Seismic Analysis of Structures
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Lecture – 24
Response Spectrum Method of Analysis (Contd.)

In the previous lecture, we discussed about the extension of the response spectrum method of analysis to multi support excitation, then for cascaded analysis and for the non-classically damped systems. After that, we discussed about a very useful seismic analysis method, which is widely used in almost all countries for designing the structures for earthquake forces and it is also given in the seismic course of all countries.

The seismic coefficient method as such has a certain limitations in the sense that it does not take into account the all participation of all modes of the structure into the response.

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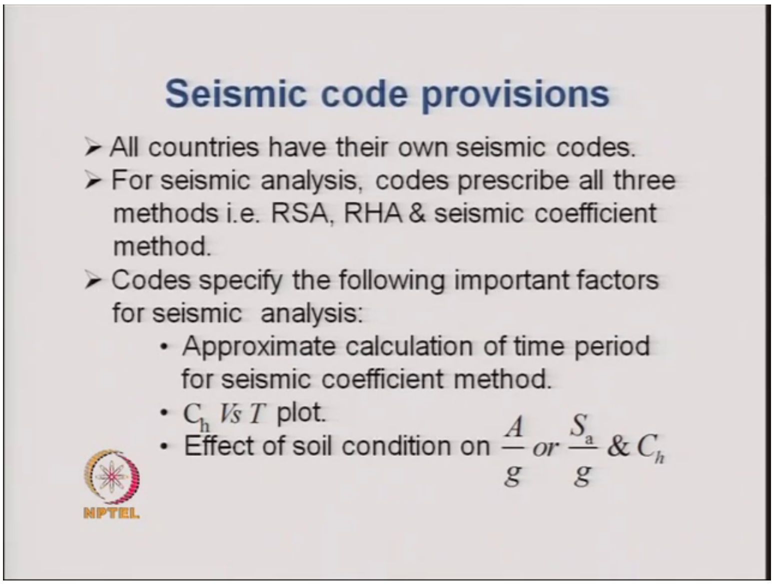

Secondly, it is based on to some extent the empirical formulas which are difficult to support completely theoretically; however, the basis of those formulas can be justified to some extent. In spite of that, the seismic coefficient method has been found to be very popular with the engineers. They try to analyze most of the structures for the in spite of that the seismic coefficient method of analysis has become very popular with the earthquake engineers and for most of the structures specially for building structures they use seismic coefficient method for finding out the forces for which they would design the structures for earthquake.

Almost all countries have their own seismic codes and in that code there are several recommendations. For the seismic analysis and design of structures the codes specifically give recommendations for 3 kinds of analysis that is the Response Spectrum Analysis, Response time History Analysis that is RHA and the seismic coefficient method. The codes also specify under which circumstances one should go for Response Spectrum method of Analysis and the cases where one can go ahead with seismic coefficient method. Apart from that, the recommendations are there for which the response time history analysis for a given time history record or a specified time history record has to be analyzed for structures which are designed with the help of seismic coefficient method or Response Spectrum method of Analysis.

These cases typically include the cases where we wish to study the behavior of the structure in the inelastic range and as you will see later that most of the designs that are accomplished for structures for earthquake, for that we deliberately allow the structures to get into the inelastic range for design earthquake level. Therefore, many a time the behavior of the structures in the inelastic range becomes very important.

For those situations the response time history analysis is an important consideration. The time history for which the structures are to be analyzed, depends. The time history of the ground motion could be a size specific time history of ground motion. It could be a time history of a ground motion which has developed maximum amount of damage in the past in that particular region or one can construct a time history of ground motion for a given response spectrum or a given power spectral density function of response. All of them, we have studied when we are discussing the inputs for ground motion, we have seen that how one can construct a response spectrum compatible time history of ground motion or a power spectral density function compatible time history of ground motions.

Thus, in the seismic codes we have all the 3 kinds of analysis and depending upon the structures and the need, we use either one of them or all 3 of them. The structures which are to be designed are to be safe against earthquake that remains the final goal.

Codes specify the following important factors for seismic analysis; the first one is the approximate calculation of time period for seismic coefficient method. Then it provides a seismic coefficient versus the time period plot. The third one that is specifies is the effect of soil condition on A by g or the spectral acceleration normalized with respect to g and the seismic coefficient. The approximate calculation of time period is generally associated with the use of seismic coefficient method. The reason is that in the case of seismic coefficient method, the entire method is thought to be an equivalent static analysis, unlike, the response spectrum analysis where it is partly dynamic and partly static.

The dynamic part consists of finding the time period or the frequencies of the structure and the mode shapes. Once those are calculated, then rest of the things turn out to be a static analysis. In the case of the seismic coefficient method the entire thing is conceived as a static analysis and therefore, the time period of the structure is obtained with the help of a empirical equation rather than finding them out from a dynamic analysis. The C h versus T plot shows that the seismic coefficient depends on the time period of the structure and this time period is calculated using this approximate method.

The effect of soil condition is extremely important in seismic design that we have discussed when again we were discussing about the effect of the soil condition on the seismic waves, that is, as the seismic waves pass from the rock bed to the surface passing through the soil then the properties of the soil modify the ground motions that are caused at the surface of the ground. Therefore, the spectral acceleration that we use for designing the structure should take into account the local soil effect.

Generally, we divide the soil affect into 3 conditions number one is the hard soil, then we categorize as a medium soil, then we consider a soft soil. So, for these 3 categories of the soil the spectral acceleration or the C h value which is obtained for different time periods they do vary. Therefore, we have different curves for the different soil conditions.

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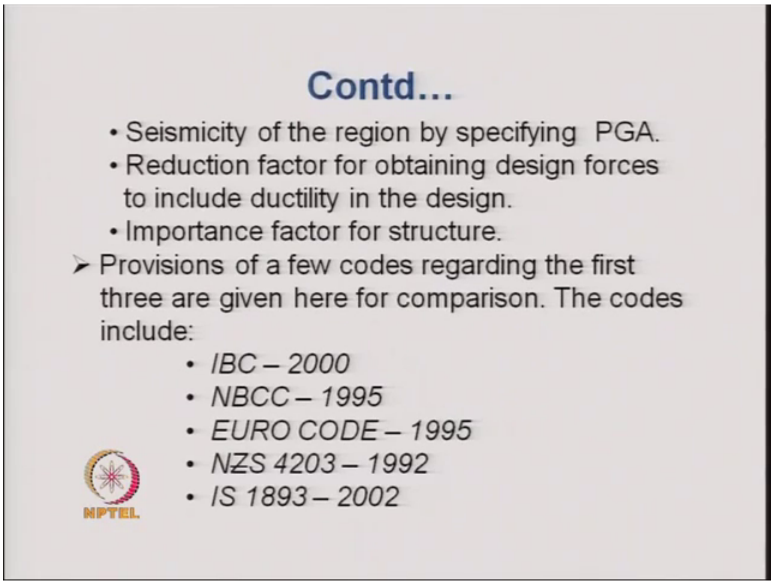

The next important thing that is specified by the code is the seismicity of the region by specifying peak ground accelerations.

This is done by dividing the entire country into different zones and each zone we specify an expected value of peak ground acceleration and the structures are designed for that peak ground acceleration while designing the structures in that specific region. The reduction factor is a very important criteria that is included in the seismic design of structures to include ductility in the design. The basis for this is that we want the structures to go into the inelastic range at the design earthquake level.

When it goes to the inelastic range then we permit a certain amount of inelasticity into the design, that is, after it has yielded we allow some kind of displacement to take place in the inelastic range. The amount of displacement or excursion that the structures do take after the yielding that is governed by what is known as the ductility factor. Now this ductility factor is again or in turn is dependent on the reduction factor that is utilized for the design.

Finally, we have the importance factor included into the seismic designs of structures. The importance factor provides relative importance to different type of structures, for example, a nuclear power plant design must be more safe than a residential building or any other structures. Therefore, the factor of safety that is taken into account for a designing a nuclear power plant is more than other structures. So, that is achieved by providing an importance factor to the seismic design coefficient or the response spectrum ordinates by multiplying them with the help of some importance factor.

So, these are the salient features of the code provisions in almost all codes of the world and we look into these things when we study the code.

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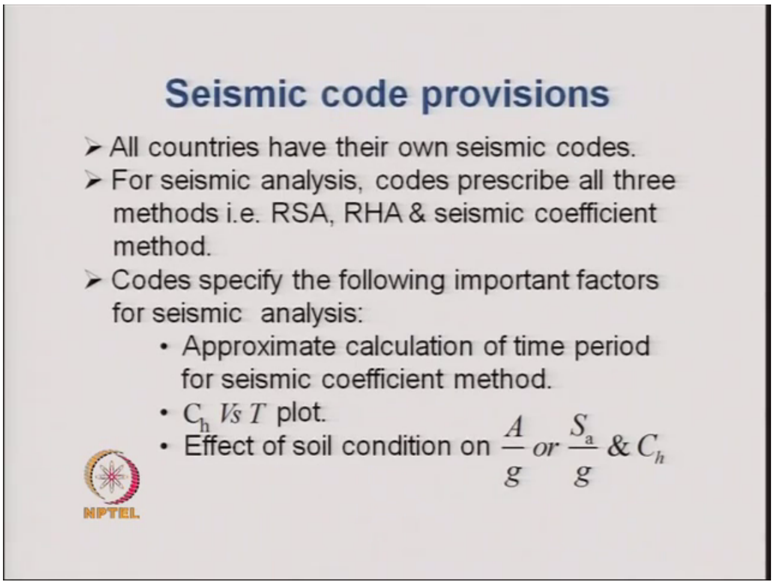

Here, we are not going to discuss all issues, we will be discussing only about the first 3; that is, the approximate calculation of time period for seismic coefficient method, then we look into C h versus time period plot and the effect of soil condition on A by g or S a by g and C h with T. So, our discussion here would be mainly centered around these 3 things which are given in the code.

The other things, that is, the seismicity, the reduction factor and the importance factor, they are dependent on the specific country and the factor of safety, that is considered in different parts of the world in designing the structures. The reduction factor has of course, some kind of communality, in the sense, that how much we allow the structures to go into the inelastic range for that we have some kind of common what you call decision in all the codes. Therefore, a reduction factor which is given in a code does not vary much if we compare it with other codes.

Similarly, the importance factors also do not vary much from one code to the other. The seismicity of the region is something which is a country specific. Each country, depending upon it is seismicity; that means, the severity of the earthquake that has taken place in the past based on that each country has their own seismic zonation map and from that zonation map they decide about the peak ground acceleration that is to be used.

So, here we will be mainly discussing about the first 3 factors for these codes that is IBC-2000 that is International Building Code, NBCC that is the National Building Code Of Canada, then EURO CODE, then New Zealand code and finally, the IS code. The main idea over here is to show that what kind of differences that are there in the 3 factors that I have said before and what are the kind of commonness that each one of these codes have with respect to those 3 factors.

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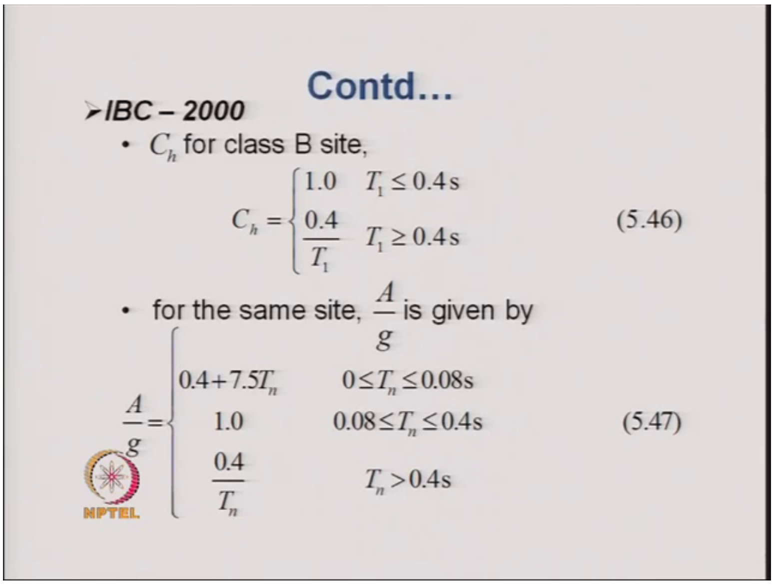

First, let us take the International Building Code. The seismic coefficient C h for class B site is given by equation 5.46. The class B site specifies certain zone in which peak ground acceleration is specified and for that the C h values are shown by this equation.

For other classes for example, class A or class C these values may differ. One can see that the value of C h it remains same up to a time period of 0.4 second that is from 0 to time period of 0.4 second it remains unity C h value as unity. Then, after the time period of 0.4 second the C h value falls down non-linearly or inversely proportional to the time period. For the same site that is for the class B site the S a by g or the spectral acceleration normalized with respect to the g value, is given by the formula A by g is equal to the 0.4 plus 7.5 T n that is the T n is the natural period, then S a by g is equal to one and S a by g is equal to 0.4 by T n.

That is, for T n greater than 0.4 second it is again inversely proportional to the time period. For the segment of time period 0.08 to 0.4 seconds it remains equal to 1 whereas, for very small time period that is up to 0.08 second it is 0.4 plus something. Now, if you compare this S a by g value with C h value, we can see that they are more or less the same, that is the last 2 of S a by g that is 1 and 0.4 by T n that is same as the value of C h within the time period range that is specified.

Therefore, it is expected that the C h value and the S a by g value for different time periods or the plots of them against T will be nearly the same that we will see later.

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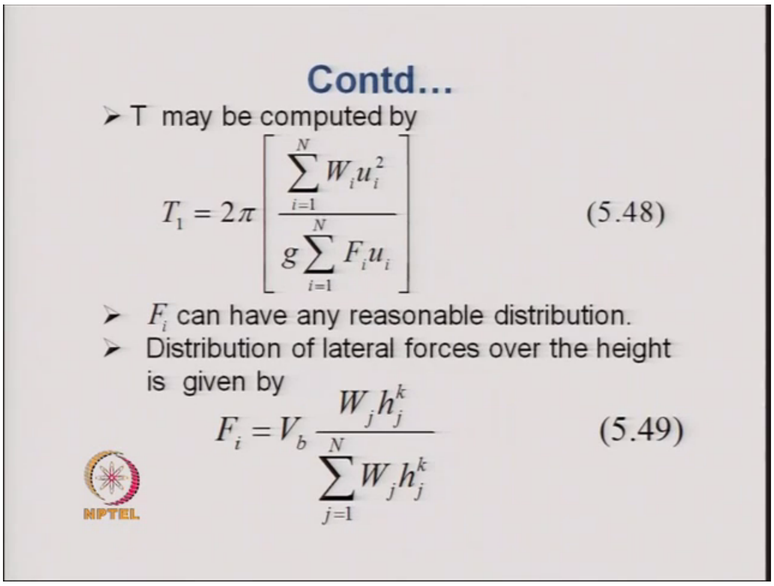

Next, the time period maybe computed by an equation 5.48 and this is given for the seismic coefficient method. If you are using the seismic coefficient method for finding out the seismic forces induced in different members of the structures, obtained by seismic coefficient method of analysis, then the time period that is calculated is by this formula.

So, this formula is known also as a Rayleigh's formula. Here, W i indicates the floor weights, F i is an arbitrary load which is distributed along the height, but this distribution should have a reasonable distribution, preferably, this distribution is taken as the distribution of the first mode of the structure or similar such kind of distribution. So, the load the arbitrary load F i distributed in that particular fashion. When it is applied to the structure it produces a displacement of u i at each floor, that is the meaning of u i in this equation 5.48 and with the help of this equation one can find out the time period for the structure and use the seismic coefficient method.

Next comes the distribution of the lateral force over the height. This we have discussed at length when we are discussing the seismic coefficient method and we have shown that the base shear that we obtain that can be distributed using a formula which is a given by equation 5.49. The k value or that is the power which is the or the height raised to the power k, that k value varies and some codes straight away provides the k value that is 1 k value for all cases. In some codes we have different k values for different time period region.

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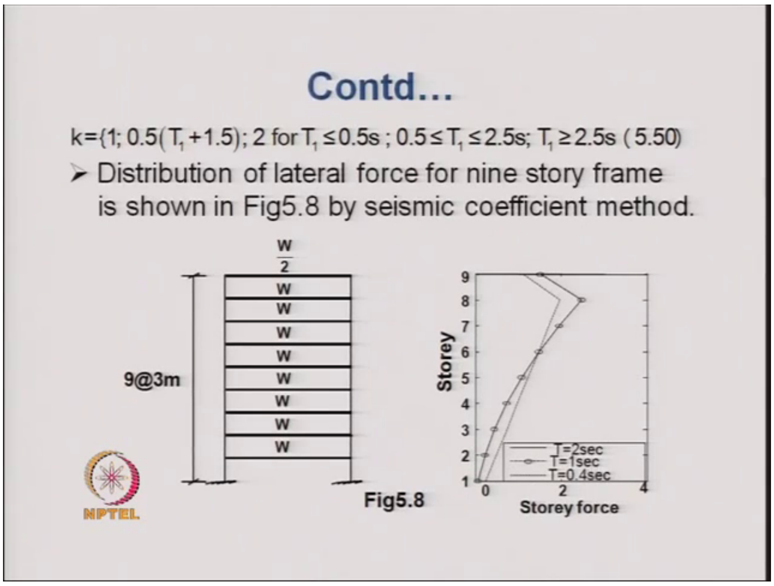

For example in this particular case that is uniform the International Building Code the 3 values of k are recommended, that is one, then next is 0.5 into T 1 plus 1.5 and the next one is 2 they are valid the first one is valid for time period less than 0.5 second that is k is taken as unity for time period less than 0.5 second. For time periods between 0.5 seconds to 2.5 second the value is taken as 0.5 into T 1 plus 1.5 and when T 1 is greater than 2.5 second, then the value of k is taken as 2.

In the last case, one can see that the power is raised to that is the variation is a quadratic variation along the height. Distribution of the lateral force for a 9 story frame using the distribution that was shown by the equation 5.49 that was shown that was computed and is shown in figure 5.8, mind you that the base shear that was calculated was obtained by multiplying the total weight of the structure by the seismic coefficient C h obtained for the structures period calculated by the Rayleigh's approximate method.

The figure shows that the variation of the storey forces that is the forces which is distributed along the height of the building that varies non-linearly for time period is equal to 1 second, for a time period is equal to 0.4 second and 2 second these variations are mildly non-linear. There is a kink at the eighth floor level; this kink has come because of the sudden change in the weight at the top floor level, one can see that at the top floor level the force the weight of the structure is reduced to half that is why this kind of kink is seen in the distribution.

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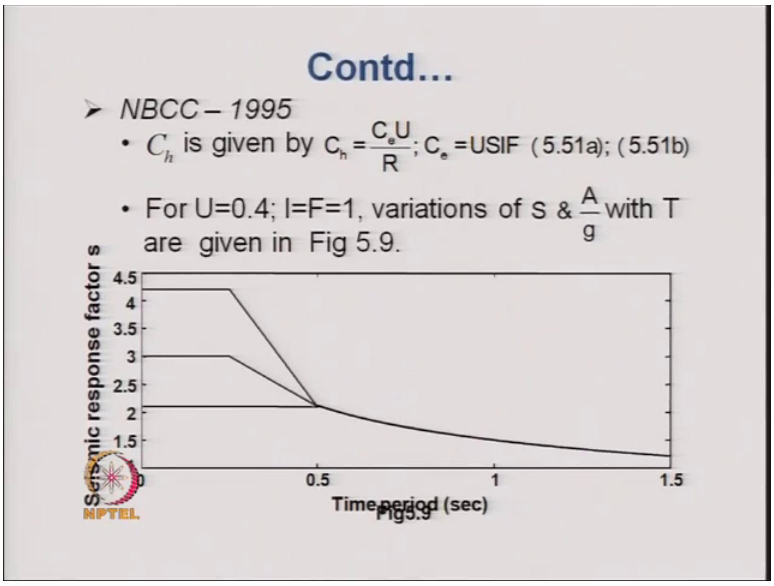

Next comes the National Building Code of Canada 1995. Here the seismic coefficient C h value is given by C e into U divided by R, where R is the reduction factor. In the previous case we have not talked about the reduction factor. Here also although R is there; that means, the seismic forces which are calculated by the formula is reduced by a reduction factor that the same thing is done in the case of IBC, but we have not shown that. Here, in the formula it is straight away given; however, we will not discuss about the factor R.

We will discuss about the C factor C, U is a scaling factor, the C e is given as it is not U, it will be small vSIF, where I and F they are the importance factor for the structure and for v is equal to 0.4, it is not U is equal to 0.4, it should be v is equal to 0.4 and for I is equal to F is equal to 1. The variation of S and A by g with T are shown in this particular figure. The figure shows that the first figure that is figure 5.9 shows the seismic response factor S versus time period T.

And the next figure, in the next figure we will show the variation of A by g with T. We can see that the seismic response factor has a branching. In the initial stage there are 3 branches; first branch is the acceleration prone region that is Z a greater than Z v the middle one is the acceleration proneness and the velocity proneness both of them are same that is Z a is equal to Z v and the third line corresponds Z a less than Z v. So, in the up to a point or 0.5 time period, these branching specifies the kind of zoning that is considered if the zone is a acceleration prone zone, then we take the top curve. If it is both acceleration and the velocity both of them are nearly have the same importance then we take the middle curve and the last curve is taken where the acceleration is relatively less important compared to the velocity.

And after 0.5 all the curves they merge together and we have one curve showing the variation of the seismic response for the factor S with T.

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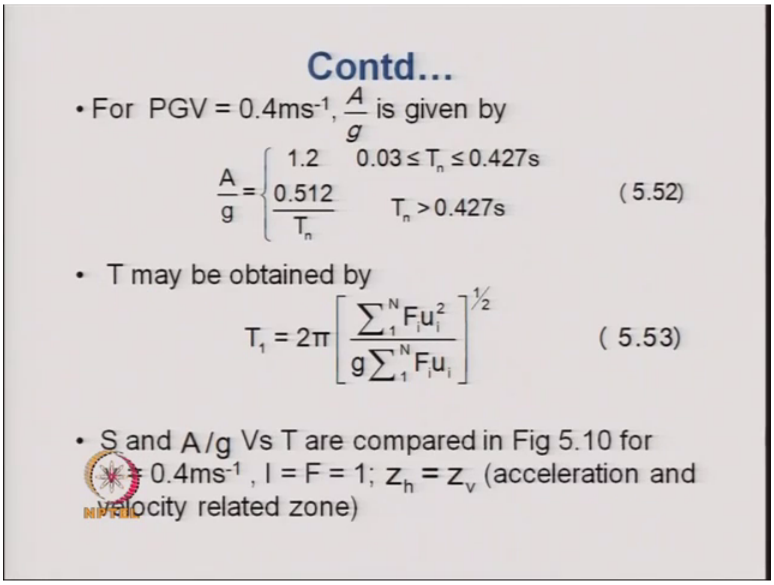

For the peak ground velocity of 0.4 meter per second the A by g or the spectral acceleration normalized with g is given by this equation that is equation 5.52 it is equal to 1.2 for a time period which is less than 0.427 and greater than 0.03 that is for the initial portion of the time period and for larger time period that is time period greater than 0.427 the S a by g is given as 0.512 by T n. Again, here we can see that it is inversely proportional to the time period.

Similarly, for other zones we will have a different values of PGV and for that S a by g values we will be given by different equations. T maybe that is the time period may be obtained again by the Rayleigh's approximate formula. These time period is used when you are using the seismic coefficient method of analysis.

The S and A by g versus time are compared for V is equal to 0.4 meter per second that is for peak ground velocity is equal to 0.4 meter per second and for I is equal to F is equal to 1 and Z h is equal to Z v that is acceleration and velocity related zones.

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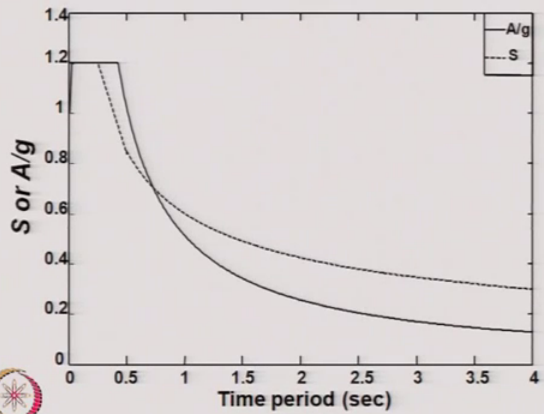

Fig5.10

For that the curve is shown over here and one can see that the S curve that basically gives a higher value than S a by g.

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- Distribution of lateral forces is given by

$$F_i = (V_b - F_t)\frac{W_i h_i}{\sum_{i=1}^{N} W_i h_i} \qquad (5.54)$$

$$F_t = \begin{cases} 0 & T_1 \le 0.7\,s \\ 0.07 T_1 V_b & 0.7 < T_1 < 3.6\,s \\ 0.25 V_b & T_1 \ge 3.6\,s \end{cases} \qquad (5.55)$$

So for as, the distribution of the lateral force is concerned, the equation that is used is somewhat different than the International Building Code. In the International Building Code we had W i H i to the power k and the value of k could be 1, could be 2 and could be in between 1 and 2, but here it is the value of k is simply is equal to 1, that is why it is W i H i divided by sum of W i H i over all the force.

The second difference is that on the left hand side in place of V b it is V b minus F t where F t is given a value of 0 for time period less than 0.7 second is equal to 0.07, V for time period ranging from 0.7 to 3.6 second and for time period greater than 3.6 second it is 0.25 V b.

One can see that only for a very high time period that is for the large time period the V b get reduced here to three fourth V b in the calculation of F i, otherwise for the time period up to 3.6 second the value of V b remains nearly equal to V b up to 0.7 second. It is exactly V b and up to 3.6 second there is a slight reduction in the value of the V b and V b we will calculate with the help of the seismic coefficient method, that is multiplying the total weight of the building by the seismic coefficient.

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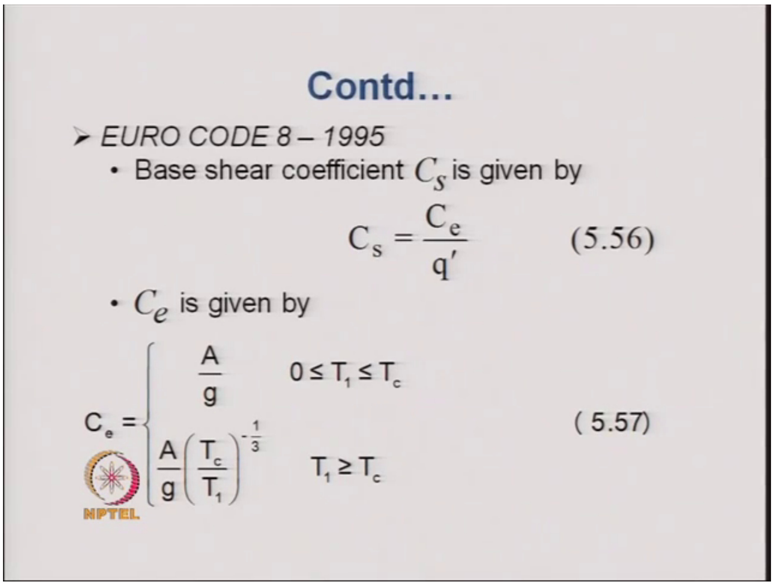

Next comes the Euro Code. In the Euro Code, the base shear coefficient is called C s is a given by C e divided by q dashed, again the q dashed over here represents the reduction factor. So, we will not talk about it. So, we will be concentrating on C e. C e is given by this equation 5.57 we can see that for the time period between 0 to T c. The value of C e is equal to same as S a by g. The T c indicates the upper limit for the straight line portion of the curve and greater than T c, that is, for time period greater than T c the value of C e is equal to S a by g multiplied by T c divided by T 1 to the power minus one third.

So, we expect that much non-linearity coming into picture for greater time period.

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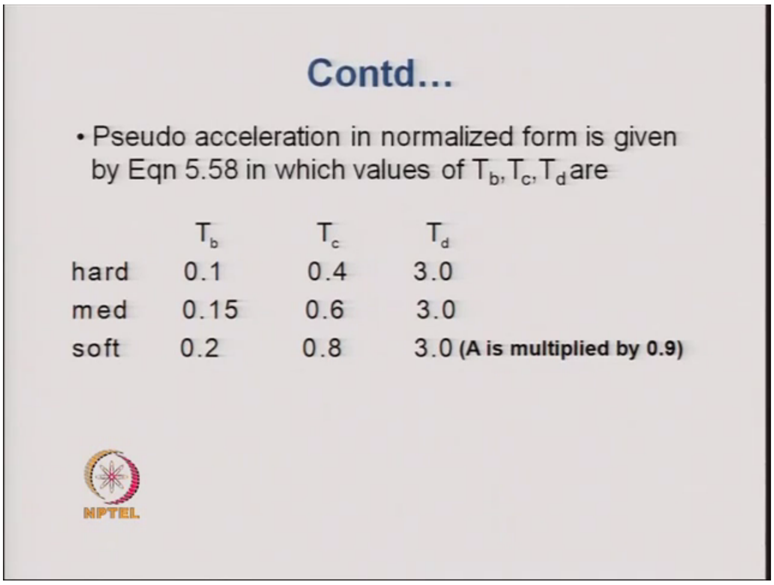

The Pseudo acceleration when it is normalized with respect to g that is what we are calling in A by g or S a by g is given by equation 5.58 that I will show next, but before that let me talk about that 3 PDS, T b, T c, T d for hard medium and soft soil. T c as I told you is the upper limit of the time period up to which the S a by g curve remain straight, remains a straight line, a horizontal straight line.

Now, T b and T c are again some other periods, that will be clear from the picture of S a by g, but these values important thing is that are different for different soil conditions. So, therefore, the nature of the curve that we see for the hard medium and soft soil they differ.

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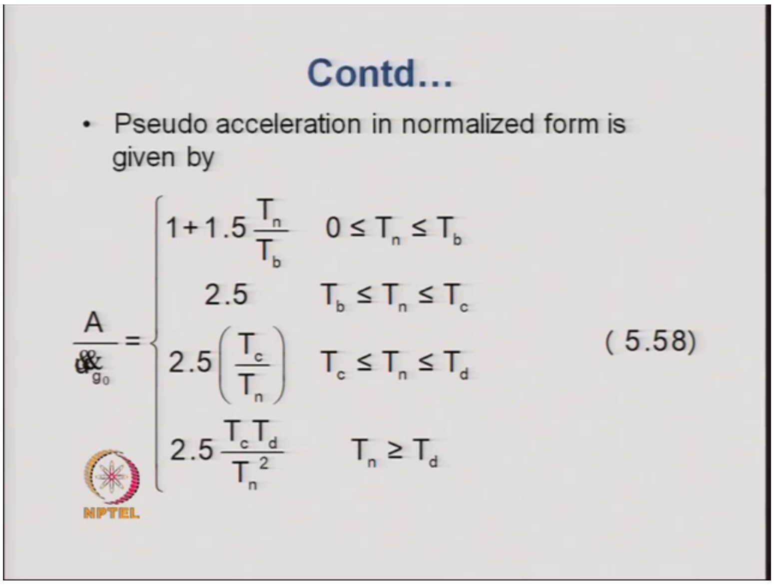

When we try to plot the pseudo acceleration and normalized with respect to g we called is A by g 0 that is given as 1 plus 1.5 T n by T b, so long as T n is lying between 0 to T b. So, it is the starting of the horizontal portion, T b is the starting of the horizontal portion of the spectral acceleration and up to that the radiation is a given by the first equation. And we can see that from T b to T c the value of A by g 0 or S a by g 0 is remains equal to 2.5 constant, that is, it becomes a horizontal line. Then from T c to T d the value is a 2.5 multiplied by T c by T n. Again here you can see that it is inversely proportional to the time period.

Finally, when T n is greater than T d then the value becomes 2.5 into T c into T d divided by T n square. That is a non-linearity is further increased in this particular region.

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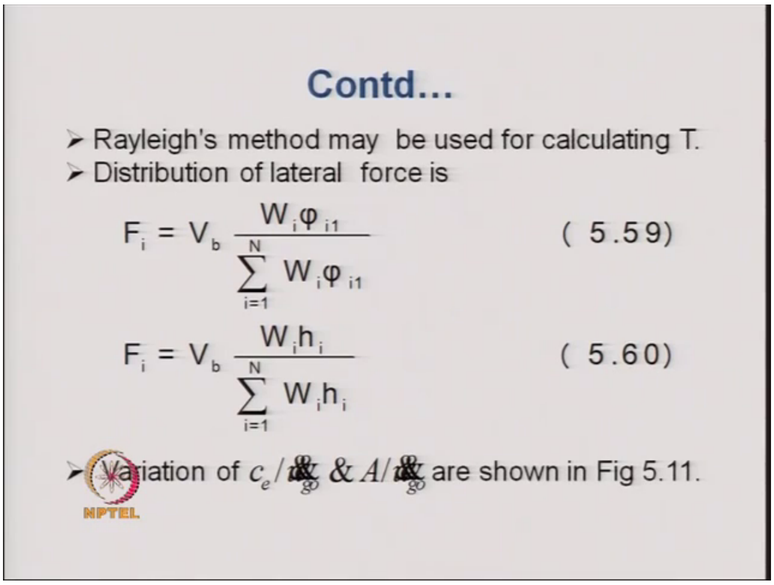

The Rayleigh's method is used for calculating the time period T. Note that many of the codes apart from prescribing the Rayleigh's method for calculating the time period T, they also provide some other empirical equations for calculating the time period T. Distribution of the lateral force follow the same pattern as the National Building Code of Canada that is h i having a power of unity that is k is equal to 1.

Now, in this case either one can use the second formula which is W i into h i, it is height dependent variation or one can use the first equation also, where phi i 1 etcetera, that depends upon the mode shape coefficient in the first mode. 1 indicates the first mode i indicates the floor. So, either of this formula can be used for obtaining the value of F i at different storey levels.

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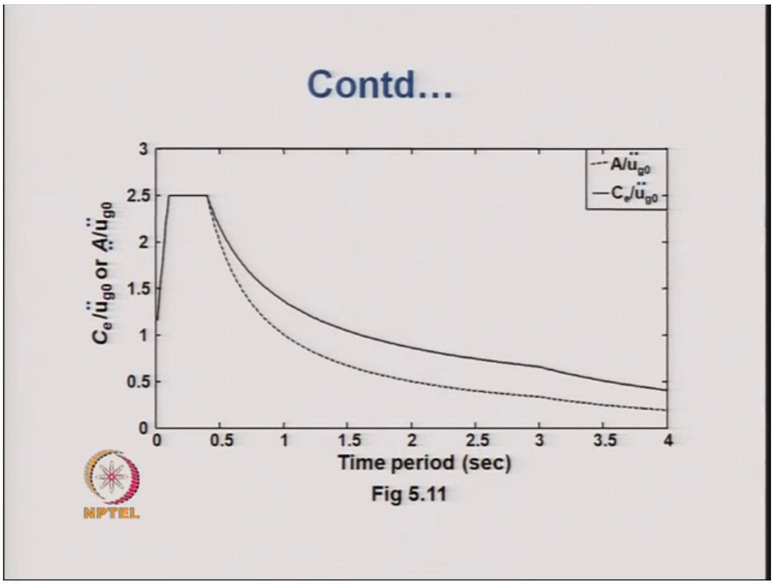

This is the comparison of the C e, the seismic coefficient value, normalized with the ground acceleration and the spectral acceleration normalized with the ground acceleration and one can see that the spectral acceleration is generally lower than the seismic coefficient in the higher time period range.

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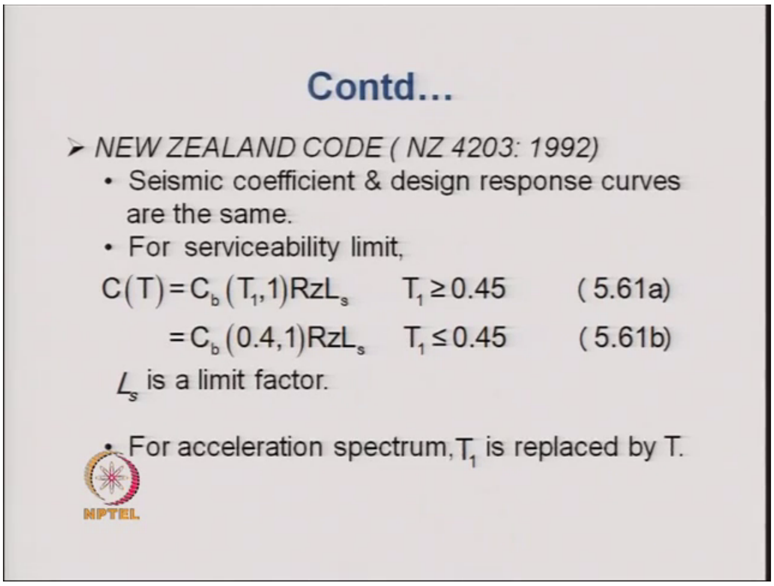

Next comes the NEW ZEALAND CODE. In the NEW ZEALAND CODE the seismic coefficient and design response curves, they are the same. They do not change. That is the C h value and the S a by g value curves, they are the same. However, when we are designing the structure then we have a situations which are called a serviceability limit situation and the ductility situation that is the structures going into the inelastic range and the C T value that is calculated that is used for calculating the forces that is given by C b T 1 comma 1 multiplied by R z L s.

Now, the R z and L s they are basically some factors; z is the zone factor, L s is a limit factor and R is the reduction factor. Now T 1 comma n 1 means that the period and u 1 means mu is equal to 1 that is a condition where the system or the structure remains in the elastic range. That is why it is called a serviceability limit condition. If we wish to design the structure to take it into the non-linear range during earthquake then I have the coefficient corresponding to different value of mu; that is mu is equal to 2, mu is equal to 3, mu is equal to 4, depending upon the ductility that you wish to incorporate in the design.

So, we have the curve, that is seismic coefficient curve and design response curve, specified for different ductility ratios.

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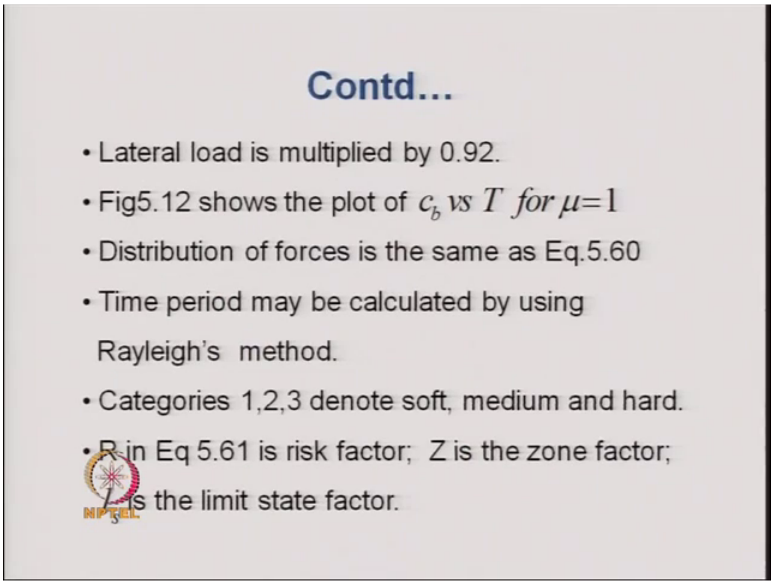

The other features of this code, is a lateral load is multiplied by a factor of 0.92. Then figure in the subsequent figure 5.12 we will show the plot of c b versus T for mu is equal to 1. Distribution of forces is the same as equation 5.60, that is, the same distribution W i h i divided by W i h i summation over all the storeys. Then the time period may be calculated again using the Rayleigh's approximate method. Categories 1, 2, 3 they denote the soft, medium and hard.

R in the equation I said basically as reduction factor that is wrong, R is a risk factor and Z is the zone factor and L s is the limit state factor.

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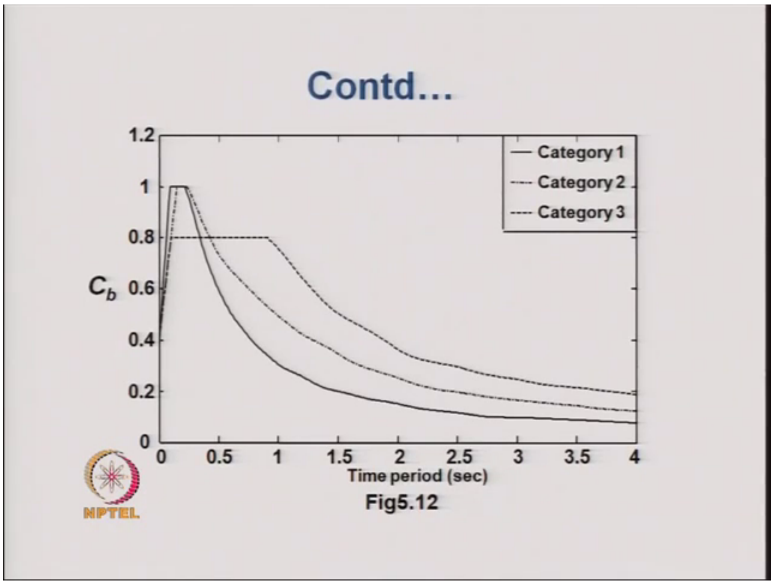

So, for the categories 1, 2, 3 we have different kinds of curves. Here the C b versus the time period curve is shown for category 1, 2 and 3. And one can see that the, for category 1 the value is less whereas, for category 3 the value is more and they are for the hard medium and soft soils. So, the C b versus a time period curve that varies with the soil condition.

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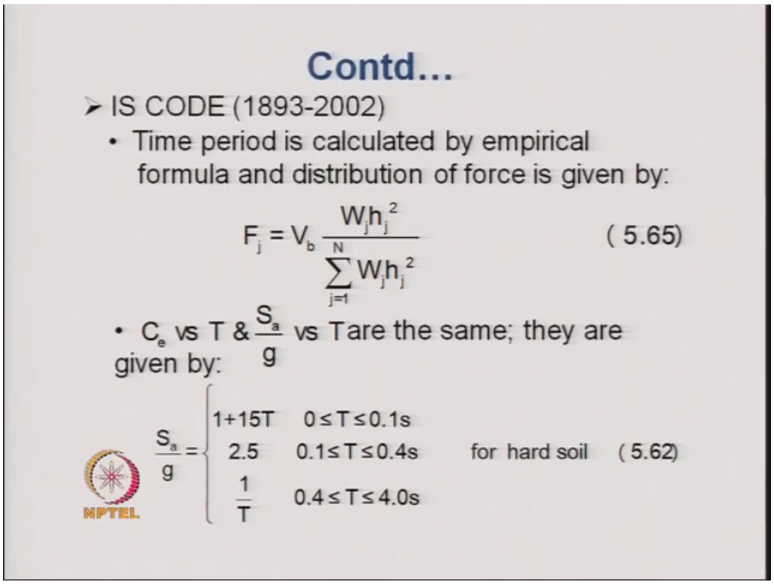

Next comes, our IS CODE. In the IS CODE, the time period is calculated by an empirical formula and these empirical formula is different than the Rayleigh's approximate formula. The distribution of forces is given by equation 5.65. Here the k value is taken as 2. The V b is calculated using the seismic coefficient method. The seismic coefficient and S a by g, they are the same or that is a variation of C e and S a by g versus T they are the same and the S a by g for the hard soil is given by equation 5.62. Here, again, we can see that there is a range that is 0.1 to 0.4 second time period. The S a by g value remains same, that is, it is a horizontal straight line and greater than 0.4, time period. The S a by g in is inversely proportional to the time period.

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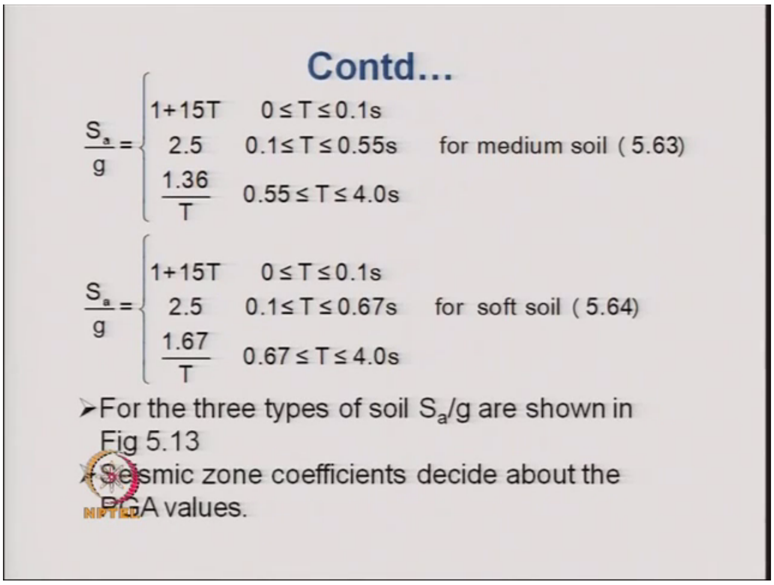

For the medium soil these values are little bit changed. The central portion that is of the curve that remains constant that is 2.5 and the initial phase that also remains same, that is 1 plus 1.5T. Only the last segment of the curve that changes it is 1 by T for hard soil. For medium soil it is 1.36 by T and for soft soil it is 1.67 by T.

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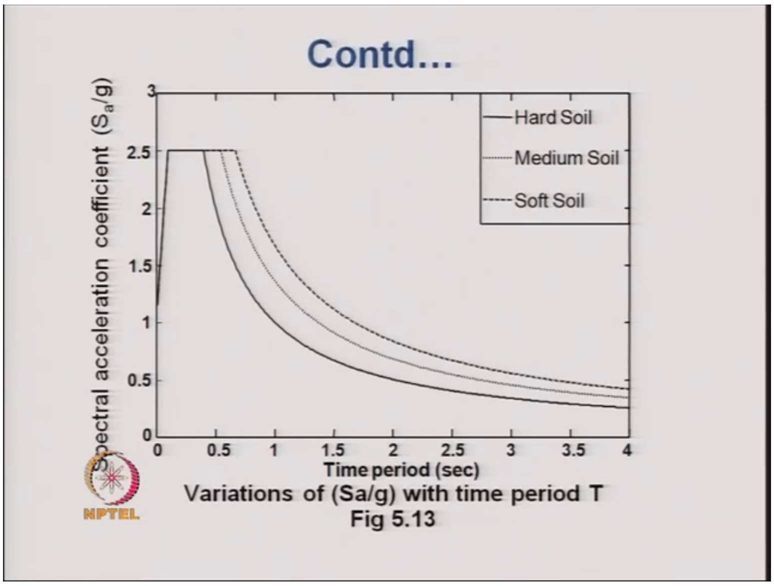

For the 3 types of the soil the S a by g are shown over here and one can see that for soft soil and for greater time period the S a by g coefficient is more. That is, for the soft soil we expect more amplification to take place.

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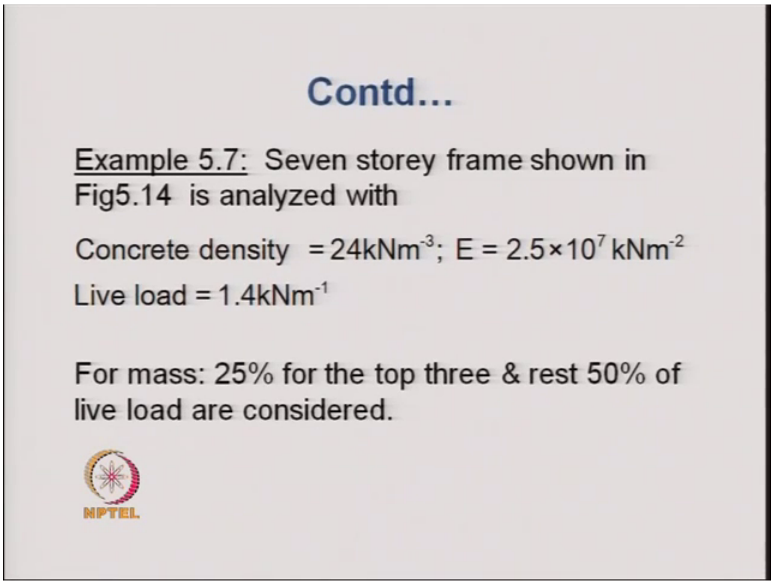

An example problem is solved using the code provisions, in the codes that I have just discussed. It is a 7 storey frame which is shown in this figure.

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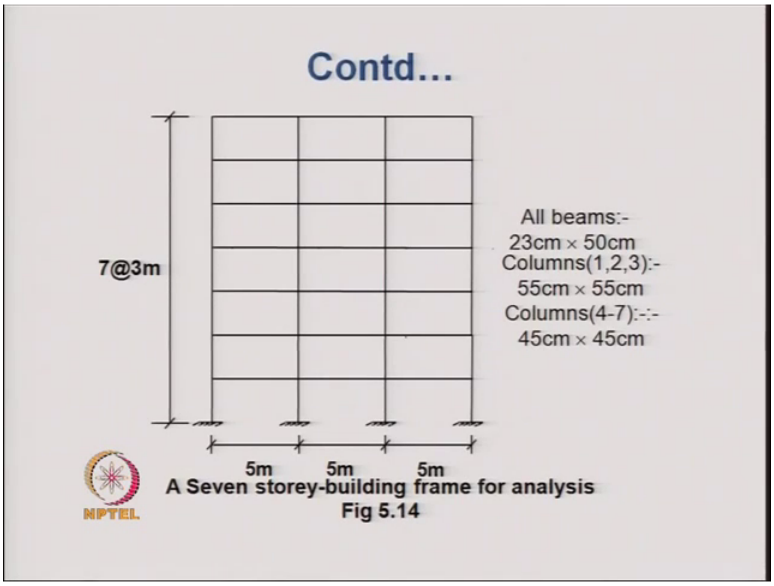

The all beams they have a dimension of 23 centimeter by 50 centimeter. Columns from 1, 2, 3, first 3 floors we have 55 centimeter by 55 centimeter column size and for the upper floors the column size are 45 centimeter by 45 centimeter.

So, this is a 7 storey frame made of concrete and this is designed for or analyzed for modules of elasticity of 2.5 into 10 to the power 7, concrete density 24 kilo Newton per meter cube and live load is taken as 1.4 kilo Newton per meter. For calculating the masses apart from the dead load 25 percent of the live load is considered for the top 3 floors and for rest of the floors 50 percent of the live load are considered in the calculation of the mass.

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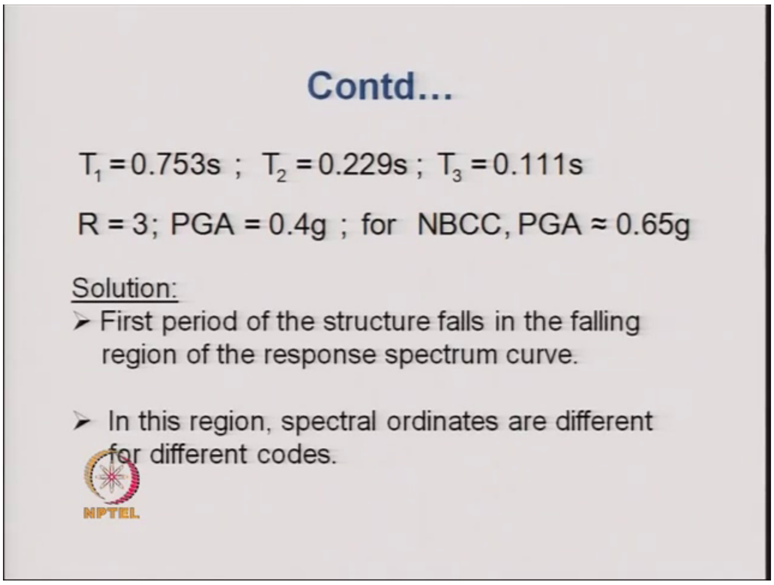

The time period that we were calculated we can get 7 time periods. Out of that the first 3 time periods are shown, that is, 0.753 second, 0.229 second and then 0.0111 second.

So, one can see that the time periods they are quite widely spaced. Therefore, we expect that the SRSS rule and CQC rule we will provide nearly the same kind of result. The reduction factor that we have considered uniformly for all the codes has 3, peak ground acceleration is taken as 0.4 g. For that purpose we have normalized all the C h versus T or S a by g versus T values first and then multiplied those normalized curves with the help of 0.4 g.

For NBCC, that it comes out to be that normalization comes out to provide an equivalent PGA of 0.65 g rather than 0.4 g. So, that is a coming because of the normalization effect. First period of the structure falls in the following region of the response spectrum curve. So, that is the most important thing and we have seen that in the following portion of the curve, different curves given by different codes they differ substantially. That is the observation, therefore, if the first time period falls in the following zone, then the differences that may arise due to the nature of the curve S a by g curve or the C h curve, that would be deflected in the response values or in obtaining are different response quantities of interest.

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Table 5.3: Comparison of results obtained by different codes

Codes	Base shear (KN)				1st Storey Displacement (mm)				Top Storey Displacement (mm)			
	SRSS		CQC		SRSS		CQC		SRSS		CQC	
	3	all	3	all	3	all	3	all	3	all	3	all
IBC	33.51	33.66	33.52	33.68	0.74	0.74	0.74	0.74	10.64	10.64	10.64	10.64
NBCC	35.46	35.66	35.46	35.68	0.78	0.78	0.78	0.78	11.35	11.35	11.35	11.35
NZ 4203	37.18	37.26	37.2	37.29	0.83	0.83	0.83	0.83	12.00	12.00	12.00	12.00
Euro 8	48.34	48.41	48.35	48.42	1.09	1.09	1.09	1.09	15.94	15.94	15.94	15.94
Indian	44.19	44.28	44.21	44.29	0.99	0.99	0.99	0.99	14.45	14.45	14.45	14.45

The results are summarized over here IBC is the International Building Code, NBCC is a National Building Code of Canada, NZ is the New Zealand code, Euro 8 that is the Euro Code and the Indian code. The comparison for the base shear, the first storey displacement and the top storey displacement are shown in this table. Firstly, one can see that when we take only the first 3 modes and when we consider all the modes, the results do not vary much that is what we expected because the time periods are well separated therefore, the values will remain more or less same.

Next, what we observed that IBC, NBCC and New Zealand code they give one kind of result, whereas, the Euro Code and the Indian code they give a higher value of the response. Therefore, we can see that the values obtained using different codes, they could be different and out of the 5 that we have compared the Euro Code provides the maximum values. Therefore, it is more conservative, whereas, the IBC and the NBCC code are more or less the same that is comparable.

So, let me summarize here, that we have discussed the code provisions with respect to 3 important parameters that is the calculation of the time period, distribution of the load and the effect of the soil condition on the S a by g or the C h value given in different codes. And we can see that depending upon the code this values can differ, specially in the case, when the time period or the fundamental time period is in the region, where the spectral acceleration curve or C h curve is falling down, that is, the variation of C h or S a by g with T is in the falling range.