondary system. So, from that we get the damping matrix after obtaining the values of alpha and beta. So, once we get this matrix damping matrix for the primary system, we put it over here it will be 3 by 3 matrix. Similarly, one can obtain the damping matrix for the secondary system, say it is also having a 3 degree of freedom. So, using a modal damping ratio of 2 percent we construct the C s matrix. In the same fashion as we have done for the primary system and this C s is the damping matrix for the secondary system. The coupling between the primary system and the secondary system that, we set to zero because that is explicitly known. So, thats how one can construct a C matrix and this C matrix is non-classically damped because we have got 2 types of damping in the system.

Once we get the damping matrix for the entire system then, one can consider the entire system together. That is, the system would be a six system of 6 degree of freedom we write down the stiffness matrix corresponding to this 6 degrees of freedom that becomes the k matrix. The mass matrix becomes a diagonal mass matrix having 6 masses. So, with the help of these mass matrix and the stiffness matrix, we obtain the un damped mode shapes and frequencies with the help of k and m, and once we get these un damped frequencies and mode shapes then, one can use the decoupling try to decouple the equation of motion by writing phi t C phi, phi t k phi and phi t m phi.

Of course, will be diagonal phi t C phi will not become a diagonal matrix, note that here, the phi matrix will be a 6 by 6 size. This is also a 6 by 6 matrix and since it is non-classically damp therefore, phi t C phi 1 be a diagonal matrix, there be of diagonal terms. By ignoring of diagonal terms that is by diagonalizing the phi t C phi matrix, one can decouple the entire equation of motion and from the diagonal terms of the damping matrix, one can obtain the modal damping ratios in each mode of vibration.

Once we get that then one can use the usual response spectrum method of analysis because we know the modal damping for each mode of vibration. Therefore, one can obtain the necessary spectral ordinates for acceleration for a given damping and given time period. Once we get the equivalent set of lateral forces in each mode a vibration then, you can apply them and carry out the usual static analysis performed in the case of response spectrum method of analysis.

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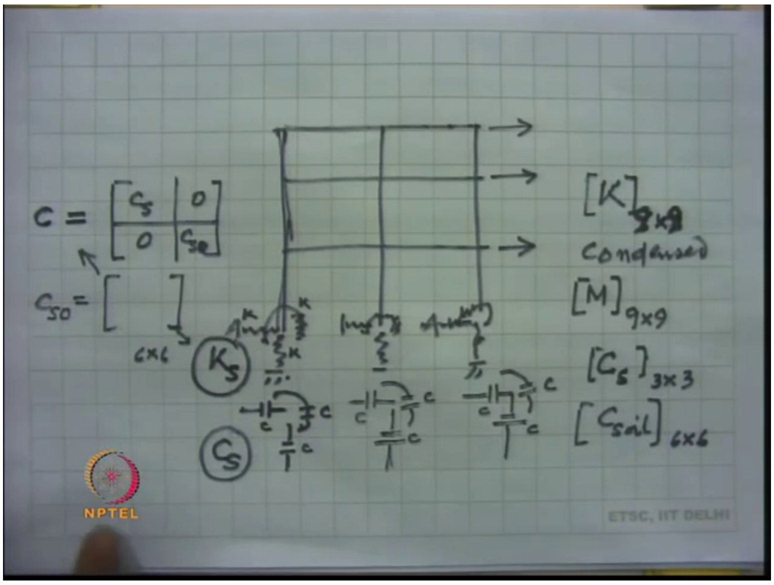

Another problem of this time this type quite often is encountered in structural engineering that is the problem of soil structure interaction where, the effect of the soil is replaced by springs and dashpots. For example, here, this is a frame which has got 3 non support degrees of freedom. At the supports the springs indicate the soil stiffness and they indicate the soil damping.

Generally, the vertical spring stiffness is are made very high. So that, effectively one can assume that these base cannot go down or it is the base degree of freedom in the vertical direction is locked; we only permit the lateral movement of the bases. Also the rotational springs can be introduced in order to take care of the rotation that can takes place because of the flexibility of the foundation.

So, in that case, the k matrix for the structure would be this 3 translation plus these 3 translation the base plus these 3 rotations. So that becomes the k matrix and one can obtain this k matrix by usual condensation procedure. The mass matrix also will be 9 by 9, the rotational masses we can give some rotational masses to the system or also we can set them to zero if you wish to ignore it. Then the damping matrix for the entire system consists of two parts, one is the structures damping matrix which will be 3 by 3 and which can be obtained from the damping specified damping ratio for the structure say it is if it is concrete then, it will be 5 percent damping, for that 5 percent damping one can obtain the damping matrix for the superstructure.

The other part of the damping matrix is the C soil and these C soil matrices can be obtained from the damping coefficient provided for the soil system. Generally, this soil damping matrix is a diagonal matrix. The coupling between the structure and the soil damping matrices, this is again set to zero or once we have these damping matrix then, as before we can multiply this damping matrix or be multiply this damping matrix with phi t and post multiplied by phi, that is phi t C phi is obtained and phi t C phi is not generally a diagonal matrix. So we ignore the diagonal terms and from the diagonal terms, we obtain the modal damping in each modal vibration and then, carry out the usual response spectrum method of analysis.

Note that, while performing the Response spectrum method of analysis, we retain these springs at the bases, that is this lateral springs and the rotational springs and they are retained and then performed the static analysis corresponding to each mode of vibration with its equivalent static lateral load.

Next we come to seismic coefficient method.

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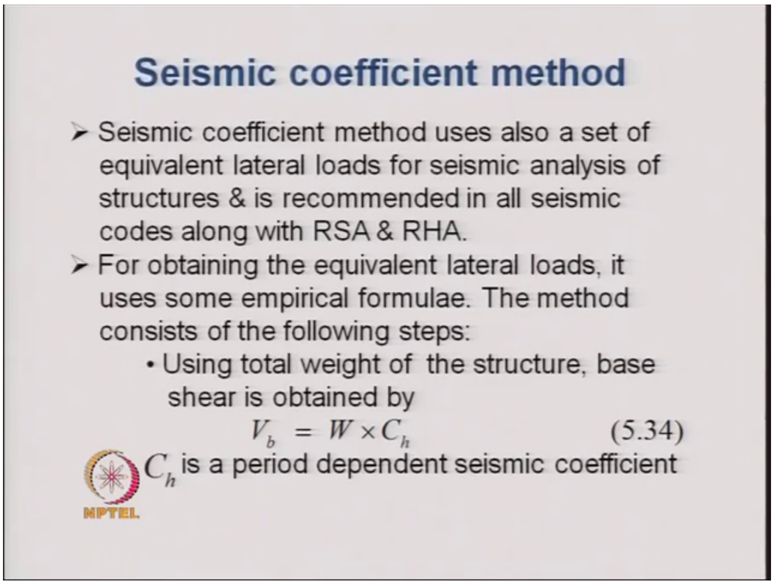

So far, we were discussing about the Response spectrum method of analysis for or the non-classically damped system, for classically damped system, for multi support excitation, for single support excitation and also how we performed the response spectrum method of analysis for cascaded system.

Now we discuss another very popular method for obtaining the design forces in structures for the earthquake and this is obtained using what is known as the Seismic coefficient method. Seismic coefficient method also prescribes a set of equivalent static lateral load and that equivalent static lateral load is applied on to the structure. Then, the structure is analyzed statically to find out its responses and those responses are assumed to be equal to the maximum forces that the structure would have experienced if the same earthquake for which the response spectrum is constructed was applied to the structure.

Note that the response spectrum of acceleration can also be used here in order to obtain the seismic coefficient that will discuss little later and however, not necessarily that the seismic coefficient is extracted from the response spectrum or acceleration response spectrum of earthquake, but sometimes the seismic coefficient is somewhat different than the coefficient that we obtain from the response spectrum.

So in those cases both the response spectrum and the seismic coefficient values for different time periods that are prescribed. In most of the codes of practice both response spectrum method of analysis and the seismic coefficient method of analysis are prescribed for earthquake analysis and design of structures. In fact, Seismic coefficient method of analysis is more popular compared to the Response spectrum method of analysis because it is purely a static analysis. We do not have to even obtain the time period or the frequency of the structure and the mode shapes of the structure in the case of seismic coefficient method.

The time period of the structure is obtained with the help of some empirical formula and the mode shape or the distribution of the load along the height of the structure that is obtained with the help of again some empirical recommendation. Therefore, we do not require the mode shape for the structure. The method consists of the following step using total weight of the structure, the base shear is obtained by the simple formula given in equation 5.34 that is V b is equal to weight of the structure multiplied by C h that is a called the seismic coefficient, which is generally prescribed and this seismic coefficient is time period dependent. Then, we obtain the base shear using this equation sorry first we obtain the base shear using this equation that is 5.34.

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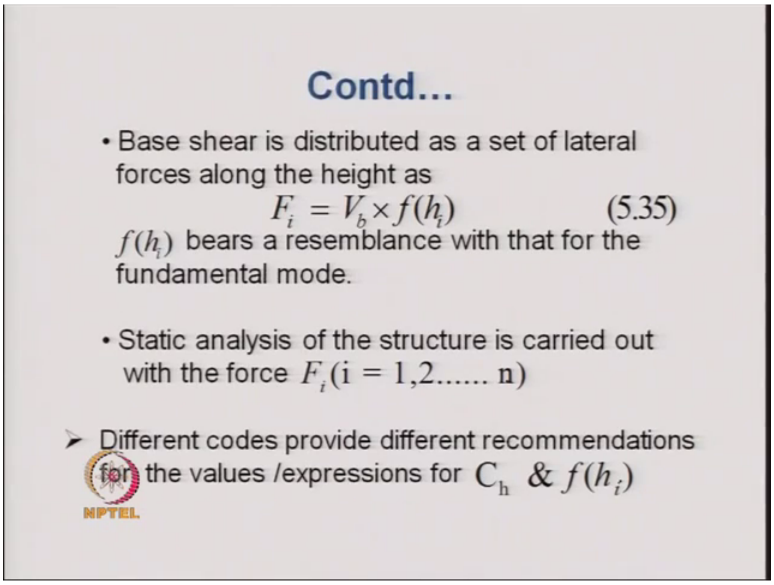

Then once we get the base shear then, this base shear is distributed as a set of lateral forces around the height of the structure and for obtaining these loads at different floor levels or along the height of the building, this formula 5.35 is used. There we multiplied V b by a function of height of the floor and this function of height of the floor that is f h i bears a resemblance with that for the fundamental mode of the structure. They are not the same, but it has a resemblance.

Thus, we can see that here the loads at different floor levels F i that we obtain from the total base shear. The loads at different floors that we obtain from this formula, if they are added together then, they would finally, will be equal to V b that is the base shear. After we have obtain this force F i for different floors then, this force is applied to the structure statically in order to obtain the values of the internal forces in different members of the structure and those forces may be combined along with other forces that is the forces due to the dead load or live load.

Then, we obtain the design forces for individual members of the structure. Many most of the time both Seismic coefficient method and in the Response spectrum method of analysis, the load that is obtained in each mode of vibration in the case of Response spectrum method of analysis, the load F i that we obtained in the seismic coefficient method of analysis is divided by a factor r which is called a Reduction factor. These reduction factor reduce the total load which is coming on to the structure due to earthquake, it is intentionally done.

So that, under the actual earthquake the structure undergoes and elastic excursion and this inelastic excursion is a very important thing for all earthquake analysis of structures because we wish to design the structure for certain ductility. This effect of ductility and the reduction factor will be discussed later on when will be discussing about the non linear analysis of structures due to earthquake.

Different course provide different recommendations for the values or expressions of C h and a function of h i.

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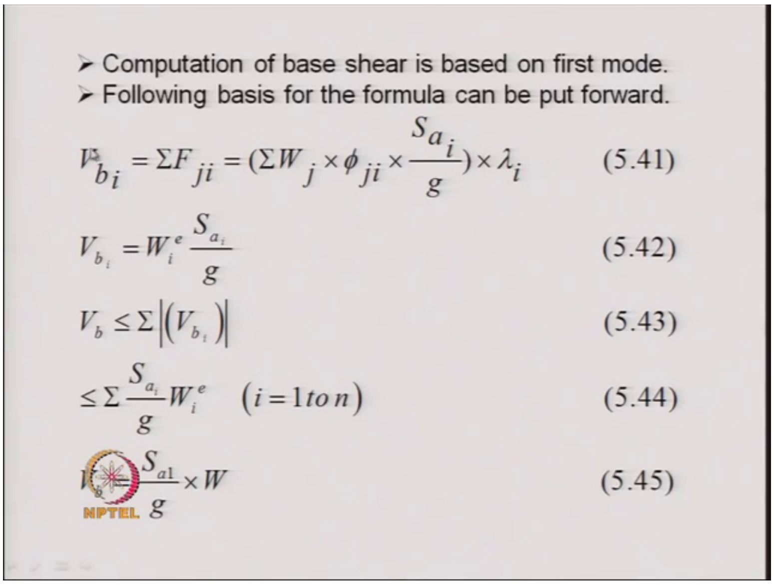

The distribution on lateral force which is given in most of the code cannot be fully justified from the theoretical point of view. But, some kind of justification can be put forward for this distribution. If we look at the first mode lateral load, in the case of Response spectrum method of analysis then, it will be like this, the mode participation factor for the first mode this is rho 1 W j is the weight of the j-th floor corresponding to that there is a mode shape coefficient in the first mode and that case multiplied by S a 1 by g, that is the spectral acceleration for the first mode, that is normalized with respect to g. So that, it finally, becomes mass times acceleration W by g becomes the mass and mass time acceleration becomes the force. So this is a typical lateral force that we obtain in each mode of vibration in the Response spectrum method of analysis.

Now, if we concentrate only on the first mode of vibration then, this spectral acceleration the mode shape and the mode participation factor all of them correspond to the first mode only and with the help of these formula one can find out a force at different floor levels j. Now we can obtain the ratio of the force acting at any particular floor level and the total force, the total force means the base shear. So these ratios can be expressed like this that is W j multiplied by phi j 1 and this is a summation of W j and phi j 1 over all the floors giving the value of base shear. So, the floor load that is for the j-th floor the lateral load can be expressed as the base shears that is, this is nothing but is equal to base shear. V b multiplied by W j into phi j 1 divided by sum summation of W j and phi j 1.

In the next step, we see that this particular formula is converted into W j into h j and at the bottom in place of W j into phi j 1, we write W j into h j. This means that we are assuming a linear variation of the deflection of the structure at the first mode or a triangular variation with a value of unity at the top of the floor. So if you, if we assume that at the top of the floor the unit value is there for a first mode then, at other floors the value of the mode shape coefficient will be h j divided by the total height of the building and since the total height of building is there both in the numerator and the denominator that cancels out and as a result of that we get this particular formula.

Now up to this one can justify in that the assumption which is made is that the first mode shape of the building is a linear mode shape or a triangular variation. Now in the codes we get the distribution of the load at different floor levels is obtained with the help of this kind formula that is base shear is multiplied by W j into h j to the power k and here also it is h j to the power k, k is a coefficient and this coefficient if it is 1 then its linear variation and if it is other than one then, the variation become non linear that is the in that case the first mode shape is assumed to be a non linear variation.

So in the case of k is equal to two, it becomes a quadratic variation. Similarly, for k is equal to 3 or k is equal to point 5, we get different kinds of non-linear variation of the first mode shapes. In the course of practice, the values of k that is adopted that is also prescribed. So to some extent one can justify the distribution of the lateral forces in the Seismic coefficient method of analysis.

Next the formula that we use for calculating the base shear, that also could be justified to some extent from the theoretical point of view. If we look at the expression for the base shear in any particular mode then, this is the sum of the lateral forces that is obtained at different floor levels in the i-th mode and in turn that can be written in this particular fashion, which we just discussed. Here the lambda i is the i-th mode participation factor, S a i denotes the spectral acceleration corresponding to the i-th mode.

Now this can be written as V b i. Now can be written as W i e into S a i by g. W i e is called the effective weight or effective modal weight for the structure and this is nothing but sum of the W j phi j i multiplied by lambda i. So, this is not summation, this is W j multiplied by phi j i multiplied by lambda i that gives W i e. That is the effective modal weight in the i-th mode.

Now, if we carry out the abs sum technique then, the base shear that we obtain in the i-th mode, the absolute value of this, that is simply added for all the modes and that gives an upper bound to the base shear, actual base shear because we have discussed before that abs sum method provides a conservative estimate of the response quantities. So thus, the if V b is the actual base shear then, that actual base shear will be less than this value that is abs sum value of or abs sum combination of the base shears.

So this V b I, now is replaced by this expression and as a result of that we can write down S a i by g into W i e. If we assume that the spectral acceleration remains same in all modes and is equal to that of the first mode then, we can replace S a i by g by S a 1 by g and that will remain constant for all modes and then, this can be taken out and the then the summation is only for W i e and sum of the W e i that is effective weight at any particular mode or summed up over all the modes will result in the total weight of the building.

So, the base shear now can be written as V b is equal to S a 1 by g multiplied by W and the seismic coefficient method prescribes the value of V b to be equal to W multiplied by a seismic coefficient. Now if the seismic coefficient happens to be equal to S a 1 by g then, we see that the V b the base shear that is used in the case of seismic coefficient method is nothing but the base shear that we obtained for the structure by considering only the first mode. So that is how one can justify the calculation of the base shear for seismic coefficient method and in many codes of practice the seismic coefficient C h and spectral acceleration they are the same.

As a result of that the base shear calculation that is obtained in the Seismic coefficient method that is equivalent to the first mode base shear in the case of Response spectrum method of analysis.

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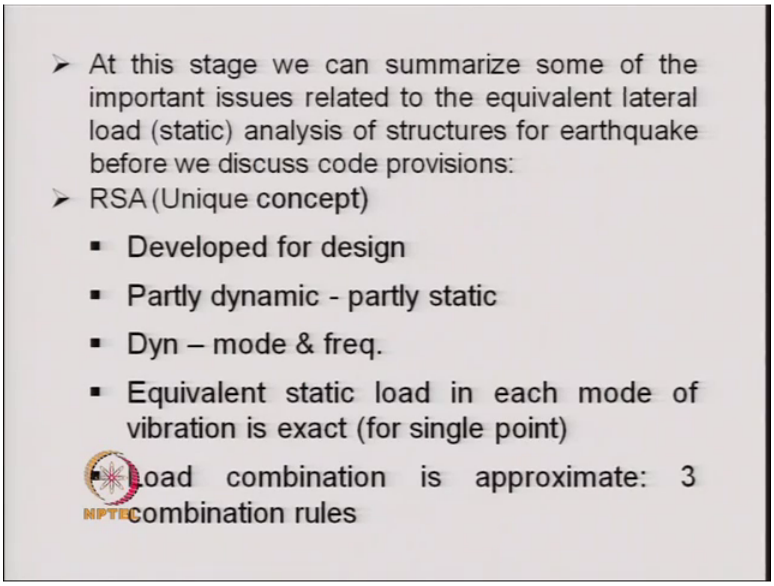

At this stage, we can summarize some of the important issues related to the equivalent lateral load analysis of structures for earthquake. Before we go to the next section of our discussion, that is the Seismic code provisions first thing, that we discussed is the about the Response spectrum method of analysis, we said that it is an unique concept, which is there in earthquake engineering. Response spectrum method of analysis was developed specifically for designing the structures for earthquake.

This is a partly dynamic and partly is static because the first part of the analysis requires the evaluation of the mode shapes and frequencies which are the dynamic analysis of the structure and once we have the mode shapes and frequencies then rest of the part is a static part. Equivalent static load that is obtained in each mode of vibration is exact for single point excitation system and it can be obtained from the fundamental equations of the multi degree freedom system.

However, when you combine the static load effects in each mode of vibration to find out the total response there are the load combination is approximate and there are 3 combination rules. These 3 combination rules are the abs sum, SRSS, CQC.

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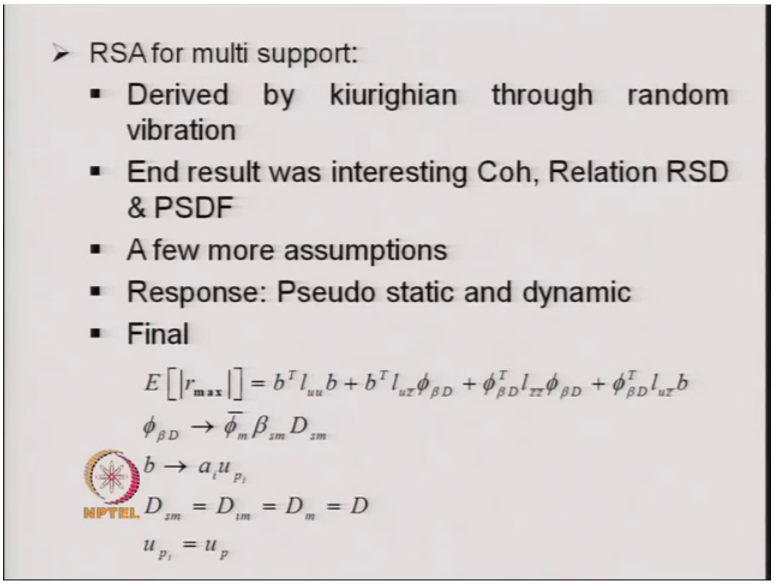

The abs sum rule provides a conservative estimate of the responses and SRSS rule provides a better result in case of the o l separated frequencies of the structure. When we have closely spaced frequencies then, we obtain in place of SRSS, CQC rule in which the modal correlation effect is taken into consideration.

After that, we discuss about the response spectrum method of analysis extended for multi support excitation. It was first derived by Kiurighian through random vibration approach. But, the end result was very interesting. It requires additionally a coherence function and the relationship between response spectrum of displacement with power spectral density function. Then, we discussed how response spectrum method of analysis can be utilized for the secondary system and response spectrum method of analysis can be utilized for non classical damp system.

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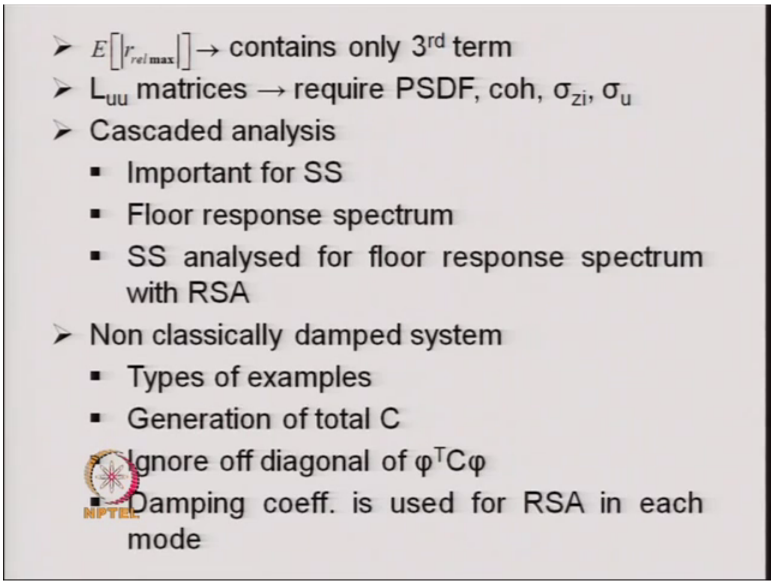

After that we discussed the basis of the Seismic coefficient method of analysis. As it is quite popular with the earthquake engineering community and it has been observed that the seismic coefficient method of analysis is essentially based on the first mode response. However, the mode shape of the first mode is approximated by different formulas and the time period of the first mode is calculated by some empirical formula.