Seismic Analysis of Structures
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Lecture – 25
Inelastic Seismic Response of Structures

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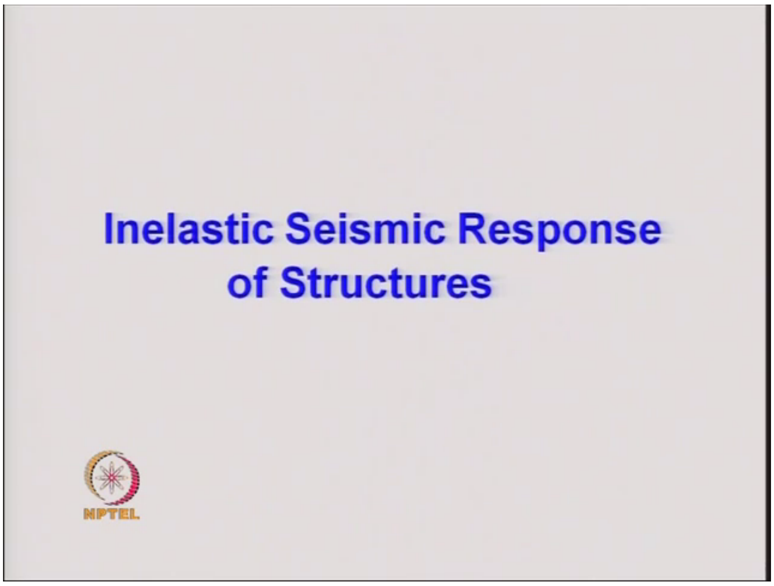

In the previous lectures, we discussed about the linear dynamic analysis of structures for earthquake excitation that is under the earthquake excitation the structures remained within the elastic range and therefore, the linear analysis was adopted that is all the what aspects of the linear analysis was maintained; however, the structures can undergo inelastic response under earthquake excitation.

The reason for this is the current design philosophy that is adopted that we will discussed shortly, but the inelastic excursion of the structures during earthquake requires some understanding about the inelastic dynamic behavior of structures and for that one has to perform an inelastic seismic response of structures for given earthquake.

In the previous lectures, we have discussed about the linear dynamic analysis of structures for specified ground motion in that we considered both time domain and frequency domain analysis, then we had taken up the response of the structure for the earthquakes modelled as a random process that is a stationary random process in that we perform the spectral analysis to obtain the response after that we consider the response spectrum method of analysis which is widely popular amongst earthquake engineers because it converts the problem partly to a dynamic and partly to a static problem.

The dynamic problem which is involved is trivial in the sense that one has to only find out the frequencies and mode shapes of the structures rest of the solution consist of static analysis only also in that connection, we discussed about seismic coefficient method which is a purely equivalent and static analysis for all this cases the responses where in the elastic range and the forces that are computed or the internal forces that is computed for different members, they are utilized for designing the structures for earthquake.

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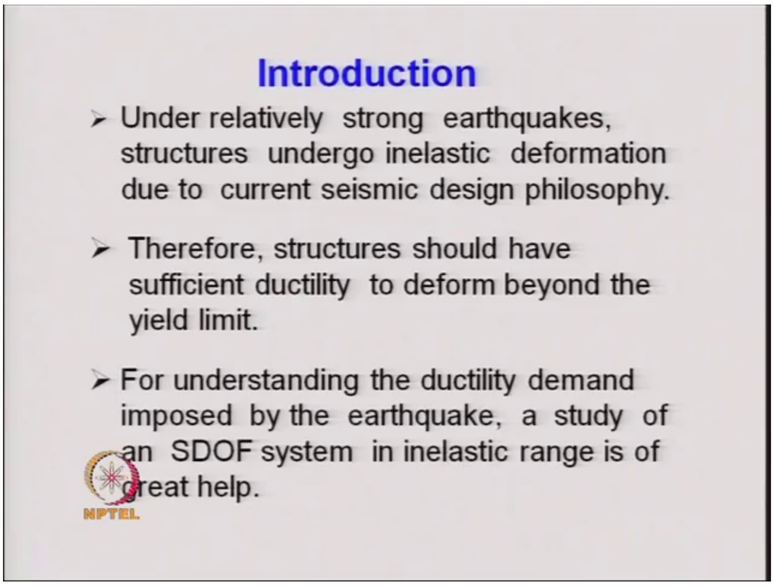

Now, in this few lectures would be discussing about the inelastic response of structures why this is required that I had mentioned before, but let us try to see some more reasons a behind these inelastic response analysis of structures for earthquake under relatively strong earthquakes structures undergo inelastic deformation due to current seismic design philosophy the seismic design philosophy that we have in our code that is based on 3 very important parameters that is the stiffness strength and the ductility.

Stiffness controls the response. So, long the structure is in the elastic range after it yields then the ductility comes into picture, but when the structure yields, it depends upon the yield strength of the structure and that is how the yield strength becomes an important parameter for the inelastic analysis of the structures for earthquake, then after the yielding, we allow the structure to deform in the inelastic range and this deformation that is the maximum deformation that it undergoes after the yielding that basically controls the behavior or inelastic behavior of the structure.

How much deformation is allowed is a tentatively decided at the time of design by way of some factors, but unless one performs an inelastic analysis one cannot judge; how much each member or each joint at the plastic hinges undergo plastic deformation beyond the yield point. So, for that it is very much necessary the seismic design philosophy as such is a dual design philosophy in the codes that is we assume that for the design earthquake the structure we will undergo the inelastic excursion that is the structure we will have plastifications at different cross sections.

However those plastifications or inelastic deformations that takes place at the joints or in the members are within certain limit and can be easily repaired or the structures can be retrofitted the reason for allowing the structure to go into the inelastic range is that we want first economy in the design. Secondly, by allowing the structure to go into the inelastic range, we allow more dissipation of energy to take place in the structure as a result of that effective earthquake loading onto the structure is reduced the other part of the seismic design is that the structure should have enormous amount of inelastic deformation or there will be a considerable amount of damages will take place in the structure.

Under the extreme earthquake, but the structure will not collapse. So, that is the life is not threatened life of the people are not threatened. So, these are the 2 dual design philosophy that we have in both we can see that we allow the structure to go into the inelastic range and therefore, we have one should understand the inelastic behavior of the structure. Secondly, the structures should have sufficient ductility to deform beyond the yield limit.

So, apart from understanding in the behavior of the structure in the inelastic range, we should make sure that the structure do have the sufficient amount of ductility and that ductility should be able to respond to the ductility demand imposed by the earthquake. So, for that also oh we require inelastic analysis to be performed.

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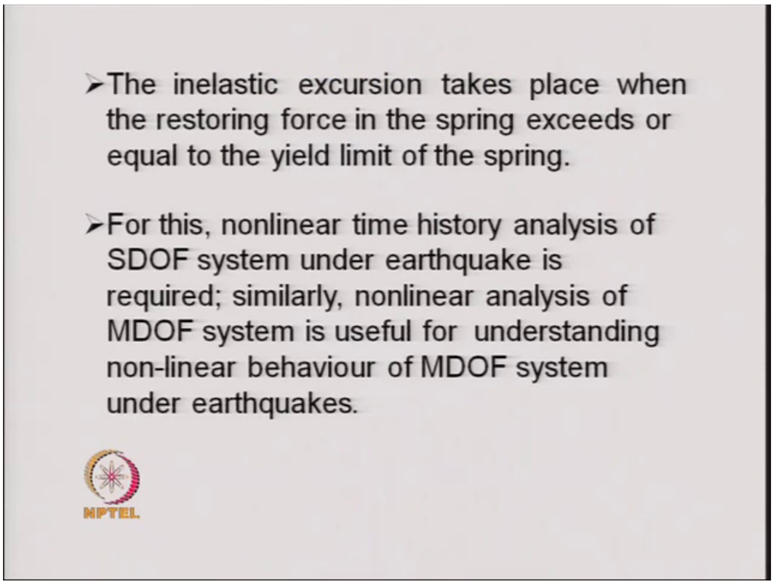

Generally the ductility demand imposed by the earthquake is understood with the help of an inelastic analysis of a single degree freedom system and that is what we will start with; however, the non-linear time history analysis of the single degree of freedom system under earthquake is not only used for understanding the ductility of the material or the system, but also to understand some more important thing about the inelastic resistance of structures for earthquake and that we understand with the help of the so called inelastic response spectrum.

Similarly, for the multi degree of freedom system the non-linear analysis is useful in understanding the non-linear behavior of multi degree of freedom system under earthquakes and therefore, the need for the non-linear analysis or inelastic analysis of multi degree of freedom systems under earthquake is also an important issue in many cases one, if we are wanting to obtain the seismic risk of a structure then one has to go up to the collapse of a structure and therefore, a non-linear analysis may have to be a performed for the seismic risk analysis of structures.

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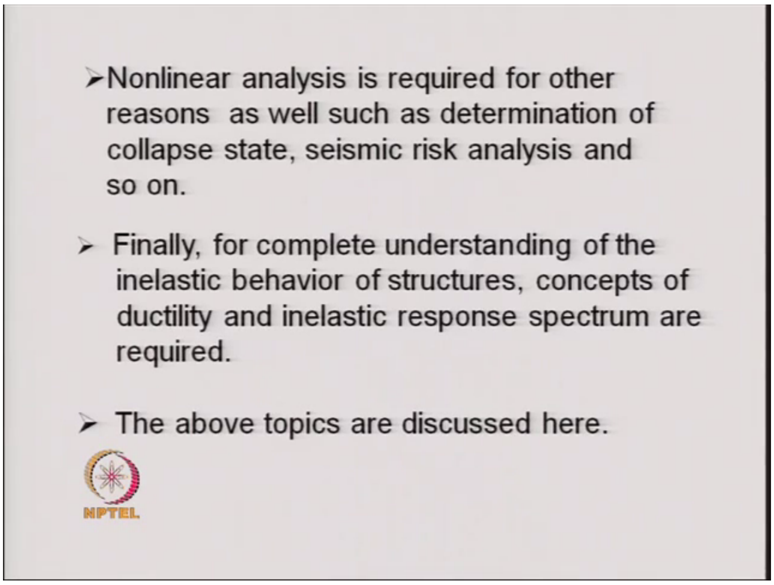

The inelastic response spectrum that is required to understand the inelastic resistance of the structures to earthquake that a construes an integral part of the inelastic response analysis of structures and we will specifically look into the ductility and inelastic response spectrum of earthquake subsequently while discussing this topic on the inelastic response analysis of structures.

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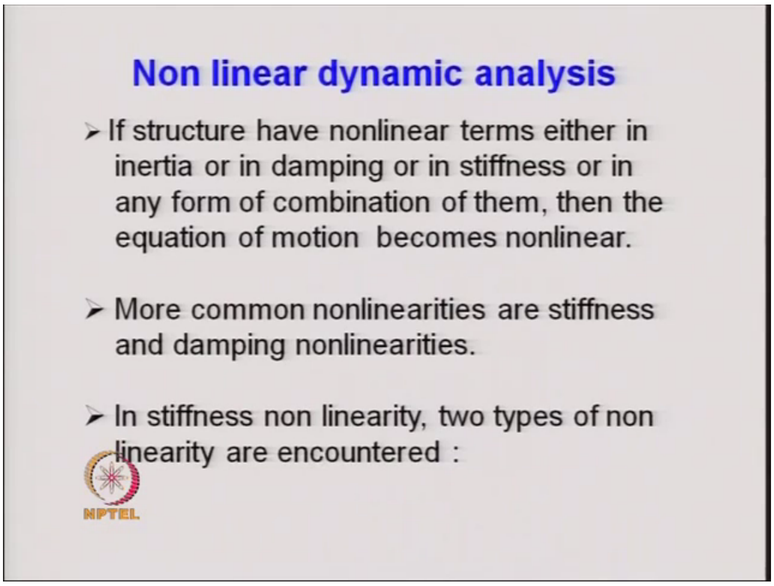

Now, what we mean by the non-linear dynamic analysis over here is that if the structure have non-linear terms either in inertia or in damping or in stiffness or in any form of combination of them, then equation of motion becomes non-linear; however, the more common nonlinearities that arise are due to the stiffness nonlinearity and the damping nonlinearity the inertia nonlinearity is hardly encounter. So, here we will be mostly talking about the nonlinearities arising due to the stiffness.

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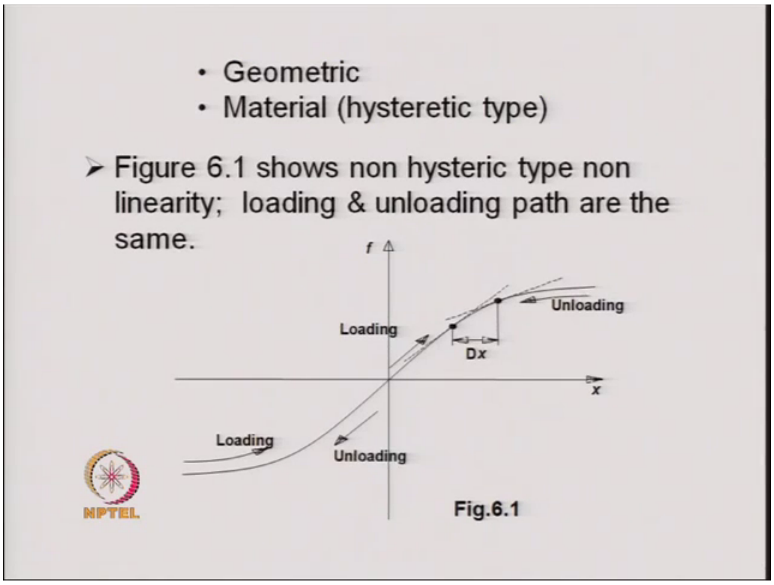

The stiffness nonlinearity can be you know 2 types; one is geometric nonlinearity other is the material nonlinearity or as the or what is known also as the hysteretic type figure 6.1 shows the non hysteretic type nonlinearity this generally arises because of the geometric nonlinearity; in this case of non hysteretic type of nonlinearity loading and unloading path are the same as you can see in this figure that is this figure you can see that the loading path and the unloading path over this line they remains the same and at any instant of time t during the analysis the stiffness of the system is a dictated by the slope of this curve at a particular point.

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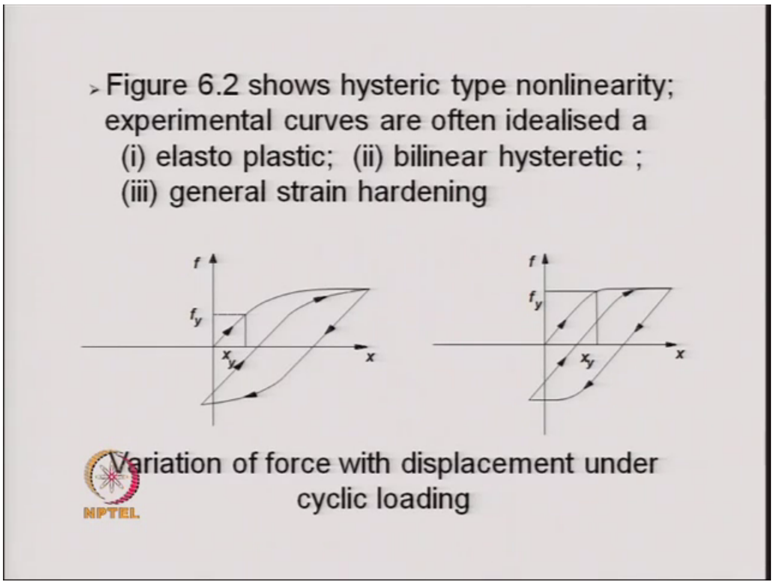

And this slope changes at every instant of time t as a result of that the stiffness changes and we call the analysis to be a non-linear analysis and it is performed using an incremental technique the hysteretic type of nonlinearity is are shown in this figures the experimental curves that we get from the inelastic behavior of the structure during testing is of this type because of the nonlinearity the loading and unloading paths are different and as a result of that it forms an hysteretic loop and the area under the hysteretic loop all of you know is equivalent to the energy that is dissipated.

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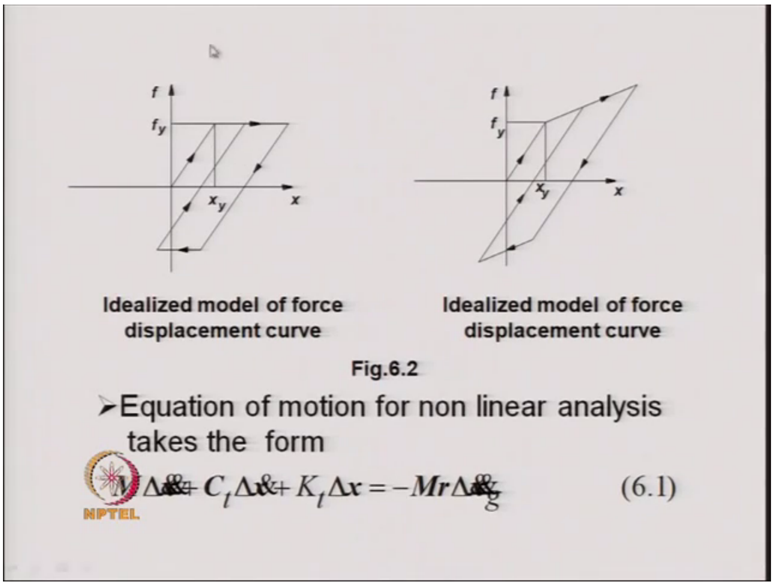

The experimental curves are idealized like this the most common idealization that is used for the analysis is the elastoplastic analysis that is in the beginning part it is elastic till the yield point and once the yield takes place, it becomes fully plastic that is under that constant stress the displacement or the strains continuously occurred and then there is an unloading. So, this is a bilinear model in that we have an initial stiffness and an after the yield point the stiffness changes that it follows this the curve which is the top line and this has a less stiffness than the initial stiffness and when it under goes what you unloading, then it undergoes a displacement or deformation which makes a slope which is parallel to the initial stiffness or the slope which we get in the load or deformation curve at the initial stage.

The mode general type of the strain hardening or the load deflation curve that we get is the general strain hardening curve and here; we have a curve that represents the load deformation behavior till it undergoes unloading and at the state of unloading the slope over here that is the slope of the load deformation curve is parallel to the initial slope or initial tangent to the curve.

The equation of motion for non-linear analysis is done in the incremental form here equation six point one shows the typical equation that we write for the solution of the non-linear problem here the same equation that we used for your linear analysis we use the same type of equation, but x x dot and x double dot, they are replaced by delta x delta x dot and delta x double dot and C t and K t represents the stiffness and the damping we changes and at the instant of time t; it has a stiffness K t and the C t is the damping at instant of time t.

So, they are called the instantaneous values of the stiffness and the damping coefficient and the meaning of the incremental form of analysis is that during delta x displacement we assume the stiffness to remain constant during that interval and the damping also to remain constant during that interval and the entire analysis for this interval is carried out in the same way as we obtain the analysis for the linear system or in other words during this interval a linear analysis is performed.

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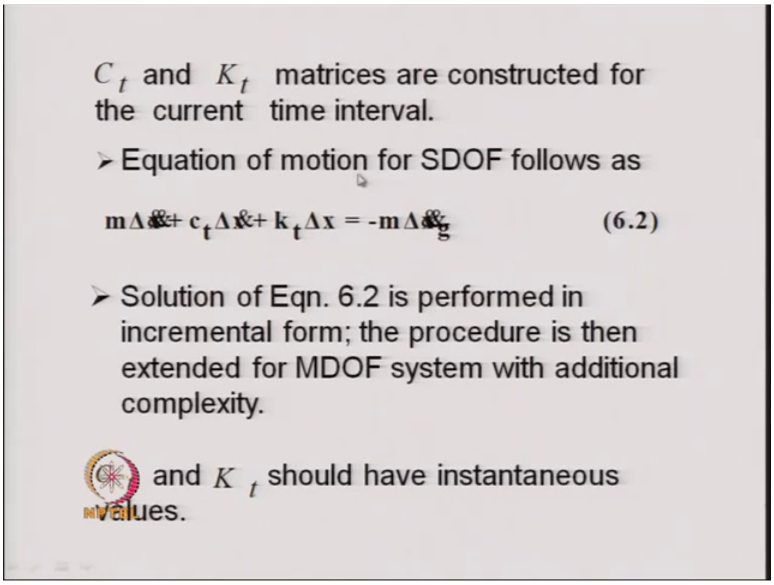

The equation of motion for single degree of freedom system from the previous equation follows like this m is the mass and C t and K t are the instantaneous values of the stiffness and damping and here we write minus m delta x double dot g that is the incremental acceleration or ground acceleration that takes place within the interval delta x note that delta x corresponds to a time interval of delta t that is in the time interval of delta t a displacement of delta x takes place which is an unknown quantity to be found out.

Once we are able to solve all this equation by some numerical technique, then the same kind of solution can be extended for multi degree freedom system; however, for generating the K t and C t for multi degree freedom system involves some kind of complexity that we will address later.

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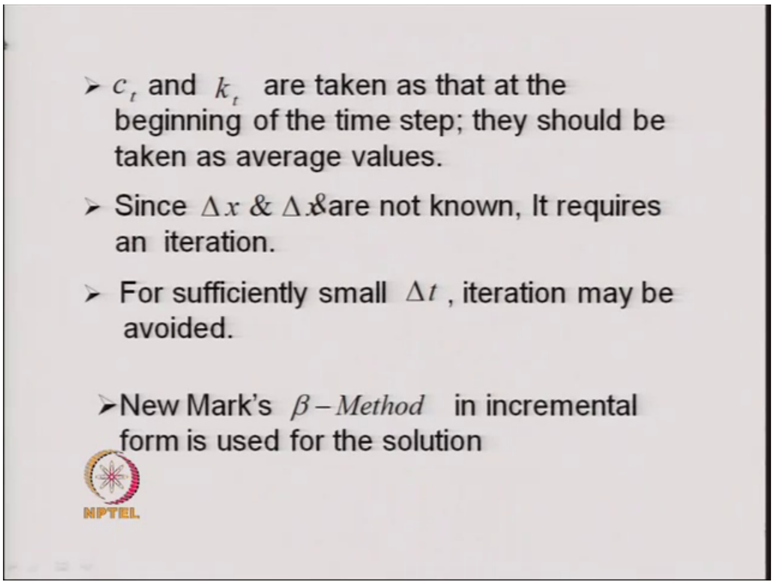

If we go for the analysis in which we do not have the hysteretic type of nonlinearity that is we have the general or geometric nonlinearity that we had shown before in this curve then we can see that during the interval delta x or during this interval of time delta t the stiffness changes from this value or the from this value to this value.

Ah therefore, if we wish to obtain a reasonably good solution to the problem then one should take an average value of the stiffness which should be equal to this stiffness plus stiffness divided by 2 since we do not know this point; therefore, we cannot find out this average stiffness and if we wish to include this average stiffness then one should go for an iteration; iteration in the sense that we start with an initial stiffness over here, then solve the problem and that will give a value of delta x and once we get the value of delta x, then we can find out this point and at this point, we can find out the stiffness then add this stiffness with the previous stiffness divided by 2 and take the average stiffness and make a solution again that will give another value of delta x and that is how we can carry out the analysis in an iterative fashion.

Unless we get the convergence; however, for small value of delta t this iteration maybe avoided and in most of the cases we take sufficiently small value of delta t. So, that the stiffness that we take at the initial point and that is at time x t with that stiffness only we calculate the value of delta x Newmarks beta method in the incremental form is used for the solution.

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$$\Delta\dot{x} = \Delta t\,\ddot{x}_k + \delta\,\Delta t\,\Delta\ddot{x} \qquad (6.3)$$

$$\Delta x = \Delta t\,\dot{x}_k + \frac{(\Delta t)^2}{2}\ddot{x}_k + \beta(\Delta t)^2\Delta\ddot{x} \qquad (6.4)$$

$$\Delta\ddot{x} = \frac{1}{\beta(\Delta t)^2}\Delta x - \frac{1}{\beta\Delta t}\dot{x}_k - \frac{1}{2\beta}\ddot{x}_k \qquad (6.5)$$

$$\Delta\dot{x} = \frac{\delta}{\beta\Delta t}\Delta x - \frac{\delta}{\beta}\dot{x}_k + \Delta t\left(1-\frac{\delta}{2\beta}\right)\ddot{x}_k \qquad (6.6)$$

$$\bar{k}\Delta x = \Delta p \qquad (6.7)$$

$$\bar{k} = k_t + \frac{\delta}{\beta\Delta t}c_t + \frac{1}{\beta(\Delta t)^2}m \qquad (6.8a)$$

$$\Delta\bar{p} = -m\Delta\ddot{x}_g + \left(\frac{m}{\beta\Delta t} + \frac{\delta}{\beta}c_t\right)\dot{x}_k + \left[\frac{m}{2\beta} + \Delta t\left(\frac{\delta}{2\beta}-1\right)c_t\right]\ddot{x}_k \qquad (6.8b)$$

$$x_{k+1} = x_k + \Delta x \qquad \dot{x}_{k+1} = \dot{x}_k + \Delta\dot{x} \qquad \ddot{x}_{k+1} = \ddot{x}_k + \Delta\ddot{x} \qquad (6.9)$$

That is described over here this is the equation or other the these are the 2 cardinal equations or relationships that are used in developing the Newmarks beta method, we discussed these 2 cardinal equations when we are discussing about the Newmarks beta method of solution for the linear system. So, they are the difference was that these variables were x not delta x here it is delta x. So, we write down delta x dot to be is equal to delta t multiplied by x dot k plus delta delta t delta x double dot; similarly we write down delta x to be is equal to delta t multiplied by x dot k plus delta t square into x double dot k plus beta delta t square into delta x double dot.

Now, from these equation 6.41 can find out or find out delta x double dot in terms of delta x x dot k and x double dot k, then substituting these value of delta x double dot into the first equation, one can get an expression for delta x dot and these a delta x dot is again expressed in terms of delta x x dot k and x double dot k since x k x dot k and x double dot k, they are known we have to only find out delta x, then we can substitute the values of delta x double dot and delta x dot to the equation of equilibrium dynamic equation equilibrium they represented by 6.2 and so, here delta x double dot and the delta x dot.

They are now written in terms of delta x and finally, we get an expression which is given over here by way of equation 6.7 which is it in a k bar delta x is equal to delta p. So, the entire left hand side now is it in terms of an unknown variable delta x only where k bar is given by this and in this you can see that we know C t we know K t because they are the instantaneous stiffness and damping at time t and m is known therefore, entire k bar can be calculated and delta p that consists of the incremental earthquake acceleration minus m delta x double dot g plus these terms in these terms x dot k and x double dot k they are known because at the initial at time t, these values are already known.

So, one can solve this equation to get the value of delta x and once we get the value of delta x then the it can be substituted in its equation 6.3 to obtain the values of delta x dot and then delta x double dot and one can find out x k plus 1 is equal to x k plus delta x x dot k plus 1 is equal to x dot k plus delta x dot and so on the acceleration is generally not obtained by this equation because if we obtain the acceleration by this equation and if we substitute this value of acceleration at the k plus 1 at time station that is at t plus delta t time station and look into the equation of equilibrium, then right hand side and the left hand side may not match and there could be some error.

So, in order to remove that error what is done is that we obtain s k plus 1 we obtain x dot k plus 1 and then substitute this into the equation of equilibrium at k plus oneth time station on the right hand side the acceleration or the ground acceleration at k plus oneth time station is known therefore, from the equation of motion we can get the value of x double dot k plus 1. So, that basically compensates for any error that can immerge in the equilibrium equation

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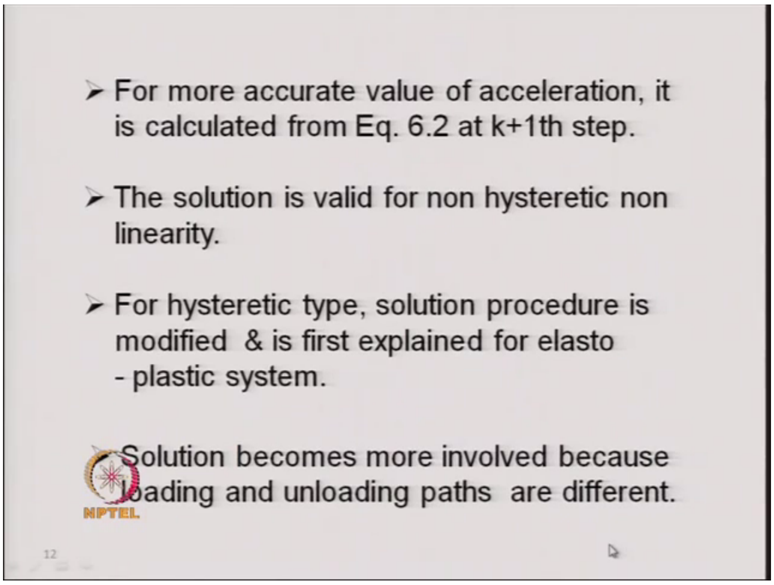

Now this kind of solution is valid for non hysteretic non-linearity, but if you are going to utilize these type of incremental solution for hysteretic non-linearity or hysteretic type of non-linearity then the solution procedure is modified and it will be explained with the help of the most simple type of idealization of the hysteretic type of non-linearity that is elastoplastic system.

Now, here the solution becomes more involved because one has to constantly tress the loading and unloading paths or in other words one has to constantly see the responses at every instant of time t in order to find out whether the state of the system is in the loading path or in the unloading path.

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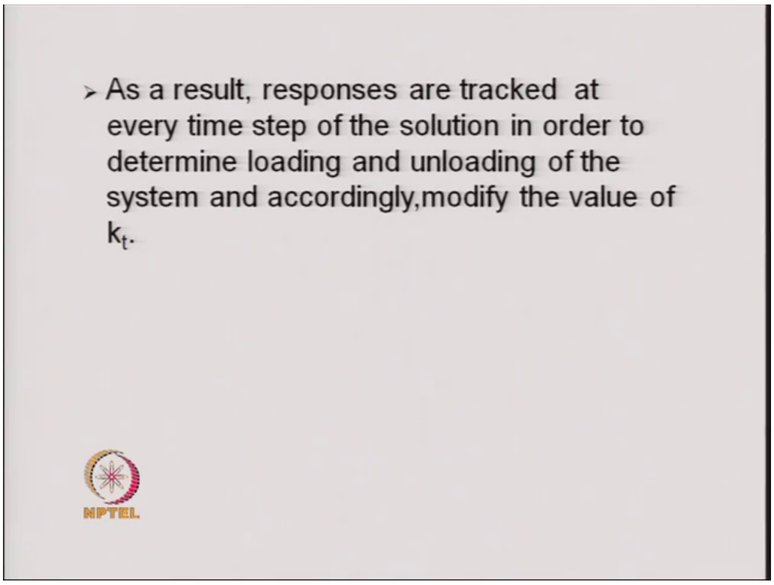

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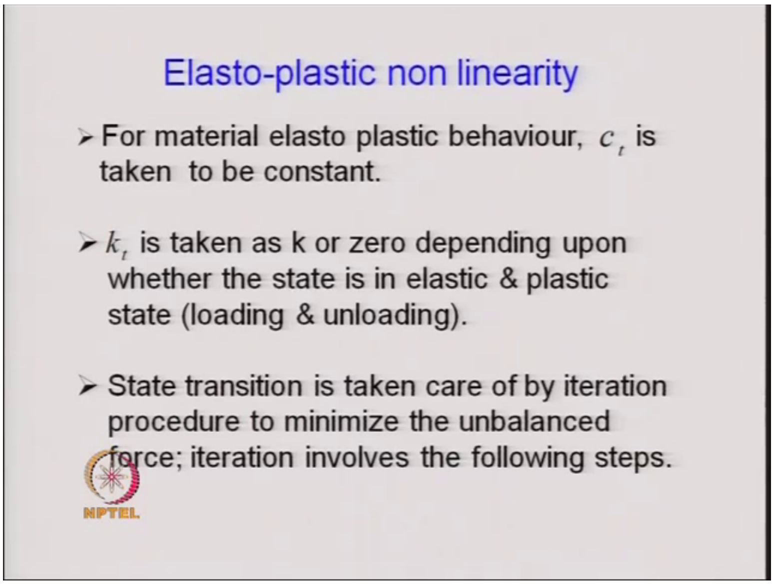

Therefore, what we do we constantly track the responses of the system at every time step, when we use the elastoplastic non-linearity and we use the same incremental solution then we only concentrate in the material non-linearity and we assume that the damping remains constant that is C t is constant at all times.

However any variation of C t if desired can also be taken into consideration into the analysis. So, for as the K t is concerned in the elastoplastic assumption either the K t value will be equal to the k value which is the elastic part of the curve or k value would be 0 that is the plastic part; that means, the horizontal part of the force deformation curve.

So, therefore, depending upon whether the state is in elastic or in the plastic state and both in the loading and unloading conditions the K t value should be properly taken into account either the K t value will be equal to k or 0 and we have to constantly monitor it now the most crucial part of the solution is that when the state of the system moves from one state to the other that is what is called the state transition takes place.

In that case we carry out some iteration to many minimize the unbalanced data. Now the iteration involves typically the following steps.

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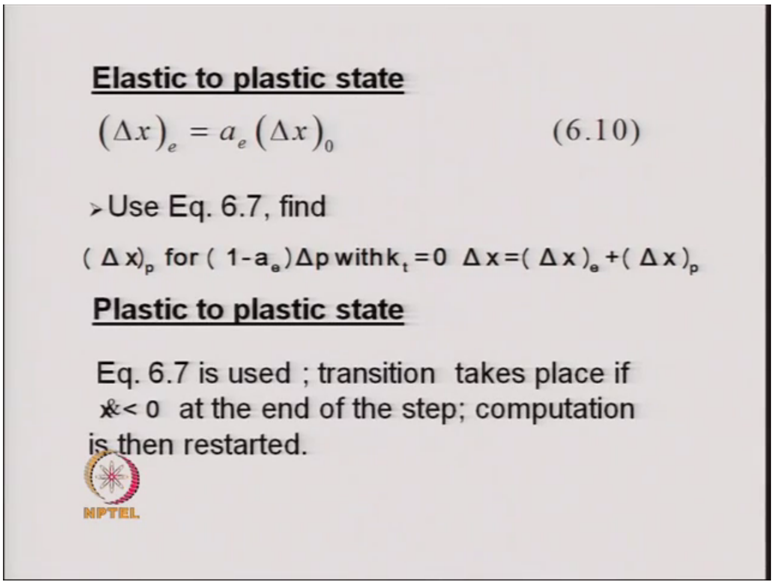

Say we are moving from elastic to plastic state that is we have a displacement and for that particular displacement we check whether the spring force that is the force spring stiffness or k multiplied by the displacement that is equal to the yield limit or not if it is not in the or less than the yield limit then it is purely in the elastic range.

Now, when we come to the near the yield point then the incremental solution in the first place may give a value of delta x which is represented by delta x 0 and we find that when we add up the value of delta x 0 with the value of the displacement at the previous time step then at time x t plus delta t or x k plus 1 that value multiplied by the initial or stiffness or the stiffness of the system that product gives a value which is more than the yield strength or the specified yield strength or in other words, the solution x k plus 1 that exceeds the yield point.

Now, in an elastoplastic system the solution point beyond the yield point does not or displacement beyond the yield point does not exist therefore, one is to bring it down. So, in order to bring it down what we do we find a scaled value of delta x e and this scale value of delta x e say is equal to a e multiplied by delta x 0; that means, the total displacement that we get in the first solution that is multiplied by a factor a e once we multiply this by a factor a e to get the value of delta x e, then one can write down x k plus delta x e multiplied by the entire thing multiplied by the k value that should be equal to the yield strength or the specified yield value.

Now, once we write that then from that equation one can find out the value of a e or in other words we can scale this delta x 0 by multiplying it by a factor a e such that if I add up that value of scaled delta x and added to the value of the displacement at the previous time station then the resulting displacement multiplied by the stiffness of the spring would be equal to the yield limit or yield strength of the system.

So, from that one can get the value of a e, then what we do that we say that delta x e is one part of the solution and the next part of the solution is obtained by is again using within 6.7 that is the incremental solution, but in that what is done is that K t is made equal to not k, but is equal to 0 because we say that rest of the solution will lie in the plastic zone and the loading that will be there on the right hand side will be equal to 1 minus a e into delta p because the a e into delta p part of the loading corresponds to the displacement delta x e.

Therefore rest of the loading that is it. In fact, utilized in finding out the value of delta x p. So, in finding out the delta x p value using the incremental equation what we do on the right hand side the delta p that is the incremental loading that we have shown here yes in this equation 6.7 the delta p is there these delta p is replaced by one minus e into delta p and the K t is set to 0 and from there we can get the value of delta x by solving that equation the that delta x is called delta x p.

Then finally, the value of delta x for that increment is equal to delta x e plus delta x p. So, that is how we take care of the transition state; that means, a transition taking from elastic to plastic state. Now when the system moves in the plastic to plastic state we constantly go on examining the value of the velocity. So, long as the velocity of the system is positive; that means, the system still moving in the one from one plastic state to other plastic state at the point of transition that is when it unloads then the value of the velocity becomes negative.

So, near the transition point if we solve the equation or incremental equation 6.7 by setting K t is equal to 0, then we get a value of delta x delta x dot and delta x double dot in the first instant and that value of delta x dot if it is added to the value of the velocity x dot of the previous time station, then it gives a final velocity which turns out to be negative that shows that the unloading has taken place at some point in between delta t.

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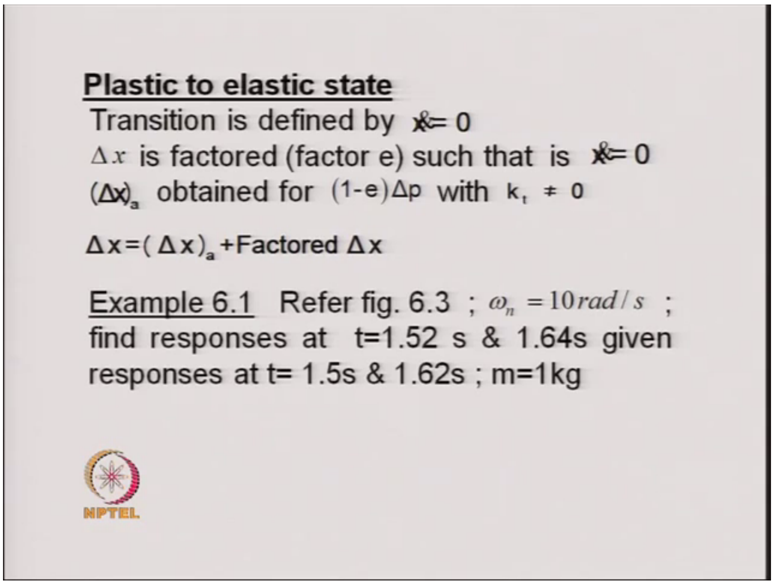

So, we have to trace that point the transition from the plastic to elastic state takes place when the x dot becomes equal to 0.

In order to trace the transition point what is done is that the delta x that is obtained over the time delta t that is a factor. So, that the displacement at the transition point is equal to a into delta x plus the value of the displacement at the kth time station that is x k plus a into delta x then one can obtain the velocity at the transition point as a into delta x dot plus x dot k.

Since the value of the velocity at the transition point is equal to 0, then we set a into delta x dot plus x dot k to be is equal to 0 from there we can find out the value of a as minus a minus x dot k divided by delta x dot, once we get the value of a, then one can find out the displacement of the transition point and at the transition point the unloading takes place. So, long the transition does not take place that is up to the transition point the value of the stiffness is equal to 0 and when the unloading takes place then the stiffness is equal to K t or K t is equal to k.

So, what we do that we find out a proportion for the loading delta p over the increment of time delta t and we set a into delta p as the portion of the loading that takes the system up to the transition point and then one minus a into delta p that is the remaining load on that basically the unloading takes place. So, therefore, we write down an equation with a variable delta x 2 and in that we provide K t to be is equal to k and on the right hand side, we write down the load as one minus a into delta p.

The solution of the single degree freedom system we this loading and the stiffness K t is equal to k provides us a value of delta x 2 then the total displacement at the k plus oneth time step becomes equal to x k plus a into delta x plus delta x 2. So, that is how the transition point from plastic to elastic state is taken care of; now the elastic to plastic; plastic to plastic and plastic to elastic these states do also occur on the negative yielding site.

So, as the system unloads from the positive yielding, then this unloading continues till we come to a transition point where the negative yielding takes place and the system transits from elastic state to plastic state and we take care of this transition as before that is the way we have taken it in the case of the positive yielding, then the system move from plastic to plastic and then from plastic to elastic the transition form from plastic to elastic is again taken care of the way, we have taken care of the this transition point in the case of positive yielding and after the plastic state to elastic state that transition takes place on the negative yielding site, then the system is reloaded and it continues till it comes to the positive yielding site and again there is a transition from the elastic state to plastic state.

Now, this kind of solution or the iterative solution to take care of the transition point many a time is avoided by making the delta t sufficiently small. So, that even if we miss the transition points then also the response that we compute do not become very much erroneous because the errors that are obtained they are of very small magnitude therefore, many a time, we solve this problem by taking a smaller increment of delta t value and carrying out the entire analysis.

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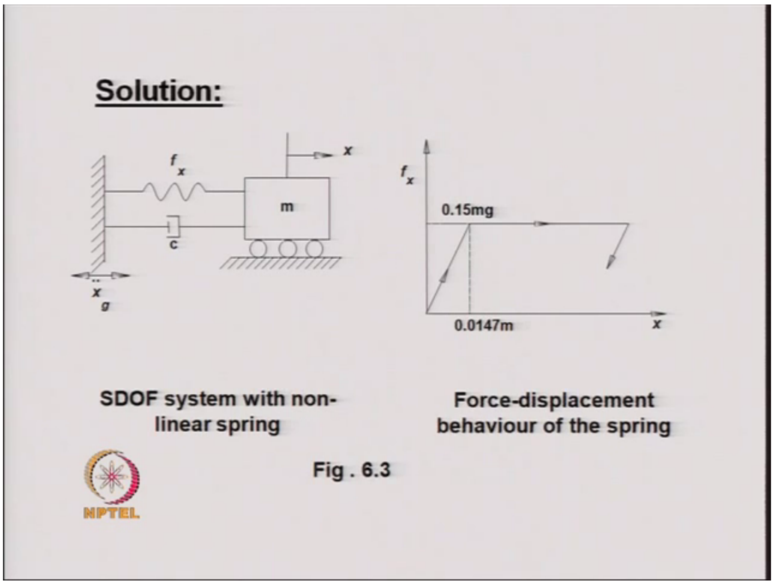

However that requires a more time lets clear this concept with the help of an example there is a single degree of freedom system and in this single degree of freedom system these f x the force which is developed in the spring for that this is the curve or in other words this is a non-linear spring and the force deformation behavior of the curve is given by this, it has a yield point or a yield value of f x and the corresponding yield displacement, it can get unloaded at any point of time and C t is assumed to be constant.

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$t = 1.5s;\ x = 0.01315m;\ \dot{x} = 0.1902m/s;$

$\ddot{x} = 0.46964\ m/s^2;\ f_x = 1.354\ ;\ c_t = 0.4Nsm^{-1};$

$k_t = 100Nm^{-1}$

$\bar{k} = 10140\ Nm^{-1}$

$\Delta \ddot{x}_g = -0.00312g$

$\Delta p = 37.55\ N$

$\Delta x = 0.0037m;\ \Delta \dot{x} = -0.01ms^{-1};\ \Delta f = k_t \Delta x$

$(f_x)_{t+\Delta t} = 1.7243N > 0.15mg$

$(f_x)_t + e\Delta x k_t = 0.15mg\ (e = 0.3176)$

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In this problem it is given that at t is equal to 1.5 second the value of x x dot and x double dot, they are given also the value of f x is given and C t is a given which is constant the K t that is the stiffness which is shown over here the stiffness that is given; now with this given values, we see that the it is a displacement and the velocity both of them are positive or in other words this is the increasing portion of the displacement and the f x value initial f x value is 1.354 which is less than 0.15 m g mass times acceleration.

Therefore the yielding has not taken place. So, what we do we find out the value of k bar keeping the value of K t is equal to k and we obtain the solution from that we get the value of delta p and corresponding value of the delta x and the delta x dot and we calculate delta f delta f becomes equal to K t multiplied by delta x. Now when we add this delta f to this initial value of the f x that is the value of the spring force at time t then the spring force at time t plus delta t that becomes equal to 1.7243 n or that is greater than 0.15 mass times acceleration mass is taken as unity.

So, therefore, we can see that in the in the first time the solution that basically provides a value of delta x such that the yield limit is exceeded. So, ones the yield limit is exceeded then we have to bring it down and or the pull it back and therefore, we factor the displacement delta x by multiplying the value of delta x by e. So, we write down f x t plus e into delta x multiplied by K t that will become equal to 0.15 m into g.

So, that the factor that the e factor is made such that the yield value is not exceeded. So, from this one can obtain the value of e and that value of e is equal to 0.3176.

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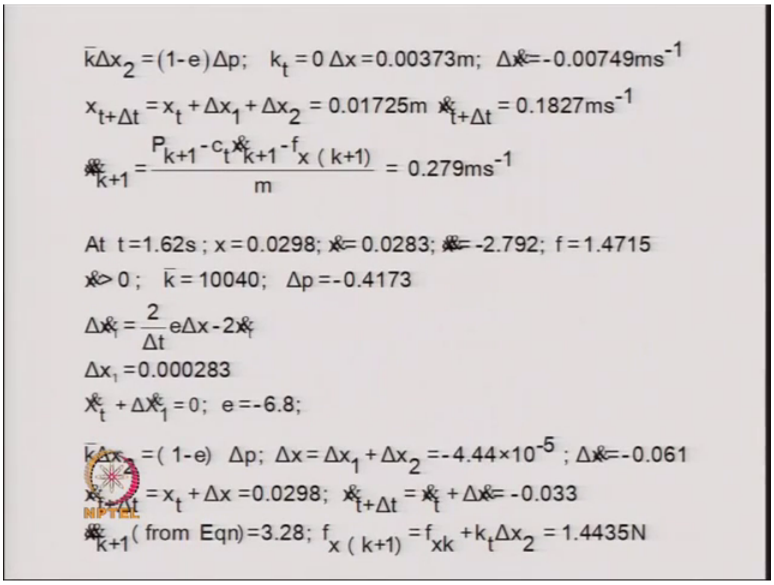

Once we get the value of e, then one can find out the value of 1 minus e into delta p that becomes on in the right hand side and we do the second part of the solution, we write down the second part of the solution as k bar into delta x t is equal to 1 minus c into delta p and in that the K t is set to 0 because this part of the solution is on the horizontal portion of the load deformation curve that is in the plastic range therefore, K t is said to 0 and the solution provides a value of delta x to be this and delta x dot to be this.

Once we get the value of delta x and delta x dot then we write down x t plus delta t that is equal to initial value of x t, then the first value first portion of the delta x displacement that is the factor displacement and plus delta x 2 that we get from the this solution. So, this gives you the final value of the x t plus delta t. Similarly, one can find out the value of x dot t plus delta t that becomes equal to 0.1827 and after you have obtain the value of x t plus delta t and x dot t plus delta t or in other words x k plus 1 x dot k plus 1 x k plus 1 then one can find out x double dot k plus 1 by solving the equation of motion at k plus oneth time station.

So, here we are not using any more the implemental equation, but the full equation at the k plus oneth time station is utilized to find out the value of x dot x double dot k plus 1 and that value is 0.279 meter per second square meter per second square. So, this is again done in order to really minimize the error that can come out of the satisfying the equilibrium equation at any instant of time t.

At time t is equal to 1.62 second, we have this is the displacement given this is the velocity given and the velocity given these acceleration given and the force basically is equal to 1.4715 that is equivalent to 0.15 m g. So, we see that x dot is greater than 0. So, it is increasing in the plastic state. So, k bar we calculate by setting K t is equal to 0 and this is a value of k bar and delta p is known we can obtain the value of delta p by knowing that delta x double dot g and x dot k x k etcetera and once we get that from that we get a value of delta x.

Now, that particular value of delta x is found to provide a value which provide which gives a value of the final velocity which becomes less than 0 so; that means, a transition point has been crossed or in other words unloading from the plastic state to the elastic state at occurred. So, once that has occurred then we find out a revised value of delta x 1 that is a factor value of the delta x and that value of delta x 1 is obtained as this and then by setting the value of the velocity at the transition point to be equal to 0 that is a x dot t plus delta x dot one that we obtained from this factoring right for that this plus this that is equal to 0 because at the point of transition as I told you the velocity is equal to 0.

So, that provides a value of e is equal to minus 6.8 and once we get the value of e, then one can have the second part of the solution here k bar into delta x 2 becomes equal to 1 minus p into delta p and since this part of the solution lies in the elastic range the first part of the factor part of the solution was in the plastic range. So, this part of the solution is in the elastic range and. So, therefore, we use in this equation for finding out the value of k bar the K t is said to is equal to k and then we get a value of delta x 2 and delta x 2, once we know the value of delta x 2, we can get the value of delta x 1 and plus delta x 2 to be is equal to the final value of delta x.

Similarly, one can get a value final value of delta x dot and once you get the values of delta x and delta x dot from there one can get the value of x t plus delta t and x dot t plus delta t and once we know them then one can find out the value of the x double dot t plus delta t by satisfying the equation at t plus delta t time station or its at k plus oneth time station also one can find out the value of the force induced into the spring that will be equal to f f x k that is at the kth time station which was equal to 1.4715 and that multi plus K t into delta x 2 because. So, long it was in the transition zone the value of f x is equal to 1.4715 and when we add the elastic part of the force that is K t into delta x 2.

Then we get the force to be equal to 1.4435 which is less than the yield stress as would be expected.

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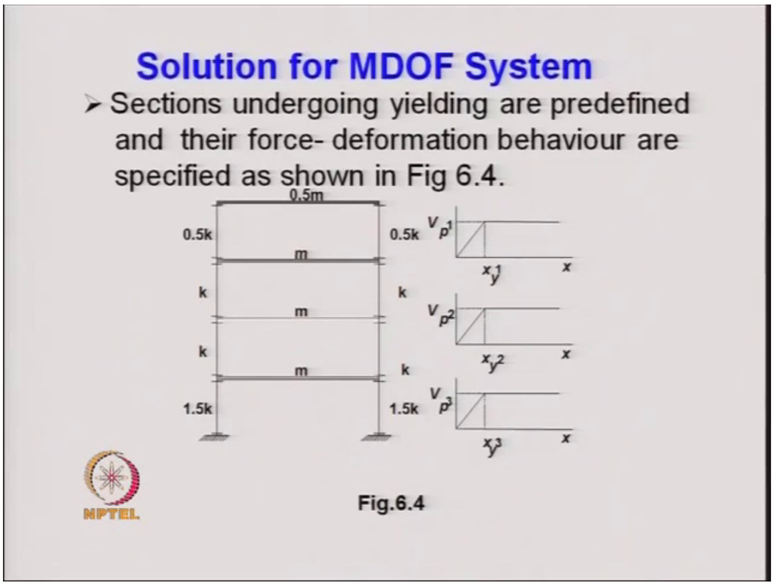

So, that is how one can carry out a solution incrementally and can take care of the transition point.